Algorithm Analysis – Spring 2025

CS/DSA 4413

Midterm 2

Date given: 03/24/2025 **Date due: 03/28/2025**

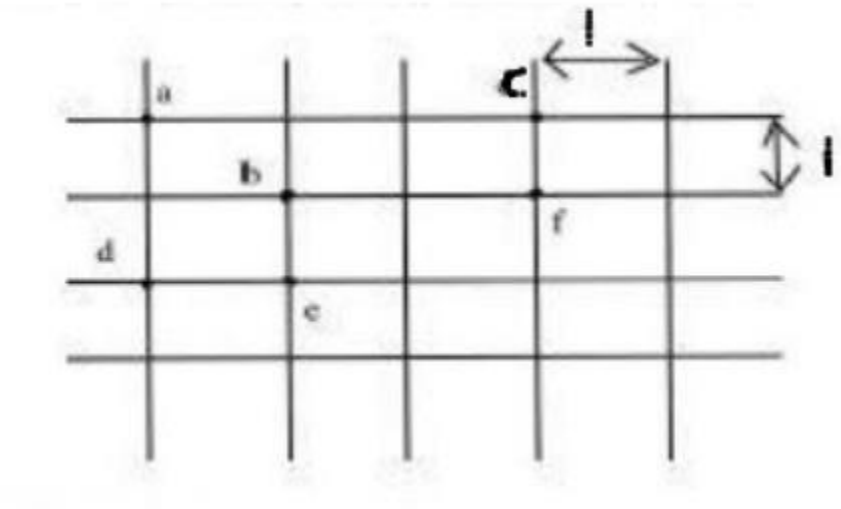
1. (a) Write a program to generate N-pairs (u_i , v_i), $1 \le i \le N$ uniformly distributed in the range [0,1). Plot these N pairs on a unit square where the i^{th} point has coordinates (u_i , v_i), $1 \le i \le N$ and $N = 10^3$.

(b) Use the Monte Carlo method used in the class, estimate the value of $\pi/4$ using N pairs of samples: $N = 10^3 , 10^4 , 10^5 , 10^6$. Plot the estimates vs N.

2. Derive a recurrence for the number P(n) of ways of parenthesizing an expression with n atoms. Compute and plot P(n) vs n for $2 \le n \le 20$.

3. Derive a recurrence for the average number L(n), of rounds needed to elect a leader in a city with n people. Compute and plot L(n) vs n.

4. Consider a set of six cities named a through f laid out on a uniform grid of grid length being unit in both X and Y directions.



(a) Compute the pair-wise distances between $(6 \times 5)/2 = 15$ distinct pairs of cities and build the 6×6 symmetric, weight matrix.

(b) Compute the MST and its cost using the two methods discussed in the class.

(c) Compute all pair shortest paths between these six cities.

5. The well-known Stirling's approximation s(n) to n! is given by

$$s(n) = \sqrt{2\Pi n}\left(\frac{n}{e}\right)^n$$

a) Compute and plot the error
e(n) = n! – s(n)
for 2 ≤ n ≤ 20

b) Compute and plot the relative error
$re(n) = \frac{e(n)}{n!}$
for 2 ≤ n ≤ 20

c) Comment on what you observe.

Remarks:

1. Each question carry 10-points.

2. Include a copy of the program and the time stamp as a proof that you ran so that we can check it.

3. An example: Distance between a and f: d (a, f) = $\sqrt{1^2 + 3^2} = \sqrt{10}$