Colby Frison   Math-3333 || Homework 2   7/15/2025

Problem 1. Suppose A and B are two matrices of the same size and X is a column vector such that AX and BX are defined.
State whether following statements are true of false. If true, briefly justify the statement. If false, provide a counterexample.

(c) If A is symmetric and $A = 2B^T$ then B is also symmetric.

Yes, the statement provided is true, to prove this we must show $B=B^T$

symmetry means $A=A^T$, so we can sub A^T for A

↳ $A^T = 2B^T \underset{\text{Simplify}}{\Rightarrow} (A^T)^T = (2B^T)^T \Rightarrow A = 2B$

now we have $A=2B$, since we know $A=2B^T$ we can now sub that in

↳ $2B^T = 2B \xrightarrow[\text{Simplify}]{} \frac{2B^T}{2} = \frac{2B}{2} \Rightarrow B^T = B$

We have now shown $B=B^T$ meaning B is symmetric ∴ the initial statement is true ■

(f) If A is a square matrix then $A + A^T$ is always symmetric.

Yes, the provided statement is true, to prove this we need to show $A+A^T = S = S^T$

we'll call S a square matrix that $= A + A^T$

If we apply a transpose to both sides we get:

$S = A + A^T \Rightarrow (S)^T = (A+A^T)^T \Rightarrow S^T = (A)^T + (A^T)^T \Rightarrow S^T = A^T + A$

we now have $S^T = A^T + A$, but we know $A^T + A = A + A^T$ since matrix addition order is irrelevant

↳ so $S = A + A^T = A^T + A = S^T$ which can be simplified to $S = S^T$ meaning S is symmetric.

∴ we have found $A+A^T$ is always symmetric since $A + A^T = (A+A^T)^T = A^T + A$ ■

Problem 8. Find the inverse using elementary row operations.

(1) $\begin{bmatrix} -1 & 1 & 2 \\ 0 & 2 & -1 \\ 0 & 1 & -1 \end{bmatrix} = D$

To do this we must use elementary row operations to make this given matrix into an I_3

We then apply these operations in the same order to an I_3

$$\begin{bmatrix} -1 & 1 & 2 \\ 0 & 2 & -1 \\ 0 & 1 & -1 \end{bmatrix} \xrightarrow[R_2 \to R_2 + (-2)R_3]{R_1 \to -1R_1} \begin{bmatrix} 1 & -1 & -2 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \xrightarrow[R_1 \to R_1 + (2)R_2]{R_3 \to R_3 + R_2} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{R_1 \to R_1 + R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

← I_3 so we know there is an inverse

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} -1 & 2 & -4 \\ 0 & 1 & -2 \\ 0 & 1 & -1 \end{bmatrix} \longrightarrow \begin{bmatrix} -1 & 3 & -5 \\ 0 & 1 & -2 \\ 0 & 1 & -1 \end{bmatrix} \longrightarrow \boxed{\begin{bmatrix} -1 & 3 & -5 \\ 0 & 1 & -1 \\ 0 & 1 & -2 \end{bmatrix}} = D^{-1}$$