COMP90086 Computer Vision

Week 2B

Image Filtering - Frequency Filtering

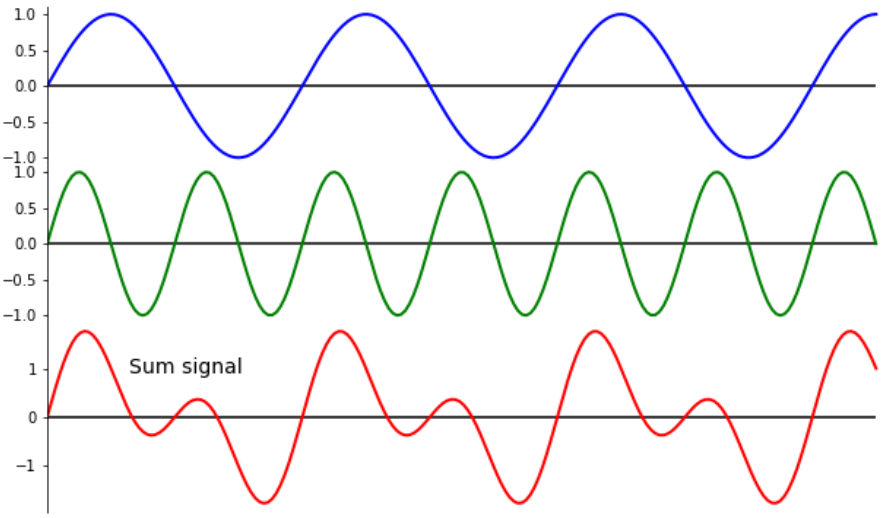
Lecture Notes summarized by Neo

Semester 2 2021

1 Fourier Analysis (1D)

1.1 Signals 信号

Any signal or pattern can be described as a sum of sinusoids. 所有信号都可以表示成很多正弦函数的叠加（和）。



- 红色的复杂信号，可以分解成蓝色正弦函数信号与绿色正弦函数的叠加
- **一张图片上截取一行 Pixel，可以转换成一段信号，所以叫 1D**

1.2 Sinusoids (Sine Waves) 正弦函数

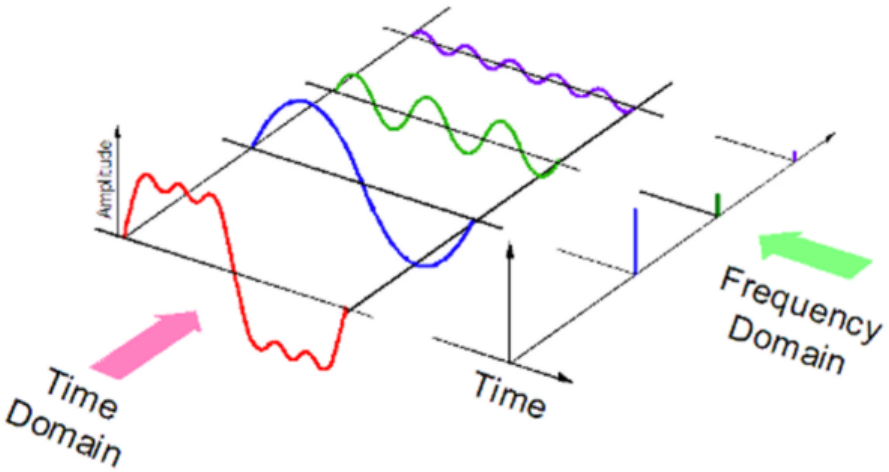
$$y = A\sin(wx + \varphi)$$

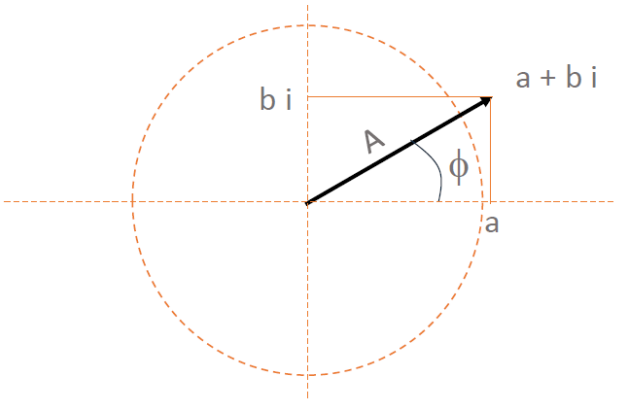
where

- A is amplitude
- w is frequency
- φ is phase

1.3 Fourier Transform 傅里叶变换

Fourier transform decomposes signal into component frequencies.
傅里叶变换是一种线性积分变换，用于信号在时域（或空域）和频域之间的变换。





- Frequency Domain:
  - The axis is frequency
  - Values are complex numbers
  - Magnitude = amplitude of the sinusoid
  - Angle = phase of the sinusoid

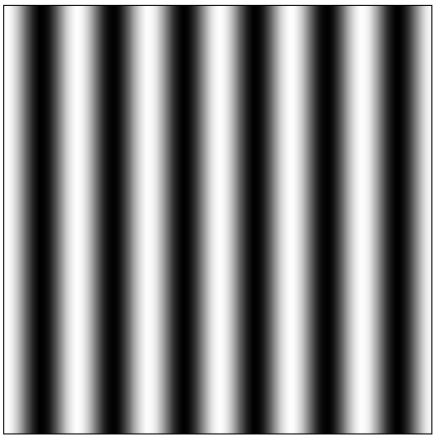
Formula (Not assessable)

$$F(w) = \int_{-\infty}^{\infty} f(x) e^{-2i\pi wx} dx$$

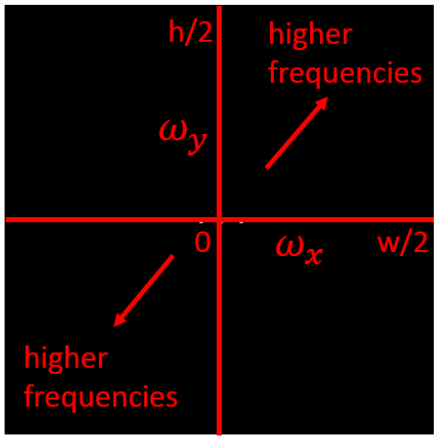
一般有包可以直接用，例如 scipy.fft(1D), scipy.fft2(2D), scipy.fftn(3D+)

2 Fourier Analysis (Image 2D)

在 2D 图像中，x 轴与 y 轴可分别做傅里叶变换并放到右图以 0freq 为中心的轴中。



Image

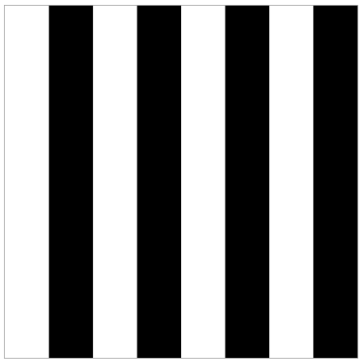
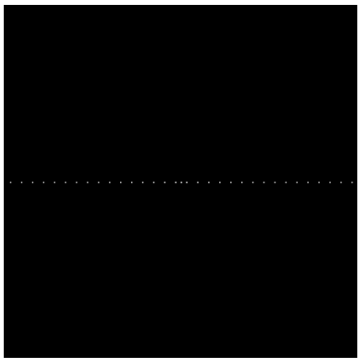


Fourier transform
(magnitude)

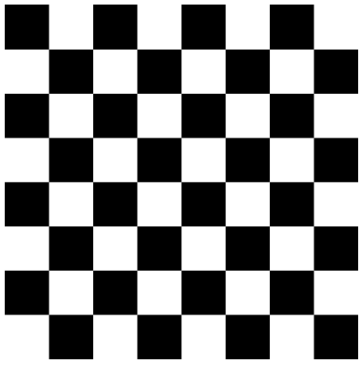
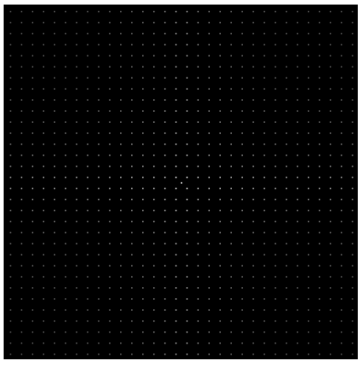
频率图中坐标表示不同的频率，白点的亮度代表频率的大小。

Examples

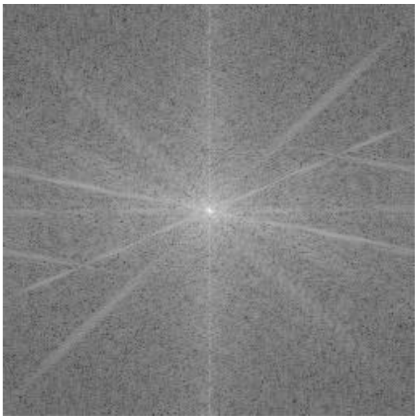
- 只有横坐标有 frequency

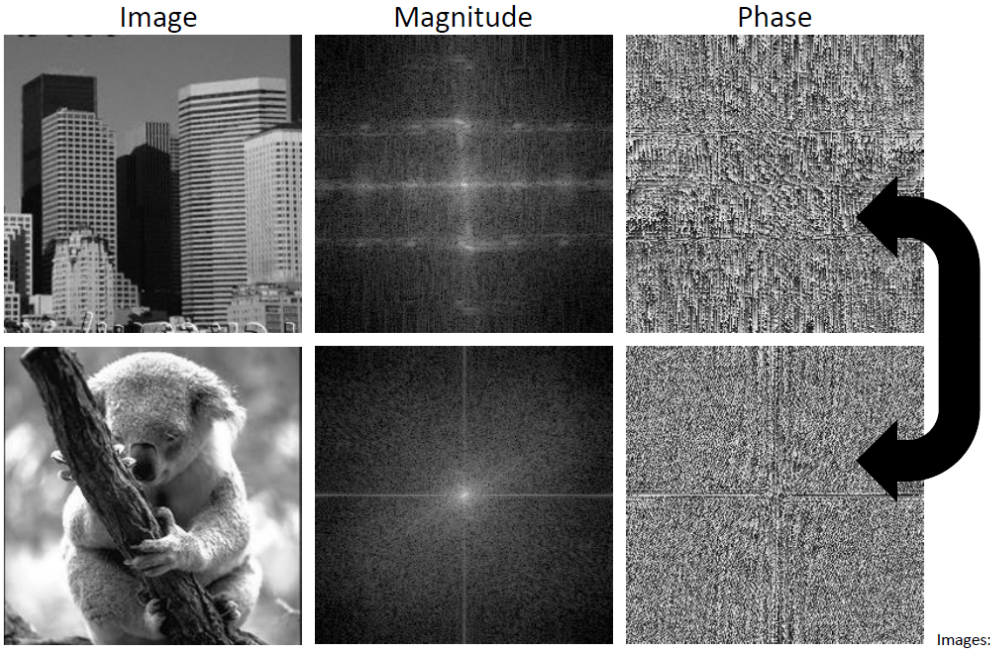
- 横竖坐标都有 frequency

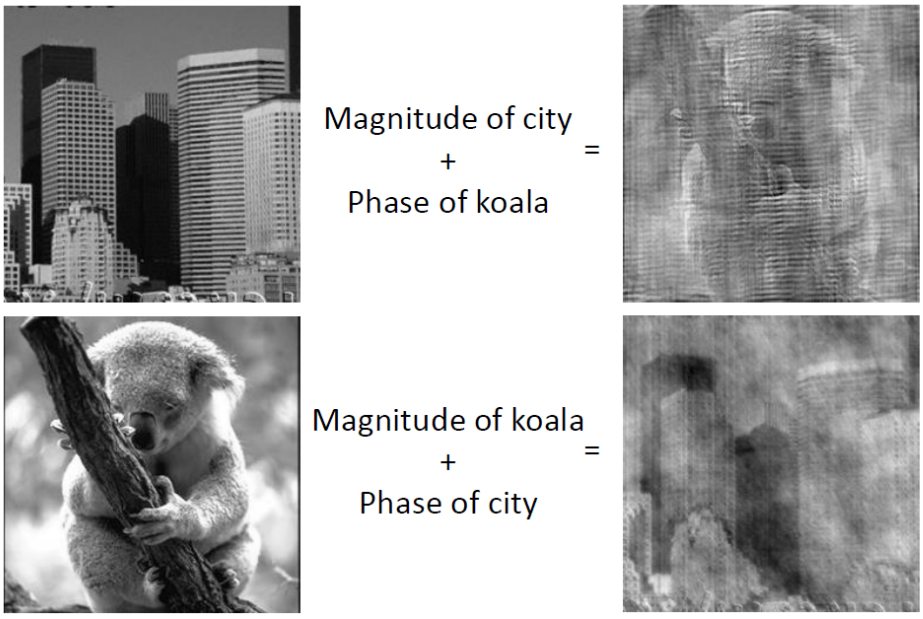



- 真实的图片（大部分图片都类似，会有很多非常低的频率出现）

2.1 Magnitude and Phase





- Any image can be represented by its Fourier transform
- Fourier transform = for each frequency, magnitude (amplitude) + phase
- Magnitude captures the holistic "texture" of an image, but the edges are mainly represented by Fourier phase

3 Frequency Filtering

Operations in the spatial domain have equivalent operations in frequency domain
每个在空域的操作都有对应在频域的操作。

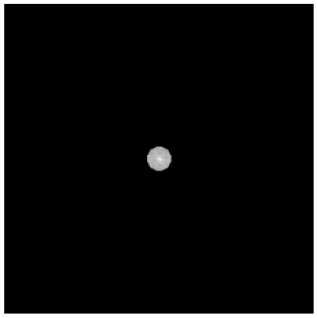
> Convolution in spatial domain = multiplication in frequency domain
> 空域卷积 = 频域相乘

3.1 Bandpass Filter

A filter that removes a range of frequencies from a signal.

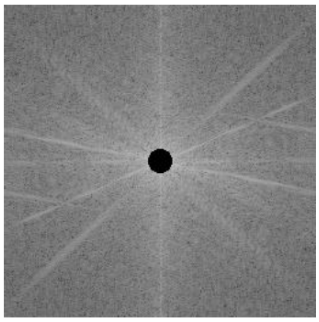
3.2 Low Pass Filter

Keep low spatial frequencies, remove high frequencies.

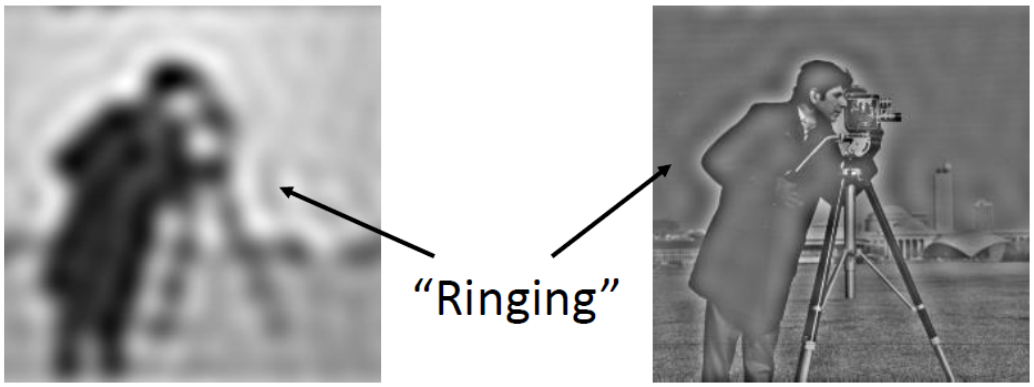




3.3 High Pass Filter

Keep high spatial frequencies, remove low frequencies.

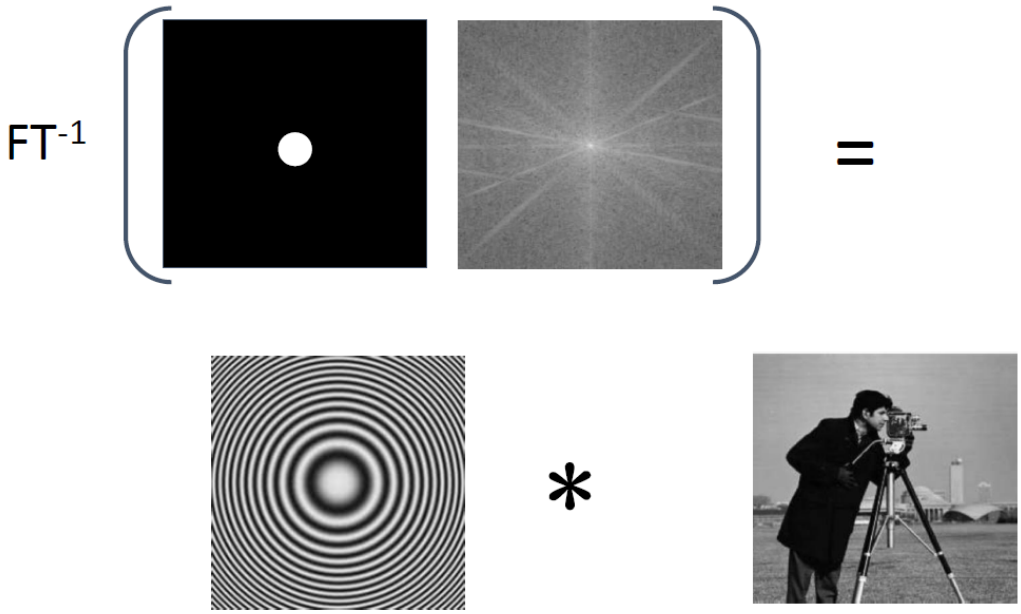




3.4 Filter Artefacts

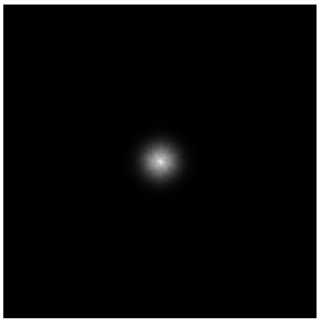


我们会发现直接使用 Low/High Pass 的效果并不好，会使图片出现水滴状的效果，原因是因为没有使用“圆滑”的 filter。

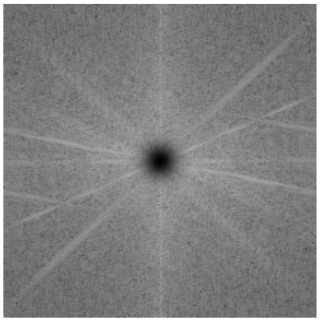
What we are doing: 为了解决这种问题可以使用更为圆滑的 Gaussian filters。



3.5 Gaussian Low Pass Filter


3.6 Gaussian High Pass Filter


4 Applications

- Image compression 图像压缩
  - Human visual system is not very sensitive to contrast in high spatial frequencies
  - Discarding information in high spatial frequencies doesn’ t change the “look” of an image
- Image forensics 图像取证
- Texture & scene representation
- Shape representation