HW 6: System Identification

General instructions: Submit a ZIP file containing a PDF document and a folder. The PDF document contains your answers to each problem while the folder contains the m-files you used to generate the figures. Students are expected to work independently and refuse to discuss their codes and thought process with other students.

Part I: Consider the ARX(2,1) model with colored noise:

$$y(n) = -a_1 y(n-1) - a_2 y(n-2) + b_1 x(n-1) + v(n),$$

where $v(n) = e(n) + 0.25e(n-1)$ is a colored noise and $e(n) \sim \mathcal{N}(0, 0.0625)$.

(a) Simulate the ARX(2,1) process given above when the true values of the parameters are $a_1 = -0.8$, $a_2 = +0.2$, and $b_1 = 1.25$ from $n = 4$ to $n = 1000$. The input should be $x(n) = [sin(2\pi 0.01n) + 0.5 * cos(2\pi 0.05n)]u[n]$ Label your plot properly.

(b) Similar to the sample code, generate the histogram plot of the IV estimate and standard LS estimate using $M = 1000$ Monte Carlo trials. There should be three subplots for a_1, a_2, and b. Draw a red dashed line to mark the true value. Label your plots properly. You may modify the sample code to do this task.

(c) Similar to the sample code, plot the measured $y(n)$, true model, standard LS prediction, and IV prediction in a single figure. The plot should be from $n = 4$ up to $n = 1000$. Label your plot properly. You may modify the sample code to do this task.

Part II: Consider the ARMAX(2,1,1) model:

$$y(n) = -a_1 y(n-1) - a_2 y(n-2) + b_1 x(n-1) + ce(n-1) + e(n),$$

where $e(n) \sim \mathcal{N}(0, 0.0625)$.

(a) Simulate the ARMAX(2,1,1) process given above when the true values of the parameters are $a_1 = -0.4$, $a_2 = +0.3$, $b_1 = 1$, and $c = 0.35$ from $n = 3$ to $n = 1000$. The input should be $x(n) = [sin(2\pi 0.01n) + 0.3 * cos(2\pi * 0.03n)]u[n]$ Label your plot properly.

(b) Similar to the sample code, generate the histogram plot of the PEM estimate and standard LS estimate using $M = 1000$ Monte Carlo trials. There should be three subplots for a_1, a_2, b, and c. Draw a red dashed line to mark the true value. Label your plots properly. You may modify the sample code to do this task.

(c) Similar to the sample code, plot the measured $y(n)$, true model, standard LS prediction, and PEM prediction in a single figure. The plot should be from $n = 3$ up to $n = 1000$. Label your plot properly.