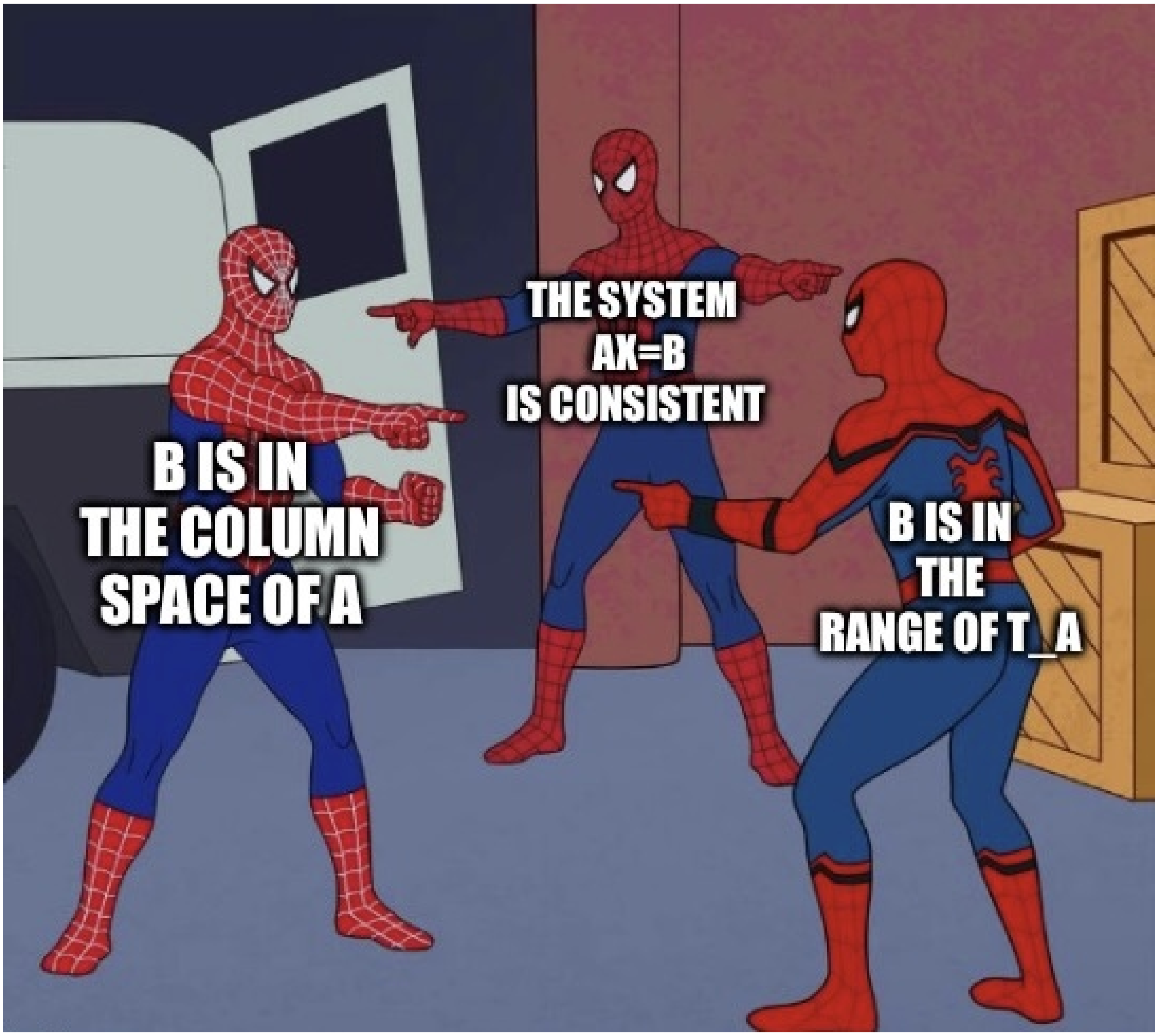
THE SYSTEM
AX=B
IS CONSISTENT
B IS IN
THE COLUMN
SPACE OF A
B IS IN
THE
RANGE OF T_A

**MATH 3333 PRACTICE PROBLEM SET 2**

**Problem 1.** Suppose $A$ and $B$ are two matrices of the same size and $X$ is a column vector such that $AX$ and $BX$ are defined.
State whether following statements are true of false. If true, briefly justify the statement. If false, provide a counterexample.

(a) If $A + B = \mathbf{0}$ then at least one of $A$ and $B$ should be $\mathbf{0}$.
**Solution:** False. We can take $A$ to be any non zero matrix and $B$ to be $-A$. For an example if $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$ then $A + B = 0$ but neither $A$ nor $B$ is a zero matrix.

(b) If $A = B^T$ then $A$ and $B$ are both square matrices.
**Solution:** False. There are many examples. Take $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$. Then $A = B^T$ but neither are square.

(c) If $A$ is symmetric and $A = 2B^T$ then $B$ is also symmetric. **Solution:** True. We have $A = 2B^T$

$\Rightarrow A^T = (2B^T)^T = 2(B^T)^T = 2B$

Since $A$ is symmetric, $A^T = A$. So from the last equation, we have $A = 2B$

Since it is given, $A = 2B^T$, this means $2B = 2B^T \Rightarrow B = B^T \Rightarrow B$ is symmetric.

(d) If $A$ and $B$ are symmetric then so is $2A - 5B$. **Solution:** True. Since $A$ and $B$ are symmetric, we have $A^T = A$ and $B^T = B$. So $(2A - 5B)^T = 2A^T - 5B^T = 2A - 5B$. Hence $2A - 5B$ is symmetric.

(e) If $A$ and $B$ are skew-symmetric then so is $2A - 5B$. **Solution:** True. Since $A$ and $B$ are symmetric, we have $A^T = -A$ and $B^T = -B$.

So $(2A - 5B)^T = 2A^T - 5B^T = 2(-A) - 5(-B) = -(2A - 5B)$. Hence $2A - 5B$ is skew-symmetric.

(f) If $A$ is a square matrix then $A + A^T$ is always symmetric. **Solution:** True.
$(A + A^T)^T = A^T + (A^T)^T = A^T + A = A + A^T$

(g) If $A$ is a square matrix then $2A + 3A^T$ is always symmetric. **Solution:** False. To see this we can take the transpose of $2A + 3A^T$.
$(2A + 3A^T)^T = 2A^T + 3(A^T)^T = 2A^T + 3A$ which is not equal to $2A + 3A^T$ in general.

As a counter example, take $A = \begin{bmatrix} 0 & 1 \\ 0 & 0. \end{bmatrix}$. Then $2A+3A^T = \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}$ which is not symmetric.

**Problem 2.** Suppose $A$ is a matrix and $X$ is a column vector such that $AX$ is defined.
State whether following statements are true of false. If true, briefly justify the statement. If false, provide a counterexample.

(a) If $AX$ has a zero entry then $A$ has a row of zeros. **Solution:** False. One counter example is $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$ and $X = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(b) If $A$ has a row of zero then $AX$ should have a zero entry. **Solution:** True. If row i of $A$ is all zero then the dot product between that row and $X$ will be zero. Hence the entry $i$ of the column vector $AX$ will be 0.

(c) If $AX = 0$ and $X \neq 0$ then $A = 0$. **Solution:** False. One counter example is $A = \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}$ and $X = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(d) If $V = AX$ then $V$ is in the range of $T_A$ where $T_A$ is the transformation induced by $A$. **Solution:** True. $T_A(X) = AX$. If $V = AX$ then $T_A(X) = V$. Hence $V$ is in the range of $T_A$.

(e) If $V = AX$ then $V$ is in $col(A)$. **Solution:** True. If $X = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix}$ then $V = AX = x_1A_1 + x_2A_2 + \dots x_nA_n$ where $A_i$ means the column $i$ of $A$.(see class notes on Sep. 1 Hence $V$ is a linear combination of the columns of $A$. Hence it's in $col(A)$.

**Problem 3.** Let $A = \begin{bmatrix} 4 & 5 & 1 & -1 \\ 2 & 0 & 3 & 6 \\ 1 & -1 & 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 0 \\ -4 & 0 & 2 \\ 1 & 5 & 0 \\ 2 & -1 & 5 \end{bmatrix}$ and $V = \begin{bmatrix} 1 & -2 & 2 \end{bmatrix}^T$

Compute the following whenever they are defined.

(a) $AV$ **Solution:** Undefined

(b) $A^T V$ **Solution:** $\begin{bmatrix} 2 \\ 3 \\ -5 \\ -5 \end{bmatrix}$

(c) $A(A^T V)$ **Solution:** $\begin{bmatrix} 23 \\ -41 \\ -21 \end{bmatrix}$

(d) $BV + 6A$ **Solution:** Undefined

(e) $-2A^T V + BV$ **Solution:** $-2\begin{bmatrix} 2 \\ 3 \\ -5 \\ -5 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ -9 \\ 14 \end{bmatrix} = \begin{bmatrix} -5 \\ -6 \\ 1 \\ 24 \end{bmatrix}$

**Problem 4.** In each case, determine all values of $s$ and $t$ such that the given matrix is symmetric.

(a) $\begin{bmatrix} s & t \\ st & 1 \end{bmatrix}$

**Solution:** If this matrix is symmetric, then $st = t$. This implies either $t = 0$ or if not $s = 1$. So the matrix is symmetric in the following cases.

- $t = 0$ and $s$ can be anything.
- $t \neq 0$ and $s = 1$

(b) $\begin{bmatrix} s & 2s & st \\ t & -1 & s \\ t & s^2 & s \end{bmatrix}$

**Solution:** We have the following equations. $2s = t, st = t$ and $s = s^2$ $s = s^2 \Rightarrow s = 0$ or $s = 1$. **Case 1 : s=0** In this case, the first equation becomes $t = 0$.
$s = 0, t = 0$ satisfies the second equation.
**Case 2 : s=1**. In this case, the second equation becomes $2 = t$. Then $s = 1, t = 2$ satisfies the second equation.
Hence the matrix is symmetric for following values of $s$ and $t$.

- $s = 0, t = 0$
- $s = 1, t = 2$

**Problem 5.**

(a) Consider the function $f : \mathbb{R}^4 \to \mathbb{R}^2$ given by

$$f(\mathbf{X}) = \begin{bmatrix} 2x_1 - x_3 \\ x_2 + 3x_3 + x_4 \end{bmatrix} \qquad \text{where } \mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Find a matrix $A$ such that $f = T_A$

**Solution:** $A = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 1 & 3 & 1 \end{bmatrix}$

(b) Consider the function $g : \mathbb{R}^2 \to \mathbb{R}^3$ given by

$$g(\mathbf{X}) = \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix} \qquad \text{where } \mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Find a matrix $B$ such that $g = T_B$

**Solution:** $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

(c) Suppose $h : \mathbb{R}^4 \to \mathbb{R}^3$ is given by $h = g \circ f$. Compute $h(X)$ where $\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ and then find a matrix $C$ such that $h = T_C$.

**Solution:** Take $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$.

Then $h(X) = g(f(X)) = g\left( \begin{bmatrix} 2x_1 - x_3 \\ x_2 + 3x_3 + x_4 \end{bmatrix} \right) = \begin{bmatrix} 2x_1 - x_3 \\ x_2 + 3x_3 + x_4 \\ 0 \end{bmatrix}$

Hence $C = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(d) How is $C$ related to $A$ and $B$ ? **Solution:** $C = BA$

**Problem 6.** Suppose $f : \mathbb{R}^2 \to \mathbb{R}^4$ is given by $f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 \\ -x_1 + 7x_2 \\ 0 \\ x_2 \end{bmatrix}$.

(a) Find two column vectors $v_1$ and $v_2$ in $\mathbb{R}^4$ such that range of $f$ is the span $\{v_1, v_2\}$.

**Solution:** $f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 \\ -x_1 + 7x_2 \\ 0 \\ x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 7 \\ 0 \\ 1 \end{bmatrix}$.

Hence we can take $v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -1 \\ 7 \\ 0 \\ 1 \end{bmatrix}$

(b) If $V = 7v_1 + 11v_2$ then find a column vector $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ in $\mathbb{R}^2$ such that $f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = V$

**Solution:** $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$

**Problem 7.** Let $A$ be an $m \times n$ matrix and $T_A$ be the transform it induces. That is the function $T_A : \mathbb{R}^n \to \mathbb{R}^m$ is given by $T_A(X) = AX$.
Prove the following statements.

(a) *$T_A$ is an injective function if and only if the linear system $AX = b$ has a unique solution for any $m \times 1$ column vector $b$ in $col(A)$.*
**Solution:** Take any $b$ in $col(A)$. We proved in the class, range of $T_A$ (set of outputs of $T_A$) is the $col(A)$. So $b$ is an output of $T_A$.

$T_A$ is injective
$\Leftrightarrow$ There is only one input maps to any output.
$\Leftrightarrow$ For any $b$ in $col(A)$ there is only one $X$ such that $T_A(X) = b$
$\Leftrightarrow$ For any $b$ in $col(A)$ there is only one $X$ such that $AX = b$
$\Leftrightarrow$ For any $b$ in $col(A)$ the linear system $AX = b$ has a unique solution.

(b) *$T_A$ is a surjective function if and only if any $m \times 1$ column vector $b$ is in the column space of $A$.* **Solution:** If you have not seen the term "surjective" before : A map is surjective if the range of the map (the set of outputs) is equal to the target space (co-domain).

$T_A$ is surjective
$\Leftrightarrow$ The range of $T_A$ is the co-domain $\mathbb{R}^m$
$\Leftrightarrow$ Any $m \times 1$ column vector $b$ is in the range of $T_A$
$\Leftrightarrow$ Any $m \times 1$ column vector $b$ is in the column space of $A$

(c) If $T_A$ is both injective and surjective, what can you say about the solutions to the linear system $AX = b$ for any $m \times 1$ column vector $b$ ? **Solution:** Combining (a) and (b), we can say if $T_A$ is both injective and surjective, then the linear system $AX = b$ has a unique solution for any $m \times 1$ column vec

**Problem 8.** Find the inverse using elementary row operations.

(1) $\begin{bmatrix} -1 & 1 & 2 \\ 0 & 2 & -1 \\ 0 & 1 & -1 \end{bmatrix}$

**Solution:**

$$\begin{bmatrix} -1 & 1 & 2 \\ 0 & 2 & -1 \\ 0 & 1 & -1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 \to -R_1 \begin{bmatrix} 1 & -1 & -2 \\ 0 & 2 & -1 \\ 0 & 1 & -1 \end{bmatrix} \qquad \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \to R_2/2 \begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 1 & -1 \end{bmatrix} \qquad \begin{bmatrix} -1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3 \to R_3 - R_2 \begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} \qquad \begin{bmatrix} -1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix}$$

$$R_1 \to R_1 + R_2 \begin{bmatrix} 1 & 0 & -\frac{5}{2} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} \qquad \begin{bmatrix} -1 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix}$$

$$R_2 \to R_2 - R_3 \begin{bmatrix} 1 & 0 & -\frac{5}{2} \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} \qquad \begin{bmatrix} -1 & \frac{1}{2} & 0 \\ 0 & 1 & -1 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix}$$

$$R_1 \to R_1 - 5R_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} \qquad \begin{bmatrix} -1 & 3 & -5 \\ 0 & 1 & -1 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix}$$

$$R_3 \to -2R_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & 3 & -5 \\ 0 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix}$$

Hence the inverse of the given matrix is $\begin{bmatrix} -1 & 3 & -5 \\ 0 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix}$

(2) $\begin{bmatrix} 1 & 0 & 7 & 5 \\ 0 & 1 & 3 & 6 \\ 1 & -1 & 5 & 2 \\ 1 & -1 & 5 & 1 \end{bmatrix}$

**Solution:** $\begin{bmatrix} 1 & 0 & 7 & 5 \\ 0 & 1 & 3 & 6 \\ 1 & -1 & 5 & 2 \\ 1 & -1 & 5 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$R_3 \to R_3 - R_1$ and $R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 0 & 7 & 5 \\ 0 & 1 & 3 & 6 \\ 0 & -1 & -2 & -3 \\ 0 & -1 & -2 & -4 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 + R_2$ and $R_4 \to R_4 + R_2$

$$\begin{bmatrix} 1 & 0 & 7 & 5 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix}$$

$R_1 \to R_1 - 7R_3,\quad R_2 \to R_2 - 3R_3$ and $R_4 \to R_4 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 & -16 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad \begin{bmatrix} 8 & -7 & -7 & 0 \\ 3 & -2 & -3 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$R_1 \to R_1 - 16R_4,\quad R_2 \to R_2 - 3R_4$ and $R_3 \to R_3 + 3R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad \begin{bmatrix} 8 & -7 & 9 & -16 \\ 3 & -2 & 0 & -3 \\ -1 & 1 & -2 & 3 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$R_4 \to -R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 8 & -7 & 9 & -16 \\ 3 & -2 & 0 & -3 \\ -1 & 1 & -2 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Hence the inverse of the given matrix is $\begin{bmatrix} 8 & -7 & 9 & -16 \\ 3 & -2 & 0 & -3 \\ -1 & 1 & -2 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

**Problem 9.** Solve the system of linear equations given below using your answers to the previous problem.

(1) $-x_1 + x_2 + 2x_3 = 1$
$2x_2 - x_3 = 0$
$x_2 - x_3 = -1$

**Solution:** The linear system can be written as $AX = b$ where $A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & 2 & -1 \\ 0 & 1 & -1 \end{bmatrix}$,

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

In problem 8(a), we saw $A$ has an inverse.

Hence the solution for this system can be given as $X = A^{-1}b = \begin{bmatrix} -1 & 3 & -5 \\ 0 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

(2) $x_1 + 7x_3 + 5x_4 = 0$
$x_2 + 3x_3 + 6x_4 = 1$
$x_1 - x_2 + 5x_3 + 2x_4 = 2$
$x_1 - x_2 + 5x_3 + x_4 = 1$

**Solution:** The linear system can be written as $AX = b$ where $A = \begin{bmatrix} 1 & 0 & 7 & 5 \\ 0 & 1 & 3 & 6 \\ 1 & -1 & 5 & 2 \\ 1 & -1 & 5 & 1 \end{bmatrix}$,

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}$

In problem 8(b), we saw $A$ has an inverse.
Hence the solution for this system can be given as

$$X = A^{-1}b = \begin{bmatrix} 8 & -7 & 9 & -16 \\ 3 & -2 & 0 & -3 \\ -1 & 1 & -2 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ -5 \\ 0 \\ 1 \end{bmatrix}$$

**Problem 10.** Let $A$ be a $4 \times 4$ matrix. The following elementary row operations are applied on $A$ to obtain the matrix $B$.

**First row operation :** $R_2 \rightarrow R_2 - 3R_1$

**Second row operation :** $R_3 \leftrightarrow R_2$

(1) Find two elementary matrices $E_1$ and $E_2$ such that $E_2 E_1 A = B$
**Solution:** By performing the first row operation on $4 \times 4$ identity we obtain
$E_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

and performing the second row operation on $4 \times 4$ identity yields $E_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

For $E_1$ and $E_2$ as above, we have $E_2 E_1 A = B$

(2) Find two elementary matrices $E_3$ and $E_4$ such that $E_4 E_3 B = A$
**Solution:** If $E_2 E_1 A = B$ then we multiply both sides from left, first by $E_2^{-1}$ and then by $E_1^{-1}$. So we have $A = E_1^{-1} E_2^{-1} B$.

Inverse of an elementary matrix is also an elementary matrix. Hence we can take $E_3 = E_2^{-1}$ and $E_4 = E_1^{-1}$.

To compute what these inverses are, we need to perform 'inverse' row operations on $I$.

The 'inverse' of the second row operation is $R_3 \leftrightarrow R_2$. Hence $E_3 = E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

The 'inverse' of the first row operation is $R_3 \rightarrow R_2 + 3R_1$. Hence $E_4 = E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(3) If $B = I$ then compute $A$.
**Solution:** In part (2) we showed $A = E_4 E_3 B$.
If $B = I$ then $A = E_4 E_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$