Problem 1 - Solution

1. From the variety of sensors available on the market, the best compromise in terms of accuracy (high), cost (low), possibility for depth measurements (present) and frequency of measuring (high) is offered by capacitance probes. They are based on the dielectric method and enable to measure bulk permittivity by measuring the time needed to charge a capacitor, which uses the soil as a dielectric.

2. To determine the volumetric soil-water content (θ in m^3 m^{-3}) from the relative permittivity of the bulk soil ϵ_r, we need a calibration curve. We will use here the dieletric mixing model, which first needs to be worked out. We end up with

$$\theta = \frac{\sqrt{\epsilon_r} - [1 + (\sqrt{\epsilon_{rs}} - 1)(1 - \phi)]}{8} \tag{1}$$

where ϵ_{rs} is the relative permittivity of the solid fraction. Calculated θ values from 1 January 2021 to 29 September 2022 can be found in `Solution_Problem_1.xlxs`. As said a value of 4 was taken for ϵ_{rs}. Note that erroneous data need to be filtered out. These include soil-water content values that are higher than the soil's porosity or negative values of soil-water content.

Note that ϵ_{rs} of soil organic matter is lower than that of minerals (around 1.5, whereas minerals typically have a relative permittivity of 4). You might correct ϵ_{rs} accordingly, by calculating the volume weighted average relative permittivity of the solid fraction now consisting of minerals and organic matter. This will result in:

$$\epsilon_{rs} = X_{w,m} \frac{\rho_s}{\rho_{s,m}} \epsilon_{rs,m} + X_{w,om} \frac{\rho_s}{\rho_{s,om}} \epsilon_{rs,om} \tag{2}$$

where $X_{w,m}$ and $X_{w,om}$ is the mineral and organic matter mass fraction, respectively, expressed as a fraction of the total solid mass, ρ_s is the particle density, $\rho_{s,m}$ and $\rho_{s,om}$ is the density of the minerals and organic matter, respectively, and $\epsilon_{rs,m}$ and $\epsilon_{rs,om}$ is the permittivity of the minerals and organic matter, respectively. For example, in the topsoil of our forest, soil-organic matter content was 0.078 kg kg^{-1} (7.8%). This gives us a ϵ_{rs} value of 3.7. Inserting this in Eq. (1), results in θ values that are 0.0038 m^3 m^{-3} or 0.4 vol% higher than if the SOM effect would be neglected: not accounting for SOM thus results in an underestimation of θ with 0.4 vol%. Taking SOM in account would thus slightly increase the accuracy of our measurements.

3. To calculate soil-water storage till -80 cm depth in units of mm (typical unit used in water balances), we simply multiply the θ values recorded at a given depth with the depth increments they represent (also expressed in mm), but only till 80 cm. If θ values are missing in the dataset (e.g., because of a corrupt sensor), we might consider

to make an interpolation using θ values taken above and below the sensor. A closer look to our dataset suggests that in our case this would not yield a reasonable estimate given that the water content at -40 cm depth (the depth where we have missing data in the period of interest in the forest) is mostly lower than that above (-10 cm) and below (-80 cm) it. A better estimate could be derived here by linear interpolation from the beginning to the end of the period with missing data; the real value at 19 May 2022 might, however, be somewhat larger (cfr. the peak at -10 cm; yet, the peak at -40 cm is most probably much less pronounced, and our estimate might thus be reasonable). See `Solution_Problem_1.xlxs`.

4. From 19 April to 7 May 2022, we clearly notice diurnal changes in water content at -15 cm in all three land uses: there are no changes during nighttime, while water content decreases during daytime. This indicates a presence of roots at that depth, with roots taking up water during daytime and not during nighttime when stomata (in leaves) are closed. At -80 cm depth, we do not see such pattern in the cropland and grassland, indicating that there is no root activity at that depth. In the forest, we see some very small diurnal changes. There might be taproots at that depth in the forest, but the density of hair roots which are primarily responsible for water uptake seems negligible. See `Solution_Problem_1.xlxs`.

5. Drainage at a given depth (in our case at -80 cm depth) does not take place when there is no change in water content at that depth with time, unless there is a constant input of water (by rain, irrigation, capillary rise or evaporation) which would result in steady state water flow (Chapter 9) which was not the case here. In Chapter 10, we will, when introducing the water conservation, mass balance or continuity equation, demonstrate that indeed the change in soil-water content with time is equal to minus the change in flux of water over depth (minus a root water uptake term). Inspection of our results show that there was no substantial change in water content at -80 cm depth in the cropland, forest and grassland within the given period. This means that there was (almost) no root activity there. As said, there might be taproots at that depth in the forest, but the density of hair roots which are primarily responsible for water uptake seems negligible. See `Solution_Problem_1.xlxs`.

6. Given that their was no drainage at -80 cm depth, and that also runoff, irrigation and lateral subsurface was zero, we can now calculate evapotranspiration from the change in soil-water storage ($\Delta W = W_{end} - W_{initial}$) and precipitation:

$$ET \;=\; P \;-\; \Delta W \tag{3}$$

This is ET of the top 80 cm; if there would have been water uptake from deeper layers, this should be added. With the data we have, however, we cannot measure that. Within the given period (of 19 days), grassland shows the highest ET values.

Within the given period, the grassland has already well developed roots, while the soil surface is not really protected against wind and sunlight. For your information, the *actual* ET of 44.0 mm calculated under grassland approaches best the *potential* ET recorded during that period in the nearby Waregem meteo station (see `waterinfo.be`, which was 57.4 mm or 3.0 mm per day). The lowest ET was found in the forest, where temperature, wind velocity and radiation are lower, while relative humidity higher (resulting in a lower actual ET). The ET of the cropland showed an intermediate value: it is also exposed to wind and sunlight, while its root system is not yet well developed. It should also be noted that actual ET is typically lower than potential ET, since in the calculation of the latter, an optimally watered (no water stress) short clipped grass is considered. See `Solution_Problem_1.xlxs`.