CS 3823 - Theory of Computation: Homework Assignment 1

Fall 2025 **Due:** Friday, September 12, 2025

Related Reading. Chapter 0 and Chapter 1.1
Instructions. Near the top of the first page of your solutions please list clearly **all** the members of the group (please see the syllabus for the collaboration policy) who have created the solutions that you are submitting. Listing the names of the people in the group implies their full name and their 4x4 IDs. Alternatively, you can use the space below and provide the relevant information in case you submit the solutions using this document.

Student Information for the Solutions Submitted

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Grade

Exercise	Pages	Your Score	Max
1	2		4
2	3		4
3	4-5		12
4	6-7		12
5	8		4
6	9		4
Total	2-9		40

Additional Help and Resources. Did you use help and/or resources other than the textbook? Please indicate below.

1 Set Theory [4 points]

Let A be the set $\{x, y, z\}$ and B be the set $\{a, b, x\}$.

(i) Is A a subset of B and why?

(ii) What is $A \cap (B \setminus A)$?

(iii) What is $A \times B$?

(iv) What is the powerset of B?

i) Is A a subset of B and why?

$\boxed{\text{No, } A \not\subseteq B}$

For A to be a subset of B all elements of A must be an element of B

$A = \{x, y, z\}$

$x \in B$

$\left.\begin{array}{l} y \notin B \\ z \notin B \end{array}\right\} \rightarrow$ 2 elements from set A are not in B

$\therefore A \not\subseteq B$

ii) What is $A \cap (B \setminus A)$

$A \cap \underbrace{(B \setminus A)}$

elements in B that are not in A

$B = \{a, b, x\}$ $\Rightarrow B \setminus A = \{a, b\}$

$A = \{x, y, z\}$

$\rightarrow A \cap \{a, b\}$

$\underbrace{\{x, y, z\} \cap \{a, b\}}_{\text{no intersection}} \Rightarrow \boxed{A \cap (B \setminus A) = \{\} = \emptyset}$

iii) What is $A \times B$

$A \times B = \{(a, b) \mid a \in A \text{ ∴ } b \in B\}$

$\boxed{A \times B = \{(x,a), (x,b), (x,x), (y,a), (y,b), (y,x), (z,a), (z,b), (z,x)\}}$

iv) What is the powerset of B

The powerset of B, $P(B)$, is the set of all subsets of B

$B = \{a, b, x\}$, so its subsets are:

0 elements: $\emptyset$

1 elements: $\{a\}, \{b\}, \{x\}$

2 elements: $\{a, b\}, \{a, x\}, \{b, x\}$

3 elements: $\{a, b, x\}$

$\Rightarrow \boxed{P(B) = \{\emptyset, \{a\}, \{b\}, \{x\}, \{a, b\}, \{a, x\}, \{b, x\}, \{a, b, x\}\}}$

2 Induction [4 points]

Prove by induction on k that for all integers $k \geqslant 4$ it holds that $k! > 2^k$.

Prove that for all $k \geq 4$ $k! > 2^k$

Base case: $k = 4$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$2^4 = 16$ $\Rightarrow 24 > 16$

$\therefore$ the base case $(k=4)$ holds

Inductive hypothesis: Assume for some integer $k \geq 4$, $k! > 2^k$

Inductive step:

★ We need to show $(k+1)! > 2^{(k+1)}$

$(k+1)! = (k+1) \cdot k!$ → $k! > 2^k$

$(k+1) \cdot k! > (k+1) \cdot 2^k$

$(k+1)! > (k+1) \cdot 2^k$

Now we need to show $(k+1) \cdot 2^k > 2^{k+1}$ as that would prove → $a > b > c$ ↓ $a > c$

$2^{k+1} = 2 \cdot 2^k \Rightarrow (k+1) \cdot 2^k > 2 \cdot 2^k$ → divide both sides by 2^k

↳ $k+1 > 2$ ⇒ this is true when $k \geq 2$.

⇒ Since $k \geq 4$ the statement $k+1 > 2$ holds

Therefore,

$(k+1)! > (k+1) \cdot 2^k > 2^{k+1}$

↓

$(k+1)! > 2^{k+1}$

$\boxed{\therefore \text{ By induction, } k! > 2^k \text{ for all integers } k \geq 4}$

3 Drawing State Diagrams [12 points; 4 points each]

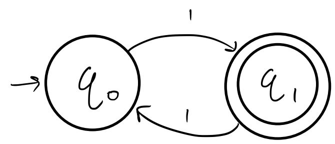
Draw state diagrams for DFAs recognizing the following languages:

(i) $L_1 = \{w \mid \text{length of } w \text{ is odd}\}, \Sigma = \{1\}$.

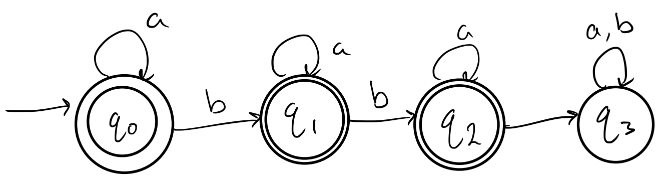
(ii) $L_2 = \{w \mid w \text{ has at most two occurrences of the symbol } b\}, \Sigma = \{a, b\}$.

(iii) $L_3 = \{w \mid w \text{ starts with an } 1 \text{ or ends with a } 0\}, \Sigma = \{0, 1\}$.

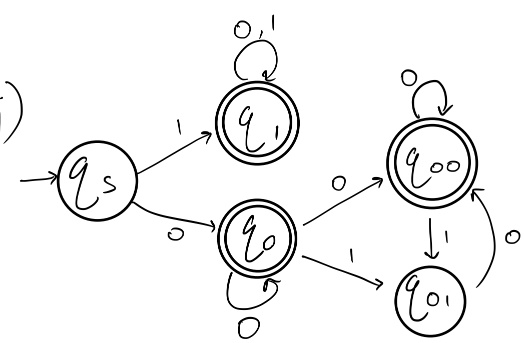
i)



ii)



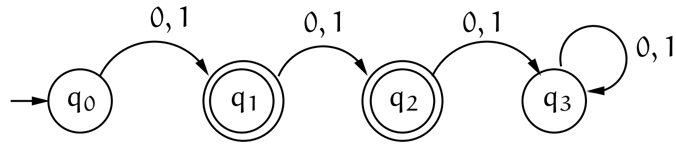
iii)



4 Interpreting State Machines [12 points]

Let $\Sigma = \{0, 1\}$. For each of the following DFAs explain what language they recognize.

(i) **[6 points]** Please see the DFA of machine M_1 below. For this machine M_1, also give its formal



description as a 5-tuple. You do not need to do this for the machine M_2 that follows in part (ii).

$q_0 = \varepsilon$ = not accepted

q_1 = word of length 1 = accepted
q_2 = word of length 2 = accepted

both 1,0 in transition so symbol is not relevant

$\Rightarrow L(M_1) = \{w \mid w \text{ is of length } 1 \text{ or } 2\}$

q_3 = word of length 3 = not accepted
↳ trap state

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0, 1\}$

$q_0 = q_0$

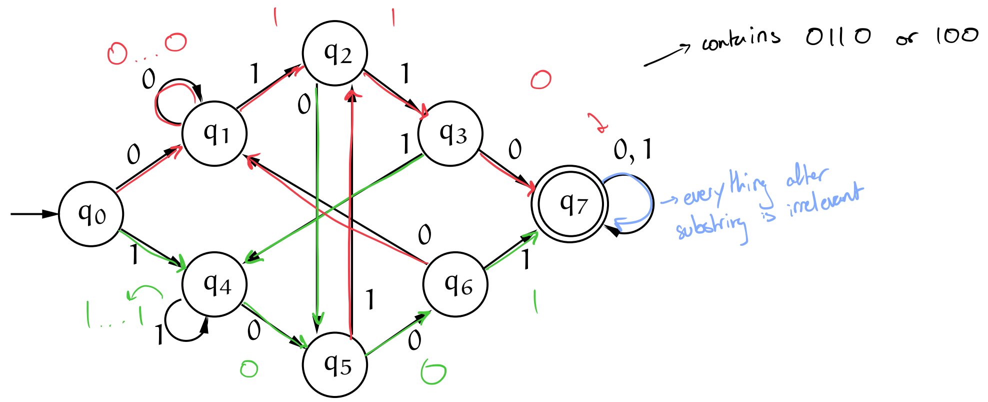
$F = \{q_1, q_2\}$

δ =

state	1	0
q_0	q_1	q_1
q_1	q_2	q_2
q_2	q_3	q_3
q_3	q_3	q_3

(continuation of exercise 4)

(ii) **[6 points]** Please see the DFA of machine M_2 below.



so $L(M_2) = \{ w \in \{0,1\} \mid w \text{ contains } 0110 \text{ or } 1001 \text{ as a substring} \}$

5 Closure [4 points]

Let A and B be regular languages. Show that $A \setminus B$ is also regular.
Recall that $A \setminus B = \{x \in A \mid x \notin B\}$. In other words, this operation removes from A all the strings that are also in B.

Proof:

Since A and B are regular, there exist deterministic finite automata (DFAs) $M_A = (Q_A, \Sigma, \delta_A, q_{0A}, F_A)$ and $M_B = (Q_B, \Sigma, \delta_B, q_{0B}, F_B)$ that recognize A and B, respectively.

We construct a DFA $M = (Q, \Sigma, \delta, q_0, F)$ for $A \setminus B$ as follows:

- $Q = Q_A \times Q_B$ (the product states),
- $\delta((p, q), a) = (\delta_A(p, a), \delta_B(q, a))$ for all $(p, q) \in Q$ and $a \in \Sigma$,
- $q_0 = (q_{0A}, q_{0B})$,
- $F = \{(p, q) \in Q \mid p \in F_A \text{ and } q \notin F_B\}$.

Correctness:

For any string $x \in \Sigma^*$, let $\delta^*(q_0, x) = (p, q)$, where $p = \delta_A^*(q_{0A}, x)$ and $q = \delta_B^*(q_{0B}, x)$. Then:

- $x \in A$ iff $p \in F_A$,
- $x \in B$ iff $q \in F_B$.

Thus, $x \in A \setminus B$ iff $p \in F_A$ and $q \notin F_B$, which is exactly when M accepts x. Therefore, M recognizes $A \setminus B$, proving it is regular.

6 Assigned Reading Question [4 points]

Please read the Quanta article *Computation Is All Around Us, and You Can See It if You Try*, by Lance Fortnow. The article is available at the link below:

https://www.quantamagazine.org/computation-is-all-around-us-and-you-can-see-it-if-you-try-20240612/

After reading this article, please answer the question below.

Question: Does Lance Fortnow believe that randomness is unpredictable? Justify your answer.

Fortnow discusses Laplace's Demon, a theoretical all-knowing entity that can accurately comprehend the motion of all things & thus predict the outcome of any random occurrence. He then connects this idea to a recent recipient of the Turing award, Avi Wigderson, who achieved a connection between randomness and incredibly complex computation; to which he reveals his belief that randomness is "is just computation we cannot predict." Towards the end he discusses the entering of an era where computation can be used to manage randomness, leading to confirmation in the assumption that Fortnow believes randomness is not unpredictable,only that it is unpredictable to us based on our abilities.