MSc in CSTE
High Performance Technical Computing

Assignment

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Hand in date: 21/1/18 (FT), 4/2/18 (PT), 09:30am

1 Introduction

In this assignment you are asked to examine the application of distributed memory parallel programming techniques for the numerical solution of partial differential equations. In order to do this, we will consider the following problem.

Consider the first order wave equation:

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} = 0$$

where u, the speed of sound, is $250m/s$. Assume that a disturbance is introduced in a one-dimensional long tube of length $L = 400m$ with both ends closed. The imposed boundary conditions are:

$$\begin{aligned} x = 0 \qquad & f(0,t) = 0 \\ x = L \qquad & f(L,t) = 0 \end{aligned}$$

Assume that at time $t = 0$, a disturbance of half sinusoidal shape has been generated. The initial condition is shown in Figure 1 and specified as:

$$\begin{aligned} f(x,0) &= 0 && 0 \leq x \leq 50 \\ f(x,0) &= 100\left\{sin\left[\pi\left(\frac{x-50}{60}\right)\right]\right\} && 50 \leq x \leq 110 \\ f(x,0) &= 0 && 110 \leq x \leq L \end{aligned}$$

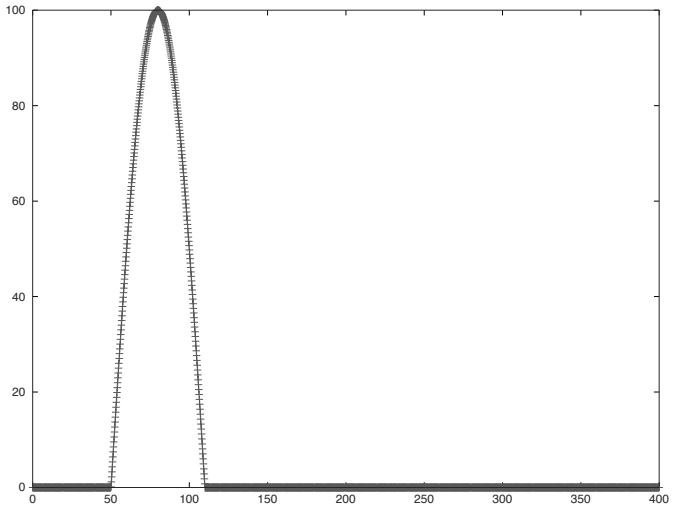

Figure 1: Initial condition at t=0 sec

2 HPC Tasks

1. First devise a way to parallelise and then analyse your algorithms for the following methods:

   - Explicit Upwind FTBS (Forward time, Backward space)
   - Implicit Upwind FTBS (Forward time, Backward space)
   - Implicit FTCS (Forward time, Central space)

   For each method the following sets of step sizes are to be used:

   (a) $\Delta x = 0.5m$ and $\Delta t = 0.002sec$

   (b) $\Delta x = 0.5m$ and $\Delta t = 0.001sec$

   (c) $\Delta x = 0.5m$ and $\Delta t = 0.0005sec$

   In all cases the solution is to be printed and plotted for all x locations every $0.1sec$ time intervals from $0.0sec$ to $0.5sec$.

2. Implement your algorithms in a program written in a programming langugage of your choice, between C, C++, FORTRAN, and coupled with MPI.

3. Compare the numerical solutions obtained by the serial programs and the parallel programs with the analytical solution.

4. Measure the cost of communication in the boundary exchange and the cost of computing a time-step for each process.

5. Measure the performance of your serial and parallel codes and discuss. How does the parallel program performance compare to the theoretical one? Is the performance of your parallel program the expected/theoretical one?

6. Replace your own linear system solver with an appropriate one from an external mathematical library. Study how the performance of a different implementation of the linear system solver affects the overall performance of an implicit scheme for the solution of the prescribed PDE.

7. Based on your results, reason on the problem sizes deemed necessary in order for MPI parallelisation to become efficient for the above numerical algorithms.

3 Source Code and Report Requirements

The source program will need to compile on Crescent using the Intel compilers and Intel MPI. Your simulations for the above tasks should be performed on Crescent using the queueing system and working nodes.

Write a report to present and discuss your findings. The report should be no less than 1,500 words and must not exceed 3,000 words. The report can contain any number of figures/tables, however all figures/tables should be numbered and discussed. The report should include a description of the design of your solution explaining your choices. The source code and a sample scheduler script should be included as Appendices to the report.

4 Assignment Submission

The source code and the sample scheduler script must be submitted electronically via the **Blackboard submission point** by 9:30am on the 21st January (full-time students) or the 4th February (part-time students).

The report should be submitted electronically via the **TurnItInUK submission point** by the prescribed deadline, for the assignment submission to be considered complete.

5 Marking

The assignment will be assessed based on the following marking scheme:

- 20% Introduction, methodology, conclusions
- 40% Source code, design
- 30% Discussion and analysis of the results
- 10% Report structure, presentation, references