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CS-4413 || Homework 7

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1. $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ Multiply this by using Strassen's and verify that you get the same result if you use the standard method.

Strassen method:

Compute products

$P_1 = a_{11}(\underbrace{b_{12} - b_{22}}_{S_1})$

$P_2 = (\underbrace{a_{11} + a_{12}}_{S_2})b_{22}$

$P_3 = (\underbrace{a_{21} + a_{22}}_{S_3})b_{11}$

$P_4 = a_{22}(\underbrace{b_{21} - b_{11}}_{S_4})$

$P_5 = (\underbrace{a_{11} + a_{22}}_{S_5})(\underbrace{b_{11} + b_{22}}_{S_6})$

$P_6 = (\underbrace{a_{12} - a_{22}}_{S_7})(\underbrace{b_{12} + b_{22}}_{S_8})$

$P_7 = (\underbrace{a_{11} - a_{21}}_{S_9})(\underbrace{b_{11} + b_{12}}_{S_{10}})$

Result matrix C:

$C_{11} = P_5 + P_4 - P_2 + P_6$
$C_{12} = P_1 + P_2$
$C_{21} = P_3 + P_4$
$C_{22} = P_1 + P_5 - P_3 - P_7$

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

$C_{11} = P_5 + P_4 - P_2 + P_6 = a_{11}b_{11} + a_{12}b_{21}$

$C_{12} = P_1 + P_2 = a_{11}b_{12} + a_{12}b_{22}$

$C_{21} = P_3 + P_4 = a_{21}b_{11} + a_{22}b_{21}$

$C_{22} = P_5 + P_1 - P_3 - P_7 = a_{22}b_{22} + a_{21}b_{12}$
$= a_{21}b_{12} + a_{22}b_{22}$

From textbook
Page 81
section 4.2

Standard Method:

Same thing

$$C = AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{21} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

2. Solve $T(n) = 7T(n/2) + n^2$ when $n = 2^k$, $T(1) = 1$

Using master Theorem:

$a = 7$
$b = 2$ $\rightarrow \log_b a = \log_2 7$
$f(n) = n^2$

compare $f(n) = n^2$ & $n^{\log_b a}$
2 vs $\log_b a$

$\log_2 7 \approx 2.8$
$\downarrow$
$2 < 2.8$
$\downarrow$
Case 1 $\Rightarrow T(n) = \Theta(n^{\log_b a})$

$= \Theta(n^{\log_2 7})$

3. Solve $M(n) = 7M(n/2)$ where $M(1) = 1$

$M(n) = 7M(n/2)$
$M(2^k) = 7M(2^k/2) \Rightarrow 7M(2^{k-1})$
$= 7^2 M(2^{k-2})$
$= 7^k M(1) \Rightarrow 7^k \cdot 1$

$M(2^k) = 7^k$ $\quad n = 2^k \Rightarrow k = \log_2 n$

$M(2^{\log_2 n}) = 7^{\log_2 n} \Rightarrow M(n) = n^{\log_2 7}$

$\therefore M(n) = \Theta(n^{\log_2 7})$

you could also use same exploration as last since the recursive part is the same and still is dominant since $n^{\log_2 7} > 0$ with a large n