COMP90086 Computer Vision
Week 2

Image Filtering

Lecture Notes summarized by Neo

Semester 2 2021

1 Convolution

1.1 Pixel Operator (one to one pixel mapping)

Computes an output value at each pixel location, based on the **input pixel value**.

$$g(i,j) = h(f(i,j)), \text{ where}$$

- (i,j) is the pixels of the image,
- g is the output image,
- f is the input image and
- h is the filtering function.

Example

- $g(i,j) = 0.5(f(i,j))$
  降低图片亮度，每个 Pixel 数值减半（Pixel 数越高越亮）。
- Gamma Correction 伽马矫正
  Pixel Operator 的一种，使用更为复杂的**非线性**方程。

1.2 Local Operator (many to one pixel mapping)

Computes an output value at each pixel location, based on **a neighbourhood of pixels around the input pixel**.

e.g. Sharpening （锐化）:
Finding the average color of the pixels around each pixel in a specified radius, and then contrasting that pixel from that average color.

2 Linear Filtering (used in local operator)

Output pixel’ s value is a weighted sum of a neighbourhood around the input pixel.

2.1 Cross-Correlation 互相关

$$g(i,j) = h(u,v) \oplus f(i,j), \text{ where}$$

- (i,j) is the pixels of the image,
- g is the output image,
- f is the input image,
- h is the kernel and
- $\oplus$ is the cross-correlation operator.

$$g(i,j) = \sum_{u,v} f(i+u, j+v) h(u,v)$$

2.2 Convolution 卷积

$$g(i,j) = h(u,v) * f(i,j), \text{ where}$$

- (i,j) is the pixels of the image,
- g is the output image,
- f is the input image,
- h is the kernel and
- $*$ is the convolution operator.

$$g(i,j) = \sum_{u,v} f(i-u, j-v) h(u,v)$$

Cross-Correlation Example

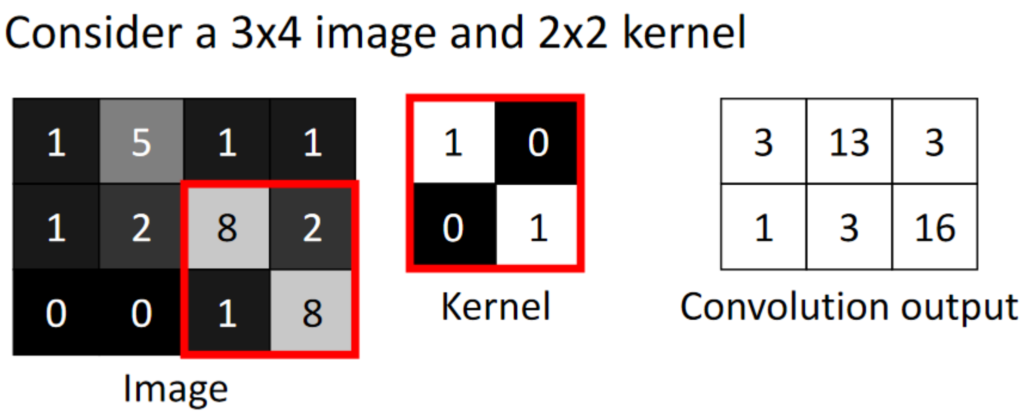


Figure 1: Cross-Correlation of 3*4 image with 2*2 kernel output 2*3 image

> 由于使用 Kenel 会导致原图像的边框无法转换，最后的图像会比原图像小。
> 右下角 Pixel：$8 * 1 + 2 * 0 + 1 * 0 + 8 * 1 = 16$

2.3 Convolution with Colour Images

在处理 RGB（或其他格式）彩色图片时，把**原图分成 3 个不同颜色的 Channel**（RGB）并分别应用不同（或相同）的 Kernel，最后把结果合并。

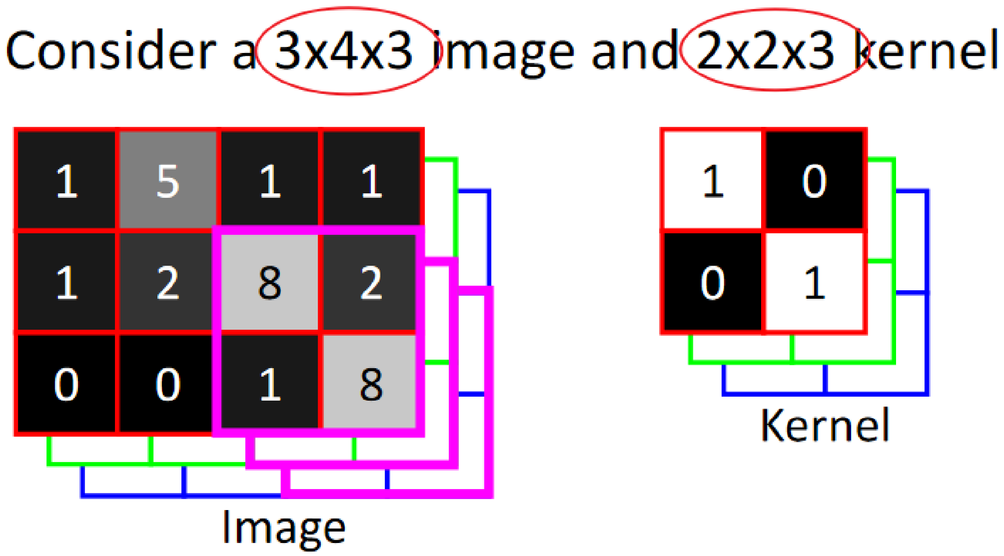


Figure 2: Separate RGB image into three channels

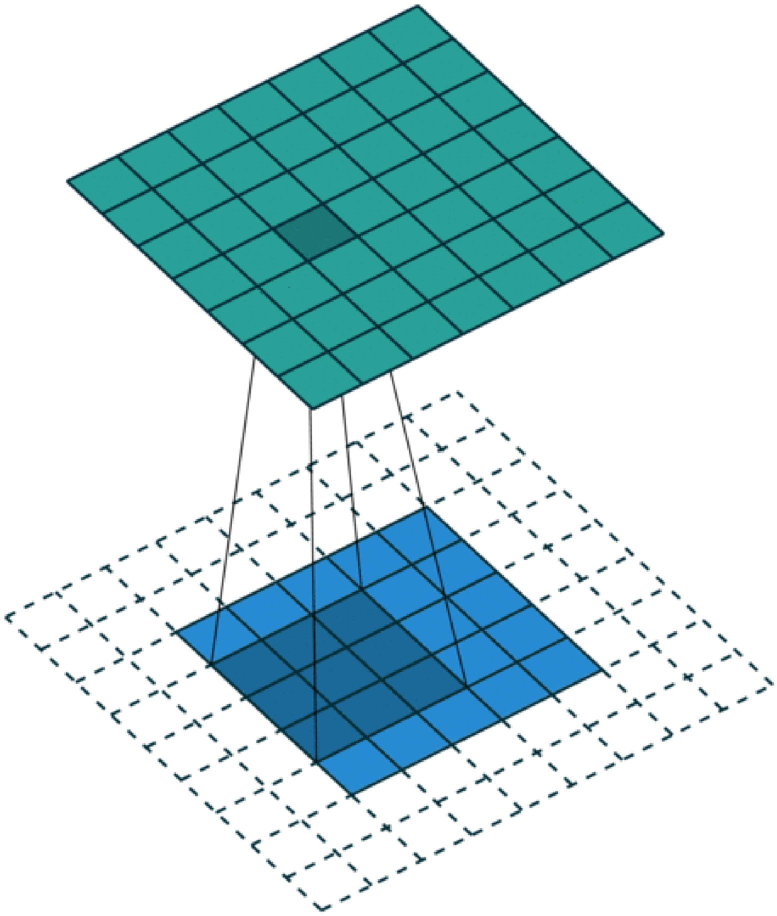


Figure 3: Example with the green grid being the original image and the blue one is the result

2.4 Cross-correlation v.s. Convolution

Overlay filter on image for Corss-correlation and **flip filter** horizontally and vertically for Convolution.
注：使用 **Convolution** 时要把 Kernel **上下前后颠倒**。

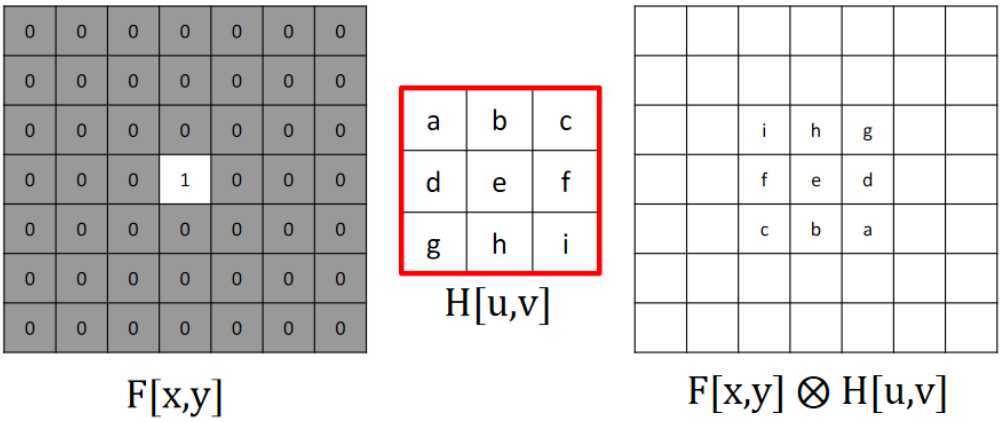


Figure 4: Cross-correlation

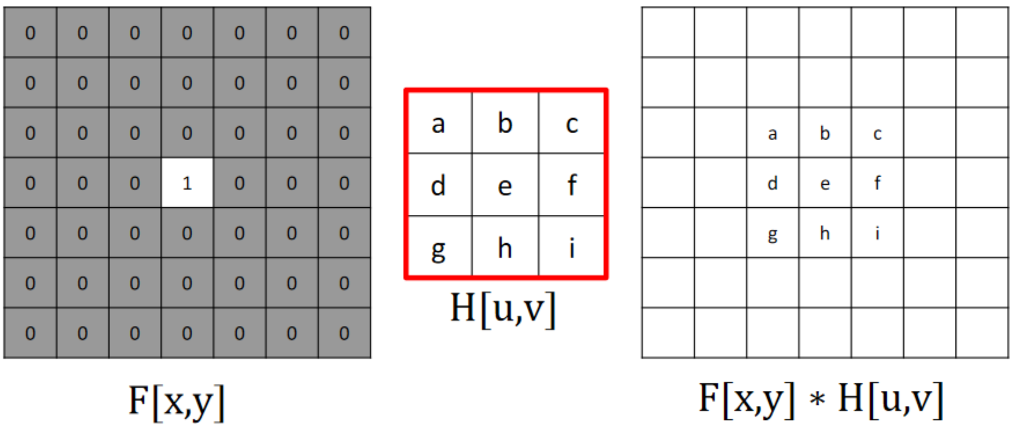


Figure 5: Convolution

Cross-correlation

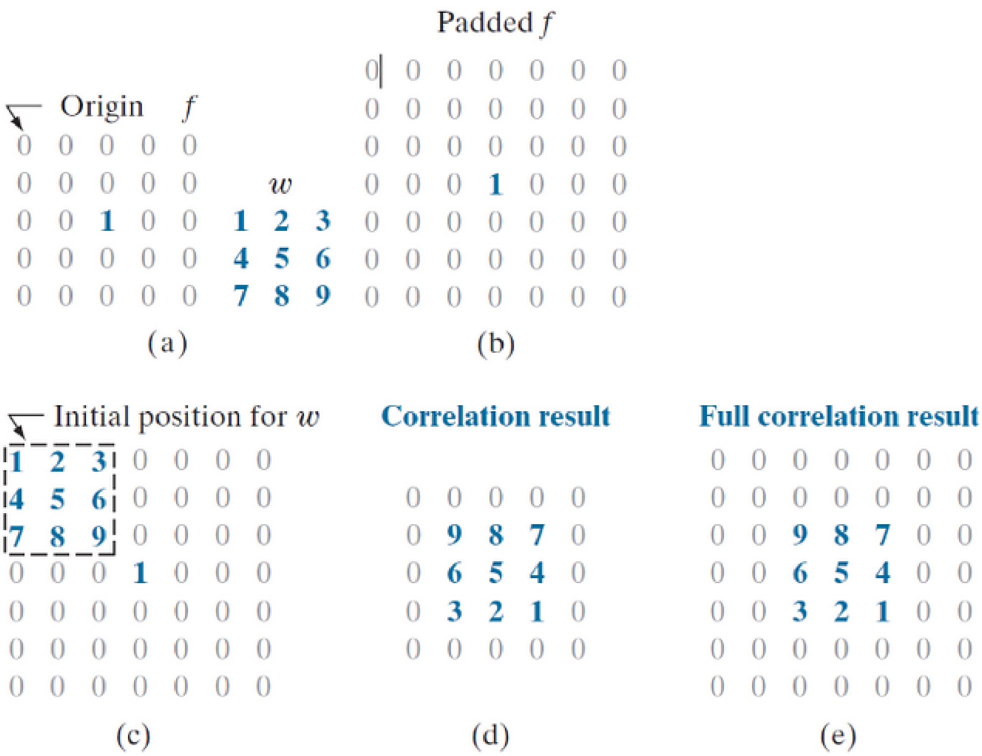


Figure 6: Cross-correlation

Convolution

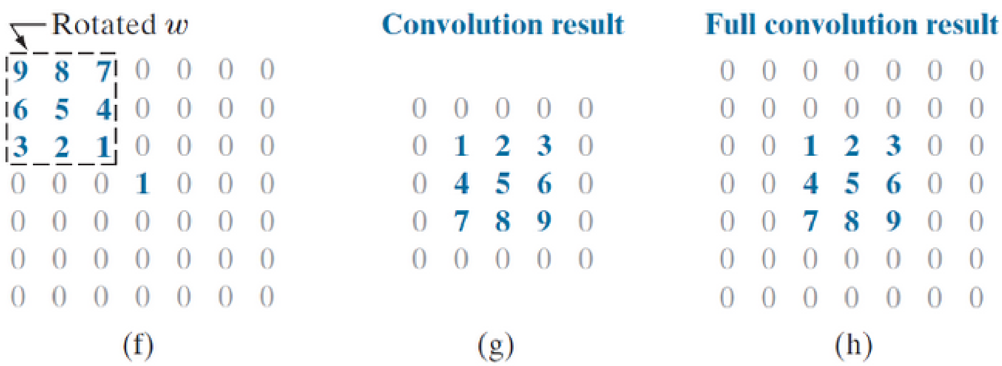


Figure 7: Convolution

3 Common Filters 常见的 Filter，与原图像做卷积

3.1 Original (No effect)

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

3.2 Shift Left

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

3.3 Sharpening

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

3.4 Gaussian Blur 高斯模糊

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

图像的高斯模糊过程就是**图像与正态分布做卷积**。

3.5 Sobel Operator 索伯算子

使用以下两种 Kernel 与图像做卷积可以**探测边界**。

- Detect Horizontal Edges

$$\begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

- Detect Vertical Edges

$$\begin{bmatrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{bmatrix}$$

3.6 Some other filters

- Average/blur filters: average pixel values, blur the image
- Sharpening filters: subtract pixel from surround, increase fine detail
- Edge filters: compute difference between pixels, detect oriented edges in image

4 Properties of Linear Filtering

- Commutative: $f * h = h * f$
- Associative: $(f * h1) * h2 = f * (h1 * h2)$
- Distributive over addition: $f * (h1 + h2) = (f * h1) + (f * h2)$
- Multiplication cancels out: $kf * h = f * kh = k(f * h)$

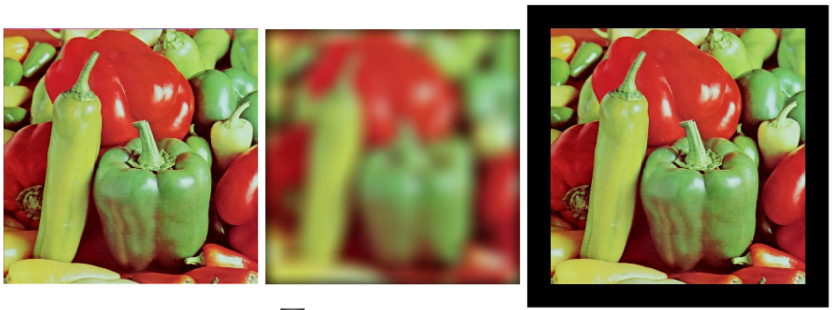
5 Efficient Filtering

通常使用一个复杂的 Filter 的效率要比连续使用简单的 Filter 效率要高。

6 Border Handling

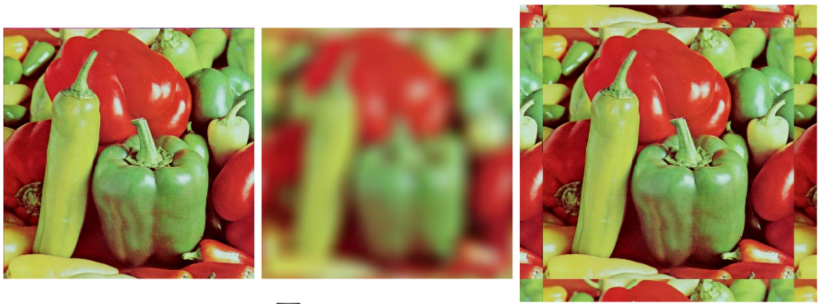
因为使用卷积会导致最外一圈的 Pixel 消失，需要一些手段来“恢复”边界。

- Pad with constant value



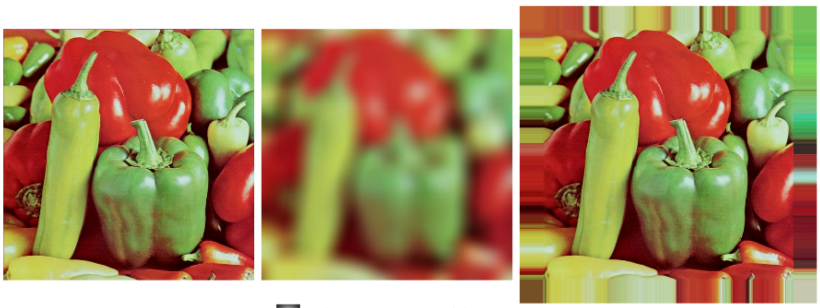
Filter: Gaussian blur

- Wrap image



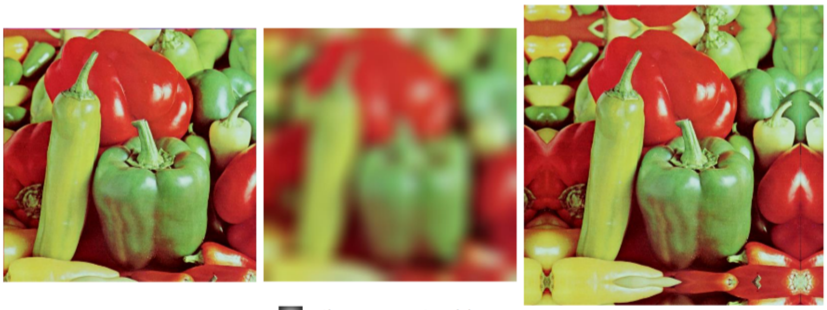
Filter: Gaussian blur

- Clamp / replicate the border value



Filter: Gaussian blur

- Reflect image



Filter: Gaussian blur