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Snake Optimizer: A novel meta-heuristic optimization algorithm

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ABSTRACT

In recent years, several metaheuristic algorithms have been introduced in engineering and scientific fields to address real-life optimization problems. In this study, a novel nature-inspired metaheuristics algorithm named as Snake Optimizer (SO) is proposed to tackle a various set of optimization tasks which imitates the special mating behavior of snakes. Each snake (male/female) fights to have the best partner if the existed quantity of food is enough and the temperature is low. This study mathematically mimics and models such foraging and reproduction behaviors and patterns to present a simple and efficient optimization algorithm. To verify the validity and superiority of the proposed method, SO is tested on 29 unconstrained Congress on Evolutionary Computation (CEC) 2017 benchmark functions and four constrained real-world engineering problems. SO is compared with other 9 well-known and newly developed algorithms such as Linear population size reduction-Success-History Adaptation for Differential Evolution (L-SHADE), Ensemble Sinusoidal incorporated with L-SHADE (LSHADE-EpSin), Covariance matrix adaptation evolution strategy (CMAES), Coyote Optimization Algorithm (COA), Moth-flame Optimization, Harris Hawks Optimizer, Thermal Exchange optimization, Grasshopper Optimization Algorithm, and Whale Optimization Algorithm. Experimental results and statistical comparisons prove the effectiveness and efficiency of SO on different landscapes with respect to exploration–exploitation balance and convergence curve speed. The source code is currently available for public from: https://se.mathworks.com/matlabcentral/fileexchange/106465-snake-optimizer

1. Introduction

Real-world optimization problems existed in almost all scientific fields and engineering applications can be easily converted to optimization problems [1–5]. These problems have many difficulties and challenges such as nonlinear, multi-objective, discontinuity, high dimensionality, uncertainties, and non-convex regions [6–8].

Optimization techniques can be classified into 2 categories: mathematical programming and metaheuristics. The formal category includes traditional mathematical methods such as integer programming, mathematical programming, Newton & quasi-Newton fails to solve such problems due to its complexity [9–12].

The other category: metaheuristics algorithms (MA) are inspired by imitating natural behavior or phenomena. MA is able to find optimal/near-optimal solutions due to its advantages such as ease of implementation, flexibility, avoiding traps in local optima, and can be used as a black box [13–16].

During the last decade, an enormous number of stochastic algorithms have been proposed. In literature, there are many MA classifications. For example, MA algorithms have been divided into 3 classes: physical algorithms, evolutionary algorithms, and Swarm Intelligence (SI) [17]. A border MA classification has been done by Hussien et al. [18] where MA is classified to Evolutionary Algorithm, Chemistry & Physics-based, Swarm-based algorithms, Music-based algorithms, Math-based algorithms, Sport-based algorithms, Plant-based algorithms, and Human-based. In this study, we classify MA to 4 categories: Evolutionary Algorithms, Swarm-Algorithms, Chemical & physical algorithms and Human-based algorithms as shown in Fig. 1.

Generally speaking, there is no standard/unique classification for MA. In this study, we categorize MA into 4 different classes: evolutionary algorithms, swarm intelligence algorithms, physical & chemical algorithms, and human-based algorithms.

Evolutionary-based algorithm (EA): refers to the stochastic population-based algorithms inspired by nature by employing some genetics rules such as selection, crossover, mutation, and elimination. Examples of EAs include Genetic Algorithm (GA) [19], evolutionary strategies (ES) [20], differential evolution (DE) [21], biogeography-based optimization (BBO) [22].

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Nomenclature

A_m	Male ability to find the food
A_f	Female ability to find the food
$f_{i,f}$	Fitness of ith individual in female group
$f_{i,m}$	Fitness of ith individual in male group
$f_{rand,f}$	Fitness of $X_{rand,f}$
$f_{rand,m}$	Fitness of $X_{rand,m}$
FF	Female fighting ability
FM	Male fighting ability
Q	Food Quantity
$rand$	Random number between 0 and 1
$Temp$	Temperature
X_{max}	Problem Upper Bound
X_{min}	Problem Lower Bound
$X_{rand,f}$	Random position in female group
$X_{rand,m}$	Random position in male group
$X_{worst,f}$	Worst individual in female group
$X_{worst,m}$	Worst individual in male group
Avg	Average
CEC	Congress on Evolutionary Computation
$CMAES$	Covariance matrix adaptation evolution strategy
COA	Coyote Optimization Algorithm
GOA	Grasshopper Optimization Algorithm
HHO	Harris Hawks Optimizer
$IEEE$	Institute of Electrical and Electronics Engineers
$L-SHADE$	Linear population size reduction-Success-History Adaptation for Differential Evolution
$LSHADE-EpSin$	Ensemble Sinusoidal incorporated with L-SHADE
max	Maximum
Med	Median
MFO	Moth-flame Optimization
min	Minimum
SO	Snake Optimizer
STD	Standard Deviation
TEO	Thermal Exchange optimization
WOA	Whale Optimization Algorithm

Physical & chemical-based algorithms simulate physical phenomena or chemical laws existed in the universe. This class contains many algorithms such as Simulated Annealing (SA) [23], Gravitational search algorithm (GSA) [24], chemical reaction optimization (CRO) [25], Atom Search Optimization (ASO) [26], vortex search algorithm (VSA) [27], water evaporation optimization (WEO) [28], big bang-big crunch algorithm (BB-BC) [29], charged system search (CSS) [30], magnetic optimization algorithm (MOA) [31], thermal exchange optimization (TEO) [32], lighting search algorithm [33,34], Multi-verse optimizer (MVO) [35], Nuclear reaction optimization (NRO) [29], Falcon Optimization Algorithm (FOA) [36], Lévy Flight Distribution (LFD) [37], Billiards-inspired optimization algorithm (BOA) [38] and Henry Gas Solubility Optimization (HGSO) [39].

Swarm Intelligence (SI) algorithms are widely known due to their characteristics such as decentralized, shape-formation, and self-organization. These algorithms inspired by natural colonies such as birds, insects, fishes, horses, etc. The most popular examples of SI algorithms are Particle Swarm Algorithm (PSO) [30], Ant Colony Optimization (ACO) [40], and Artificial Bee Colony (ABC) [41]. Literature has many newly proposed SI algorithms such as Cuckoo Search (CS) [42], Bat Algorithm (BA) [43], Firefly Algorithm (FA) [44], Flower Pollination Algorithm (FPA) [45], Crow Search Algorithm (CSA) [46], Krill Herd (KH) [47], Elephant Herding Algorithm [48], Tabu search (TS) [49,50], Gray wolf optimizer (GWO) [51], Ant Lion Optimizer (ALO) [52,53], Whale Optimization Algorithm (WOA) [2,54], Salp Swarm Algorithm (SSA) [55,56], Grasshopper Optimization Algorithm (GOA) [57], Harris Hawks Optimization (HHO) [58,59] Squirrel Search Algorithm [60], Emperor Penguin Optimizer (EPO) [61], Seagull Optimization Algorithm (SOA) [62], Spotted Hyena Optimizer (SHO) [63], Aquila Optimizer (AO) [64], Manta Ray Foraging Optimization (MRFO) [65], Pathfinder Algorithm (PFA) [66], Barnacles mating optimizer (BMO) [67], Slime Mold Algorithm (SMA) [68], Supply–Demand-Based Optimization (SDO) [69], Competitive Swarm Optimizer (CSO) [70], Fitness Dependent Optimizer (FDO) [71], Virus Colony Search [72] and Side-Blotched Lizard Algorithm [73].

The last category of metaheuristics algorithms presented in this study is Human-based algorithms which contain algorithms inspired by human beings, including physical and non-physical activities such as thinking and social behavior. This class contains many algorithms such as teaching–learning based optimization (TLBO) algorithm [74], ideology algorithm [75], socio evolution and learning optimization (SELO) algorithm, [76], cognitive behavior optimization algorithm (COA) [77], human mental search (HMS) [78], Poor and Rich Optimization (PRO) [79], Student Psychology based Optimization algorithm (SPBO) [80], Search and Rescue optimization (SAR) [81] and Arithmetic Optimization Algorithm [82].

The aforementioned algorithms can be classified into 2 categories: single-based and population-based algorithms. Single-based algorithms contain algorithms like SA [23], TS [49], VSA [27]. However, single-based algorithms have better exploitation capabilities, they fall easily in local suboptimal regions. Population-based algorithms start by generating a set of solutions randomly, then the population is enhanced over iterations. Population-based techniques have many benefits/advantages when compared with single-based ones such as widen search space scope, population diversity, assisting each other to bypass local optima, and sharing information between individuals which results in great exploration abilities.

The most 2 important features in MA which can be considered as key features are exploration and exploitation. Exploration means the algorithm's ability to search the entire space to find a new good solution that is very far from the current one, whereas exploitation means the algorithm's ability in finding new solutions around promising areas. In fact, it is very difficult to have a good balance between these 2 factors. Some studies [83,84] have been introduced in order to create proper trade-off them. Using fine-tune between exploration and exploitation is required to find near optimal solutions.

On the other hand, The No Free Lunch theory (NFL) states that no metaheuristic algorithm is able to solve all optimization tasks. In other words, the optimization algorithm can have brilliant results in a specific class of problems and at the same time fails to solve other classes of problems [85].

The above-mentioned facts motivate and encourage us to propose a novel population-based algorithm with swarm intelligence characteristics to solve global optimization problems. The proposed algorithm called Snake Optimizer (SO) is inspired by the mating behavior of the snakes. To the best of our knowledge, there is no such study exist in the literature and this is the first time to propose such a behavior.

The remainder of this paper is organized as follows: Section 2 provides some natural facts about snakes, whereas Section 3

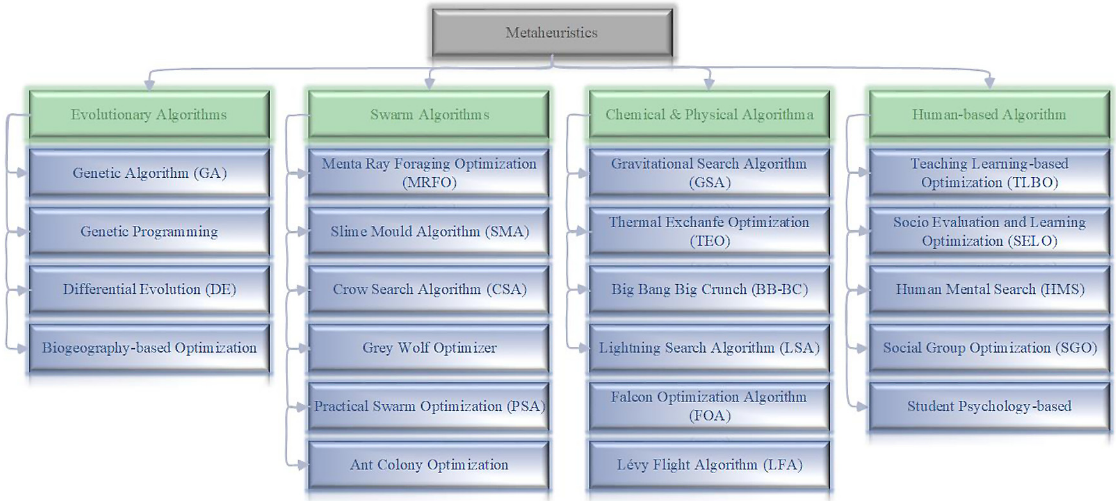

Fig. 1. Metaheuristics classification.

presents the inspiration source and mathematical model of SO. Sections 4 and 5 presented the experimental results and discussion using 30 IEEE CEC benchmark functions and 4 engineering problems. Section 6 shows the conclusion and future work.

2. Snake

Snakes are amazing creatures that belong to reptiles. They have legless elongated bodies as shown in Fig. 2. Moreover, as all squamates, they are cold-blooded vertebrates (Ectothermic). Almost all species of snakes have skulls with many joints which enable them to swallow the prey even if the prey size is larger than their heads. The discovered number of snakes reaches 3600 species that belong to 520 genera, and they were classified into 20 families. The most interesting thing in snakes' life is their unique behavior in mating.[1] Female snakes have many reproduction features (multiple mating, seasonality, and reproductive mode). Females can manipulate both genotypes (mate choice and sperm competition) and phenotypes (physiological thermoregulation and nest-site choosing).

Male–male rivalry has many forms such as mate guarding and female mimicry. Compact bouts have a very strong impose in selecting the winner mating [86,87].

3. Snake Optimization (SO)

This section includes the inspiration and the mathematical model

3.1. Snake mating behavior

The occurrence of mating between both males and females is governed by some factors. Snakes mate in late spring and the early summer where the temperature is low i.e cold area, but the mating process does not depend only on temperature but also on the availability of food. If the temperate is low and the food is available; Competitor males will fight each other to attract the female's attention. The female has the decision to mate or not. If the mating occurs, the female starts laying its eggs in a nest or burrow and it leaves though as soon as the eggs emerge. Mating behavior is shown in Fig. 3.

[1] Interested readers can refer to the following documentary videos: (a) https://bit.ly/38Ayoe1, (b) https://bit.ly/2KRLWKf (c) https://bit.ly/2KDBdmF.

3.2. Inspiration source

The SO is for "Snake Optimization" that inspired by the mating behavior of snakes, the mating occurs if the temperature is low and the food is available, otherwise the snakes will only search for food or eat the exiting food, depend on this information, we will consider the search process has two phases, exploration or exploitation. The exploration represents the environmental factors, that are the cold place and food, are not exist in this case the snake searches only for food in its surroundings.

For exploitation, this phase includes many transition phases to get the global more efficient. In the case of the food is available but the temperature is high, the snakes will focus only on eating the available food. Finally, if the food is available and the area is cold, this leads to the occurrence of the mating process; the mating process has cases, the fight mode or mating mode. In the fight mode, each male will fight to get the best female and each female will try to select the best male. In the mating mode, the mating occurs between each pair related to the availability of food quantity. If the mating process occurs during search space there is a probability the female lays eggs that will hatch into new snakes.

3.3. Mathematical model and algorithm

The pseudocode of SO is shown in Fig. 4, the sections involved are explained in detail as the following:

3.3.1. Initialization

Like all metaheuristics algorithms, SO starts by generating a random population in uniform distribution to be able to begin the optimization algorithm process. The initial population can be obtained using the next equation:

$$X_i = X_{min} + r \times (X_{max} - X_{min}) \quad (1)$$

where X_i is the position of ith individual, r is a random number between 0 and 1, and X_{min}, X_{max} are the lower and upper bounds of the problem respectively.

3.3.2. Diving the swarm into two equal groups males and females

In this study, we assume that the number of male is 50% and the number of females is 50%. The population is divided to 2 groups: male group and female one. To divide the swarm use the following 2 Eqs. (2) and (3).

$$N_m \approx N/2 \quad (2)$$

(a) Snake

(b) Compact bouts between snakes

(c) Snakes laying eggs

Fig. 2. Snakes in nature.

$$N_f = N - N_m \tag{3}$$

where N is the number of individuals, N_m refers to the male individual numbers and N_f refers to the female individual numbers.

3.3.3. *Evaluate each group & defining temperature and food quantity*

- Find the best individual in each group and get the best male ($f_{best,m}$) and best Female ($f_{best,f}$) and the Food position (f_{food}).
- The Temperature *Temp* can be defined using the following equation

$$Temp = exp(\frac{-t}{T}) \tag{4}$$

where t refers to the current iteration and T refers to the maximum number of iterations.

- Defining Food quantity (Q) The food quantity can be obtained using the following equation

$$Q = c_1 * exp(\frac{t-T}{T}) \tag{5}$$

where c_1 is constant and equals 0.5

3.3.4. *Exploration phase (no food)*

If $Q < $ Threshold (Threshold $= 0.25$) the snakes search for food by selecting any random position and update their position respect to it. To model exploration phase, the following

$$X_{i,m}(t+1) = X_{rand,m}(t) \pm c_2 \times A_m \times ((X_{max} - X_{min}) \times rand + X_{min}) \tag{6}$$

Fig. 3. Snake mating.

where $X_{i,m}$ refers to ith male position, $X_{rand,m}$ refers to position of random male, *rand* is a random number between 0 and 1 and A_m is the male ability to find the food and can be calculated as follows:

$$A_m = exp(\frac{-f_{rand,m}}{f_{i,m}}) \tag{7}$$

where $f_{rand,m}$ is the fitness of $X_{rand,m}$ and $f_{i,m}$ is the fitness of ith individual in male group and C_2 is constant and equals 0.05

$$X_{i,f} = X_{rand,f}(t+1) \pm c_2 \times A_f \times ((X_{max} - X_{min}) \times rand + X_{min}) \tag{8}$$

where $X_{i,f}$ refers to ith female position, $X_{rand,f}$ refers to position of random female, *rand* is a random number between 0 and 1 and A_f is the female ability to find the food and can be calculated as follows:

$$A_f = exp(\frac{-f_{rand,f}}{f_{i,f}}) \tag{9}$$

where $f_{rand,f}$ is the fitness of $X_{rand,f}$ and $f_{i,f}$ is the fitness of ith individual in female group.

3.3.5. Exploitation phase (food exists)

If $Q >$ Threshold

If the temperature $>$ Threshold (0.6) (hot)

The snakes will move to the food only

$$X_{i,j}(t+1) = X_{food} \pm c_3 \times \text{Temp} \times \text{rand} \times (X_{food} - X_{i,j}(t)) \tag{10}$$

where $X_{i,j}$ is the position of individual (male or female), X_{food} is the position of the best individuals, and c_3 is constant and equals 2.

If the temperature $<$ Threshold (0.6) %cold

The snake will be in the fight mode or mating mode

Fight Mode

$$X_{i,m}(t+1) = X_{i,m}(t) + c_3 \times FM \times rand \times (Q \times X_{best,f} - X_{i,m}(t)) \tag{11}$$

where $X_{i,m}$ refers to ith male position, $X_{best,f}$ refers to the position of the best individual in female group, and FM is the fighting ability of male agent.

$$X_{i,f}(t+1) = X_{i,f}(t+1) + c_3 \times FF \times rand \times (Q \times X_{best,m} - X_{i,F}(t+1)) \tag{12}$$

where $X_{i,f}$ refers to ith female position, $X_{best,m}$ refers to the position of the best individual in male group, and FF is the fighting ability of female agent.

FM and FF can be calculated from the following equations:

$$\text{FM} = exp(\frac{-f_{best,f}}{f_i}) \tag{13}$$

$$\text{FF} = exp(\frac{-f_{best,m}}{f_i}) \tag{14}$$

where $f_{best,f}$ is the fitness of the best agent of female group, $f_{best,m}$ is the fitness of the best agent of male group, and f_i is the agent fitness.

Mating Mode:

$$X_{i,m}(t+1) = X_{i,m}(t) + c_3 \times M_m \times rand \times (Q \times X_{i,f}(t) - X_{i,m}(t)) \tag{15}$$

$$X_{i,f}(t+1) = X_{i,f}(t) + c_3 \times M_f \times rand \times (Q \times X_{i,m}(t) - X_{i,f}(t)) \tag{16}$$

where $X_{i,f}$ is the position of ith agent in female group and $X_{i,m}$ is the position of ith agent in male group and M_m&M_f refers to the mating ability of male and female respectively and they can be calculated as follow:

$$M_m = exp(\frac{-f_{i,f}}{f_{i,m}}) \tag{17}$$

$$M_f = exp(\frac{-f_{i,m}}{f_{i,f}}) \tag{18}$$

If Egg hatch, select worst male & Female and replace them

$$X_{worst,m} = X_{min} + rand \times (X_{max} - X_{min}) \tag{19}$$

$$X_{worst,f} = X_{min} + rand \times (X_{max} - X_{min}) \tag{20}$$

where $X_{worst,m}$ is the worst individual in male group, $X_{worst,f}$ is the worst individual in female group. The flag direction operator $\pm$ which is also called diversity factor, gives possibility to increase or decrease positions' solution to give high opportunities to change the direction of agents that results a good scan of the given search space in all possible directions. This parameter generated randomly to achieve randomization aspect hat is essential in any meta heuristic algorithm. The idea of this operator is not new, many metaheuristic algorithms were used it before but in two separated equations like foraging behavior (Eq 2.1) in Hunger games search (HGS) [88] algorithm that published recently. HGS algorithm uses two identical equations but with change in operator $\pm$ and the swaps between them is randomly. Other algorithms uses one equation that has a parameter changes from negative value to positive value like "A" parameter in WOA algorithm.

Checking terminating conditions

The process will continue for a number of iteration from step 2, if the criterion is satisfied the process will be terminated.

Algorithm 1 Snake Optimizer Algorithm

Initialize Problem Setting (Dim, UB, LB, and Pop_Size(N), Max_Iter(T), Curr_Iter t
Initialize the population randomly
Divide population N to 2 equal groups N_m and N_f using Eqs. (2) and (3).
while ($t \leq T$) do **do**
  Evaluate each group N_m and N_f
  Find best male $f_{best,m}$
  Find best male $f_{best,f}$
  Define $Temp$ using Eq. (4).
  Define food Quantity Q using Eq. (5).
  if ($Q < 0.25$) **then**
    Perform exploration using Eqs. (6) and (8)
  else if ($Q > 0.6$) **then**
    Perform exploitation Eq. (10)
  else
    if ($rand > 0.6$) **then**
      Snakes in Fight Mode Eqs. (11) and (12)
    else
      Snakes in Mating Mode Eqs. (17) and (18)
      Change the worst male and female Eqs. (19) and (20)
    end if
  end if
end while
Return best solution.

4. Experiments & results

4.1. Results on CEC 2017

4.1.1. Benchmark functions & compared algorithms

This section evaluates and demonstrates the powerfulness and effectiveness of our proposed algorithms. In this study, we have used 30 unconstrained benchmark functions obtained from CEC 2017 [89]. These functions cover 4 types of benchmark landscapes: unimodal, multimodal, hybrid, and composite. A full description of each function accompanied by its optimal solution is shown in Table 2. Due to the stochastic nature of the metaheuristics algorithm, the results change in every execution. Here, the experiments have been performed 30 times for each algorithm. The parameter setting of each algorithm is given in Table 3. Several state-of-art algorithms have been used to solve the same benchmark problems including L-SHADE, LSHADE-EpSin, MFO, HHO, TEO, GOA, and WOA.

Table 1 shows the experimental setup conditions used in the experiments. All these experiments have been performed using MATLAB 2015a on 8.1 Windows Operating System with 6 GB RAM memory and core i5 CPU.

4.1.2. Numerical results & discussions

In this subsection, we show and analyze the statistical results of SO and other seven state-of-art algorithms using 30 CEC 2017 functions. Table 4 shows the results of SO and other comparative algorithms in terms of mean (average), best (min), worst (max), median, and standard deviation (STD). from this table, it can be seen that SO outperform other mentioned metaheuristic algorithms. It is notable that SO has the best average results in 22 functions from 29 ones and the second-best in 3 other functions whereas it ranked third in other 2 functions. GOA has ranked first in only 3 functions however, each of MFO, and HHO has achieved the best average in only 2 functions. On the other hand, SO has ranked first in 11 functions in standard deviation (STD) and has the second-best STD values in other 8 functions. Also, LSHADE-EpSin has recorded the second-best algorithm in terms of STD values as it ranked first in 7 functions. Each of L-SHADE, HHO, and TEO has ranked first twice.

The most 2 important factors in metaheuristics are exploration and exploitation. Exploration can be defined as the ability of search algorithm in finding new solutions in the far neighborhood areas whereas exploitation can be defined as the ability of algorithm in finding new solutions in already existed promising area. It is worth mentioning that the good algorithm should have a good balance between these 2 factors. Figs. 11, 12, and 13 show the exploration and exploitation results in SO algorithm. It can be seen from these figures that SO began with low exploitation and high exploration in the first half of iterations however, it transformed to exploitation stages later(in the 2nd half of iterations).

4.1.3. Scalability test

This subsection discusses and evaluates the proposed algorithm in high dimensional problems $Dim = 50$ and low dimensional one $Dim = 10$ with respect to other algorithms. Table 7 shows the experimental results on 2 dimensions values 10 & 50 in terms of average and STD. Regarding 10 dimensions, SO has ranked first in 17 functions out of 29 functions and has ranked the second-best in other 4 functions in average values whereas regarding STD values, SO has ranked first in 9 functions and achieved the second-best in other 8 functions. On the other hand, SO has ranked the best average and best STD in 50 dimensions in 22 and 10 functions respectively.

4.1.4. Convergence & statistical analysis

Figs. 5, 6, and 7 show the convergence curves of all functions when $Dim = 30$. Obviously, it can be observed that SO has a speed convergence when compared with state-of-art algorithms. Moreover, Figs. 14, 15, and 16 show the convergence curve when $Dim = 50$.
Figs. 8, 9, and 10 show the box plot of all functions when $Dim = 30$.

In order to validate SO results statistically, Wilcoxon Rank Sum (WRS) has been used to perform a nonparametric test [90]. Tables 5 shows WRS results on 30 Dimensions whereas Table 8 shows WRS results on 10 & 50 Dimensions respectively. According to the results of these 2 tables, it can be conducted that SO is

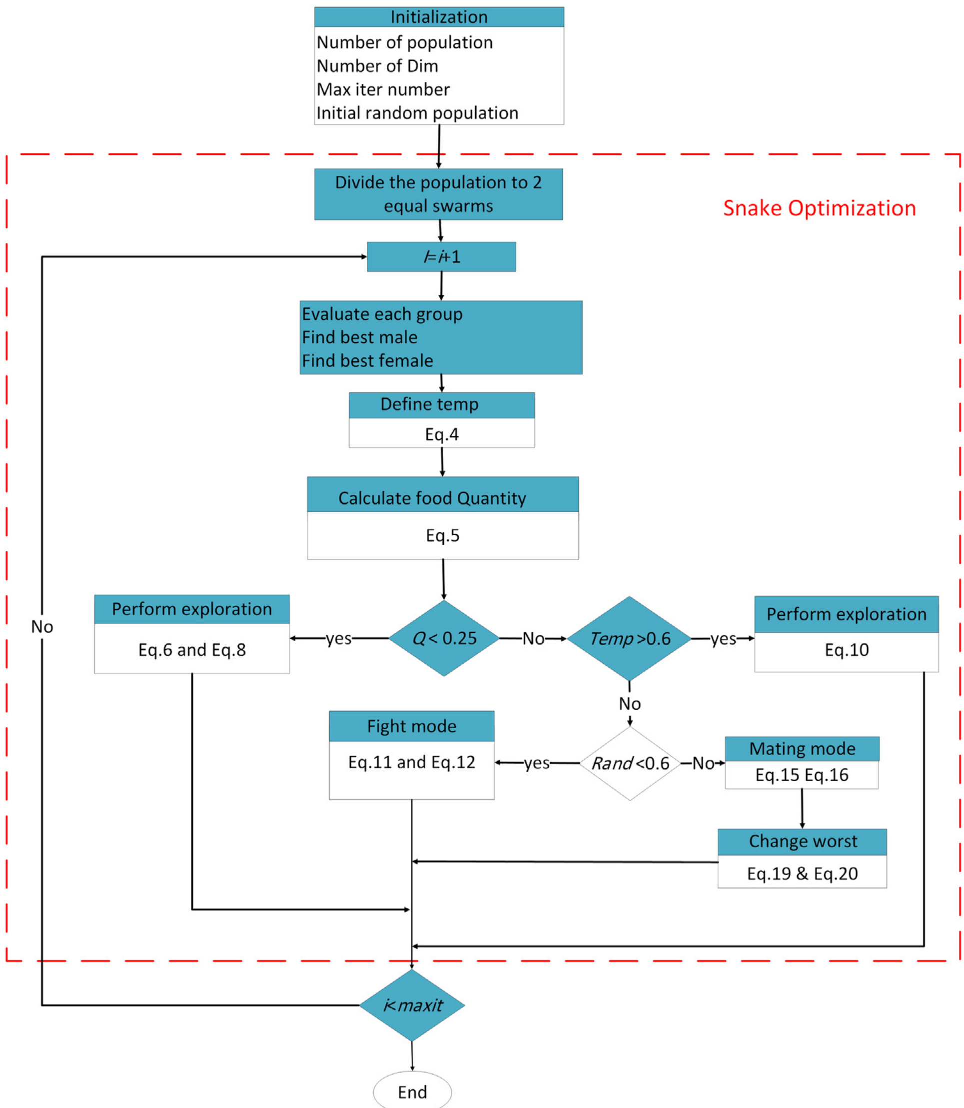

Fig. 4. Snake optimization flow chart.

still better than other algorithms. Moreover, table Table 6 shows the results of Friedman test.

Furthermore, a Frideman test has been carried out to rank all algorithms. From this table, it can be conducted that SO is better than all comparative algorithms.

4.2. Experiments on CEC 2020

In this subsection, we used 10 functions from CEC 2020 and compared SO performance with Covariance Matrix adaptation Evolution Strategy (CMAES) and Coyote Optimization Algorithm (COA) besides the other algorithms mentioned above: Moth-flame Optimization, Harris Hawks Optimizer, Thermal Exchange optimization, Grasshopper Optimization Algorithm, Whale Optimization Algorithm, Linear population size reduction-Success-History Adaptation for Differential Evolution (L-SHADE), and Ensemble Sinusoidal incorporated with L-SHADE (LSHADE-EpSin). Results of this experiments are given in Table 9, from which we can observe that SO has ranked first in 6 functions out of 10. Moreover, SO achieved the second best results in 3 functions (F2, F6, and F10) and ranked third in F9. Figs. 17 and 18 show the convergence curve and box plot between So and other competitive algorithms. Also, Table 10 shows the wilicxon rank sum results between SO and other algorithms.

Table 1
Experiments parameters settings.

No.	Parameter name	Value
1	Population size	30
2	Dim	30
3	Max number of iteration	500

5. Engineering problems

In this subsection, 4 constrained problems are used to prove efficiency of SO. These problems are Speed Reducer Design, Welded Beam Design, Pressure Vessel Design, and Tension/compression Spring Design. The above mentioned problems have many inequalities constraints, So we use penalty functions in which the metaheuristic algorithm has achieved a good values if it violates one of these constraints.

5.1. Speed reducer design problem

The first engineering problem considered in this study is the speed reducer problem. The goal of this problem is to find the minimum speed reducer weight based on some constraints [91]:

Table 2
Benchmark functions.

No.	Types	Name	Opt
F1	Unimodal	Shifted and Rotated Bent Cigar Function	100
F2		Shifted and Rotated Sum of Different Power Function	200
F3		Shifted and Rotated Zakharov Function	300
F4	Multimodal	Shifted and Rotated Rosenbrock's Function	400
F5		Shifted and Rotated Rastrigin's Function	500
F6		Shifted and Rotated Expanded Scaffer's F6 Function	600
F7		Shifted and Rotated Lunacek Bi-Rastrigin Function	700
F8		Shifted and Rotated Non-Continuous Rastrigin's Function	800
F9		Shifted and Rotated Lévy Function	900
F10		Shifted and Rotated Schwefel's Function	1000
F11	Hybrid	Hybrid function 1 (N = 3)	1100
F12		Hybrid function 2 (N = 3)	1200
F13		Hybrid function 3 (N = 3)	1300
F14		Hybrid function 4 (N = 4)	1400
F15		Hybrid function 5 (N = 4)	1500
F16		Hybrid function 6 (N = 4)	1600
F17		Hybrid function 6 (N = 5)	1700
F18		Hybrid function 6 (N = 5)	1800
F19		Hybrid function 6 (N = 5)	1900
F20		Hybrid function 6 (N = 6)	2000
F21	Composition	Composition function 1 (N = 3)	2100
F22		Composition function 2 (N = 3)	2200
F23		Composition function 3 (N = 4)	2300
F24		Composition function 4 (N = 4)	2400
F25		Composition function 5 (N = 5)	2500
F26		Composition function 6 (N = 5)	2600
F27		Composition function 7 (N = 6)	2700
F28		Composition function 8 (N = 6)	2800
F29		Composition function 9 (N = 3)	2900
F30		Composition function 10 (N = 3)	3000
Search range:[−100, 100]		Dimension: = 50	

Table 3
Meta-heuristic algorithms parameters settings.

Alg.	Parameter	Value
L-SHADE	Pbest	0.1
	Arc rate	2
MFO	t	[−1, 1]
	b	1
HHO	E_0	[−1, 1]
TEO	c_1	2
	c_2	2
GOA	c_{max}	1
	c_{min}	0.00004
WOA	a	2
LSHADE-EpSin	Pbest	0.1
	Arc rate	2
CMAES	α	2
COA	N_p	20
	N_c	5

- Gear teeth Stress
- Stress of the surface
- Shafts transverse deflections
- Shafts Stresses

Speed reducer is considered as a challenging benchmark engineering problem as it has 7 design variables ($z_1 - z_7$). Table 11 shows the results of the proposed algorithm SO compared with other state-of-art algorithm. It can be noticed that SO achieves optimal solutions at $z^* = (3.4976, 0.7000, 17.0000, 7.3000, 7.8000, 3.3501, 5.2857)$ with fitness value (2.995542437084133e+03). The results prove that SO outperform other algorithms. Also, the convergence curve proves that SO has a better convergence compared to other algorithms as shown in Figure Fig. 19. Moreover Table 12 shows the statistical comparison between SO and other counterparts. The mathematical equations of this problem is shown below :

$$\begin{aligned}\text{Min } f(\vec{z}) = {} & 0.7854 z_1 z_2^2 \left(3.3333 z_3^2 + 14.9334 z_3 - 43.0934\right) \\ & - 1.508 z_1\end{aligned}$$
$$\left(z_6^2 + z_7^2\right) + 7.4777\left(z_6^3 + z_7^3\right) + 0.7854\left(z_4 z_6^2 + z_5 z_7^2\right)$$

Subject to:

$$g_1(\vec{z}) = \frac{27}{z_1 z_2^2 z_3} - 1 \le 0$$
$$g_2(\vec{z}) = \frac{397.5}{z_1 z_2^2 z_3} - 1 \le 0$$
$$g_3(\vec{z}) = \frac{1.93 z_4^3}{z_2 z_3 z_6^4} - 1$$
$$g_4(\vec{z}) = \frac{1.93 z_5^3}{z_2 z_3 z_7^4} - 1 \le 0$$
$$g_5(\vec{z}) = \frac{1}{110 z_6^3}\sqrt{\left(\frac{745 z_4}{z_2 z_3}\right)^2 + 16.9 \times 10^6} - 1 \le 0$$
$$g_6(\vec{z}) = \frac{1}{85 z_7^3}\sqrt{\left(\frac{745 z_5}{z_2 z_3}\right)^2 + 157.5 \times 10^6} - 1 \le 0$$
$$g_7(\vec{z}) = \frac{z_2 z_3}{40} - 1 \le 0$$
$$g_8(\vec{z}) = \frac{5 z_2}{z_1} - 1 \le 0$$
$$g_9(\vec{z}) = \frac{z_1}{12 z_2} - 1 \le 0$$
$$g_{10}(\vec{z}) = \frac{1.5 z_6 + 1.9}{z_4} - 1 \le 0$$
$$g_{11}(\vec{z}) = \frac{1.1 z_7 + 1.9}{z_5} - 1 \le 0$$

Table 4
The comparison results of all algorithms over 30 functions using CEC2017 and Dim = 30.

F		SO	L-SHADE	MFO	HHO	TEO	GOA	WOA	LSHADE-EpSin
F1	Avg	**4.65E+07**	1.14E+10	1.03E+10	3.97E+08	6.23E+10	8.23E+07	5.32E+09	5.56E+10
	Min	**6.84E+06**	7.22E+09	1.20E+09	1.87E+08	5.59E+10	3.74E+07	4.20E+09	5.15E+10
	Max	**1.13E+08**	1.88E+10	3.23E+10	1.17E+09	7.27E+10	2.63E+08	9.95E+09	6.69E+10
	Med	**3.76E+07**	1.04E+10	5.27E+09	3.88E+08	6.33E+10	6.11E+07	5.15E+09	5.48E+10
	STD	**3.28E+07**	3.58E+09	9.69E+09	2.46E+08	6.16E+09	6.07E+07	1.71E+09	5.81E+09
F2	Avg	NA	NA	NA	NA	NA	NA	NA	NA
	Min	NA	NA	NA	NA	NA	NA	NA	NA
	Max	NA	NA	NA	NA	NA	NA	NA	NA
	Med	NA	NA	NA	NA	NA	NA	NA	NA
	STD	NA	NA	NA	NA	NA	NA	NA	NA
F3	Avg	7.36E+04	1.81E+05	1.91E+05	**5.47E+04**	9.10E+04	5.63E+04	2.83E+05	1.37E+05
	Min	6.29E+04	1.06E+05	1.18E+05	5.04E+04	8.84E+04	**4.40E+04**	2.32E+05	1.21E+05
	Max	8.50E+04	2.45E+05	3.43E+05	**6.45E+04**	9.69E+04	9.83E+04	4.00E+05	1.66E+05
	Med	7.38E+04	1.81E+05	1.74E+05	5.61E+04	9.30E+04	**5.18E+04**	2.84E+05	1.37E+05
	Std	6.31E+03	4.18E+04	5.61E+04	6.45E+03	**4.82E+03**	1.93E+04	7.07E+04	1.65E+04
F4	Avg	**5.56E+02**	1.84E+03	1.71E+03	7.26E+02	2.33E+04	5.60E+02	1.36E+03	1.24E+04
	Min	**4.95E+02**	9.51E+02	5.38E+02	6.30E+02	1.77E+04	5.15E+02	1.07E+03	1.03E+04
	Max	**6.28E+02**	3.06E+03	5.71E+03	9.67E+02	3.10E+04	6.70E+02	2.32E+03	1.76E+04
	Med	5.56E+02	1.78E+03	1.25E+03	7.15E+02	2.30E+04	**5.44E+02**	1.30E+03	1.24E+04
	Std	**3.69E+01**	5.70E+02	1.23E+03	1.14E+02	5.21E+03	5.97E+01	3.82E+02	2.37E+03
F5	Avg	**6.42E+02**	8.18E+02	6.87E+02	7.68E+02	9.36E+02	6.75E+02	8.57E+02	9.53E+02
	Min	**5.91E+02**	7.65E+02	6.41E+02	7.34E+02	9.14E+02	6.30E+02	8.26E+02	9.42E+02
	Max	**7.24E+02**	8.73E+02	7.66E+02	8.24E+02	1.00E+03	7.64E+02	9.40E+02	9.90E+02
	Med	**6.29E+02**	8.21E+02	6.78E+02	7.77E+02	9.33E+02	6.76E+02	8.52E+02	9.52E+02
	Std	3.94E+01	3.19E+01	3.31E+01	3.39E+01	3.33E+01	4.42E+01	3.45E+01	**1.88E+01**
F6	Avg	**6.34E+02**	6.58E+02	6.43E+02	6.66E+02	6.93E+02	6.56E+02	6.80E+02	6.92E+02
	Min	**6.13E+02**	6.34E+02	6.14E+02	6.62E+02	6.88E+02	6.41E+02	6.72E+02	6.90E+02
	Max	**6.51E+02**	6.78E+02	6.83E+02	6.78E+02	7.05E+02	6.86E+02	7.00E+02	6.99E+02
	Med	**6.35E+02**	6.58E+02	6.39E+02	6.66E+02	6.93E+02	6.56E+02	6.82E+02	6.92E+02
	Std	8.66E+00	1.05E+01	1.93E+01	5.32E+00	7.01E+00	1.44E+01	1.26E+01	**4.04E+00**
F7	Avg	**9.50E+02**	1.25E+03	1.16E+03	1.29E+03	1.45E+03	9.99E+02	1.32E+03	1.99E+03
	Min	**8.28E+02**	1.09E+03	8.71E+02	1.25E+03	1.41E+03	9.02E+02	1.26E+03	1.89E+03
	Max	**1.08E+03**	1.37E+03	1.64E+03	1.39E+03	1.51E+03	1.27E+03	1.50E+03	2.16E+03
	Med	**9.60E+02**	1.25E+03	1.11E+03	1.30E+03	1.44E+03	9.77E+02	1.33E+03	2.00E+03
	Std	5.81E+01	7.84E+01	2.17E+02	7.29E+01	**3.98E+01**	1.02E+02	8.50E+01	1.05E+02
F8	Avg	**9.38E+02**	1.10E+03	9.96E+02	9.83E+02	1.16E+03	9.67E+02	1.09E+03	1.20E+03
	Min	**8.90E+02**	1.05E+03	9.24E+02	9.58E+02	1.14E+03	9.30E+02	1.05E+03	1.18E+03
	Max	**1.00E+03**	1.16E+03	1.08E+03	1.04E+03	1.21E+03	1.10E+03	1.15E+03	1.24E+03
	Med	**9.31E+02**	1.11E+03	9.89E+02	9.78E+02	1.16E+03	9.55E+02	1.10E+03	1.20E+03
	Std	2.91E+01	**2.47E+01**	4.19E+01	2.95E+01	2.95E+01	4.62E+01	4.17E+01	2.54E+01
F9	Avg	**3.92E+03**	8.28E+03	7.40E+03	8.67E+03	1.08E+04	7.46E+03	1.18E+04	1.63E+04
	Min	**1.96E+03**	3.29E+03	4.08E+03	7.64E+03	1.02E+04	5.33E+03	8.66E+03	1.52E+04
	Max	**7.64E+03**	1.44E+04	1.36E+04	1.15E+04	1.27E+04	1.45E+04	2.42E+04	1.90E+04
	Med	**3.65E+03**	8.63E+03	7.27E+03	8.36E+03	1.08E+04	6.67E+03	1.05E+04	1.62E+04
	Std	1.49E+03	2.64E+03	1.90E+03	**1.19E+03**	1.22E+03	2.72E+03	4.11E+03	1.50E+03
F10	Avg	**4.23E+03**	9.45E+03	5.60E+03	6.09E+03	9.26E+03	5.68E+03	7.36E+03	8.81E+03
	Min	**3.37E+03**	8.65E+03	4.03E+03	5.67E+03	8.97E+03	5.38E+03	6.94E+03	8.65E+03
	Max	**5.53E+03**	1.01E+04	7.77E+03	7.41E+03	1.01E+04	7.33E+03	8.50E+03	9.13E+03
	Med	**4.19E+03**	9.45E+03	5.65E+03	6.13E+03	9.31E+03	5.60E+03	7.38E+03	8.81E+03
	Std	5.43E+02	4.50E+02	8.93E+02	6.02E+02	4.66E+02	5.71E+02	8.04E+02	**1.91E+02**
F11	Avg	**1.57E+03**	9.89E+03	7.35E+03	1.57E+03	1.76E+04	1.61E+03	9.41E+03	9.65E+03
	Min	**1.32E+03**	3.57E+03	1.43E+03	1.44E+03	1.06E+04	1.45E+03	6.90E+03	7.64E+03
	Max	2.24E+03	1.98E+04	2.12E+04	**2.07E+03**	1.12E+05	2.17E+03	1.99E+04	1.29E+04
	Med	1.55E+03	8.35E+03	5.74E+03	**1.53E+03**	1.26E+04	1.58E+03	8.46E+03	9.97E+03
	STD	2.19E+02	4.65E+03	6.44E+03	1.93E+02	2.24E+04	**1.88E+02**	4.16E+03	2.03E+03
F12	Avg	**5.08E+06**	6.83E+08	1.39E+08	8.52E+07	1.87E+10	3.22E+07	4.73E+08	7.72E+09
	Min	**1.01E+06**	2.82E+08	2.06E+06	2.09E+07	1.66E+10	8.46E+06	2.36E+08	6.45E+09
	Max	**1.33E+07**	1.25E+09	5.25E+08	3.34E+08	2.48E+10	1.04E+08	1.51E+09	1.02E+10
	Med	**3.10E+06**	6.88E+08	2.21E+07	4.98E+07	1.86E+10	1.80E+07	3.32E+08	7.65E+09
	STD	**4.09E+06**	2.88E+08	2.00E+08	9.17E+07	2.91E+09	3.37E+07	3.63E+08	1.26E+09
F13	Avg	**4.17E+04**	1.89E+08	1.50E+08	1.09E+06	2.02E+10	1.22E+05	2.00E+07	4.21E+09
	Min	**8.10E+03**	2.44E+07	2.35E+04	7.05E+05	1.78E+10	6.55E+04	3.47E+06	3.42E+09
	Max	**1.36E+05**	6.95E+08	2.91E+09	1.81E+06	2.86E+10	2.53E+05	1.73E+08	5.73E+09
	Med	**3.84E+04**	1.42E+08	1.21E+05	1.09E+06	2.15E+10	1.07E+05	9.32E+06	4.30E+09
	STD	**2.79E+04**	1.67E+08	6.50E+08	3.69E+05	6.18E+09	6.37E+04	3.75E+07	1.06E+09

(*continued on next page*)

Table 4 (*continued*).

F		SO	L-SHADE	MFO	HHO	TEO	GOA	WOA	LSHADE-EpSin
F14	Avg	3.39E+05	1.58E+06	8.12E+05	1.30E+06	3.36E+07	**7.02E+04**	3.26E+06	1.46E+06
	Min	**4.39E+03**	2.83E+04	3.22E+04	3.14E+05	1.21E+07	3.99E+04	5.44E+05	9.22E+05
	Max	2.43E+06	8.04E+06	3.26E+06	5.97E+06	1.14E+08	**1.61E+05**	1.15E+07	2.47E+06
	Med	1.54E+05	8.76E+05	7.23E+05	9.05E+05	2.08E+07	**6.51E+04**	1.70E+06	1.43E+06
	STD	5.53E+05	2.00E+06	7.82E+05	1.40E+06	3.14E+07	**3.86E+04**	3.37E+06	5.76E+05
F15	Avg	**1.05E+04**	1.84E+07	5.70E+04	1.54E+05	1.73E+09	1.16E+05	1.08E+07	4.75E+08
	Min	**2.96E+03**	1.09E+06	3.62E+03	7.05E+04	9.04E+08	4.10E+04	1.21E+06	3.69E+08
	Max	**3.73E+04**	9.20E+07	2.45E+05	4.59E+05	3.95E+09	4.06E+05	7.52E+07	8.63E+08
	Med	**7.58E+03**	1.39E+07	4.31E+04	1.22E+05	1.74E+09	7.21E+04	4.30E+06	4.60E+08
	STD	**8.37E+03**	2.13E+07	5.25E+04	1.10E+05	1.02E+09	1.06E+05	1.70E+07	1.85E+08
F16	Avg	**2.74E+03**	4.30E+03	3.19E+03	3.55E+03	9.33E+03	3.07E+03	4.38E+03	5.01E+03
	Min	**2.10E+03**	3.80E+03	2.52E+03	3.26E+03	8.05E+03	2.76E+03	3.91E+03	4.91E+03
	Max	**3.16E+03**	5.25E+03	3.99E+03	4.40E+03	1.24E+04	3.81E+03	6.02E+03	5.39E+03
	Med	**2.77E+03**	4.34E+03	3.19E+03	3.58E+03	9.22E+03	3.06E+03	4.44E+03	5.08E+03
	STD	**2.59E+02**	3.54E+02	3.69E+02	4.24E+02	1.99E+03	3.60E+02	6.17E+02	3.13E+02
F17	Avg	**2.34E+03**	2.83E+03	2.66E+03	2.68E+03	2.68E+04	2.40E+03	2.82E+03	3.30E+03
	Min	**1.89E+03**	2.35E+03	2.08E+03	2.32E+03	3.90E+03	2.24E+03	2.47E+03	3.24E+03
	Max	**2.76E+03**	3.14E+03	3.07E+03	3.54E+03	1.42E+05	2.94E+03	3.84E+03	3.59E+03
	Med	**2.33E+03**	2.91E+03	2.72E+03	2.63E+03	1.06E+04	2.37E+03	2.83E+03	3.31E+03
	STD	2.48E+02	2.38E+02	2.77E+02	3.90E+02	3.37E+04	2.06E+02	4.55E+02	**1.45E+02**
F18	Avg	**9.84E+05**	1.79E+07	3.34E+06	3.02E+06	3.82E+08	1.74E+06	1.30E+07	1.86E+07
	Min	**5.22E+04**	3.38E+05	5.13E+05	8.58E+05	4.71E+07	8.31E+05	4.12E+06	1.31E+07
	Max	**2.82E+06**	1.50E+08	1.23E+07	9.19E+06	1.61E+09	6.59E+06	4.45E+07	3.89E+07
	Med	**6.20E+05**	6.68E+06	1.95E+06	2.39E+06	2.67E+08	1.32E+06	9.16E+06	1.77E+07
	STD	**9.84E+05**	3.35E+07	3.15E+06	2.66E+06	3.97E+08	1.54E+06	1.14E+07	7.98E+06
F19	Avg	**1.39E+04**	3.16E+07	2.62E+07	2.01E+06	3.49E+09	6.75E+06	2.36E+07	5.94E+08
	Min	**2.32E+03**	4.32E+06	2.26E+03	8.64E+05	1.78E+09	3.13E+06	6.90E+06	4.42E+08
	Max	**4.43E+04**	1.13E+08	1.79E+08	6.85E+06	6.43E+09	1.56E+07	7.67E+07	1.34E+09
	Med	**1.26E+04**	2.41E+07	1.13E+05	1.82E+06	3.50E+09	5.99E+06	2.24E+07	5.25E+08
	STD	**1.16E+04**	2.63E+07	5.70E+07	1.56E+06	1.91E+09	4.13E+06	1.88E+07	2.48E+08
F20	Avg	**2.61E+03**	3.23E+03	2.75E+03	2.78E+03	3.22E+03	2.65E+03	2.98E+03	3.02E+03
	Min	**2.32E+03**	2.89E+03	2.35E+03	2.60E+03	3.09E+03	2.52E+03	2.82E+03	2.90E+03
	Max	**2.86E+03**	3.58E+03	3.14E+03	3.16E+03	3.62E+03	2.97E+03	3.28E+03	3.17E+03
	Med	**2.60E+03**	3.24E+03	2.76E+03	2.84E+03	3.21E+03	2.65E+03	2.98E+03	3.03E+03
	STD	1.58E+02	1.92E+02	2.40E+02	1.90E+02	2.16E+02	1.84E+02	1.81E+02	**1.01E+02**
F21	Avg	**2.43E+03**	2.60E+03	2.49E+03	2.60E+03	2.82E+03	2.48E+03	2.64E+03	2.71E+03
	Min	**2.39E+03**	2.55E+03	2.42E+03	2.55E+03	2.76E+03	2.45E+03	2.57E+03	2.69E+03
	Max	**2.52E+03**	2.63E+03	2.56E+03	2.81E+03	2.90E+03	2.58E+03	2.82E+03	2.75E+03
	Med	**2.43E+03**	2.60E+03	2.48E+03	2.59E+03	2.84E+03	2.48E+03	2.62E+03	2.71E+03
	STD	3.25E+01	2.40E+01	4.49E+01	6.49E+01	6.35E+01	3.71E+01	8.83E+01	**2.08E+01**
F22	Avg	**3.78E+03**	6.75E+03	5.88E+03	7.41E+03	1.03E+04	6.70E+03	7.61E+03	8.58E+03
	Min	**2.35E+03**	3.14E+03	2.67E+03	6.88E+03	9.76E+03	5.93E+03	5.74E+03	8.19E+03
	Max	**6.33E+03**	1.13E+04	8.95E+03	8.62E+03	1.12E+04	7.96E+03	9.97E+03	9.53E+03
	Med	**2.75E+03**	5.44E+03	6.40E+03	7.56E+03	1.06E+04	6.85E+03	8.30E+03	8.65E+03
	STD	1.61E+03	2.64E+03	1.87E+03	**1.23E+03**	8.15E+02	7.62E+02	1.99E+03	6.32E+02
F23	Avg	2.94E+03	2.99E+03	**2.82E+03**	3.30E+03	4.02E+03	2.88E+03	3.18E+03	3.34E+03
	Min	2.84E+03	2.93E+03	**2.76E+03**	3.18E+03	3.81E+03	2.84E+03	3.13E+03	3.30E+03
	Max	3.08E+03	3.05E+03	**2.92E+03**	3.63E+03	4.84E+03	3.03E+03	3.41E+03	3.46E+03
	Med	2.94E+03	2.99E+03	**2.81E+03**	3.29E+03	3.97E+03	2.87E+03	3.17E+03	3.34E+03
	STD	7.07E+01	**2.39E+01**	4.41E+01	1.40E+02	2.94E+02	5.43E+01	8.85E+01	4.68E+01
F24	Avg	3.09E+03	3.16E+03	**2.98E+03**	3.51E+03	4.42E+03	3.01E+03	3.31E+03	3.53E+03
	Min	2.96E+03	3.10E+03	**2.93E+03**	3.44E+03	4.17E+03	2.94E+03	3.25E+03	3.47E+03
	Max	3.25E+03	3.23E+03	**3.07E+03**	3.90E+03	4.95E+03	3.17E+03	3.51E+03	3.64E+03
	Med	3.07E+03	3.16E+03	**2.98E+03**	3.49E+03	4.43E+03	3.00E+03	3.28E+03	3.52E+03
	STD	7.05E+01	3.85E+01	**3.82E+01**	1.42E+02	3.18E+02	6.61E+01	1.03E+02	6.16E+01
F25	Avg	**2.95E+03**	3.48E+03	3.19E+03	3.00E+03	6.74E+03	2.98E+03	3.22E+03	6.66E+03
	Min	**2.91E+03**	3.13E+03	2.91E+03	2.99E+03	6.16E+03	2.94E+03	3.13E+03	6.21E+03
	Max	**3.04E+03**	4.04E+03	4.21E+03	3.07E+03	9.01E+03	3.08E+03	3.52E+03	7.91E+03
	Med	**2.94E+03**	3.45E+03	3.11E+03	3.00E+03	6.68E+03	2.98E+03	3.19E+03	6.67E+03
	STD	3.57E+01	2.53E+02	3.60E+02	**3.36E+01**	8.91E+02	4.15E+01	1.05E+02	5.83E+02
F26	Avg	6.55E+03	7.23E+03	6.10E+03	8.22E+03	1.33E+04	**5.42E+03**	8.32E+03	1.02E+04
	Min	**4.03E+03**	6.64E+03	5.32E+03	7.44E+03	1.25E+04	4.36E+03	7.40E+03	9.66E+03
	Max	**7.54E+03**	7.98E+03	7.08E+03	9.72E+03	1.56E+04	7.55E+03	1.06E+04	1.15E+04
	Med	6.40E+03	7.22E+03	6.10E+03	8.47E+03	1.33E+04	**5.61E+03**	8.46E+03	1.04E+04
	STD	8.09E+02	4.07E+02	4.86E+02	1.41E+03	1.16E+03	1.24E+03	**1.02E+03**	8.34E+02

(*continued on next page*)

Table 4 (*continued*).

F		SO	L-SHADE	MFO	HHO	TEO	GOA	WOA	LSHADE-EpSin
F27	Avg	**3.23E+03**	3.38E+03	3.25E+03	3.57E+03	5.33E+03	3.29E+03	3.53E+03	3.97E+03
	Min	**3.19E+03**	3.29E+03	3.22E+03	3.43E+03	4.87E+03	3.26E+03	3.39E+03	3.86E+03
	Max	3.50E+03	3.51E+03	**3.28E+03**	4.20E+03	6.08E+03	3.51E+03	3.76E+03	4.15E+03
	Med	**3.20E+03**	3.37E+03	3.25E+03	3.52E+03	5.31E+03	3.27E+03	3.52E+03	4.02E+03
	STD	7.76E+01	5.51E+01	1.52E+01	1.98E+02	4.53E+02	6.47E+01	1.37E+02	**1.51E+02**
F28	Avg	3.38E+03	4.34E+03	4.21E+03	3.56E+03	8.30E+03	**3.29E+03**	3.89E+03	6.94E+03
	Min	**3.24E+03**	3.65E+03	3.45E+03	3.52E+03	7.58E+03	3.27E+03	3.68E+03	6.33E+03
	Max	3.49E+03	5.57E+03	6.37E+03	3.72E+03	9.81E+03	**3.39E+03**	4.29E+03	7.86E+03
	Med	3.38E+03	4.22E+03	3.80E+03	3.55E+03	8.51E+03	**3.29E+03**	3.90E+03	6.95E+03
	STD	5.86E+01	5.09E+02	8.84E+02	8.82E+01	1.01E+03	**3.61E+01**	2.30E+02	5.18E+02
F29	Avg	4.33E+03	5.11E+03	**4.27E+03**	5.06E+03	1.60E+04	4.39E+03	5.33E+03	6.10E+03
	Min	**3.52E+03**	4.73E+03	3.87E+03	4.70E+03	7.82E+03	4.21E+03	5.03E+03	5.97E+03
	Max	4.85E+03	5.81E+03	**4.81E+03**	6.19E+03	5.59E+04	4.83E+03	6.11E+03	6.66E+03
	Med	4.39E+03	5.07E+03	**4.23E+03**	4.95E+03	1.23E+04	4.43E+03	5.40E+03	6.15E+03
	STD	2.79E+02	2.75E+02	2.53E+02	5.49E+02	1.42E+04	**2.50E+02**	5.22E+02	3.48E+02
F30	Avg	**3.72E+05**	2.86E+07	6.17E+05	1.10E+07	4.12E+09	1.31E+07	6.53E+07	4.95E+08
	Min	**3.86E+03**	1.07E+07	1.77E+04	3.53E+06	2.35E+09	6.69E+06	2.70E+07	4.22E+08
	Max	**1.68E+06**	5.91E+07	2.52E+06	4.94E+07	7.94E+09	4.46E+07	1.27E+08	6.85E+08
	Med	**2.28E+05**	2.75E+07	4.75E+05	8.26E+06	4.40E+09	1.08E+07	5.64E+07	5.07E+08
	STD	**4.62E+05**	1.41E+07	6.60E+05	1.11E+07	2.00E+09	9.56E+06	3.94E+07	1.16E+08

Table 5
Wilcoxon rank sum test results for SO against other algorithms using CEC2017 and Dim = 30.

F	L-SHADE	MFO	HHO	TEO	GOA	WOA	LSHADE-EpSin
F1	6.80E−08	6.80E−08	6.80E−08	6.80E−08	2.07E−02	6.80E−08	6.80E−08
F3	6.80E−08	6.80E−08	7.90E−08	1.66E−07	1.12E−03	6.80E−08	6.80E−08
F4	6.80E−08	5.87E−06	1.05E−06	6.80E−08	8.82E−01	6.80E−08	6.80E−08
F5	6.80E−08	2.14E−03	1.06E−07	6.80E−08	1.55E−02	6.80E−08	6.80E−08
F6	7.95E−07	1.81E−01	6.80E−08	6.80E−08	3.07E−06	6.80E−08	6.80E−08
F7	6.80E−08	3.05E−04	6.80E−08	6.80E−08	1.72E−01	6.80E−08	6.80E−08
F8	6.80E−08	4.17E−05	7.41E−05	6.80E−08	2.75E−02	7.90E−08	6.80E−08
F9	4.54E−06	3.50E−06	1.43E−07	7.90E−08	2.04E−05	6.80E−08	6.80E−08
F10	6.80E−08	6.67E−06	1.23E−07	6.80E−08	3.42E−07	6.80E−08	6.80E−08
F11	6.80E−08	9.28E−05	9.68E−01	6.80E−08	4.57E−01	6.80E−08	6.80E−08
F12	6.80E−08	3.71E−05	1.43E−07	6.80E−08	1.44E−04	6.80E−08	6.80E−08
F13	6.80E−08	1.16E−04	6.80E−08	6.80E−08	4.54E−06	6.80E−08	6.80E−08
F14	2.56E−03	3.64E−03	1.12E−03	7.90E−08	9.79E−03	6.61E−05	3.07E−06
F15	6.80E−08	7.58E−06	7.90E−08	6.80E−08	1.23E−07	6.80E−08	6.80E−08
F16	6.80E−08	2.22E−04	1.20E−06	6.80E−08	4.70E−03	6.80E−08	6.80E−08
F17	3.50E−06	1.01E−03	3.97E−03	6.80E−08	5.25E−01	5.09E−04	6.80E−08
F18	4.54E−06	9.21E−04	1.12E−03	3.94E−07	2.94E−02	3.94E−07	6.80E−08
F19	6.80E−08	1.04E−04	6.80E−08	6.80E−08	6.80E−08	6.80E−08	6.80E−08
F20	6.80E−08	4.39E−02	5.12E−03	1.66E−07	4.90E−01	2.36E−06	9.17E−08
F21	6.80E−08	9.28E−05	7.90E−08	6.80E−08	8.29E−05	7.90E−08	6.80E−08
F22	1.12E−03	2.47E−04	2.56E−07	6.80E−08	1.20E−06	8.60E−06	6.80E−08
F23	1.79E−02	2.06E−06	9.17E−08	6.80E−08	4.70E−03	1.66E−07	6.80E−08
F24	2.22E−04	3.99E−06	6.80E−08	6.80E−08	4.16E−04	2.96E−07	6.80E−08
F25	6.80E−08	6.22E−04	2.75E−04	6.80E−08	6.79E−02	6.80E−08	6.80E−08
F26	2.14E−03	8.35E−03	3.29E−05	6.80E−08	8.36E−04	3.50E−06	6.80E−08
F27	6.67E−06	1.23E−03	2.56E−07	6.80E−08	1.79E−04	2.56E−07	6.80E−08
F28	6.80E−08	1.06E−07	6.92E−07	6.80E−08	3.29E−05	1.23E−07	6.80E−08
F29	1.06E−07	1.72E−01	3.50E−06	6.80E−08	5.79E−01	1.20E−06	6.80E−08
F30	6.80E−08	1.40E−01	1.66E−07	6.80E−08	6.80E−08	6.80E−08	6.80E−08

with $2.6 \leq z_1 \leq 3.6$
$0.7 \leq z_2 \leq 0.8$
$17 \leq z_3 \leq 28$
$7.3 \leq z_4 \leq 8.3$
$7.8 \leq z_5 \leq 8.3$
$2.9 \leq z_6 \leq 3.9$
and $5 \leq z_7 \leq 5.5$

5.2. Welded beam design problem

The second engineering problem presented un this study is welded beam design which has been proposed by Coello [92] for the first time. The objective of this probem is to find the minimum of welded beam fabrication cost. The problem has 4 decision variables:

- weld thickness h.
- clamped bar length l.
- bar height t.
- bar thickness b.

The mathematical formulas to this problem is given below:

Consider $\vec{x} = [x_1 x_2 x_3 x_4] = [h\, l\, t\, b]$
Minimize $f(\vec{x}) = 1.10471 x_1^2 x_2 + 0.04811 x_3 x_4 (14.0 + x_2)$
Subject to:
$g_1(\vec{x}) = \tau(\vec{x}) - 13600 \leq 0$
$g_2(\vec{x}) = \sigma(\vec{x}) - 30000 \leq 0$
$g_3(\vec{x}) = x_1 - x_4 \leq 0$
$g_4(\vec{x}) = 0.10471 (x_1^2) + 0.04811 x_3 x_4 (14 + x_2) - 5.0 \leq 0$
$g_6(\vec{x}) = \delta(\vec{x}) - 0.25 \leq 0$

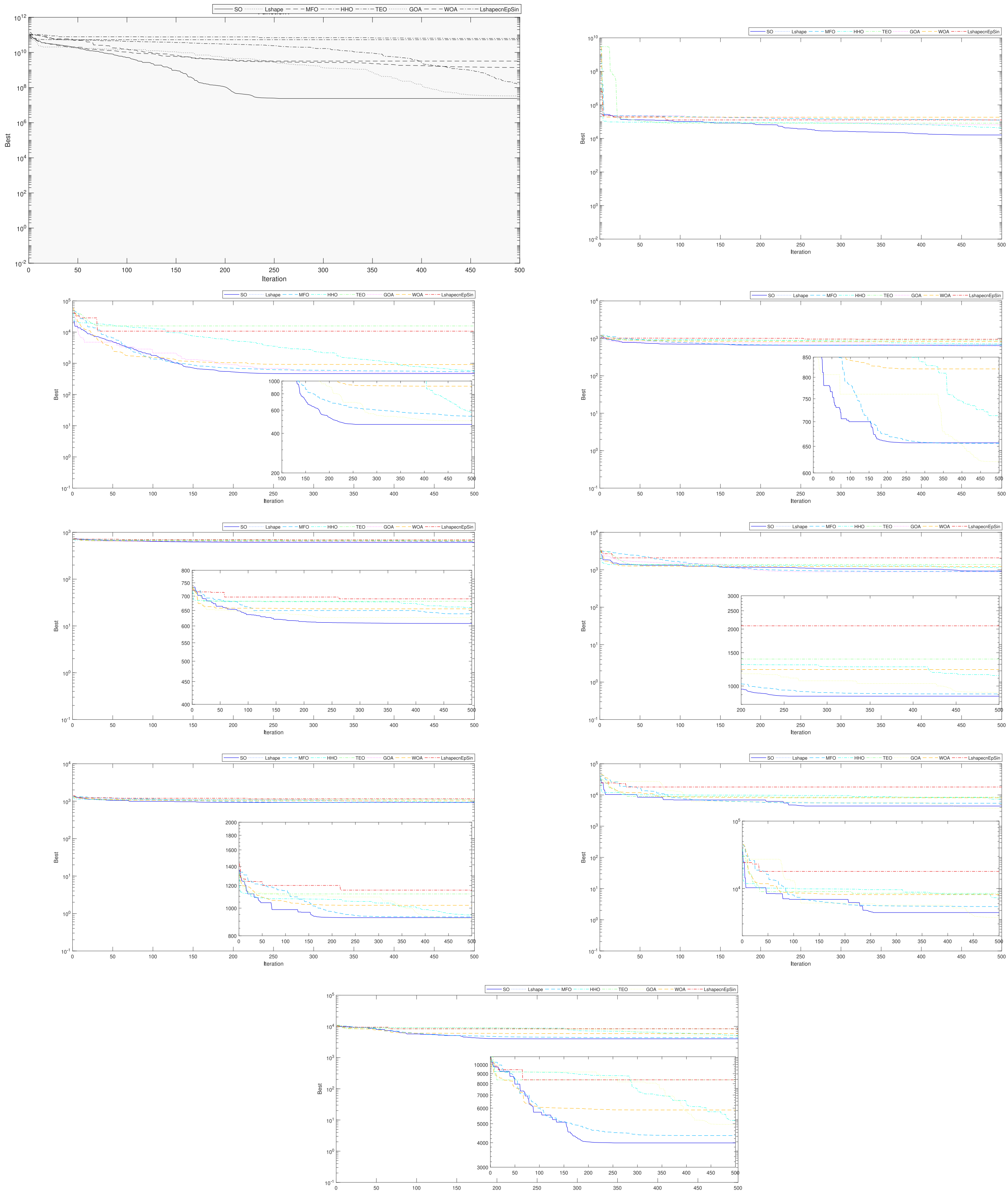

Fig. 5. Convergence curve of some functions from F1–F10 for all algorithms using CEC2017 and Dim = 30.

$$g_7\left(\vec{x}\right) = 6000 - p_c\left(\vec{x}\right) \leq 0$$

where

$$\tau\left(\vec{x}\right) = \sqrt{(\tau') + (2\tau'\tau'')\frac{x_2}{2R} + (\tau'')^2}$$

$$\tau' = \frac{6000}{\sqrt{2}x_1x_2}$$

$$\tau'' = \frac{MR}{J}$$

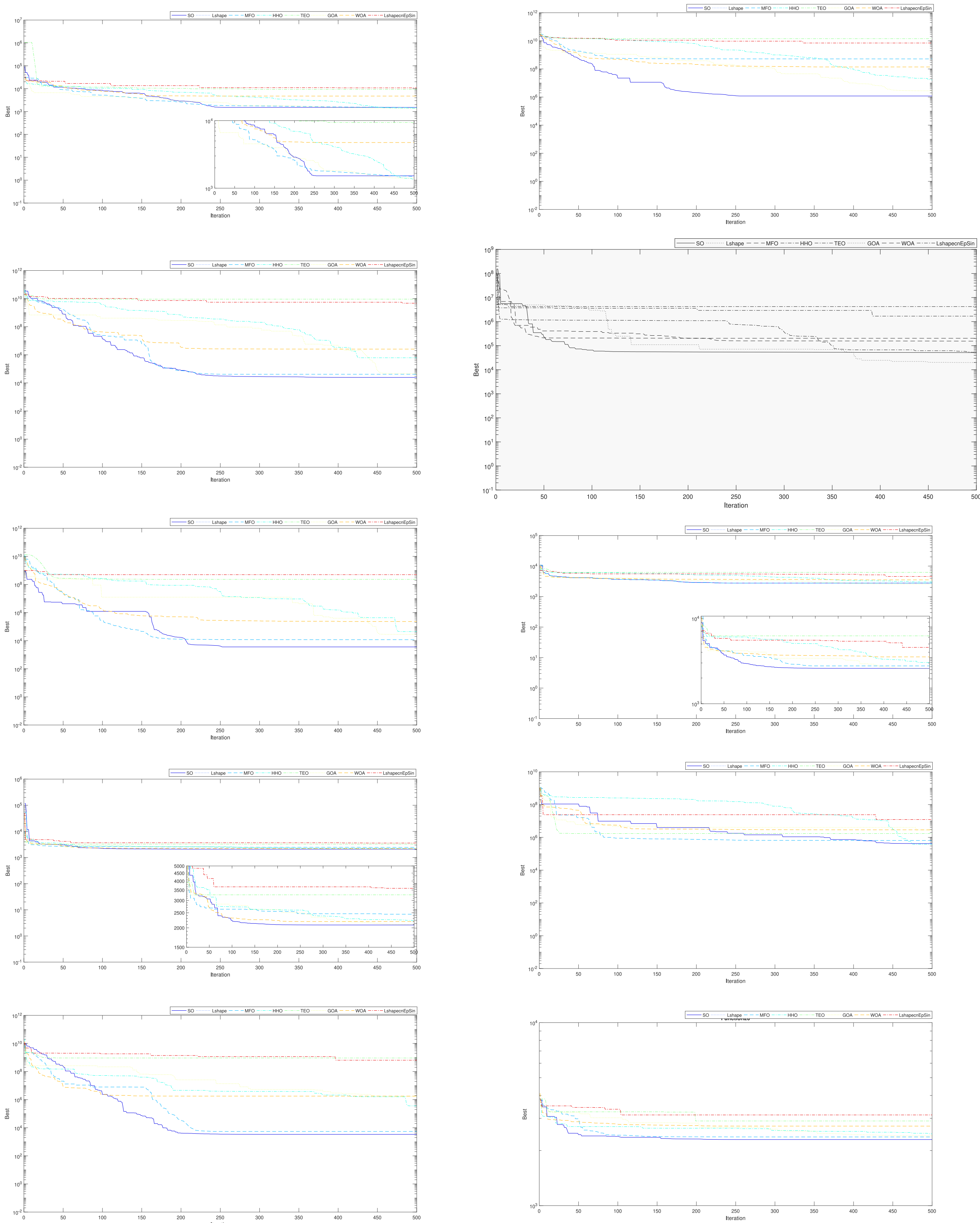

Fig. 6. Convergence curve of some functions from F11–F20 for all algorithms using CEC2017 and Dim = 30.

$$M = 6000\left(14 + \frac{x_2}{2}\right)$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}$$

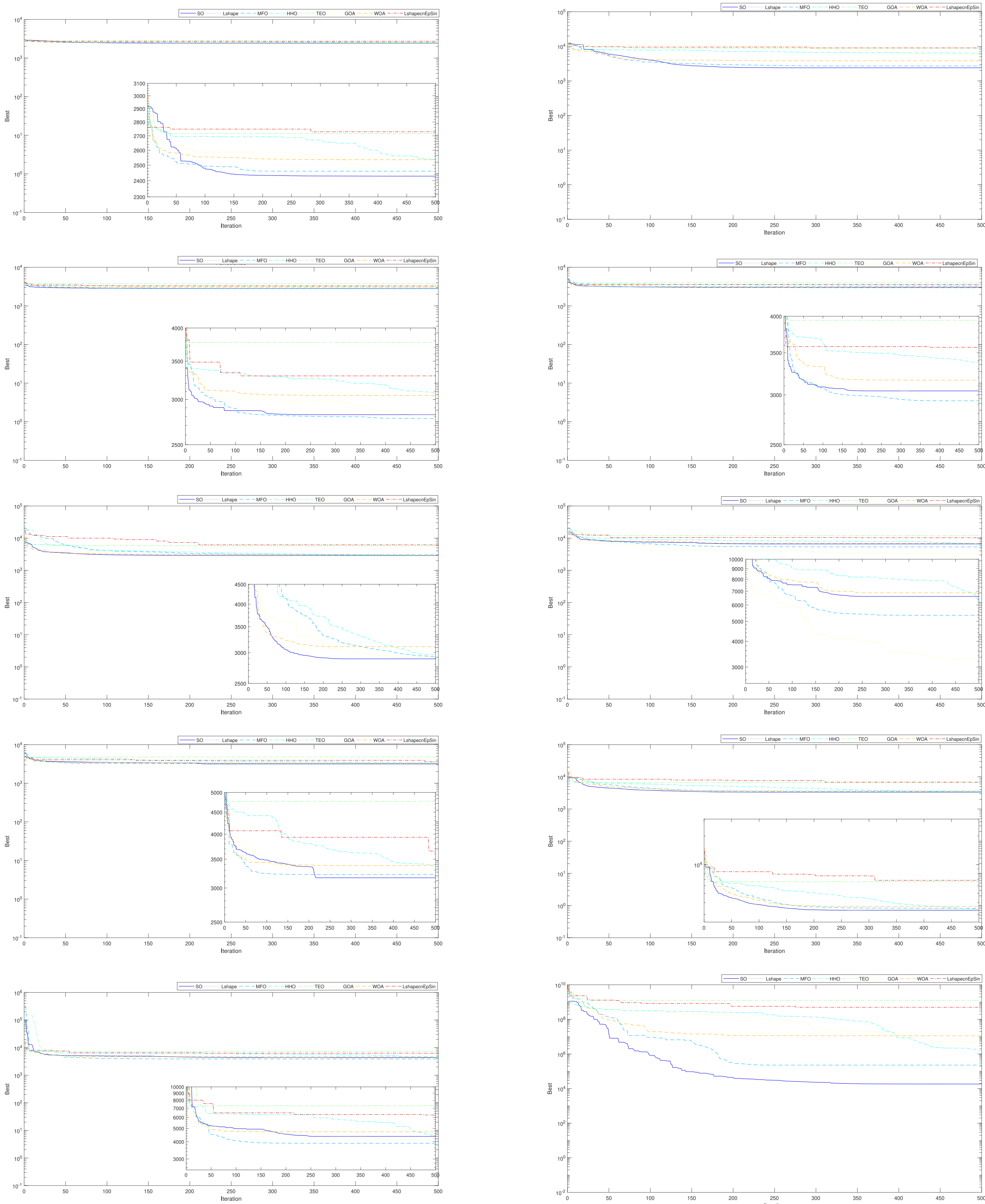

Fig. 7. Convergence curve of some functions from F21–F30 for all algorithms using CEC2017 and Dim = 30.

$$j = 2\left\{x_1 x_2 \sqrt{2}\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\}$$

$$\sigma\left(\vec{x}\right) = \frac{504000}{x_4 x_3^2}$$

$$\delta\left(\vec{x}\right) = \frac{65856000}{\left(30 \times 10^6\right) x_4 x_3^3}$$

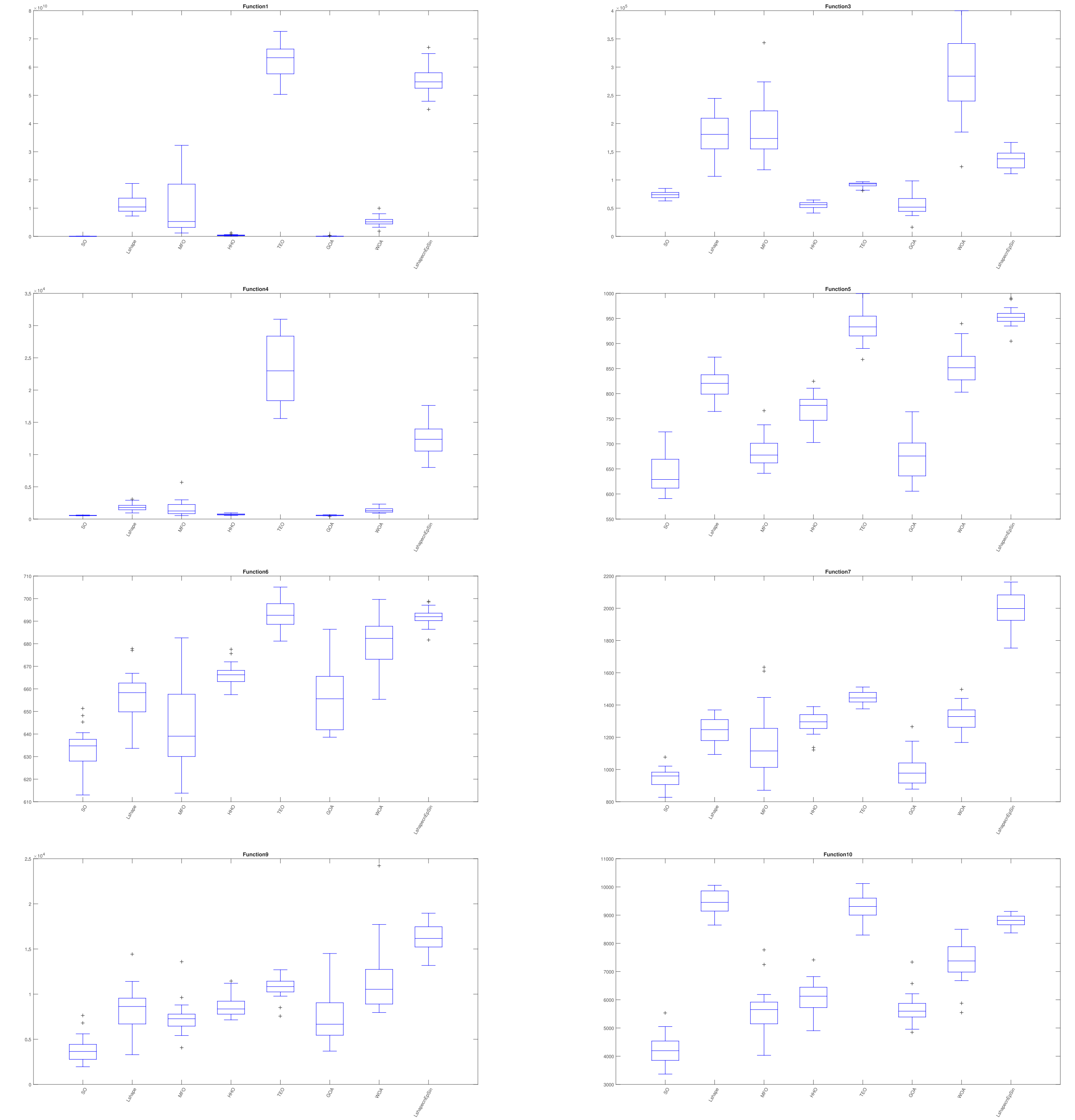

Fig. 8. Box Plot of some functions from F1–F10 for all algorithms using CEC2017 and Dim = 30.

$$p_c\left(\vec{x}\right)=\frac{4.013\left(30\times10^{6}\right)\sqrt{\frac{x_3^2x_4^6}{36}}}{196}\left(1-\frac{x_3\sqrt{\frac{30\times10^{6}}{4(12\times10^{6})}}}{28}\right)$$

with $0.1 \leq x_1, x_4 \leq 2.0\, and\, 0.1 \leq x_2, x_3 \leq 10.0$

Results of SO on welded beam is given in Table 13 where the best results is given to SO and other comparative algorithm. From this table, SO has ranked first and achieved the optimal solution $x^* = (0.2057, 3.4705, 9.0366, 0.2057)$ with fitness function equal to 1.724851930920065. Moreover Table 14 shows the statistical comparison between SO and other counterparts. Figure Fig. 20 shows the convergence curve between SO and other algorithms.

5.3. Pressure vessel design problem

Pressure vessel design is a constrained engineering problem introduced by Kannan and Kramer [93]. This problem has 4 design variables:

- Shell thickness T_s
- head thickness T_h
- Radius

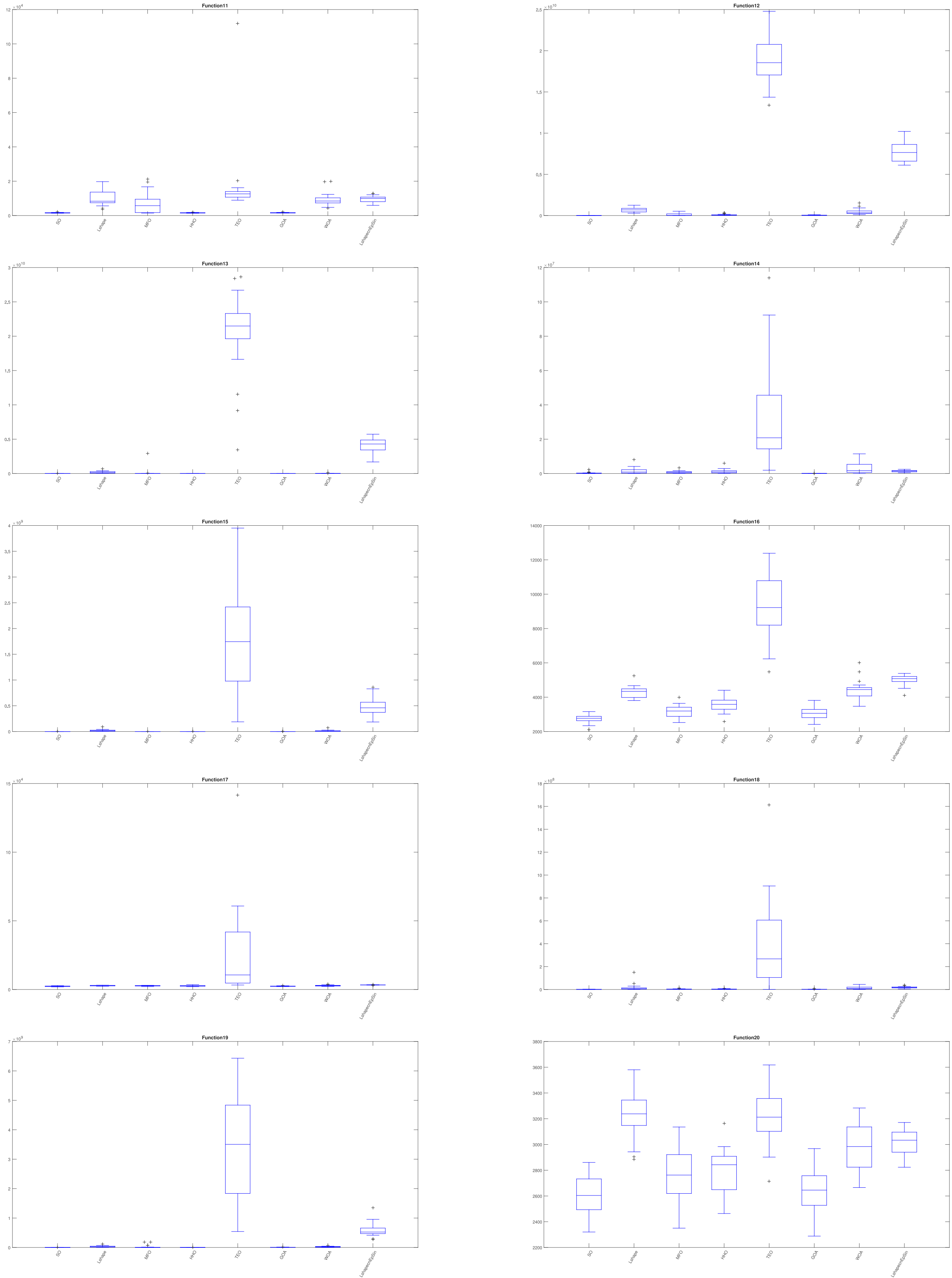

Fig. 9. Box Plot of some functions from F11–F20 for all algorithms using CEC2017 and Dim = 30.

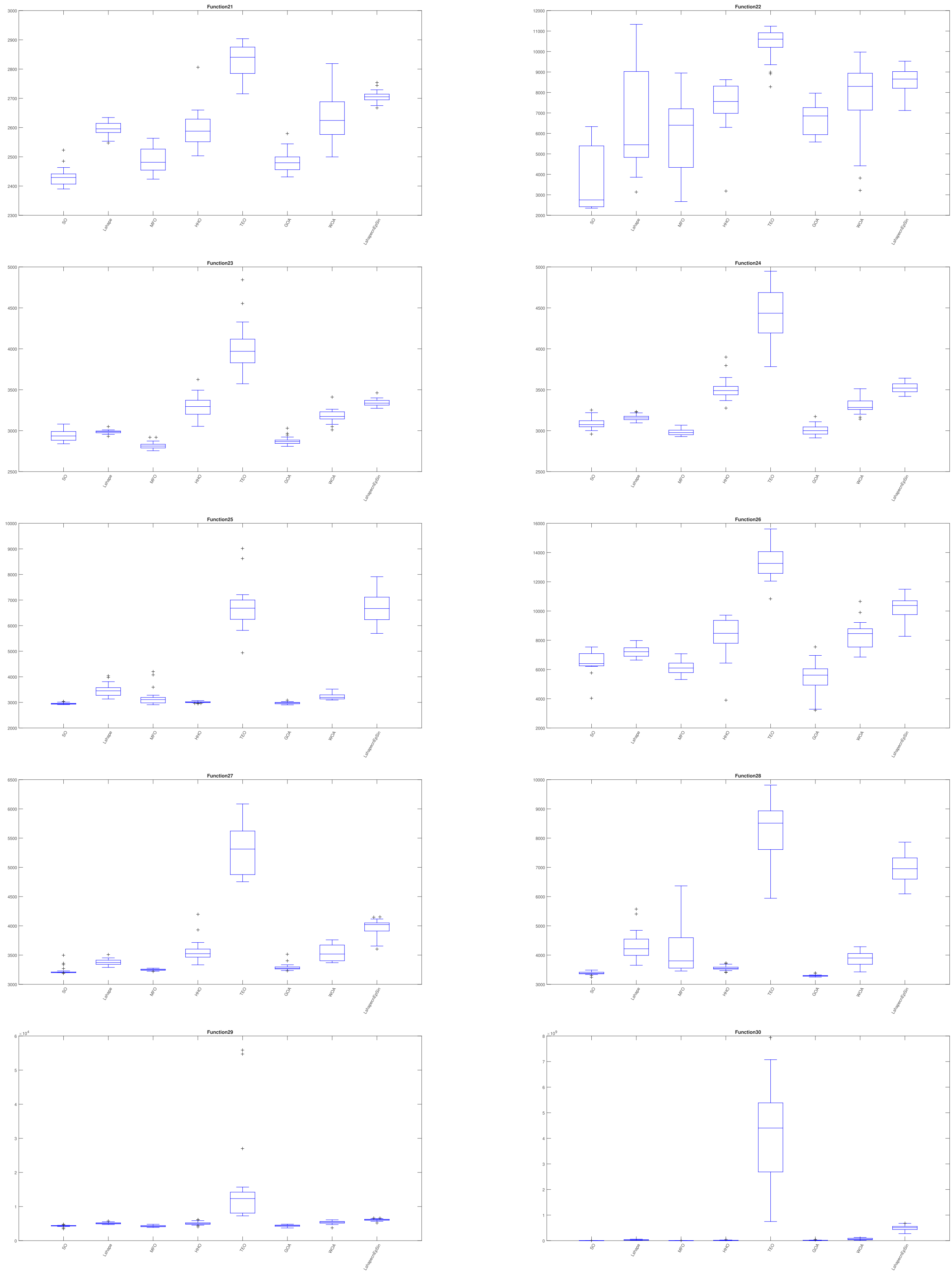

Fig. 10. Box Plot of some functions from F21–F30 for all algorithms using CEC2017 and Dim = 30.

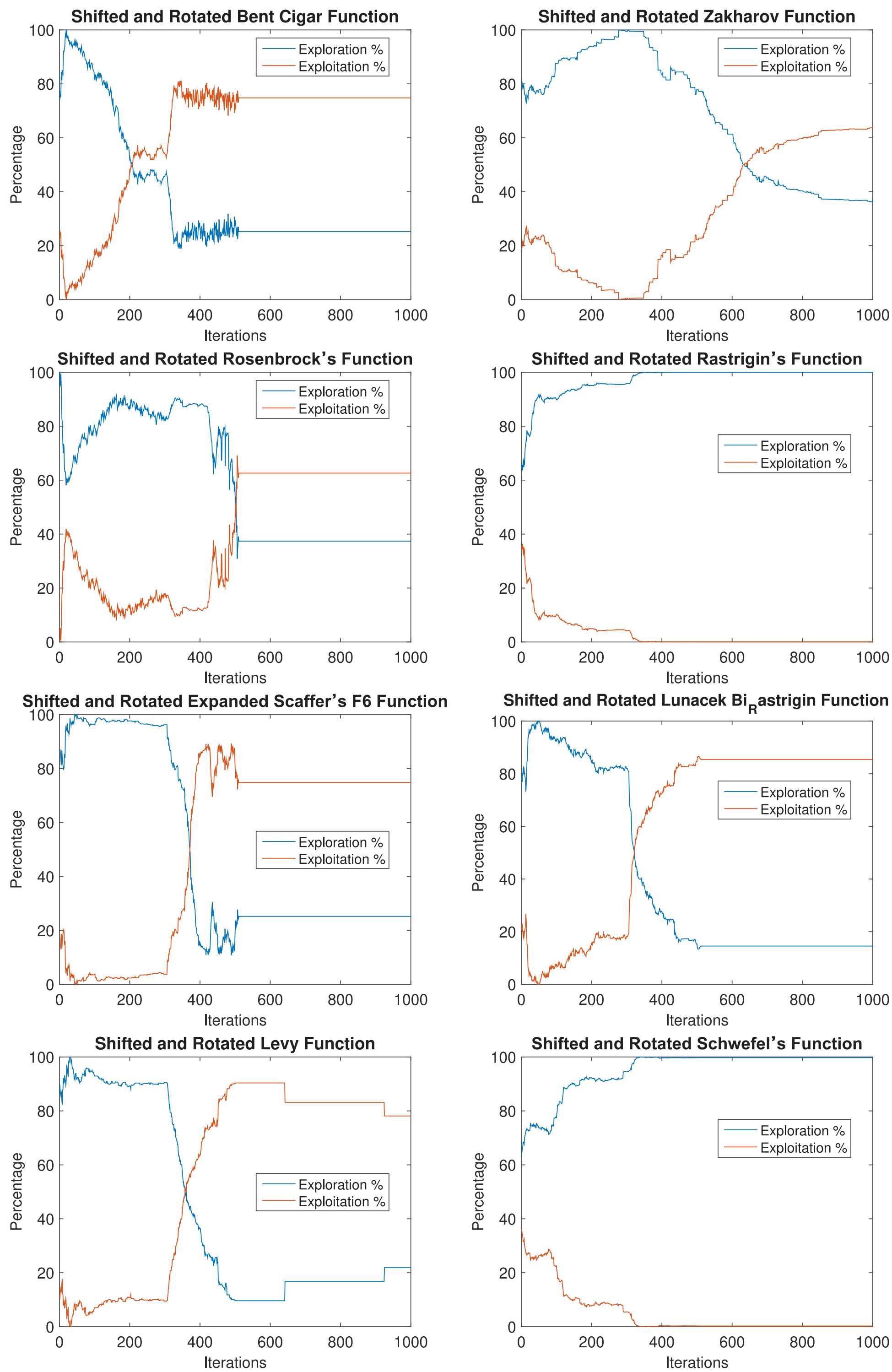

Fig. 11. Exploration and exploitation phases for the SO algorithm on samples of benchmark functions from F1–F10 for all algorithms using CEC2017 and Dim = 30.

- Cylindrical length without head L

This problem can be represented mathematically as follows:

$$\text{Minimize } f(x) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3$$

Subject to:

$$g_1(x) = -x_1 + 0.0193x$$

$$g_2(x) = -x_2 + 0.00954x_3 \le 0$$

$$g_3(x) = -\pi x_3^2 x_4 - (4/3)\pi x_3^3 + 1,296,000 \le 0$$

$$g_4(x) = x_4 - 240 \le 0$$

$$0 \le x_i \le 100, \quad i = 1, 2$$

$$10 \le x_i \le 200, \quad i = 3, 4$$

Table 15 record the comparison results of best solutions obtained from SO and other competitive algorithms. It is notable that SO achieves better results with $x^* = (0.7819, 0.3857, 40.5752, 196.5499)$ and fitness value 5.887529768474379e+03. So, we can

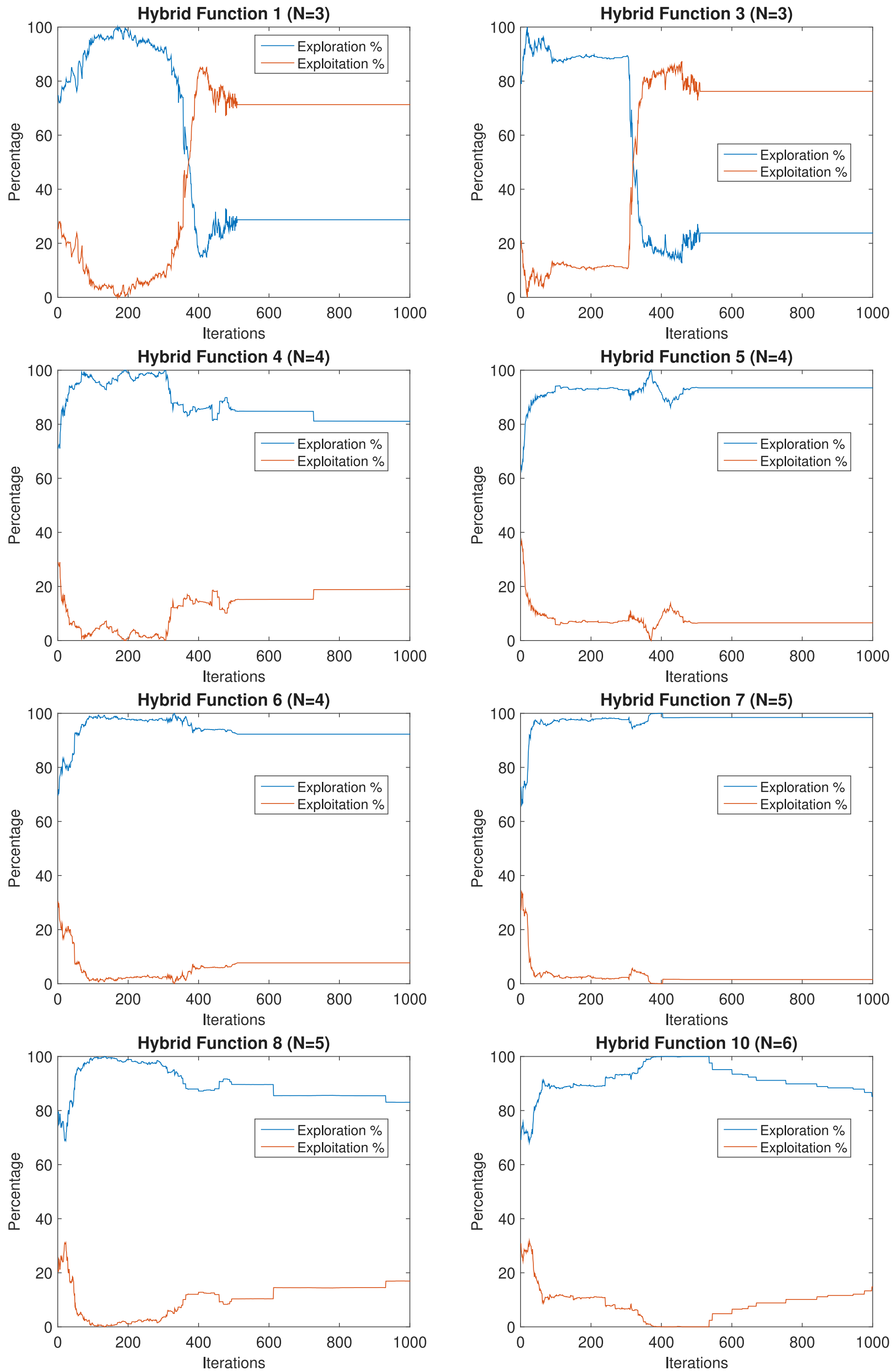

Fig. 12. Exploration and exploitation phases for the SO algorithm on samples of benchmark functions from F11–F20 for all algorithms using CEC2017 and Dim = 30.

conclude that SO is able to find the best (minimum) fabrication cost. Table 16 shows the statistical results between SO and other algorithms. From this table, it is notable that SO has better results than other algorithms in terms of Mean, best, worst, and standard deviation. Figure Fig. 21 shows the convergence curve between SO and other algorithms.

5.4. Tension/compression spring design problem

The last engineering problem presented in this study is called tension/compression spring design which is introduced by Arora [94]. This problem has 3 decision variables:

1. Wire diameter w
2. coil mean diameter d
3. length L

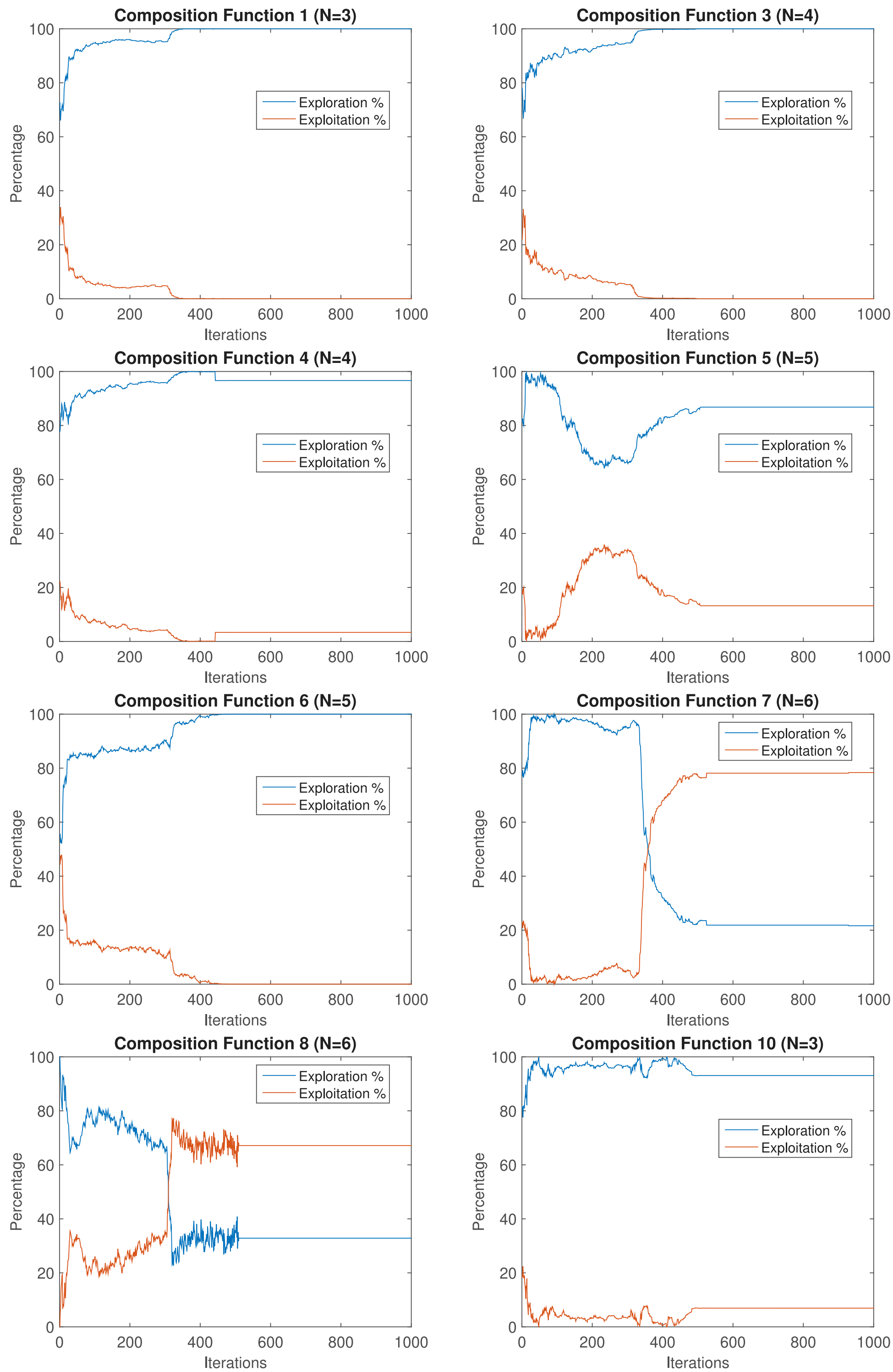

Fig. 13. Exploration and exploitation phases for the SO algorithm on samples of benchmark functions from F21–F30 for all algorithms using CEC2017 and Dim = 30.

The objective of this problem is to find the minimum weight. The mathematical formulas of this problem is described below:

Consider:

$\vec{x} = [x_1 x_2 x_3] = [d\, D\, N]$

Minimize $f\left(\vec{x}\right) = (x_3 + 2)\, x_2 x_1^2$

subject to:

$$g_1\left(\vec{x}\right) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \le 0$$

$$g_2\left(\vec{x}\right) = \frac{4x_2^2 - x_1 x_2}{12566\left(x_2 x_1^3 - x_1^4\right)} + \frac{1}{5108 x_1^2} - 1 \le 0$$

$$g_3\left(\vec{x}\right) = 1 - \frac{140.45 x_1}{x_2^2 x_3} \le 0$$

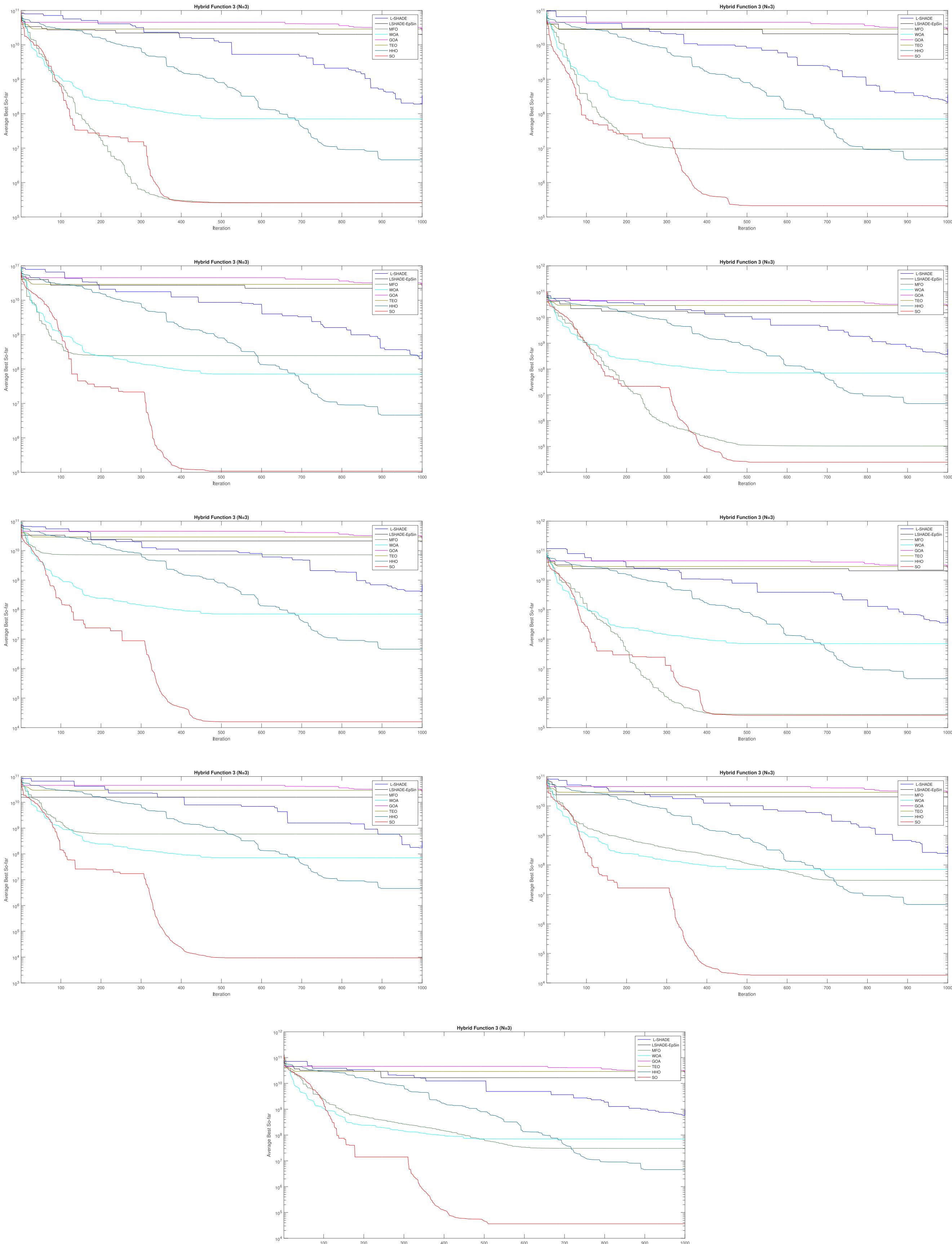

Fig. 14. Convergence curve of some functions from F1–F10 for all algorithms CEC2017 Dim = 50.

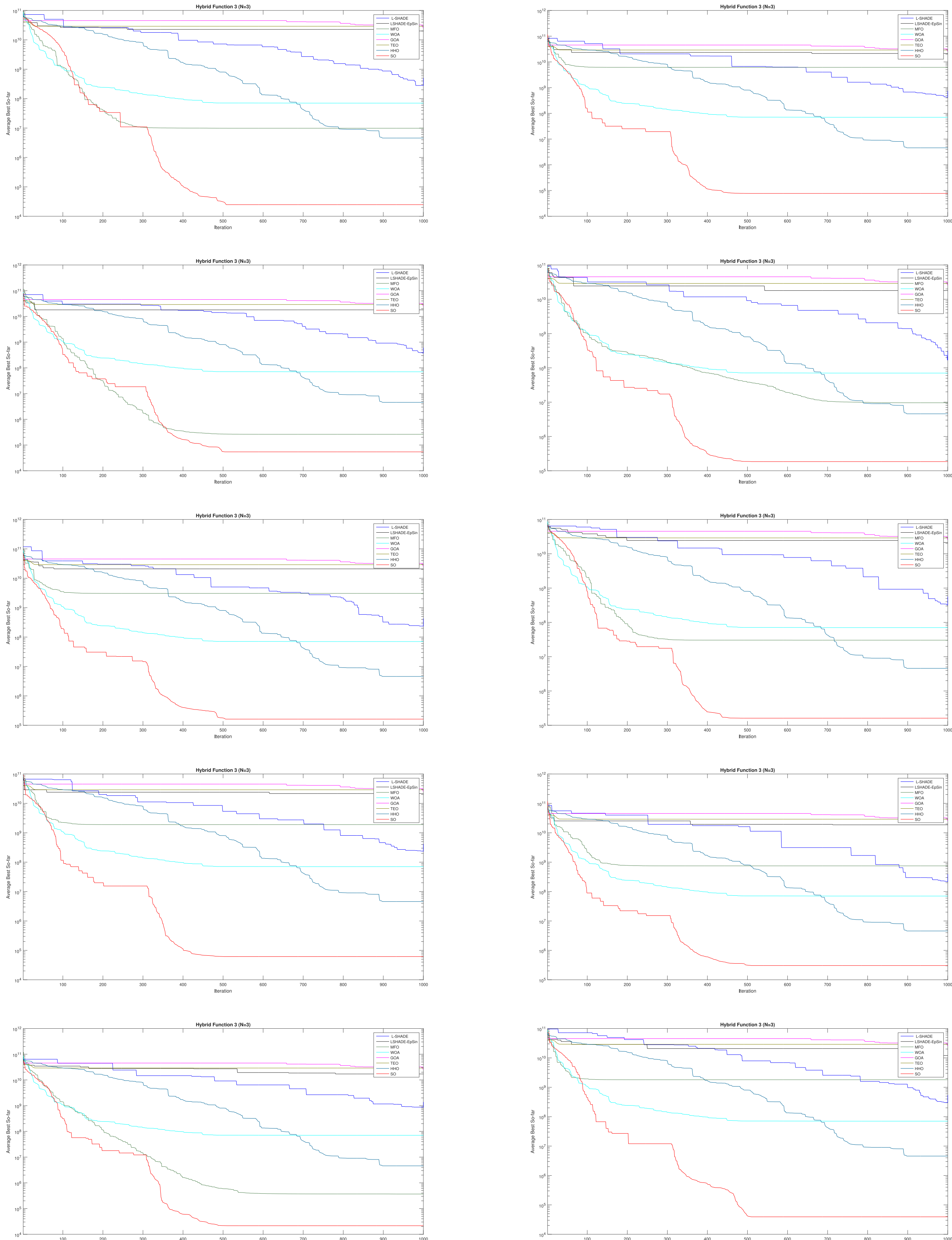

Fig. 15. Convergence curve of some functions from F11–F20 for all algorithms CEC2017 Dim = 50.

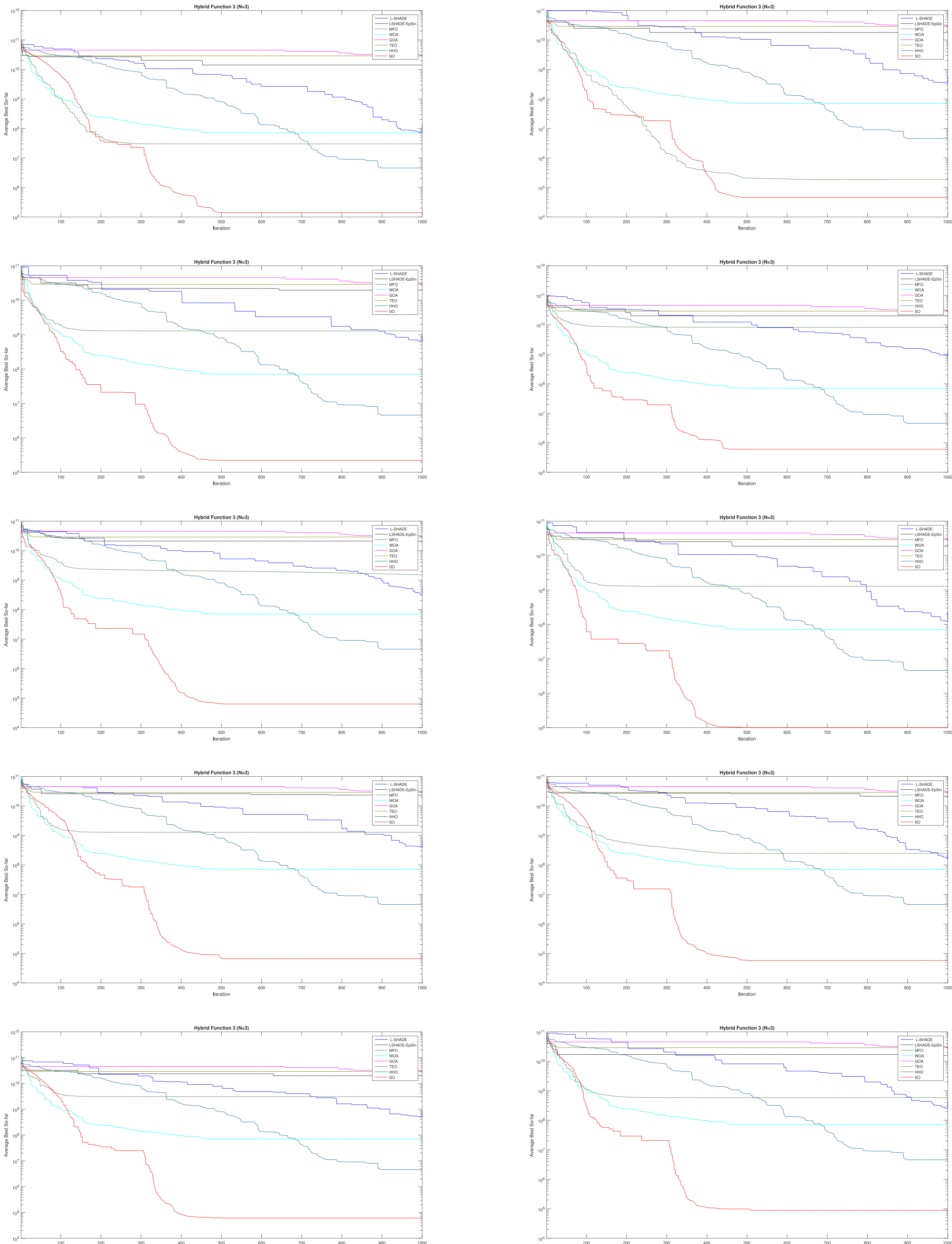

Fig. 16. Convergence curve of some functions from F21–F30 for all algorithms CEC2017 Dim = 50.

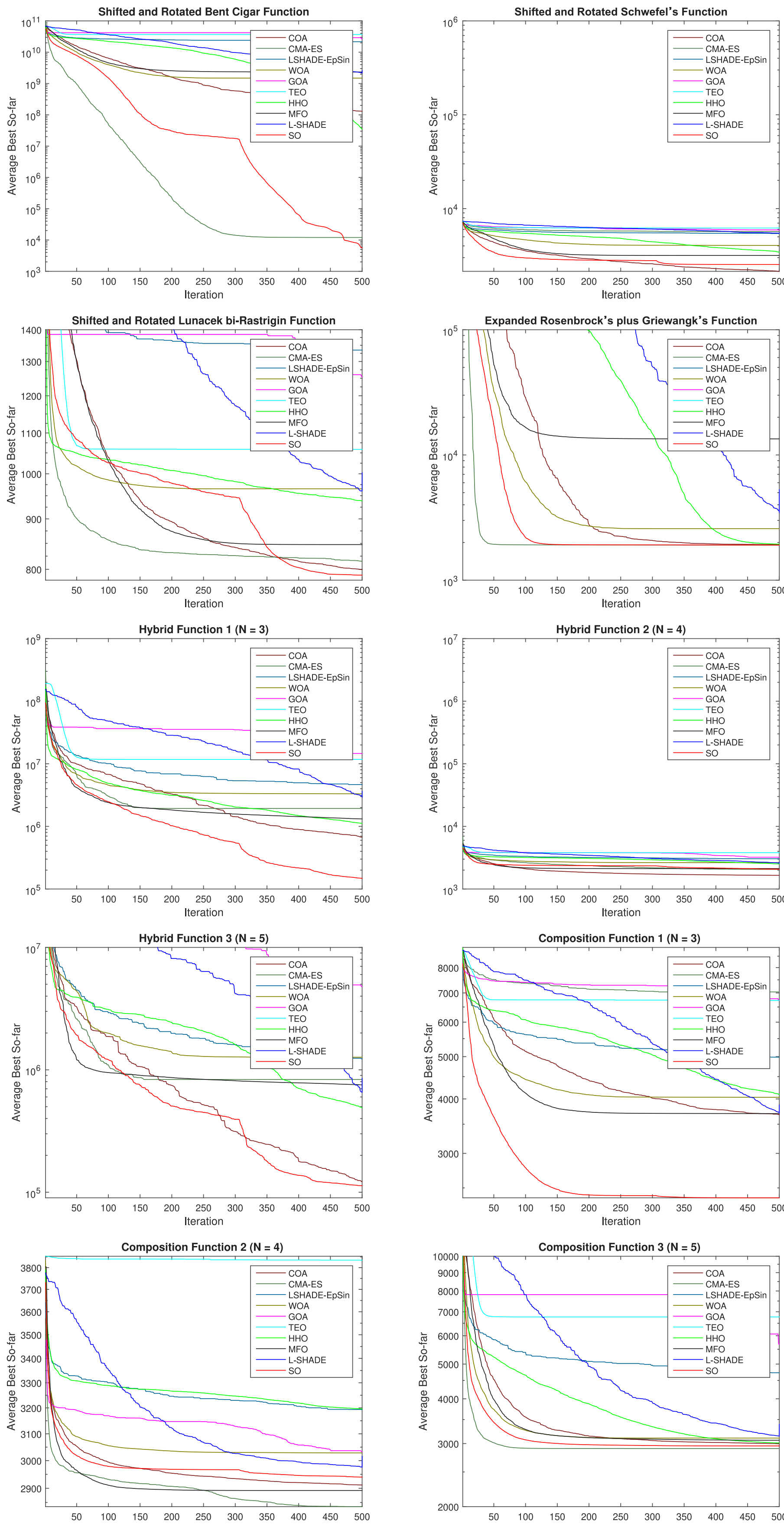

Fig. 17. Convergence curve of some functions from F1–F10 CEC2020 for all algorithms.

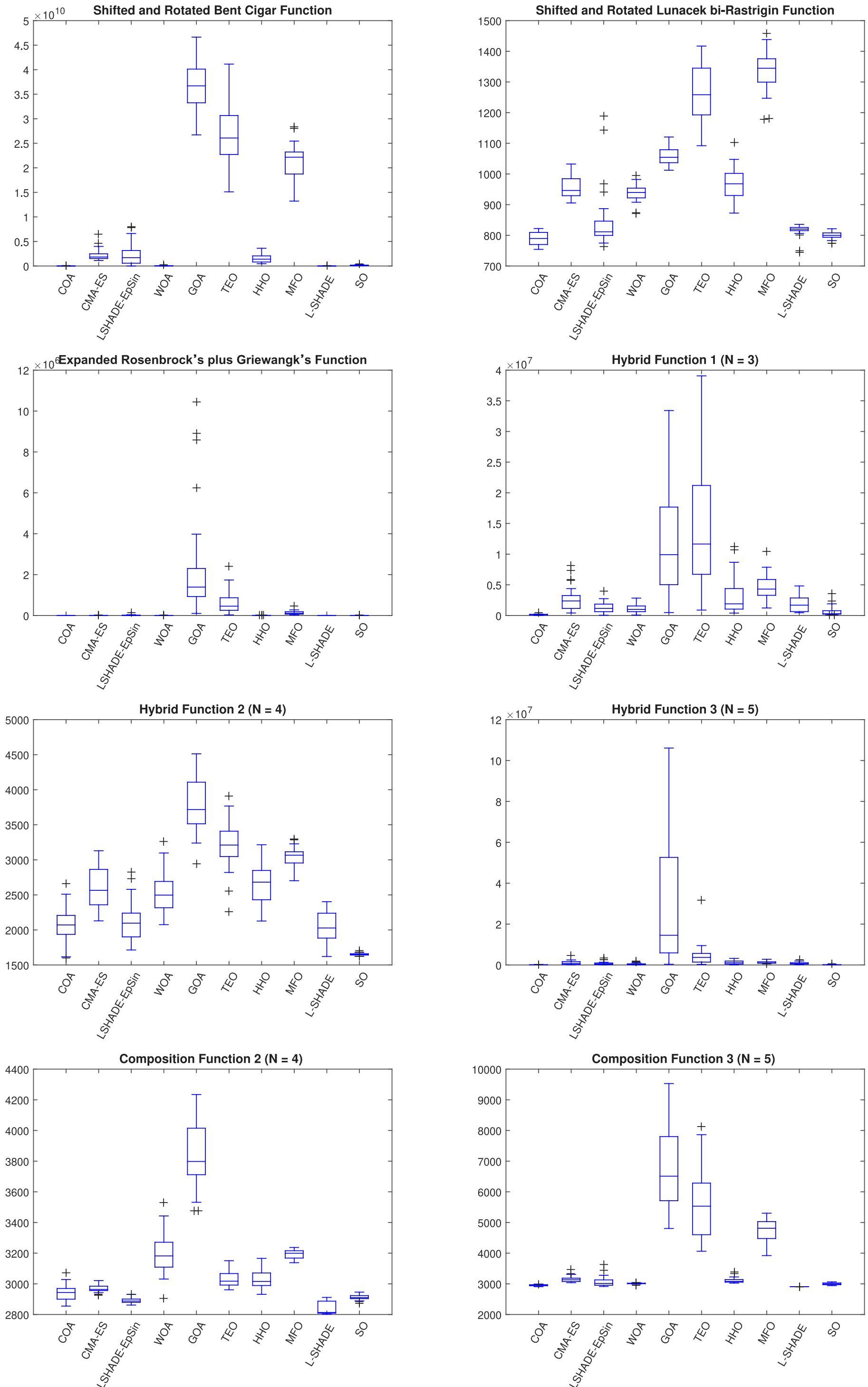

Fig. 18. Box Plot of some functions from F1–F10 CEC 2020.

$$g_4\left(\vec{x}\right) = \frac{x_1 + x_2}{1.5} - 1 \leq 0$$

with $0.05 \leq x_1 \leq 2.0, 0.25 \leq x_2 \leq 1.3, and\, 2.0 \leq x_3 \leq 15.0$

The comparison results between SO and other state-of-art algorithms is given in Table 17. From this table, it is clear that SO achieves the best results with $x^* = (0.051690, 0.356750, 11.287126)$ and objective function $f(x) = 0.012665$. Moreover, Table 18 shows the comparison statistical results in terms of median, worst, best, and average. SO also has the best statistical results. Figure Fig. 22 show the convergence curve between SO and all other compared algorithms.

6. Conclusion and possible research directions

In this work, a novel swarm-based algorithm called Snake Optimizer is proposed. The fundamental concepts of this algorithm is the mating behavior of snakes. The novel algorithm has been evaluated using 30 unconstrained functions. Moreover, 4 different real-world engineering problems have been used to test the validity of SO namely: Speed reducer design problem,

Table 6
Statistical analysis results of the comparative methods using all benchmark functions (Friedman test).

F	SO	L-SHAD	MFO	HHO	TEO	GOA	WOA	LSHADE-EpSin
F1	1	6	5	3	8	2	4	7
F3	3	6	7	1	4	2	8	5
F4	1	6	5	3	8	2	4	7
F5	1	5	3	4	7	2	6	8
F6	1	4	2	5	8	3	6	7
F7	1	4	3	5	7	2	6	8
F8	1	6	4	3	7	2	5	8
F9	1	4	2	5	6	3	7	8
F10	1	8	2	4	7	3	5	6
F11	2	7	4	1	8	3	5	6
F12	1	6	4	3	8	2	5	7
F13	1	6	5	3	8	2	4	8
F14	2	6	3	4	8	1	7	5
F15	1	6	2	4	8	3	5	7
F16	1	5	3	4	8	2	6	7
F17	1	6	3	4	8	2	5	7
F18	1	6	4	3	8	2	5	7
F19	1	6	5	2	8	3	4	7
F20	1	8	3	4	7	2	5	6
F21	1	4	3	5	8	2	6	7
F22	1	4	2	5	8	3	6	7
F23	3	4	1	6	8	2	5	7
F24	3	4	1	6	8	2	5	7
F25	1	6	4	3	8	2	5	7
F26	3	4	2	5	8	1	6	7
F27	1	4	2	6	8	3	5	7
F28	2	6	5	3	8	1	4	7
F29	2	5	1	4	8	3	6	7
F30	1	5	2	3	8	4	6	7
Mean rank	1.3703	5.3703	2.9629	3.9629	7.9259	2.2962	5.3333	8.7407
Final rank	1	6	3	4	7	2	5	8

Table 7
The comparison results of all algorithms over 10 & 50 functions using CEC2017.

F			SO	L-SHADE	MFO	HHO	TEO	GOA	WOA	LSHADE-EpSin
F1	D = 10	Avg	5.40E+03	1.51E+08	1.51E+08	1.23E+06	1.56E+10	3.58E+03	8.53E+07	3.48E+09
		Std	7.80E+03	1.15E+08	4.56E+08	7.34E+05	3.77E+09	2.99E+03	1.05E+08	8.13E+08
	D = 50	Avg	3.04E+09	4.39E+10	3.74E+10	5.15E+09	1.15E+11	4.82E+09	2.10E+10	1.41E+11
		STD	1.29E+09	1.26E+10	2.18E+10	1.38E+09	8.03E+09	1.85E+09	5.54E+09	1.10E+10
F2	D = 10	Avg	NA	NA	NA	NA	NA	NA	NA	NA
		Std	NA	NA	NA	NA	NA	NA	NA	NA
	D = 50	Avg	NA	NA	NA	NA	NA	NA	NA	NA
		Std	NA	NA	NA	NA	NA	NA	NA	NA
F3	D = 10	Avg	1.13E+03	1.45E+04	6.20E+03	8.44E+02	1.63E+04	3.13E+02	4.80E+03	1.18E+04
		Std	1.08E+03	8.96E+03	1.44E+04	2.99E+02	4.66E+03	4.03E+01	2.92E+03	3.31E+03
	D = 50	Avg	1.65E+05	3.90E+05	3.68E+05	1.72E+05	2.05E+05	2.18E+05	3.02E+05	2.80E+05
		STD	1.34E+04	9.66E+04	8.58E+04	2.16E+04	2.59E+04	3.40E+04	1.30E+05	3.32E+04
F4	D = 10	Avg	4.09E+02	4.31E+02	4.11E+02	4.39E+02	2.13E+03	4.09E+02	4.71E+02	6.25E+02
		Std	1.91E+01	2.47E+01	2.07E+01	4.13E+01	7.88E+02	1.48E+01	5.32E+01	6.88E+011
	D = 50	Avg	1.07E+03	7.99E+03	5.51E+03	2.04E+03	4.18E+04	1.11E+03	5.15E+03	4.05E+04
		STD	2.62E+02	3.32E+03	4.70E+03	6.49E+02	6.93E+03	2.15E+02	1.53E+03	5.11E+03
F5	D = 10	Avg	5.29E+02	5.55E+02	5.32E+02	5.56E+02	6.04E+02	5.29E+02	5.60E+02	5.72E+02
		Std	1.23E+01	9.90E+00	1.29E+01	2.23E+01	2.11E+01	1.38E+01	2.50E+01	8.61E+00
	D = 50	Avg	8.28E+02	1.11E+03	9.85E+02	9.39E+02	1.22E+03	9.39E+02	1.10E+03	1.35E+03
		STD	5.36E+01	4.38E+01	9.06E+01	2.85E+01	3.66E+01	4.90E+01	7.77E+01	3.96E+01
F6	D = 10	Avg	6.04E+02	6.18E+02	6.03E+02	6.41E+02	6.52E+02	6.16E+02	6.41E+02	6.41E+02
		Std	4.12E+00	5.69E+00	3.12E+00	1.03E+01	1.14E+01	1.60E+01	1.40E+01	4.18E+00
	D = 50	Avg	6.51E+02	6.75E+02	6.63E+02	6.80E+02	7.07E+02	6.74E+02	6.98E+02	7.12E+02
		STD	9.65E+00	1.13E+01	9.14E+00	4.56E+00	5.91E+00	1.17E+01	1.34E+01	5.29E+00
F7	D = 10	Avg	7.39E+02	7.89E+02	7.40E+02	7.87E+02	8.27E+02	7.37E+02	7.95E+02	8.73E+02
		Std	1.36E+01	1.50E+01	1.48E+01	1.97E+01	2.51E+01	1.36E+01	2.52E+01	2.11E+01
	D = 50	Avg	1.36E+03	1.96E+03	2.01E+03	1.90E+03	2.08E+03	1.59E+03	1.88E+03	3.51E+03
		STD	9.27E+01	1.72E+02	4.54E+02	7.32E+01	4.53E+01	1.32E+02	1.37E+02	2.26E+02
F8	D = 10	Avg	8.23E+02	8.56E+02	8.33E+02	8.29E+02	8.65E+02	8.26E+02	8.47E+02	8.71E+02
		Std	5.86E+00	9.11E+00	1.72E+01	7.24E+00	1.62E+01	1.12E+01	2.27E+01	7.31E+00
	D = 50	Avg	1.15E+03	1.42E+03	1.30E+03	1.24E+03	1.54E+03	1.25E+03	1.37E+03	1.65E+03
		STD	5.74E+01	5.10E+01	1.29E+02	3.32E+01	4.50E+01	6.50E+01	6.18E+01	3.92E+01

(continued on next page)

Table 7 (*continued*).

F			SO	L-SHADE	MFO	HHO	TEO	GOA	WOA	LSHADE-EpSin
F9	D = 10	Avg	9.29E+02	1.20E+03	9.95E+02	1.48E+03	1.63E+03	1.04E+03	1.57E+03	1.77E+03
		Std	4.08E+01	2.83E+02	2.62E+02	2.14E+02	2.09E+02	2.72E+02	6.52E+02	1.91E+02
	D = 50	Avg	1.59E+04	3.13E+04	2.17E+04	3.11E+04	3.81E+04	2.18E+04	4.07E+04	5.24E+04
		STD	5.66E+03	7.07E+03	9.02E+03	2.46E+03	2.84E+03	5.35E+03	1.41E+04	6.56E+03
F10	D = 10	Avg	1.79E+03	2.83E+03	1.91E+03	2.07E+03	3.05E+03	1.97E+03	2.24E+03	2.51E+03
		Std	2.65E+02	2.36E+02	3.56E+02	3.68E+02	2.90E+02	3.35E+02	3.44E+02	1.65E+02
	D = 50	Avg	8.03E+03	1.60E+04	8.39E+03	1.06E+04	1.59E+04	9.58E+03	1.33E+04	1.54E+04
		STD	1.47E+03	8.51E+02	8.98E+02	1.08E+03	5.90E+02	1.17E+03	9.59E+02	3.57E+02
F11	D = 10	Avg	1.13E+03	1.21E+03	1.29E+03	1.17E+03	1.06E+04	1.18E+03	1.27E+03	1.48E+03
		Std	1.86E+01	5.70E+01	2.05E+02	6.03E+01	6.90E+03	5.12E+01	8.32E+01	1.16E+02
	D = 50	Avg	3.79E+03	2.69E+04	1.27E+04	3.09E+03	2.87E+04	5.20E+03	8.26E+03	2.88E+04
		STD	1.61E+03	1.04E+04	1.03E+04	6.62E+02	3.16E+03	2.10E+03	1.78E+03	5.40E+03
F12	D = 10	Avg	6.07E+04	1.09E+07	2.49E+06	3.45E+06	1.44E+09	3.62E+06	3.42E+06	8.35E+07
		Std	1.27E+05	6.28E+06	6.19E+06	2.89E+06	9.06E+08	2.86E+06	5.22E+06	3.73E+07
	D = 50	Avg	9.62E+07	6.43E+09	7.30E+09	7.80E+08	9.94E+10	5.20E+08	4.77E+09	5.50E+10
		STD	8.20E+07	2.69E+09	5.09E+09	3.68E+08	1.00E+10	3.61E+08	2.18E+09	1.01E+10
F13	D = 10	Avg	7.10E+03	2.34E+04	1.44E+04	1.18E+04	7.92E+07	1.59E+04	1.92E+04	2.43E+05
		Std	7.73E+03	1.77E+04	1.21E+04	7.22E+03	1.34E+08	1.73E+04	1.18E+04	2.82E+05
	D = 50	Avg	1.01E+06	1.80E+09	8.45E+08	3.65E+07	6.54E+10	3.03E+05	6.36E+08	2.25E+10
		STD	1.64E+06	9.94E+08	1.21E+09	4.72E+07	1.13E+10	3.15E+05	3.83E+08	5.23E+09
F14	D = 10	Avg	2.66E+03	2.05E+03	6.35E+03	2.19E+03	1.42E+06	1.89E+03	3.06E+03	1.95E+03
		Std	2.56E+03	9.88E+02	6.91E+03	9.71E+02	5.08E+06	6.18E+02	1.67E+03	4.68E+02
	D = 50	Avg	1.73E+06	4.24E+06	1.64E+06	5.11E+06	3.18E+08	5.02E+05	7.67E+06	2.36E+07
		STD	1.43E+06	2.66E+06	9.94E+05	6.08E+06	1.99E+08	2.35E+05	6.93E+06	8.28E+06
F15	D = 10	Avg	3.52E+03	3.57E+03	1.25E+04	7.75E+03	1.75E+07	5.81E+03	9.39E+03	5.56E+03
		Std	1.67E+03	2.22E+03	1.01E+04	2.81E+03	4.98E+07	5.08E+03	9.02E+03	2.90E+03
	D = 50	Avg	4.44E+04	2.72E+08	2.81E+07	3.46E+06	1.27E+10	7.65E+04	8.28E+07	5.23E+09
		STD	2.29E+04	1.85E+08	9.44E+07	5.38E+06	4.53E+09	5.45E+04	1.02E+08	9.93E+08
F16	D = 10	Avg	1.76E+03	1.95E+03	1.77E+03	1.93E+03	2.24E+03	1.86E+03	1.94E+03	1.94E+03
		Std	1.48E+02	1.81E+02	1.23E+02	1.20E+02	1.78E+02	2.00E+02	1.54E+02	5.73E+01
	D = 50	Avg	3.60E+03	6.56E+03	4.50E+03	5.25E+03	1.26E+04	4.39E+03	6.58E+03	8.16E+03
		STD	3.95E+02	4.27E+02	4.58E+02	1.15E+03	2.31E+03	6.29E+02	1.02E+03	3.56E+02
F17	D = 10	Avg	1.79E+03	1.83E+03	1.76E+03	1.78E+03	2.02E+03	1.82E+03	1.82E+03	
		Std	8.71E+01	4.28E+01	3.83E+01	3.21E+01	1.43E+02	6.26E+01	4.16E+01	2.66E+01
	D = 50	Avg	3.31E+03	5.18E+03	4.12E+03	3.93E+03	3.97E+04	3.52E+03	4.47E+03	8.56E+03
		STD	2.90E+02	4.09E+02	9.33E+02	3.91E+02	2.64E+04	4.19E+02	6.00E+02	1.57E+03
F18	D = 10	Avg	8.56E+03	2.92E+05	1.95E+04	1.66E+04	6.02E+08	2.22E+04	1.63E+04	7.11E+05
		Std	4.90E+03	5.40E+05	1.68E+04	1.36E+04	9.59E+08	1.32E+04	1.37E+04	6.94E+05
	D = 50	Avg	6.29E+06	5.55E+07	2.12E+07	1.41E+07	5.20E+08	7.44E+06	5.93E+07	1.16E+08
		STD	5.14E+06	3.36E+07	4.20E+07	1.54E+07	2.50E+08	6.35E+06	3.74E+07	2.99E+07
F19	D = 10	Avg	6.18E+03	5.37E+03	1.18E+04	2.23E+04	7.52E+07	4.63E+03	1.23E+05	1.09E+04
		Std	6.33E+03	4.56E+03	1.21E+04	2.55E+04	2.13E+08	3.76E+03	3.37E+05	1.01E+04
	D = 50	Avg	5.31E+04	1.31E+08	1.88E+07	2.81E+06	7.55E+09	8.97E+06	1.90E+07	2.24E+09
		STD	8.79E+04	8.80E+07	5.61E+07	1.99E+06	2.32E+09	6.62E+06	1.39E+07	6.91E+08
F20	D = 10	Avg	2.07E+03	2.12E+03	2.10E+03	2.18E+03	2.32E+03	2.19E+03	2.21E+03	2.16E+03
		Std	3.97E+01	5.03E+01	6.47E+01	7.52E+01	7.78E+01	1.03E+02	9.52E+01	2.62E+01
	D = 50	Avg	3.27E+03	4.67E+03	3.70E+03	3.48E+03	4.47E+03	3.71E+03	4.08E+03	4.29E+03
		STD	3.87E+02	2.65E+02	3.49E+02	2.97E+02	2.64E+02	4.01E+02	3.24E+02	2.12E+02
F21	D = 10	Avg	2.32E+03	2.35E+03	2.33E+03	2.34E+03	2.41E+03	2.31E+03	2.32E+03	2.25E+03
		Std	1.67E+01	3.68E+01	3.07E+01	3.36E+01	3.89E+01	5.63E+01	4.78E+01	2.04E+01
	D = 50	Avg	2.64E+03	2.91E+03	2.77E+03	2.95E+03	3.38E+03	2.77E+03	3.09E+03	3.18E+03
		STD	3.83E+01	3.78E+01	9.83E+01	9.67E+01	1.22E+02	8.74E+01	1.19E+02	3.38E+01
F22	D = 10	Avg	2.30E+03	2.34E+03	2.31E+03	2.39E+03	3.50E+03	2.36E+03	2.37E+03	2.55E+03
		Std	9.97E−01	2.20E+01	2.01E+01	3.42E+02	3.70E+02	2.40E+02	2.09E+02	9.89E+01
	D = 50	Avg	1.06E+04	1.79E+04	1.01E+04	1.24E+04	1.74E+04	1.19E+04	1.48E+04	1.71E+04
		STD	2.25E+03	5.54E+02	8.87E+02	1.12E+03	5.28E+02	1.63E+03	7.65E+02	3.78E+02
F23	D = 10	Avg	2.63E+03	2.65E+03	2.63E+03	2.69E+03	2.84E+03	2.63E+03	2.66E+03	2.69E+03
		Std	1.57E+01	1.14E+01	1.43E+01	3.76E+01	7.79E+01	1.09E+01	3.26E+01	1.16E+01
	D = 50	Avg	3.34E+03	3.48E+03	3.18E+03	4.17E+03	5.32E+03	3.31E+03	3.87E+03	4.19E+03
		STD	9.28E+01	6.82E+01	5.84E+01	2.44E+02	3.91E+02	1.78E+02	2.25E+02	8.67E+01

(*continued on next page*)

Table 7 (*continued*).

F			SO	L-SHADE	MFO	HHO	TEO	GOA	WOA	LSHADE-EpSin
F24	D = 10	Avg	2.76E+03	2.78E+03	2.76E+03	2.81E+03	3.00E+03	2.75E+03	2.79E+03	2.72E+03
		Std	5.64E+01	1.41E+01	1.24E+01	8.99E+01	1.51E+02	1.23E+01	3.12E+01	.81E+01
	D = 50	Avg	3.51E+03	3.61E+03	3.27E+03	4.45E+03	6.01E+03	3.44E+03	3.92E+03	4.51E+03
		STD	1.20E+02	9.64E+01	5.94E+01	2.69E+02	2.78E+02	1.39E+02	1.68E+02	1.25E+02
F25	D = 10	Avg	2.93E+03	2.96E+03	2.94E+03	2.93E+03	3.88E+03	2.92E+03	2.97E+03	3.10E+03
		Std	2.01E+01	1.29E+01	2.72E+01	2.24E+01	2.97E+02	8.09E+01	3.09E+01	3.69E+01
	D = 50	Avg	3.46E+03	8.32E+03	6.33E+03	3.78E+03	1.71E+04	3.58E+03	4.94E+03	2.39E+04
		STD	1.54E+02	1.07E+03	2.94E+03	2.08E+02	1.48E+03	1.62E+02	5.76E+02	1.62E+03
F26	D = 10	Avg	3.39E+03	3.15E+03	3.08E+03	3.63E+03	4.53E+03	3.14E+03	3.59E+03	3.39E+03
		Std	4.11E+02	3.17E+02	2.41E+02	6.18E+02	2.92E+02	5.18E+02	5.36E+02	1.29E+02
	D = 50	Avg	1.02E+04	1.19E+04	8.63E+03	1.21E+04	1.87E+04	9.31E+03	1.57E+04	1.93E+04
		STD	1.09E+03	8.36E+02	8.87E+02	9.57E+02	8.49E+02	2.03E+03	1.45E+03	9.30E+02
F27	D = 10	Avg	3.11E+03	3.11E+03	3.10E+03	3.18E+03	3.42E+03	3.10E+03	3.15E+03	3.15E+03
		Std	1.80E+01	1.74E+01	4.87E+00	6.44E+01	8.50E+01	2.08E+01	4.58E+01	1.14E+01
	D = 50	Avg	4.08E+03	4.36E+03	3.64E+03	4.99E+03	9.23E+03	3.67E+03	5.07E+03	5.99E+03
		STD	3.04E+02	1.77E+02	1.16E+02	5.03E+02	1.07E+03	1.24E+02	9.77E+02	3.34E+02
F28	D = 10	Avg	3.32E+03	3.36E+03	3.37E+03	3.41E+03	3.97E+03	3.39E+03	3.47E+03	3.45E+03
		Std	8.60E+01	8.24E+01	7.76E+01	1.56E+02	1.45E+02	1.44E+02	1.57E+02	5.93E+01
	D = 50	Avg	4.26E+03	7.85E+03	7.66E+03	5.13E+03	1.52E+04	3.91E+03	6.23E+03	1.40E+04
		STD	3.45E+02	1.10E+03	1.65E+03	4.23E+02	1.48E+03	2.80E+02	7.78E+02	1.09E+03
F29	D = 10	Avg	3.22E+03	3.37E+03	3.25E+03	3.43E+03	3.68E+03	3.25E+03	3.37E+03	3.35E+03
		Std	4.76E+01	7.89E+01	7.39E+01	9.67E+01	1.55E+02	6.33E+01	1.23E+02	5.05E+01
	D = 50	Avg	5.15E+03	8.27E+03	5.33E+03	7.38E+03	5.61E+05	6.05E+03	9.64E+03	1.68E+04
		STD	6.46E+02	1.07E+03	4.97E+02	1.28E+03	4.60E+05	6.66E+02	1.91E+03	4.36E+03
F30	D = 10	Avg	2.91E+04	1.91E+06	1.16+06	2.47E+06	1.18E+08	5.37E+05	2.42E+06	5.30E+06
		Std	3.03E+04	2.03E+06	7.74E+05	3.16E+06	8.72E+07	6.34E+05	2.98E+06	2.15E+06
	D = 50	Avg	1.05E+07	5.28E+08	1.91E+08	1.28E+08	1.17E+10	1.49E+08	3.52E+08	4.32E+09
		STD	1.58E+07	1.83E+08	4.60E+08	5.00E+07	4.13E+09	5.49E+07	1.42E+08	8.51E+08

Table 8
Wilcoxon rank sum test results for SO against other algorithms with Dim = 10 & 50 using CEC2017.

F		L-SHADE	MFO	HHO	TEO	GOA	WOA	LSHADE-EpSin
F1	D = 10	6.80E−08	1.08E−01	6.80E−08	6.80E−08	7.35E−01	6.80E−08	6.80E−08
	D = 50	6.80E−08	6.80E−08	3.71E−05	6.80E−08	1.35E−03	6.80E−08	6.80E−08
F3	D = 10	7.90E−08	8.82E−01	8.82E−01	6.80E−08	1.20E−06	3.50E−06	6.80E−08
	D = 50	6.80E−08	6.80E−08	1.33E−01	2.69E−06	1.05E−06	3.42E−07	6.80E−08
F4	D = 10	1.05E−06	3.14E−02	6.22E−04	6.80E−08	1.64E−01	7.95E−07	6.80E−08
	D = 50	6.80E−08	1.66E−07	3.94E−07	6.80E−08	3.65E−01	6.80E−08	6.80E−08
F5	D = 10	2.06E−06	3.65E−01	1.16E−04	6.80E−08	9.68E−01	4.68E−05	1.06E−07
	D = 50	6.80E−08	9.75E−06	1.58E−06	6.80E−08	2.36E−06	6.80E−08	6.80E−08
F6	D = 10	4.54E−07	3.10E−01	6.80E−08	6.80E−08	8.36E−04	6.80E−08	6.80E−08
	D = 50	2.96E−07	4.16E−04	6.80E−08	6.80E−08	6.92E−07	6.80E−08	6.80E−08
F7	D = 10	1.43E−07	8.60E−01	1.66E−07	6.80E−08	4.73E−01	2.56E−07	6.80E−08
	D = 50	6.80E−08	2.69E−06	6.80E−08	6.80E−08	4.54E−06	6.80E−08	6.80E−08
F8	D = 10	6.80E−08	7.64E−02	6.04E−03	7.90E−08	6.55E−01	3.29E−05	6.80E−08
	D = 50	6.80E−08	1.04E−04	1.81E−05	6.80E−08	2.60E−05	9.17E−08	6.80E−08
F9	D = 10	2.56E−07	3.10E−01	7.90E−08	6.80E−08	2.98E−01	9.17E−08	6.80E−08
	D = 50	1.05E−06	2.56E−02	9.17E−08	6.80E−08	2.14E−03	1.66E−07	6.80E−08
F10	D = 10	6.80E−08	2.73E−01	1.06E−02	6.80E−08	6.39E−02	1.61E−04	1.06E−07
	D = 50	6.80E−08	1.33E−01	1.41E−05	6.80E−08	1.12E−03	7.90E−08	6.80E−08
F11	D = 10	6.01E−07	6.22E−04	1.78E−03	6.80E−08	5.90E−05	1.43E−07	6.80E−08
	D = 50	6.80E−08	1.60E−05	4.73E−01	6.80E−08	1.67E−02	2.56E−07	6.80E−08
F12	D = 10	6.80E−08	9.21E−04	1.23E−07	6.80E−08	1.23E−07	5.87E−06	6.80E−08
	D = 50	6.80E−08	4.54E−07	7.90E−08	6.80E−08	9.75E−06	6.80E−08	6.80E−08
F13	D = 10	2.47E−04	3.37E−02	9.79E−03	1.43E−07	1.06E−02	3.05E−04	1.66E−07
	D = 50	6.80E−08	8.29E−05	9.17E−08	6.80E−08	1.20E−01	6.80E−08	6.80E−08
F14	D = 10	7.15E−01	3.06E−03	3.51E−01	3.37E−02	5.43E−01	6.01E−02	2.73E−01
	D = 50	2.56E−03	7.76E−01	9.05E−03	6.80E−08	2.14E−03	1.60E−05	6.80E−08
F15	D = 10	6.95E−01	2.00E−04	3.71E−05	5.23E−07	4.99E−02	9.79E−03	3.97E−03
	D = 50	6.80E−08	9.28E−05	6.80E−08	6.80E−08	1.14E−02	6.80E−08	6.80E−08
F16	D = 10	3.64E−03	5.08E−01	4.60E−04	1.06E−07	8.10E−02	9.21E−04	4.16E−04
	D = 50	6.80E−08	3.50E−06	2.22E−07	6.80E−08	9.28E−05	6.80E−08	6.80E−08

(*continued on next page*)

Table 8 (*continued*).

F		L-SHADE	MFO	HHO	TEO	GOA	WOA	LSHADE-EpSin
F17	D = 10	3.64E−03	7.76E−01	2.62E−01	5.87E−06	2.75E−02	1.14E−02	6.87E−04
	D = 50	6.80E−08	9.28E−05	1.81E−05	6.80E−08	2.75E−02	3.94E−07	6.80E−08
F18	D = 10	2.60E−05	2.56E−02	1.08E−01	7.95E−07	5.63E−04	7.20E−02	6.80E−08
	D = 50	1.43E−07	1.64E−01	1.55E−02	6.80E−08	7.15E−01	4.54E−07	6.80E−08
F19	D = 10	5.43E−01	9.62E−02	5.63E−04	9.13E−07	9.46E−01	5.63E−04	3.97E−03
	D = 50	6.80E−08	1.14E−02	6.80E−08	6.80E−08	6.80E−08	6.80E−08	6.80E−08
F20	D = 10	3.05E−04	3.51E−01	1.10E−05	1.06E−07	3.29E−05	2.69E−06	2.69E−06
	D = 50	6.80E−08	2.34E−03	8.10E−02	9.17E−08	1.35E−03	1.80E−06	7.90E−08
F21	D = 10	4.68E−05	2.98E−01	1.04E−04	1.58E−06	4.25E−01	1.40E−01	1.43E−07
	D = 50	6.80E−08	1.10E−05	6.80E−08	6.80E−08	2.36E−06	6.80E−08	6.80E−08
F22	D = 10	6.80E−08	7.11E−03	6.80E−08	6.80E−08	1.29E−04	6.80E−08	6.80E−08
	D = 50	6.80E−08	8.39E−01	4.16E−04	7.90E−08	6.56E−03	1.60E−05	6.80E−08
F23	D = 10	5.63E−04	5.98E−01	1.10E−05	6.80E−08	2.08E−01	2.14E−03	7.90E−08
	D = 50	2.60E−05	5.17E−06	6.80E−08	6.80E−08	2.98E−01	6.80E−08	6.80E−08
F24	D = 10	1.78E−03	1.99E−01	1.35E−03	1.41E−05	1.35E−03	3.15E−02	5.56E−03
	D = 50	1.23E−02	3.94E−07	6.80E−08	6.80E−08	1.14E−01	2.56E−07	6.80E−08
F25	D = 10	6.92E−07	8.10E−02	6.17E−01	6.80E−08	6.55E−01	8.29E−05	6.80E−08
	D = 50	6.80E−08	4.54E−06	6.67E−06	6.80E−08	2.07E−02	6.80E−08	6.80E−08
F26	D = 10	7.71E−03	1.01E−03	3.37E−01	1.43E−07	3.34E−03	2.18E−01	2.29E−01
	D = 50	9.75E−06	3.71E−05	2.92E−05	6.80E−08	3.37E−01	9.17E−08	6.80E−08
F27	D = 10	6.36E−01	1.63E−03	8.29E−05	6.80E−08	9.79E−03	1.35E−03	1.20E−06
	D = 50	5.63E−04	2.69E−06	3.94E−07	6.80E−08	3.99E−06	1.44E−04	6.80E−08
F28	D = 10	6.39E−02	3.53E−02	1.44E−02	6.80E−08	2.18E−01	9.21E−04	8.60E−06
	D = 50	6.80E−08	1.25E−05	1.58E−06	6.80E−08	2.47E−04	6.80E−08	6.80E−08
F29	D = 10	1.05E−06	2.08E−01	5.23E−07	6.80E−08	1.64E−01	1.10E−05	6.92E−07
	D = 50	6.80E−08	2.98E−01	1.66E−07	6.80E−08	3.05E−04	6.80E−08	6.80E−08
F30	D = 10	7.90E−08	6.80E−08	1.23E−07	6.80E−08	1.35E−03	1.80E−06	6.80E−08
	D = 50	6.80E−08	5.63E−04	6.80E−08	6.80E−08	6.80E−08	6.80E−08	6.80E−08

Table 9
The comparison results of all algorithms over 10 functions CEC2020 using Dim = 20.

F		SO	L-SHADE	MFO	HHO	TEO	GOA	WOA	LSHADE-EpSin	CMAES	COA
F1	Avg	**5.30E+03**	2.21E+09	2.33E+09	3.44E+07	3.68E+10	2.68E+10	1.49E+09	2.16E+10	1.21E+04	1.30E+08
	Min	**1.52E+02**	1.12E+09	1.01E+04	9.78E+06	2.67E+10	1.51E+10	4.64E+08	1.32E+10	3.25E+02	3.37E+07
	Max	**1.93E+04**	6.54E+09	8.01E+09	2.13E+08	4.66E+10	4.11E+10	3.61E+09	2.83E+10	7.83E+04	3.51E+08
	Med	**3.43E+03**	1.83E+09	1.69E+09	2.37E+07	3.67E+10	2.61E+10	1.39E+09	2.22E+10	6.21E+03	1.14E+08
	STD	**4.70E+03**	1.15E+09	2.44E+09	3.79E+07	4.65E+09	6.29E+09	8.36E+08	3.40E+09	1.56E+04	7.60E+07
F2	Avg	2.54E+03	5.49E+03	3.17E+03	3.44E+03	6.21E+03	5.89E+03	4.05E+03	5.42E+03	5.69E+03	**2.15E+03**
	Min	1.67E+03	4.41E+03	2.36E+03	2.32E+03	5.49E+03	4.82E+03	3.08E+03	4.71E+03	4.76E+03	**1.58E+03**
	Max	3.37E+03	6.57E+03	4.34E+03	4.42E+03	6.83E+03	6.69E+03	5.10E+03	5.73E+03	6.29E+03	**2.54E+03**
	Med	2.57E+03	5.42E+03	2.95E+03	3.42E+03	6.28E+03	5.81E+03	4.04E+03	5.46E+03	5.74E+03	**2.23E+03**
	STD	4.30E+02	4.77E+02	5.50E+02	5.11E+02	3.64E+02	5.05E+02	5.56E+02	2.31E+02	2.98E+02	**2.51E+02**
F3	Avg	**7.89E+02**	9.57E+02	8.48E+02	9.39E+02	1.06E+03	1.25E+03	9.65E+02	1.33E+03	8.15E+02	8.00E+02
	Min	7.54E+02	9.06E+02	7.63E+02	8.70E+02	1.01E+03	1.09E+03	8.72E+02	1.18E+03	**7.46E+02**	7.73E+02
	Max	**8.22E+02**	1.03E+03	1.19E+03	9.95E+02	1.12E+03	1.42E+03	1.10E+03	1.46E+03	8.36E+02	8.21E+02
	Med	**7.90E+02**	9.46E+02	8.11E+02	9.40E+02	1.05E+03	1.26E+03	9.68E+02	1.34E+03	8.20E+02	8.00E+02
	Std	2.15E+01	3.69E+01	9.80E+01	2.89E+01	2.84E+01	9.73E+01	5.13E+01	6.55E+01	**1.97E+01**	1.19E+01
F4	Avg	**1.91E+03**	3.11E+03	1.34E+04	1.94E+03	2.38E+06	6.41E+05	2.58E+03	1.35E+05	1.91E+03	1.92E+03
	Min	**1.90E+03**	1.94E+03	1.91E+03	1.92E+03	1.02E+05	2.97E+04	1.92E+03	2.82E+04	1.90E+03	1.91E+03
	Max	**1.91E+03**	1.81E+04	1.30E+05	2.05E+03	1.04E+07	2.40E+06	5.57E+03	4.52E+05	1.91E+03	1.96E+03
	Med	**1.91E+03**	2.45E+03	3.43E+03	1.93E+03	1.39E+06	4.55E+05	2.18E+03	1.09E+05	1.91E+03	1.92E+03
	Std	**2.15E+00**	2.91E+03	2.43E+04	2.33E+01	2.65E+06	5.63E+05	9.56E+02	9.32E+04	2.44E+00	1.37E+01
F5	Avg	**1.40E+05**	2.66E+06	1.31E+06	1.12E+06	1.17E+07	1.45E+07	3.31E+06	4.65E+06	1.93E+06	6.72E+05
	Min	**5.60E+03**	4.09E+05	4.45E+04	6.86E+04	4.77E+05	8.83E+05	3.97E+05	1.22E+06	4.46E+05	4.09E+04
	Max	**4.81E+05**	8.08E+06	3.94E+06	2.83E+06	3.34E+07	3.91E+07	1.13E+07	1.05E+07	4.82E+06	3.56E+06
	Med	**1.34E+05**	2.37E+06	1.16E+06	9.90E+05	9.92E+06	1.17E+07	1.90E+06	4.30E+06	1.68E+06	2.88E+05
	Std	**9.57E+04**	1.98E+06	9.20E+05	7.25E+05	8.40E+06	1.07E+07	2.98E+06	1.88E+06	1.42E+06	7.95E+05
F6	Avg	2.06E+03	2.61E+03	2.12E+03	2.52E+03	3.79E+03	3.20E+03	2.63E+03	3.04E+03	2.06E+03	**1.65E+03**
	Min	**1.60E+03**	2.13E+03	1.71E+03	2.08E+03	2.94E+03	2.27E+03	2.13E+03	2.70E+03	1.62E+03	1.62E+03
	Max	2.66E+03	3.13E+03	2.83E+03	3.26E+03	4.51E+03	3.90E+03	3.21E+03	3.30E+03	2.40E+03	**1.70E+03**
	Med	2.07E+03	2.56E+03	2.10E+03	2.50E+03	3.72E+03	3.21E+03	2.68E+03	3.07E+03	2.03E+03	**1.65E+03**
	Std	2.69E+02	2.81E+02	2.74E+02	2.78E+02	4.02E+02	3.39E+02	3.18E+02	1.33E+02	2.34E+02	**1.94E+01**
F7	Avg	**9.80E+04**	1.05E+06	7.53E+05	4.89E+05	3.03E+07	4.77E+06	1.27E+06	1.23E+06	8.34E+05	1.22E+05
	Min	7.04E+03	8.60E+04	5.82E+04	6.81E+03	3.74E+05	3.12E+05	3.45E+04	4.66E+05	1.77E+05	**3.73E+03**
	Max	**3.66E+05**	4.49E+06	3.53E+06	1.95E+06	1.06E+08	3.18E+07	3.23E+06	2.79E+06	2.70E+06	4.74E+05

(*continued on next page*)

Table 9 (*continued*).

F		SO	L-SHADE	MFO	HHO	TEO	GOA	WOA	LSHADE-EpSin	CMAES	COA
	Med	**6.27E+04**	8.18E+05	4.80E+05	2.77E+05	1.45E+07	3.70E+06	1.10E+06	9.94E+05	6.57E+05	7.42E+04
	Std	**8.68E+04**	1.01E+06	8.13E+05	5.50E+05	3.30E+07	5.83E+06	9.33E+05	5.56E+05	5.97E+05	1.16E+05
F8	Avg	**2.37E+03**	3.26E+03	3.70E+03	4.10E+03	6.73E+03	6.73E+03	4.03E+03	4.99E+03	7.04E+03	3.68E+03
	Min	**2.30E+03**	2.52E+03	2.30E+03	2.32E+03	4.82E+03	4.83E+03	2.38E+03	3.90E+03	5.66E+03	2.33E+03
	Max	**4.43E+03**	7.08E+03	6.50E+03	6.34E+03	8.27E+03	7.84E+03	6.83E+03	5.71E+03	7.67E+03	5.84E+03
	Med	**2.30E+03**	2.95E+03	3.08E+03	4.73E+03	6.79E+03	7.14E+03	2.66E+03	5.13E+03	7.12E+03	2.48E+03
	Std	**3.89E+02**	1.11E+03	1.37E+03	1.70E+03	6.90E+02	9.64E+02	1.82E+03	4.56E+02	4.31E+02	1.50E+03
F9	Avg	2.94E+03	2.97E+03	2.89E+03	3.20E+03	3.83E+03	3.03E+03	3.03E+03	3.19E+03	**2.84E+03**	2.91E+03
	Min	2.85E+03	2.93E+03	2.86E+03	2.90E+03	3.47E+03	2.96E+03	2.93E+03	3.14E+03	**2.80E+03**	2.88E+03
	Max	3.07E+03	3.02E+03	2.93E+03	3.53E+03	4.23E+03	3.15E+03	3.17E+03	3.24E+03	**2.91E+03**	2.95E+03
	Med	2.94E+03	2.96E+03	2.89E+03	3.18E+03	3.80E+03	3.02E+03	3.02E+03	3.20E+03	**2.81E+03**	2.91E+03
	Std	5.16E+01	2.38E+01	1.87E+01	1.37E+02	2.16E+02	5.28E+01	5.75E+01	3.02E+01	**3.94E+01**	1.66E+01
F10	Avg	2.95E+03	3.16E+03	3.07E+03	3.01E+03	6.77E+03	5.68E+03	3.11E+03	4.73E+03	**2.91E+03**	3.00E+03
	Min	2.91E+03	3.04E+03	2.91E+03	2.95E+03	4.81E+03	4.06E+03	3.02E+03	3.92E+03	**2.91E+03**	2.94E+03
	Max	3.00E+03	3.48E+03	3.64E+03	3.05E+03	9.53E+03	8.13E+03	3.40E+03	5.30E+03	**2.91E+03**	3.06E+03
	Med	2.96E+03	3.13E+03	3.01E+03	3.01E+03	6.51E+03	5.53E+03	3.08E+03	4.81E+03	**2.91E+03**	2.99E+03
	Std	2.73E+01	1.04E+02	1.66E+02	2.07E+01	1.35E+03	1.21E+03	8.93E+01	3.57E+02	**1.63E−02**	3.24E+01

Table 10
Wilcoxon rank sum test results for SO against other algorithms CEC2020 Dim = 20.

F	L-SHADE	MFO	HHO	TEO	GOA	WOA	LSHADE-EpSin	CMAES	COA
F1	3.01986E−11	1.46431E−10	3.01986E−11	3.01986E−11	3.01986E−11	3.01986E−11	3.01986E−11	0.111986872	3.01986E−11
F2	3.01986E−11	2.27802E−05	3.64589E−08	3.01986E−11	3.01986E−11	4.97517E−11	3.01986E−11	3.01986E−11	0.000158461
F3	3.01986E−11	0.00039881	3.01986E−11	3.01986E−11	3.01986E−11	3.01986E−11	3.01986E−11	7.04298E−07	0.090490361
F4	3.01986E−11	2.60985E−10	3.01986E−11	3.01986E−11	3.01986E−11	3.01986E−11	3.01986E−11	0.899995037	5.49405E−11
F5	3.68973E−11	5.96731E−09	1.28704E−09	3.33839E−11	3.01986E−11	3.33839E−11	3.01986E−11	3.68973E−11	1.38525E−06
F6	1.84999E−08	0.641423523	2.19589E−07	3.01986E−11	6.06576E−11	6.52774E−08	3.01986E−11	0.982307053	1.02773E−06
F7	1.69472E−09	2.38974E−08	2.43271E−05	3.01986E−11	3.33839E−11	8.89099E−10	3.01986E−11	1.46431E−10	0.510597937
F8	4.61591E−10	2.38974E−08	1.20567E−10	3.01986E−11	3.01986E−11	1.61323E−10	4.97517E−11	3.01986E−11	1.61323E−10
F9	0.008684371	0.000132495	4.19968E−10	3.01986E−11	3.96477E−08	5.18568E−07	3.01986E−11	6.51827E−09	0.019112397
F10	3.01986E−11	0.000903069	1.41098E−09	3.01986E−11	3.01986E−11	3.01986E−11	3.01986E−11	3.01986E−11	1.63506E−05

Table 11
Comparison of optimum results for Speed Reducer problem.

Algorithm	z1	z2	z3	z4	z5	z6	z7	Cost
L-SHADE	3.3626	0.7418	26.2003	7.5158	8.2479	3.8144	5.2271	4.340278816331378e+03
LSHADE-EpSin	3.5987	0.7358	19.8452	7.4775	7.9874	3.4014	5.2777	3.305963920216824e+03
MFO	3.4976	0.7000	17.0000	7.3000	7.8000	3.3501	5.2857	2.995542437084133e+03
WOA	3.4975	0.7000	17.0000	7.4863	7.8054	3.7245	5.2853	3.684981241192847e+03
GOA	3.5859	0.7201	23.3080	7.9034	7.9067	3.4285	5.3771	4.603812570774953e+03
TEO	3.4902	0.7000	17.0495	7.4571	7.9486	3.7022	5.2148	4.827348506926367e+03
HHO	3.4981	0.7000	17.0000	7.6398	7.8000	3.3582	5.2853	3.000672075002531e+03
SO	3.4976	0.7000	17.0000	7.3000	7.8000	3.3501	5.2857	2.995542437084133e+03

Table 12
Statistical results for Speed Reducer problem.

Algorithm	Best	Mean	Worst	Std Dev
L-SHADE	4.340278816331378e+03	3.181604338514015e+04	5.997557019302970e+04	2.10E+04
LSHADE-EpSin	3.305963920216824e+03	2.279851228074023e+04	7.468396545921339e+04	2.39E+04
MFO	2.995542437084133e+03	2.995542437084135e+03	2.995542437084133e+03	1.44E−12
WOA	3.684981241192847e+02	4.125912824555334e+02	3.684981241192847e+03	1.09E+03
GOA	4.603812570774953e+03	8.435625630729317e+03	1.370798960961360e+04	4.72E+03
TEO	4.827348506926367e+03	6.621484515483270e+03	1.003853989342308e+04	2.96E+03
HHO	3.000672075002531e+03	3.893806945464781e+03	4.452585676275292e+03	7.82E+02
SO	2.995542437084133e+03	2.995542437084135e+03	2.995542437084134e+03	1.35E−12

Table 13
Comparison of optimum results for Welded beam design problem.

Algorithm	h	l	t	b	Cost
L-SHADE	0.2389	3.4067	9.6383	0.2901	2.070128606337081
LSHADE-EpSin	0.2884	3.1057	9.3491	0.2999	2.015674905110601
MFO	0.2057	3.4705	9.0366	0.2057	1.724851892759705
WOA	0.3290	2.5471	6.8078	0.3789	2.358352935363380
GOA	0.1672	5.0210	9.0397	0.2058	1.857201147866808
TEO	0.2606	4.9671	5.0117	0.6744	3.456904238077941
HHO	0.2134	3.5601	8.4629	0.2346	1.856147752533281
SO	0.2057	3.4705	9.0366	0.2057	1.724851930920065

Table 14
Statistical results for Welded beam design problem.

Algorithm	Best	Mean	Worst	Std Dev
L-SHADE	2.070128606337081	2.136086252982838	3.222617322356771	5.88E+1
LSHADE-EpSin	2.015674905110601	2.416525568752919	2.897780985683523	4.49E+1
MFO	1.724851892759705	1.936570864837374	2.251895192185770	1.56E−01
WOA	2.358352935363380	2.568541580677852	2.786238155635347	2.14E−01
GOA	1.857201147866808	3.177010726818908	5.088736631100389	1.70E+00
TEO	3.456904238077941	4.879999194650345	6.267138221585146	1.41E+00
HHO	1.856147752533281	1.930212756609781	1.975926331265177	6.47E−02
SO	1.724851930920065	1.769948593020146	2.455648905951723	1.37E−01

Table 15
Comparison of optimum results for Pressure Vessel Design.

Algorithm	x1	x2	x3	x4	Cost
L-SHADE	0.8525	0.5775	56.3105	65.7572	7.672497279309694e+03
LSHADE-EpSin	0.9330	0.6982	59.9952	47.5678	6.854519124785935e+03
MFO	0.8297	0.4093	43.0742	164.9099	5.974209217485041e+03
WOA	0.9730	0.6512	50.6804	93.0377	7.112507749066582e+03
GOA	0.9571	0.4749	49.9302	99.0053	6.333087345321548e+03
TEO	2.5816	1.4787	47.1647	148.7692	3.259329410000000e+04
HHO	0.9833	0.4758	49.9297	98.9036	6.391874545908871e+03
SO	0.7819	0.3857	40.5752	196.5499	5.887529768474379e+03

Table 16
Statistical results for Pressure Vessel Design.

Algorithm	Best	Mean	Worst	Std Dev
L-SHADE	7.672497279309694e+03	1.473849856826275e+04	3.482650395037130e+04	7.74E+03
LSHADE-EpSin	6.854519124785935e+03	1.866618640062184e+05	1.609374491455645e+06	5.00E+05
MFO	5.974209217485041e+03	6.627185468410157e+03	7.302944019595159e+03	4.03E+02
WOA	7.112507749066582e+03	1.053694900845159e+04	1.353156617329189e+04	3.23E+03
GOA	6.333087345321548e+03	6.549406315908564e+03	6.764404482600929e+03	2.16E+02
TEO	1.248596966786956e+04	2.770226637543395e+04	3.762207591975377e+04	1.34E+04
HHO	6.391874545908871e+03	6.610547974554764e+03	6.888413671892376e+03	2.54E+02
SO	5.887529768474379e+03	5.989809192860846e+03	6.247616957848920e+03	1.04E+02

Table 17
Comparison of optimum results for Tension/compression spring.

Algorithm	d	D	P	Cost
L-SHADE	0.0555	0.4706	7.4552	0.01866180328950
LSHADE-EpSin	0.0592	0.4983	8.898	0.017226746529469
MFO	0.065094637569898	0.773164831458251	2.788677437297630	0.012865412436274
WOA	0.0507	0.3339	12.7645	0.012683072995993
GOA	0.0698	0.9685	2.0000	0.018867792598292
TEO	0.0511	0.3987	7.2450	2.483100917468873e+04
HHO	0.0562	0.4754	6.6670	0.013016405984960
SO	0.0511	0.3418	12.2222	0.012672535007285

Table 18
Statistical results for Tension/compression spring.

Algorithm	Best	Mean	Worst	Std Dev
L-SHADE	0.01866180328950	0.01999219464878	0.022591632407	1.48E+02
LSHADE-EpSin	0.017226746529469	0.0162144885040	0.020033845179	1.01E+01
MFO	0.012865412436274	0.013766855214744	0.017773157740936	1.25E−03
WOA	0.012683072995993	0.014699463380212	0.017210962384699	2.30E−03
GOA	0.018867792598292	2.983481234862856e+03	8.950404708138274e+03	5.17E+03
TEO	2.483100917468873e+04	3.151156166705854e+05	7.254200245983428e+05	3.65E+05
HHO	0.013016405984960	0.014159718076957	0.016034259566512	1.64E−03
SO	0.012672535007285	0.013633985220210	0.017773157740936	1.20E−03

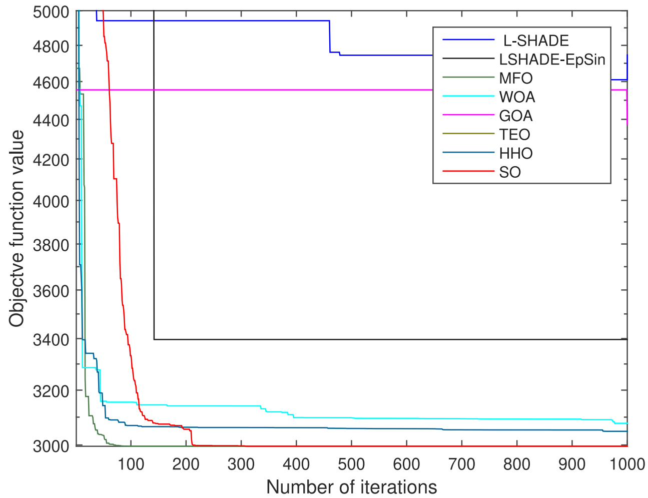

Fig. 19. Speed reducer problem.

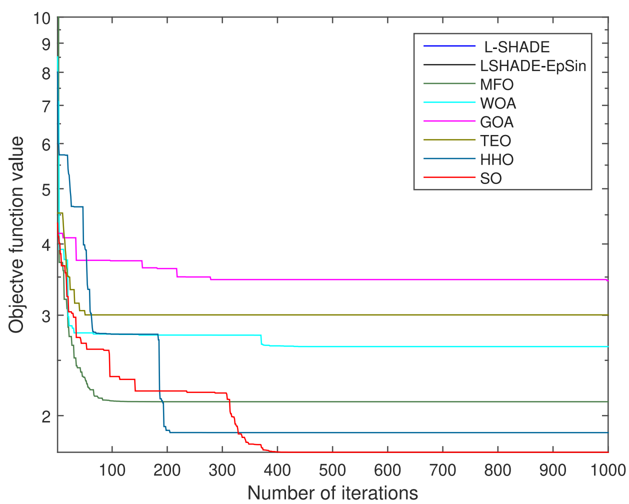

Fig. 20. Welded beam design problem.

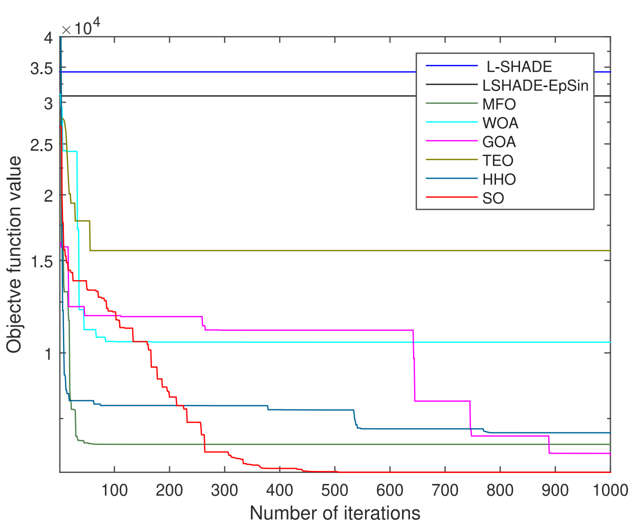

Fig. 21. Pressure vessel design.

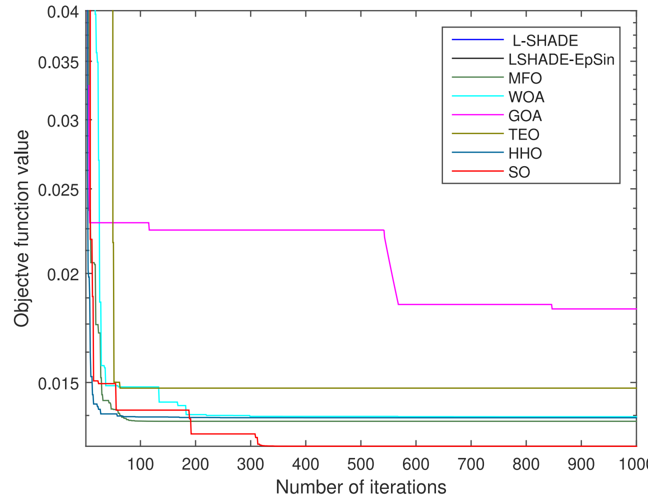

Fig. 22. Tension/compression spring design.

Welded beam design problem, Pressure vessel design problem, and Tension/compression spring design problem. The results reveal and prove the efficiency and superiority of SO compared with L-SHADE, LSHADE-EpSin, MFO, HHO, TEO, GOA, and WOA. There are many research ideas can be suggested as future work such as SO variants include binary, chaotic, multi-objective. Moreover, SO can be used to solve combinatorial optimization problems such as knapsack, dimensionality reduction, image segmentation, etc...

Compliance with ethical standards

- **Ethical approval** This article does not contain any studies with human participants or animals performed by any of the authors.
- **Informed consent** Informed consent was obtained from all individual participants included in the study.

CRediT authorship contribution statement

Fatma A. Hashim: Software, Visualization, Investigation, Methodology, Conceptualization. **Abdelazim G. Hussien:** Writing – original draft, Writing – review & editing, Visualization, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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