Colby Frison — Math-3333 || Homewor 4 — 7/28/25

Problem 9. Find a basis for $span\{u_1, u_2, u_3, u_4\}$

where $u_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, $u_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, $u_3 = \begin{bmatrix} 11 \\ 10 \\ 7 \end{bmatrix}$ and $u_4 = \begin{bmatrix} 7 \\ 6 \\ 4 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 11 & 7 \\ 2 & 2 & 10 & 6 \\ 2 & 1 & 7 & 4 \end{bmatrix} \xrightarrow{\substack{R_3 \to R_3 - 2R_1 \\ R_2 \to R_2 - 2R_1}} \begin{bmatrix} 1 & 3 & 11 & 7 \\ 0 & -4 & -12 & -8 \\ 0 & -5 & -15 & -10 \end{bmatrix} \xrightarrow{\substack{R_2 \to 1/4 R_2 \\ R_3 \to 1/5 R_3}} \begin{bmatrix} 1 & 3 & 11 & 7 \\ 0 & 1 & 3 & 2 \\ 0 & -1 & -3 & -2 \end{bmatrix} \xrightarrow{\substack{R_1 \to R_1 - 3R_2 \\ R_3 \to R_3 + R_2}} \begin{bmatrix} \boxed{1} & 0 & 2 & 1 \\ 0 & \boxed{1} & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow u_1 \text{ \& } u_2 \Rightarrow \boxed{\left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \right\}}$$

Problem 5. Let $\mathcal{V} = \mathbb{R}^+ =$ The set of all positive real numbers.
Suppose a vector addition on $\mathcal{V}$ is defined as

$$u + v = uv$$

for any u and v in $\mathcal{V}$ and a scalar multiplication is defined by

$$s.u = u^s$$

for any u in $\mathcal{V}$ and a scalar s.

(a) Determine if $\mathcal{V}$ with these operations is a vector space or not.

(b) If we keep operations the same, but change $\mathcal{V}$ to $\mathcal{V} = \mathbb{R} - \{0\} =$ The set of all the real numbers except 0, will $\mathcal{V}$ be a vector space ?

(c) If we keep operations the same, but change $\mathcal{V}$ such that $\mathcal{V} = \mathbb{R}^+ \cup \{0\} =$ The set of all the non negative numbers, will $\mathcal{V}$ be a vector space ?

a) $V = \mathbb{R}^+$ $\quad u, v, w \in \mathbb{R}^+$ $\quad s \in \mathbb{R}$

All axioms pass

(A.1) ✓ $u + v = uv \in V$

(A.2) ✓ $u + v = uv = vu = v + u$

(A.3) ✓ $(u+v) + w = (uv)w = u(vw) = u + (v + w)$

(A.4) ✓ $u + 0_v = 0_v \cdot u = u \Rightarrow \underline{0_v = 1} \Rightarrow u + 1 = 1 \cdot u = u$

(A.5) ✓ $u + (-u) = 0_v = 1 \Rightarrow (-u) = 1/u \in \mathbb{R}^n$

(S.1) ✓ $s \cdot u = u^s \in \mathbb{R}^n$

(S.2) ✓ $s \cdot (u + v) = s \cdot u + s \cdot v = u^s + v^s = u^s v^s$

(S.3) ✓ $(s + r) \cdot u = s \cdot u + r \cdot u = u^s u^r$

(S.4) ✓ $s \cdot (r \cdot u) = s \cdot (u^r) = (u^r)^s = u^{rs} = (rs) \cdot u$

(S.5) ✓ $1 \cdot u = u^1 = u$

b) $V = \mathbb{R} - \{0\}$, vector space ?

All of the additive axioms still pass, but we run into a problem witha the scalars

(S.1) $s \cdot u = u^s \in V$

↳ if u is negative & scaler s is a non integer

$.5 \cdot -2 = (-2)^{.5} = \sqrt{-2} \Rightarrow$ not a real number $\Rightarrow \notin V$

So, no $V = \mathbb{R} - \{0\}$ is not a vector space as scalar multiplication $s \cdot u = u^s$ is not well defined under $u \in V$ & $s \in \mathbb{R}$ specifically when $u < 0$ & s is a non integer causing $u^s \notin \mathbb{R}$

c) $V = \mathbb{R}^+ \cup \{0\}$

Also passes additive axioms

(S.1) since 0 is included for

$u = 0$ & $s = -1$ $\quad s \cdot u = 0^{-1}$ which is undefined.

meaning, $V = \mathbb{R}^+ \cup \{0\}$ is not a vector space as it is not closed under salar multiplication.