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Homework #2

1. Compute the size of the problem that can be solved on a machine that takes 10^{-9} sec/op in one day when $T(n) = 15n^2, 8n^3, 2^n, 3^n, n!, n\log n$. Also plot $T(n)$ vs n where $n \in [2, 20]$ on a graph.

Given:

Machine speed: 10^{-9} sec/op

Total time: 1 day = 86,400 sec

$\Downarrow$

Total operations in 1 day:

$$\frac{\text{Total time}}{\text{Time per operation}} = \frac{86400}{10^{-9}} = \underline{8.64 \cdot 10^{13} \text{ operations}}$$

To Find:

The maximum n for each $T(n)$ such that

$T(n) \le 8.64 \cdot 10^{13}$

$T(n) = 15n^2$

$15n^2 \le 8.64 \cdot 10^{13}$

$n^2 \le \frac{8.64 \cdot 10^{13}}{15} = 5.76 \cdot 10^{12}$

$n \le \sqrt{5.76 \cdot 10^{12}}$

$\downarrow$

$\boxed{n \le 2.4 \cdot 10^6}$

$T(n) = 8n^3$

$8n^3 \le 8.64 \cdot 10^{13}$

$n^3 \le \frac{8.64 \cdot 10^{13}}{8} = 1.08 \cdot 10^{13}$

$n \le \sqrt[3]{1.08 \cdot 10^{13}}$

$\approx$

$\boxed{n \le 2.21 \cdot 10^4}$

$T(n) = 2^n$

$2^n \le 8.64 \cdot 10^{13}$

$n \le \log_2(8.64 \cdot 10^{13}) \approx 46.2$

$\boxed{n \le 46.2}$

$T(n) = 3^n$

$3^n \le 8.64 \cdot 10^{13}$

$n \le \log_3(8.64 \cdot 10^{13}) \approx 29.1$

$\boxed{n \le 29.1}$

$T(n) = n!$

$n! \le 8.64 \cdot 10^{13}$

$\downarrow$ by trial

$15! = 1.3 \cdot 10^{12} \le 8.64 \cdot 10^{13}$ ✓

$16! = 2.1 \cdot 10^{13} \le 8.64 \cdot 10^{13}$ ✓

$17! = 3.6 \cdot 10^{14} > 8.64 \cdot 10^{13}$ ✗

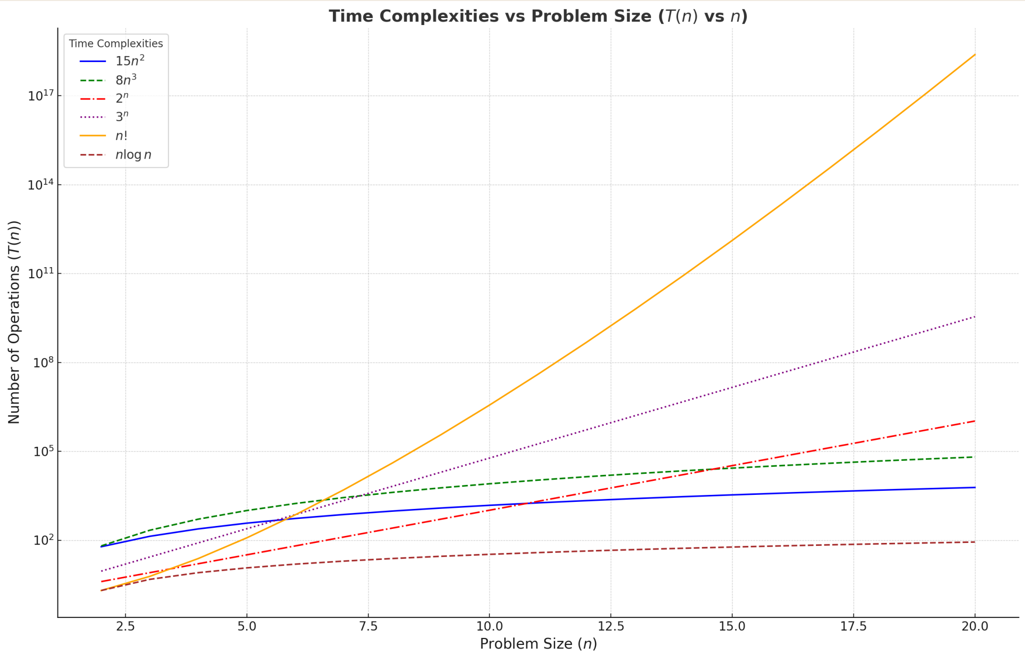
$\boxed{n \le 16}$

$T(n) = n\log n$

$n\log_2(n) \le 8.64 \cdot 10^{13}$

using a graph, the intersection was at $\approx 2.11 \cdot 10^{12}$

so $n \le 2.11 \cdot 10^{12}$



Plot made using python with the matplotlib library

2. Illustrate the operation of insertion sort, merge sort on the array A = ⟨13, 57, 19, 28, 21, 68, 17, 54⟩.

Insertion Sort

1) 13 | 57 | 19 | 28 | 21 | 68 | 17 | 54
2) 13 | 57 | 19 | 28 | 21 | 68 | 17 | 54
3) 13 | 19 | 57 | 28 | 21 | 68 | 17 | 54
4) 13 | 19 | 28 | 57 | 21 | 68 | 17 | 54
5) 13 | 19 | 21 | 28 | 57 | 68 | 17 | 54
6) 13 | 19 | 21 | 28 | 57 | 68 | 17 | 54
7) 13 | 17 | 19 | 21 | 28 | 57 | 68 | 54
8) 13 | 17 | 19 | 21 | 28 | 54 | 57 | 68 Sorted

Merge Set

13   57   19   28   21   68   17   54

13 | 57   19 | 28   21 | 68   17 | 54

13 | 19 | 28 | 57   17 | 21 | 54 | 68

13 | 17 | 19 | 21 | 28 | 54 | 57 | 68

Sorted

2. (con't) Which algorithm is better preforming and why?

Merge sort typically performs better than insertion sort as it has a time complexity of $O(n\log n)$ vs. insertion sorts $O(n^2)$ especially for large data sets.

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3. Define average case, worst case, & best-case complexity.

- **Best-case complexity**: The minimum time required for an algorithm to run, given the best possible input.
- **Worst-case complexity**: The maximum time required for an algorithm to run, given the worst possible input.
- **Average-case complexity**: The expected time required for an algorithm to run; average over all possible inputs.

Derive the average case complexity for sequential search.

Sequential search: Scan each element of the list until target element is found

the target is equally likely to be at any position so:

$$\text{average case} = \frac{1+2+\dots+n}{n} = \frac{n(n+1)/2}{n} = \frac{n+1}{2} \Rightarrow O(n)$$

Prove $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ using induction

Base case ($n=1$)

$$\sum_{i=1}^{1} i = 1 = \frac{1(1+1)}{2} \checkmark \Rightarrow$$

inductive step:

Assume $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$ holds for $k \geq 1$. Then:

$$\sum_{i=1}^{k+1} i = \sum_{i=1}^{k} i + (k+1) = \frac{k(k+1)}{2} + (k+1) = \frac{k(k+1)+2(k+1)}{2} = \frac{(k+1)(k+2)}{2} \Rightarrow \frac{(k+1)((k+1)+1)}{2} \quad \frac{n(n+1)}{2} \checkmark$$

$k^2+k+2k+2$

k^2+3k+2

$1 \; \cancel{2}$
$\quad 3$

$$\frac{(k+1)(k+2)}{2}$$

So by induction the formula holds for all $n \geq 1$

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4. Let us assume algorithm-1 runs in $T_1(n) = \frac{1}{2}n^2$ steps, algorithm-2 takes $T_2(n) = 6n\log_2(n)+6n$ steps for an input of size n. For what values of n (interval) does algorithm-2 perform better than algorithm-1

Find interval of 'n' where $T_2(n) < T_1(n)$:

$$6n\log_2(n)+6n < \frac{1}{2}n^2$$

$$\frac{6n\log_2(n)+6n < \frac{1}{2}n^2}{n}$$

$$\left(6\log_2(n)+6 < \tfrac{1}{2}n\right)\cdot 2$$

$$12\log_2(n)+12 < n$$

$n > 12\log_2(n)+12 \Rightarrow$ solve for n.

n can be found by doing trials of values for n, but to be more exact I used a graph to find 89.879 was the point at which the plots over lapped

so

Alg-2 performs better than alg-1 for $n > 89.879$

$n \in (89.879, \infty)$