

## 介绍

路网嵌入是一种用来计算最短道路距离的方案，通过记录每个节点（道路之间的交叉口）的信息，把常规的道路用多维空间向量表示，使得每个节点之间的距离更能更有效地被计算。

Road Network Embedding (RNE), proposed by Shahabi et al.\* [12], is an approach to compute shortest path distance in road networks, which bases on the LLR embedding techniques [19]. RNE transforms a road network into a higher dimensional space by assigning a sketch (i.e., a vector) to every node such that the distance between any two nodes can be efficiently approximated using only their sketches.

## 主要步骤

(1) 设 $G=(V,E)$ 为路网 (road network)

$V$ 代表道路之间的交叉节点,  $E$  (edge) 代表道路。

(2)  $V$ 有 $n$ 个, 把 $V$ 分成 $n$ 个子集如下

$V : R = \{V_{1,1}, \dots, V_{1,\alpha}, \dots, V_{\beta,1}, \dots, V_{\beta,\alpha}\}$

Let  $n = |V|$  be the size of the node set  $V$ . Define  $R$  as a set of  $O(\log^2 n)$  reference sets, which are subsets of  $V : R = \{V_{1,1}, \dots, V_{1,\alpha}, \dots, V_{\beta,1}, \dots, V_{\beta,\alpha}\}$ , where  $\alpha = O(\log n)$  and  $\beta = O(\log n)$ .

Each subset  $V_{i,j}$  is defined as a random subset of  $V$  with  $2^i$  nodes randomly chosen from  $V$ .

(3) 一个节点到一个子集 $V_{i,j}$ 的距离即该节点到这个子集的最近距离所有节点距离的最小值

The distance between node  $v$  and subset  $V_{i,j}$  is defined as  $\text{dist}(v, V_{i,j}) = \min_{w \in V_{i,j}} \text{dist}(v, w)$

(4) 对于一个节点, 把其到每个子集的最短距离记录下来, 构成一个 $S$ 集, 这个 $S$ 集就是该节点的**路网嵌入向量**, 然后我们把所有的向量放在一起, 构成路网嵌入数据集。

Based on this, each node  $v \in V$  in the road network is mapped to a sketch as follows:

$$S(v) = (S_{V_{1,1}}(v), \dots, S_{V_{1,\alpha}}(v), \dots, S_{V_{\beta,1}}(v), \dots, S_{V_{\beta,\alpha}}(v)),$$

where  $S_{V_{i,j}}(v) = \text{dist}(v, V_{i,j})$ . Note that the sketch  $S(v)$  is a  $O(\log^2 n)$ -dimension vector in the high-dimensional embedding space defined by  $R$ . Finally, denote  $\Omega = \{S(v) | v \in V\}$  as the embedded road network.

(5) 假设一个点 $u$ , 处于节点 $s$ 和 $t$ 之间, 该节点 $u$ 到某个子集 $V_{i,j}$ 之间的距离可以如下公式计算,

$\text{dist}(u, s)$ 为点 $u$ 到节点 $s$ 的距离,  $\text{dist}(u, t)$ 同理。

$$\begin{aligned}
S_{V_{i,j}}(u) &= \text{dist}(u, V_{i,j}) \\
&= \min \{ \text{dist}(u, s) + S_{V_{i,j}}(s), \text{dist}(u, t) + S_{V_{i,j}}(t) \}.
\end{aligned}$$

(6) 根据上面的分析，对于该点 $u$ ，我们也可以得到一个**路网嵌入向量**如下

$$S(u) = (S_{V_{1,1}}(u), \dots, S_{V_{1,\alpha}}(u), \dots, S_{V_{\beta,1}}(u), \dots, S_{V_{\beta,\alpha}}(u)).$$

$S(u)$  is a vector in the embedding space. For simplicity, we denote a sketch of a rider or driver  $u$  as:

$$S(u) = (S_1(u), \dots, S_{\kappa}(u)), \quad (1)$$

(7) 最终，我们可以计算任意两点 $a, b$ 的最短距离

$$\text{dist}(a, b) \approx \delta(a, b) = \max_{1 \leq j \leq \kappa} (|S_j(a) - S_j(b)|),$$