

### Aufgabe 1

$$\begin{array}{l|l} \text{I} & x_1 - 2x_2 = 4 \\ \text{II} & -x_2 - x_3 = -1 \\ \text{III} & -x_1 + x_2 + 3x_3 = -1 \quad | + \text{I} \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 - 2x_2 = 4 \\ \text{II} & -x_2 - x_3 = -1 \\ \text{III} & -x_2 + 3x_3 = 3 \quad | - \text{II} \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 - 2x_2 = 4 \\ \text{II} & -x_2 - x_3 = -1 \\ \text{III} & 4x_3 = 4 \quad | : 4 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 - 2x_2 = 4 \\ \text{II} & -x_2 - x_3 = -1 \quad | + \text{III} \\ \text{III} & x_3 = 1 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 - 2x_2 = 4 \\ \text{II} & -x_2 = 0 \quad | \cdot (-1) \\ \text{III} & x_3 = 1 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 - 2x_2 = 4 \quad | + 2 \cdot \text{II} \\ \text{II} & x_2 = 0 \\ \text{III} & x_3 = 1 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 = 4 \\ \text{II} & x_2 = 0 \\ \text{III} & x_3 = 1 \end{array}$$

## Aufgabe 2

$$\begin{array}{l|l} \text{I} & 5x_1 + 2x_2 - 2x_3 = -1 \\ \text{II} & 3x_1 + x_2 - 3x_3 = -4 \quad | \cdot 5 - 3 \cdot \text{I} \\ \text{III} & 2x_1 + x_3 = 4 \quad | \cdot 5 - 2 \cdot \text{I} \end{array}$$

$$\begin{array}{l|l} \text{I} & 5x_1 + 2x_2 - 2x_3 = -1 \\ \text{II} & -x_2 - 9x_3 = -17 \\ \text{III} & -4x_2 + 9x_3 = 22 \quad | -4 \cdot \text{II} \end{array}$$

$$\begin{array}{l|l} \text{I} & 5x_1 + 2x_2 - 2x_3 = -1 \\ \text{II} & -x_2 - 9x_3 = -17 \\ \text{III} & 45x_3 = 90 \quad | : 45 \end{array}$$

$$\begin{array}{l|l} \text{I} & 5x_1 + 2x_2 - 2x_3 = -1 \quad | + 2 \cdot \text{III} \\ \text{II} & -x_2 - 9x_3 = -17 \quad | + 9 \cdot \text{III} \\ \text{III} & x_3 = 2 \end{array}$$

$$\begin{array}{l|l} \text{I} & 5x_1 + 2x_2 = 3 \\ \text{II} & -x_2 = 1 \quad | \cdot (-1) \\ \text{III} & x_3 = 2 \end{array}$$

$$\begin{array}{l|l} \text{I} & 5x_1 + 2x_2 = 3 \quad | -2 \cdot \text{II} \\ \text{II} & x_2 = -1 \\ \text{III} & x_3 = 2 \end{array}$$

$$\begin{array}{l|l} \text{I} & 5x_1 = 5 \quad | : 5 \\ \text{II} & x_2 = -1 \\ \text{III} & x_3 = 2 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 = 1 \\ \text{II} & x_2 = -1 \\ \text{III} & x_3 = 2 \end{array}$$

### Aufgabe 3

$$\begin{array}{l|l} \text{I} & x_1 + 2x_2 - x_3 = 2 \\ \text{II} & x_1 + x_2 + 2x_3 = 9 \quad | - \text{I} \\ \text{III} & 2x_1 + 3x_2 - 3x_3 = -1 \quad | - 2 \cdot \text{I} \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 + 2x_2 - x_3 = 2 \\ \text{II} & -x_2 + 3x_3 = 7 \\ \text{III} & -x_2 - x_3 = -5 \quad | - \text{II} \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 + 2x_2 - x_3 = 2 \\ \text{II} & -x_2 + 3x_3 = 7 \\ \text{III} & -4x_3 = -12 \quad | : (-4) \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 + 2x_2 - x_3 = 2 \quad | + \text{III} \\ \text{II} & -x_2 + 3x_3 = 7 \quad | - 3 \cdot \text{III} \\ \text{III} & x_3 = 3 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 + 2x_2 = 5 \\ \text{II} & -x_2 = -2 \quad | \cdot (-1) \\ \text{III} & x_3 = 3 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 + 2x_2 = 5 \quad | - 2 \cdot \text{II} \\ \text{II} & x_2 = 2 \\ \text{III} & x_3 = 3 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 = 1 \\ \text{II} & x_2 = 2 \\ \text{III} & x_3 = 3 \end{array}$$

#### Aufgabe 4

$$\begin{array}{l|l} \text{I} & x_1 + x_2 + x_3 = 3 \\ \text{II} & 2x_1 + 4x_2 + 3x_3 = 9 \quad | - 2 \cdot \text{I} \\ \text{III} & 5x_1 + 4x_2 + 4x_3 = 8 \quad | - 5 \cdot \text{I} \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 + x_2 + x_3 = 3 \\ \text{II} & 2x_2 + x_3 = 3 \\ \text{III} & -x_2 - x_3 = -7 \quad | \cdot 2 + \text{II} \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 + x_2 + x_3 = 3 \\ \text{II} & 2x_2 + x_3 = 3 \\ \text{III} & -x_3 = -11 \quad | \cdot (-1) \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 + x_2 + x_3 = 3 \quad | - \text{III} \\ \text{II} & 2x_2 + x_3 = 3 \quad | - \text{III} \\ \text{III} & x_3 = 11 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 + x_2 = -8 \\ \text{II} & 2x_2 = -8 \quad | : 2 \\ \text{III} & x_3 = 11 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 + x_2 = -8 \quad | - \text{II} \\ \text{II} & x_2 = -4 \\ \text{III} & x_3 = 11 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 = -4 \\ \text{II} & x_2 = -4 \\ \text{III} & x_3 = 11 \end{array}$$

### Aufgabe 5

$$\begin{array}{l|l} \text{I} & 4x_1 + 3x_2 + x_3 = 13 \\ \text{II} & 2x_1 - 5x_2 + 3x_3 = 1 \quad | \cdot 2 - \text{I} \\ \text{III} & 7x_1 - x_2 - 2x_3 = -1 \quad | \cdot 4 - 7 \cdot \text{I} \end{array}$$

$$\begin{array}{l|l} \text{I} & 4x_1 + 3x_2 + x_3 = 13 \\ \text{II} & -13x_2 + 5x_3 = -11 \\ \text{III} & -25x_2 - 15x_3 = -95 \quad | : 5 \end{array}$$

$$\begin{array}{l|l} \text{I} & 4x_1 + 3x_2 + x_3 = 13 \\ \text{II} & -13x_2 + 5x_3 = -11 \\ \text{III} & -5x_2 - 3x_3 = -19 \quad | \cdot 13 - 5 \cdot \text{II} \end{array}$$

$$\begin{array}{l|l} \text{I} & 4x_1 + 3x_2 + x_3 = 13 \\ \text{II} & -13x_2 + 5x_3 = -11 \\ \text{III} & -64x_3 = -192 \quad | : (-64) \end{array}$$

$$\begin{array}{l|l} \text{I} & 4x_1 + 3x_2 + x_3 = 13 \quad | - \text{III} \\ \text{II} & -13x_2 + 5x_3 = -11 \quad | - 5 \cdot \text{III} \\ \text{III} & x_3 = 3 \end{array}$$

$$\begin{array}{l|l} \text{I} & 4x_1 + 3x_2 = 10 \\ \text{II} & -13x_2 = -26 \quad | : (-13) \\ \text{III} & x_3 = 3 \end{array}$$

$$\begin{array}{l|l} \text{I} & 4x_1 + 3x_2 = 10 \quad | - 3 \cdot \text{II} \\ \text{II} & x_2 = 2 \\ \text{III} & x_3 = 3 \end{array}$$

$$\begin{array}{l|l} \text{I} & 4x_1 = 4 \quad | : 4 \\ \text{II} & x_2 = 2 \\ \text{III} & x_3 = 3 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 = 1 \\ \text{II} & x_2 = 2 \\ \text{III} & x_3 = 3 \end{array}$$

### Aufgabe 6

$$\begin{array}{l|l} \text{I} & x_1 + 3x_2 + x_3 = 19 \\ \text{II} & -x_1 + x_2 - x_3 = -7 \quad | + \text{I} \\ \text{III} & 2x_1 + 2x_2 + x_3 = 18 \quad | - 2 \cdot \text{I} \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 + 3x_2 + x_3 = 19 \\ \text{II} & 4x_2 = 12 \quad | : 4 \\ \text{III} & -4x_2 - x_3 = -20 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 + 3x_2 + x_3 = 19 \\ \text{II} & x_2 = 3 \\ \text{III} & -4x_2 - x_3 = -20 \quad | + 4 \cdot \text{II} \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 + 3x_2 + x_3 = 19 \\ \text{II} & x_2 = 3 \\ \text{III} & -x_3 = -8 \quad | \cdot (-1) \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 + 3x_2 + x_3 = 19 \quad | - \text{III} \\ \text{II} & x_2 = 3 \\ \text{III} & x_3 = 8 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 + 3x_2 = 11 \quad | - 3 \cdot \text{II} \\ \text{II} & x_2 = 3 \\ \text{III} & x_3 = 8 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 = 2 \\ \text{II} & x_2 = 3 \\ \text{III} & x_3 = 8 \end{array}$$

### Aufgabe 7

$$\begin{array}{l|l} \text{I} & -x_1 + 7x_2 - x_3 = 5 \\ \text{II} & 4x_1 - x_2 + x_3 = 1 \quad | + 4 \cdot \text{I} \\ \text{III} & 5x_1 - 3x_2 + x_3 = -1 \quad | + 5 \cdot \text{I} \end{array}$$

$$\begin{array}{l|l} \text{I} & -x_1 + 7x_2 - x_3 = 5 \\ \text{II} & 27x_2 - 3x_3 = 21 \quad | : 3 \\ \text{III} & 32x_2 - 4x_3 = 24 \quad | : 4 \end{array}$$

$$\begin{array}{l|l} \text{I} & -x_1 + 7x_2 - x_3 = 5 \\ \text{II} & 9x_2 - x_3 = 7 \\ \text{III} & 8x_2 - x_3 = 6 \quad | \cdot 9 - 8 \cdot \text{II} \end{array}$$

$$\begin{array}{l|l} \text{I} & -x_1 + 7x_2 - x_3 = 5 \\ \text{II} & 9x_2 - x_3 = 7 \\ \text{III} & -x_3 = -2 \quad | \cdot (-1) \end{array}$$

$$\begin{array}{l|l} \text{I} & -x_1 + 7x_2 - x_3 = 5 \quad | + \text{III} \\ \text{II} & 9x_2 - x_3 = 7 \quad | + \text{III} \\ \text{III} & x_3 = 2 \end{array}$$

$$\begin{array}{l|l} \text{I} & -x_1 + 7x_2 = 7 \\ \text{II} & 9x_2 = 9 \quad | : 9 \\ \text{III} & x_3 = 2 \end{array}$$

$$\begin{array}{l|l} \text{I} & -x_1 + 7x_2 = 7 \quad | - 7 \cdot \text{II} \\ \text{II} & x_2 = 1 \\ \text{III} & x_3 = 2 \end{array}$$

$$\begin{array}{l|l} \text{I} & -x_1 = 0 \quad | \cdot (-1) \\ \text{II} & x_2 = 1 \\ \text{III} & x_3 = 2 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 = 0 \\ \text{II} & x_2 = 1 \\ \text{III} & x_3 = 2 \end{array}$$

### Aufgabe 8

$$\begin{array}{l|l} \text{I} & 2x_1 - 4x_2 + 5x_3 = 3 \\ \text{II} & 3x_1 + 3x_2 + 7x_3 = 13 \quad | \cdot 2 - 3 \cdot \text{I} \\ \text{III} & 4x_1 - 2x_2 - 3x_3 = -1 \quad | - 2 \cdot \text{I} \end{array}$$

$$\begin{array}{l|l} \text{I} & 2x_1 - 4x_2 + 5x_3 = 3 \\ \text{II} & 18x_2 - x_3 = 17 \\ \text{III} & 6x_2 - 13x_3 = -7 \quad | \cdot 3 - \text{II} \end{array}$$

$$\begin{array}{l|l} \text{I} & 2x_1 - 4x_2 + 5x_3 = 3 \\ \text{II} & 18x_2 - x_3 = 17 \\ \text{III} & -38x_3 = -38 \quad | : (-38) \end{array}$$

$$\begin{array}{l|l} \text{I} & 2x_1 - 4x_2 + 5x_3 = 3 \quad | - 5 \cdot \text{III} \\ \text{II} & 18x_2 - x_3 = 17 \quad | + \text{III} \\ \text{III} & x_3 = 1 \end{array}$$

$$\begin{array}{l|l} \text{I} & 2x_1 - 4x_2 = -2 \quad | : 2 \\ \text{II} & 18x_2 = 18 \quad | : 18 \\ \text{III} & x_3 = 1 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 - 2x_2 = -1 \quad | + 2 \cdot \text{II} \\ \text{II} & x_2 = 1 \\ \text{III} & x_3 = 1 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 = 1 \\ \text{II} & x_2 = 1 \\ \text{III} & x_3 = 1 \end{array}$$



### Aufgabe 9

$$\begin{array}{l|l} \text{I} & 3x_1 + 4x_2 + 6x_3 = -5 \\ \text{II} & 2x_1 - 3x_2 - 4x_3 = 2 \quad | \cdot 3 - 2 \cdot \text{I} \\ \text{III} & x_1 + 2x_2 - 4x_3 = 22 \quad | \cdot 3 - \text{I} \end{array}$$

$$\begin{array}{l|l} \text{I} & 3x_1 + 4x_2 + 6x_3 = -5 \\ \text{II} & -17x_2 - 24x_3 = 16 \\ \text{III} & 2x_2 - 18x_3 = 71 \quad | \cdot 17 + 2 \cdot \text{II} \end{array}$$

$$\begin{array}{l|l} \text{I} & 3x_1 + 4x_2 + 6x_3 = -5 \\ \text{II} & -17x_2 - 24x_3 = 16 \\ \text{III} & -354x_3 = 1239 \quad | : (-177) \end{array}$$

$$\begin{array}{l|l} \text{I} & 3x_1 + 4x_2 + 6x_3 = -5 \quad | - 3 \cdot \text{III} \\ \text{II} & -17x_2 - 24x_3 = 16 \quad | + 12 \cdot \text{III} \\ \text{III} & 2x_3 = -7 \end{array}$$

$$\begin{array}{l|l} \text{I} & 3x_1 + 4x_2 = 16 \\ \text{II} & -17x_2 = -68 \quad | : (-17) \\ \text{III} & 2x_3 = -7 \end{array}$$

$$\begin{array}{l|l} \text{I} & 3x_1 + 4x_2 = 16 \quad | - 4 \cdot \text{II} \\ \text{II} & x_2 = 4 \\ \text{III} & 2x_3 = -7 \end{array}$$

$$\begin{array}{l|l} \text{I} & 3x_1 = 0 \quad | : 3 \\ \text{II} & x_2 = 4 \\ \text{III} & 2x_3 = -7 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 = 0 \\ \text{II} & x_2 = 4 \\ \text{III} & 2x_3 = -7 \quad | : 2 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 = 0 \\ \text{II} & x_2 = 4 \\ \text{III} & x_3 = -3.5 \end{array}$$

### Aufgabe 10

$$\begin{array}{l|l} \text{I} & 2x_1 - 3x_2 - x_3 = 1 \\ \text{II} & 2x_2 + 3x_3 = 1 \\ \text{III} & 4x_1 + 2x_2 + 3x_3 = 6 \quad | - 2 \cdot \text{I} \end{array}$$

$$\begin{array}{l|l} \text{I} & 2x_1 - 3x_2 - x_3 = 1 \\ \text{II} & 2x_2 + 3x_3 = 1 \\ \text{III} & 8x_2 + 5x_3 = 4 \quad | - 4 \cdot \text{II} \end{array}$$

$$\begin{array}{l|l} \text{I} & 2x_1 - 3x_2 - x_3 = 1 \\ \text{II} & 2x_2 + 3x_3 = 1 \\ \text{III} & -7x_3 = 0 \quad | : (-7) \end{array}$$

$$\begin{array}{l|l} \text{I} & 2x_1 - 3x_2 - x_3 = 1 \quad | + \text{III} \\ \text{II} & 2x_2 + 3x_3 = 1 \quad | - 3 \cdot \text{III} \\ \text{III} & x_3 = 0 \end{array}$$

$$\begin{array}{l|l} \text{I} & 2x_1 - 3x_2 = 1 \quad | \cdot 2 + 3 \cdot \text{II} \\ \text{II} & 2x_2 = 1 \\ \text{III} & x_3 = 0 \end{array}$$

$$\begin{array}{l|l} \text{I} & 4x_1 = 5 \quad | : 4 \\ \text{II} & 2x_2 = 1 \quad | : 2 \\ \text{III} & x_3 = 0 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 = 1.25 \\ \text{II} & x_2 = 0.5 \\ \text{III} & x_3 = 0 \end{array}$$

### Aufgabe 11

$$\begin{array}{l|l} \text{I} & 2x_1 - x_2 + 3x_3 = 0 \\ \text{II} & -x_1 + 2x_2 - 5x_3 = 7 \quad | \cdot 2 + \text{I} \\ \text{III} & 3x_1 - 4x_2 + 7x_3 = -6 \quad | \cdot 2 - 3 \cdot \text{I} \end{array}$$

$$\begin{array}{l|l} \text{I} & 2x_1 - x_2 + 3x_3 = 0 \\ \text{II} & 3x_2 - 7x_3 = 14 \\ \text{III} & -5x_2 + 5x_3 = -12 \quad | \cdot 3 + 5 \cdot \text{II} \end{array}$$

$$\begin{array}{l|l} \text{I} & 2x_1 - x_2 + 3x_3 = 0 \\ \text{II} & 3x_2 - 7x_3 = 14 \\ \text{III} & -20x_3 = 34 \quad | : (-2) \end{array}$$

$$\begin{array}{l|l} \text{I} & 2x_1 - x_2 + 3x_3 = 0 \quad | \cdot 10 - 3 \cdot \text{III} \\ \text{II} & 3x_2 - 7x_3 = 14 \quad | \cdot 10 + 7 \cdot \text{III} \\ \text{III} & 10x_3 = -17 \end{array}$$

$$\begin{array}{l|l} \text{I} & 20x_1 - 10x_2 = 51 \\ \text{II} & 30x_2 = 21 \quad | : 3 \\ \text{III} & 10x_3 = -17 \end{array}$$

$$\begin{array}{l|l} \text{I} & 20x_1 - 10x_2 = 51 \quad | + \text{II} \\ \text{II} & 10x_2 = 7 \\ \text{III} & 10x_3 = -17 \end{array}$$

$$\begin{array}{l|l} \text{I} & 20x_1 = 58 \quad | : 2 \\ \text{II} & 10x_2 = 7 \\ \text{III} & 10x_3 = -17 \end{array}$$

$$\begin{array}{l|l} \text{I} & 10x_1 = 29 \quad | : 10 \\ \text{II} & 10x_2 = 7 \quad | : 10 \\ \text{III} & 10x_3 = -17 \quad | : 10 \end{array}$$

$$\begin{array}{l|l} \text{I} & x_1 = 2.9 \\ \text{II} & x_2 = 0.7 \\ \text{III} & x_3 = -1.7 \end{array}$$