The University of Melbourne
School of Computing and Information Systems
COMP30026 Models of Computation

Sample Answers to Tutorial Exercises, Week 8

63. For languages $L_1 = \{\texttt{ab}, \texttt{c}\}$ and $L_2 = \{\texttt{ca}, \texttt{c}\}$:

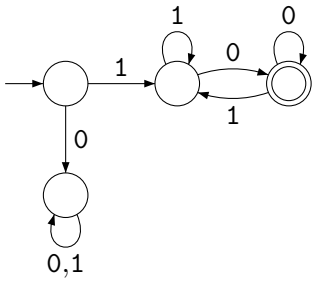
    (a) $L_1 \cup L_2 = \{\texttt{ab}, \texttt{ca}, \texttt{c}\}$

    (b) $L_1 \circ L_2 = \{\texttt{abca}, \texttt{cca}, \texttt{abc}, \texttt{cc}\}$

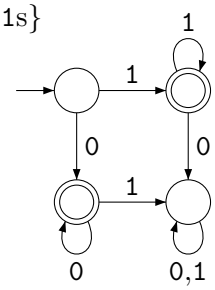
    (c) $L_1^* = \{\epsilon, \texttt{ab}, \texttt{c}, \texttt{abab}, \texttt{abc}, \texttt{cab}, \texttt{cc}, \ldots\}$

    (d) $L_1^* \backslash L_2^* = \{\texttt{ab}, \texttt{abab}, \texttt{abc}, \texttt{cab}, \ldots\}$

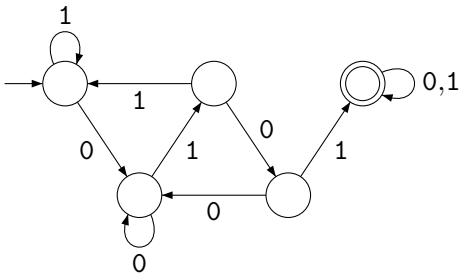
64. (a) $\{w \mid w$ begins with a `1` and ends with a `0`$\}$



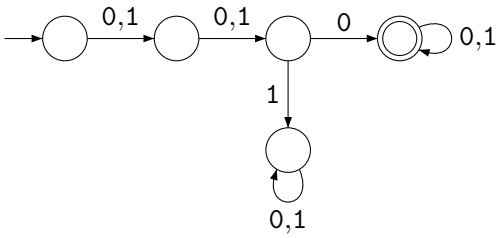
    (b) $\{w \mid w$ is not empty and contains only `0`s or only `1`s$\}$



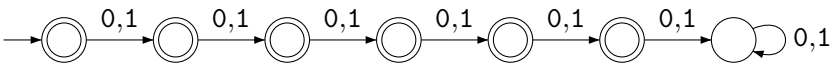
    (c) $\{w \mid w$ contains the substring `0101`$\}$



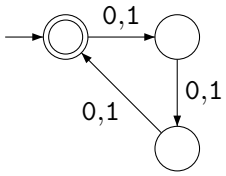
    (d) $\{w \mid w$ has length at least 3 and its third symbol is `0`$\}$



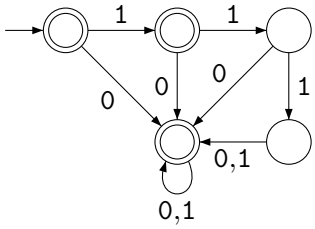
(e) $\{w \mid$ the length of w is at most $5\}$



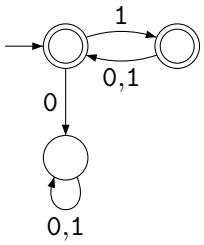
(f) $\{w \mid$ the length of w is a multiple of $3\}$



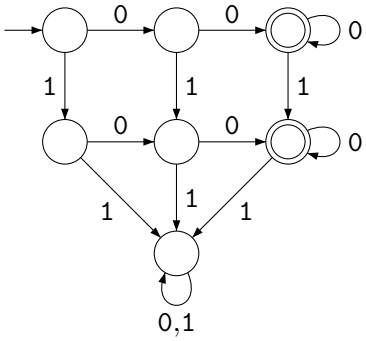
(g) $\{w \mid w$ is any string except `11` and `111`$\}$



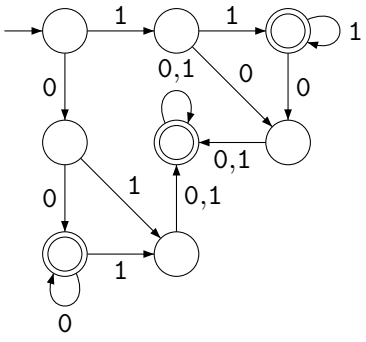
(h) $\{w \mid$ every odd position of w is a `1`$\}$



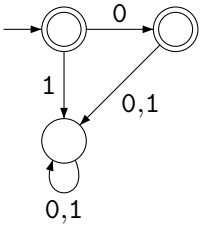
(i) $\{w \mid w$ contains at least two `0`s and at most one `1`$\}$



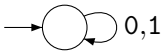
(j) $\{w \mid$ the last symbol of w is occured at least twice in $w\}$



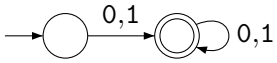
(k) $\{\epsilon, 0\}$



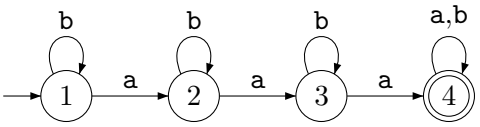
(l) The empty set

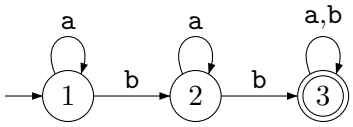


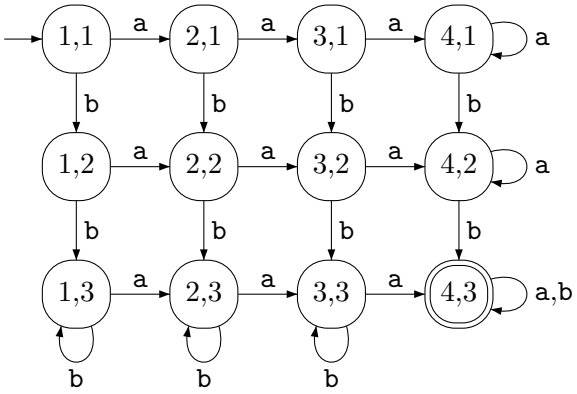
(m) All strings except the empty string



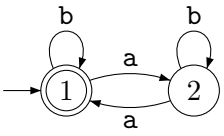
65. (a) $\{w \mid w \text{ has at least three } \texttt{a}\texttt{s}\} \cap \{w \mid w \text{ has at least two } \texttt{b}\texttt{s}\}$

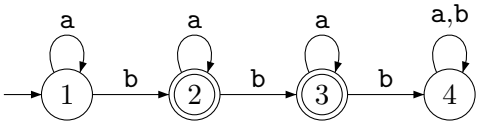


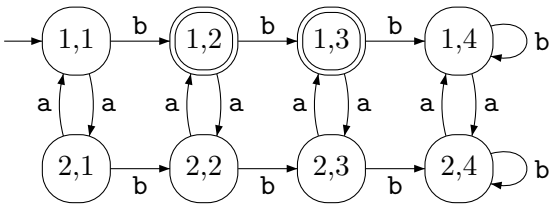




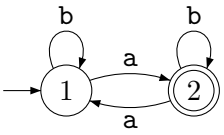
(b) $\{w \mid w \text{ has an even number of } \texttt{a}\texttt{s}\} \cap \{w \mid w \text{ has one or two } \texttt{b}\texttt{s}\}$

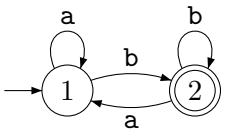


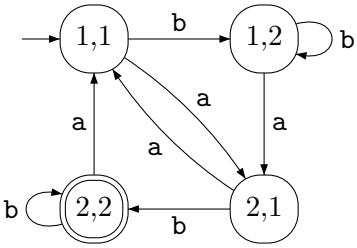




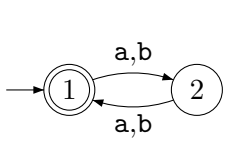
(c) $\{w \mid w \text{ has an odd number of } \texttt{a}\texttt{s}\} \cap \{w \mid w \text{ ends with } \texttt{b}\}$

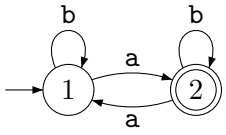


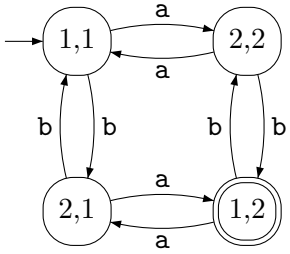




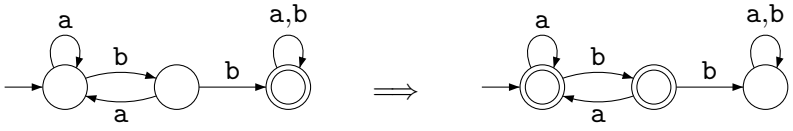
(d) $\{w \mid w \text{ has an even length}\} \cap \{w \mid w \text{ has an odd number of } \mathtt{a}\mathtt{s}\}$



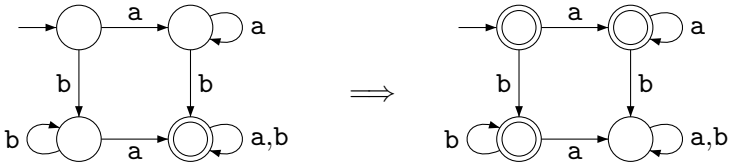




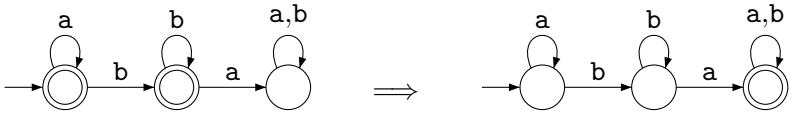
66. (a) $\{w \mid w \text{ does not contain the substring } \mathtt{b}\mathtt{b}\}$



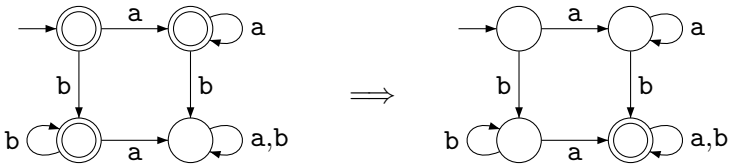
(b) $\{w \mid w \text{ contains neither the substring } \mathtt{a}\mathtt{b} \text{ nor } \mathtt{b}\mathtt{a}\}$



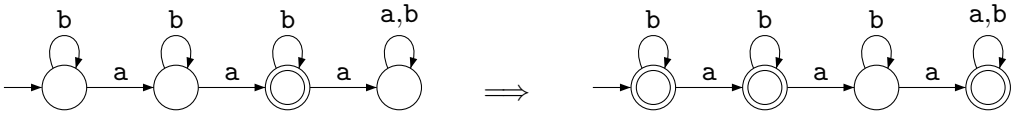
(c) $\{w \mid w \text{ is any string not in } \mathtt{A}^* \circ \mathtt{B}^*, \text{ where } \mathtt{A} = \{\mathtt{a}\}, \mathtt{B} = \{\mathtt{b}\}\}$



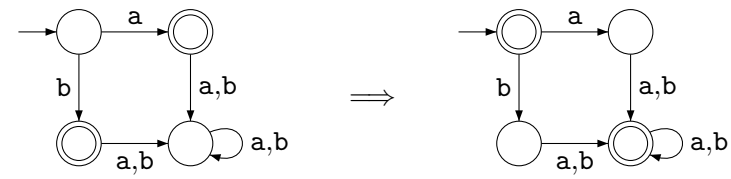
(d) $\{w \mid w \text{ is any string not in } \mathtt{A}^* \cup \mathtt{B}^*, \text{ where } \mathtt{A} = \{\mathtt{a}\}, \mathtt{B} = \{\mathtt{b}\}\}$ (compare to (b)!)



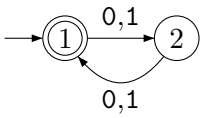
(e) $\{w \mid w \text{ is any string that doesn't contain exactly two } \mathtt{a}\mathtt{s}\}$

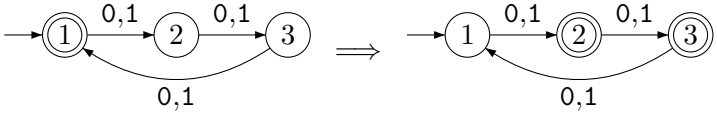


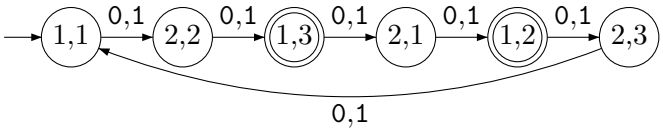
(f) $\{w \mid w \text{ is any string except } \mathtt{a} \text{ and } \mathtt{b}\}$



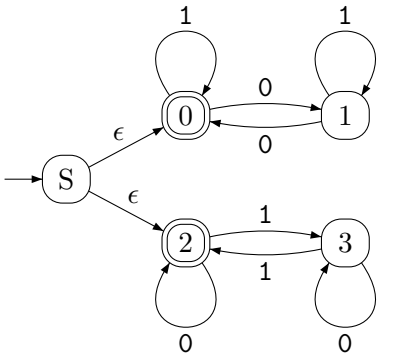
67. $\{w \mid$ the lenght of w is a multiple of 2 and is not multiple of 3$\}$



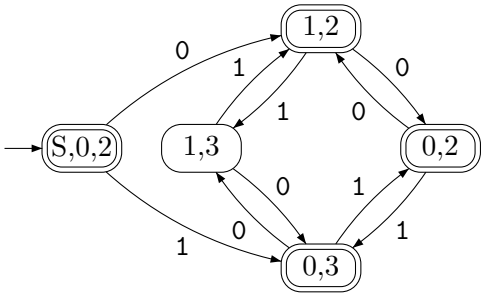




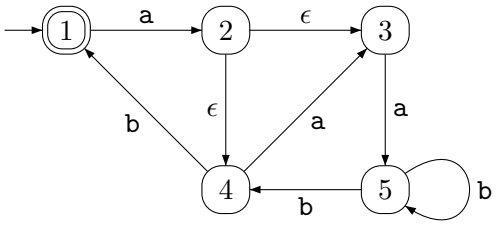
68. From this DFA:



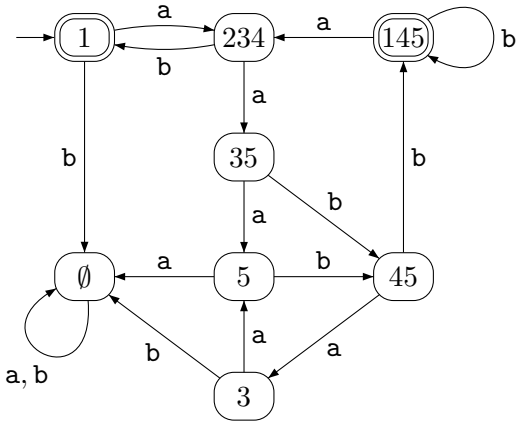
we get:



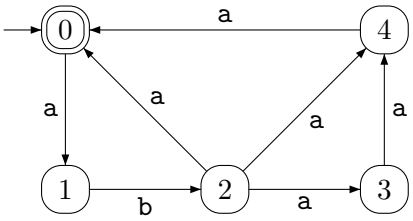
69. From this NFA:



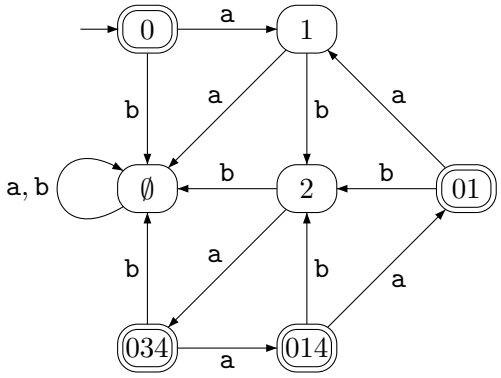
we get:



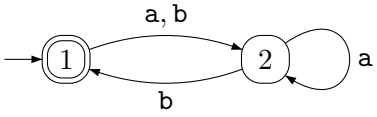
70. From this NFA:



we end up with the following DFA:



71. This is the minimal DFA:



72. This is the minimal DFA: