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Math-3333 || Homewor4

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Problem 9. Find a basis for $span\{u_1, u_2, u_3, u_4\}$

where $u_1 = \begin{bmatrix}1\\2\\2\end{bmatrix}$, $u_2 = \begin{bmatrix}3\\2\\1\end{bmatrix}$, $u_3 = \begin{bmatrix}11\\10\\7\end{bmatrix}$ and $u_4 = \begin{bmatrix}7\\6\\4\end{bmatrix}$ $\Rightarrow \begin{bmatrix}1&3&11&7\\2&2&10&6\\2&1&7&4\end{bmatrix} \xrightarrow{\substack{R_3 \to R_3 - 2R_1\\ R_2 \to R_2 - 2R_1}} \begin{bmatrix}1&3&11&7\\0&-4&-12&-8\\0&-5&-15&-10\end{bmatrix} \xrightarrow{\substack{R_2 \to -1/4 R_2\\ R_3 \to 1/5 R_3}} \begin{bmatrix}1&3&11&7\\0&1&3&2\\0&-1&-3&-2\end{bmatrix} \xrightarrow{\substack{R_1 \to R_1 - 3R_2\\ R_3 \to R_3 + R_2}} \begin{bmatrix}\boxed{1}&0&2&1\\0&\boxed{1}&3&2\\0&0&0&0\end{bmatrix} \Rightarrow u_1 \,\&\, u_2 \Rightarrow \boxed{\left\{\begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}3\\2\\1\end{bmatrix}\right\}}$

Problem 5. Let $\mathcal{V} = \mathbb{R}^+ =$ The set of all positive real numbers. Suppose a vector addition on $\mathcal{V}$ is defined as

$$u+v = uv$$

for any u and v in $\mathcal{V}$ and a scalar multiplication is defined by

$$s.u = u^s$$

for any u in $\mathcal{V}$ and a scalar s.

(a) Determine if $\mathcal{V}$ with these operations is a vector space or not.

(b) If we keep operations the same, but change $\mathcal{V}$ to $\mathcal{V} = \mathbb{R} - \{0\} =$ The set of all the real numbers except 0, will $\mathcal{V}$ be a vector space ?

(c) If we keep operations the same, but change $\mathcal{V}$ such that $\mathcal{V} = \mathbb{R}^+ \cup \{0\} =$ The set of all the non negative numbers, will $\mathcal{V}$ be a vector space ?

a) $V = \mathbb{R}^+$ $\quad u, v, w \in \mathbb{R}^+$ $\quad s \in \mathbb{R}$

All axioms pass

(A.1)✓ $u + v = uv \in V$

(A.2)✓ $u + v = uv = vu = v + u$

(A.3)✓ $(u+v)+w = (uv)w = u(vw) = u+(v+w)$

(A.4)✓ $u + 0_v = 0_v \cdot u = u \Rightarrow \underline{0_v = 1} \Rightarrow u + 1 = 1 \cdot u = u$

(A.5)✓ $u + (-u) = 0_v = 1 \Rightarrow (-u) = 1/u \in \mathbb{R}^n$

(S.1)✓ $s \cdot u = u^s \in \mathbb{R}^n$

(S.2)✓ $s \cdot (u+v) = s \cdot u + s \cdot v = u^s + v^s = u^s v^s$

(S.3)✓ $(s+r) \cdot u = s \cdot u + r \cdot u = u^s u^r$

(S.4)✓ $s \cdot (r \cdot u) = s \cdot (u^r) = (u^r)^s = u^{rs} = (rs) \cdot u$

(S.5)✓ $1 \cdot u = u^1 = u$

b) $V = \mathbb{R} - \{0\}$, vector space?

All of the additive axioms still pass, but we run into a problem witha the scalars

(S.1) $s \cdot u = u^s \in V$

↳ if u is negative & scaler s is a non integer

$.5 \cdot -2 = (-2)^{.5} = \sqrt{-2} \Rightarrow$ not a real number $\Rightarrow \notin V$

So, no $V = \mathbb{R} - \{0\}$ is not a vector space as scalar multiplication $s \cdot u = u^s$ is not well defined under $u \in V$ & $s \in \mathbb{R}$ specifically when $u < 0$ & s is a non integer causing $u^s \notin \mathbb{R}$

c) $V = \mathbb{R}^+ \cup \{0\}$

Also passes additive axioms

(S.1) since 0 is included for

$u = 0$ & $s = -1$ $\quad s \cdot u = 0^{-1}$ which is undefined.

meaning, $V = \mathbb{R}^+ \cup \{0\}$ is not a vector space as it is not closed under salar multiplication.