Quiz #0: Pre-course Questions

June 28th, 2023

Directions: you have 20 minutes to complete these questions. External references (talking to another human, going to an external website, or reading other files/book) are not permitted. You may opt not to participate, but your input is appreciated and will be used as data for research to improve courses here at Drexel and other institutions. There are no "wrong answers", and participation (or lack thereof) will have no negative impact on your grade, so please just make an honest effort to answer all four questions to the best of your ability.

#1. Consider the following mathematical function:

 $f(x) = x + 2 - x^2$, where the domain of f (i.e. all and only legal inputs) has been defined as the set of two integers $\{0, 1\}$

PersonA claims that the function f always outputs the number 2, although PersonB says this isn't true. Whom do you think is correct? Be sure to fully explain your answer in a few sentences of **plain English** (do not use logic symbols, Racket, ProofBuddy, Natural Deduction rules, Boolean Algebra, or Equational Reasoning). Imagine you have to convince a skeptic who is smart, but does not know much math/cs.

The algebraic definition of the function, x+2-x² does not always output the number 2; however with the domain being restricted to {0,1} only, the function would strictly output the number 2 for both inputs. Therefore, PersonA is right because they are speaking to the function with a specified domain, not the plain function itself.

#2. Consider the following mathematical function:

$$g(x) = \begin{cases} 1 & if \ x = 2 \\ 0 & otherwise \end{cases}$$

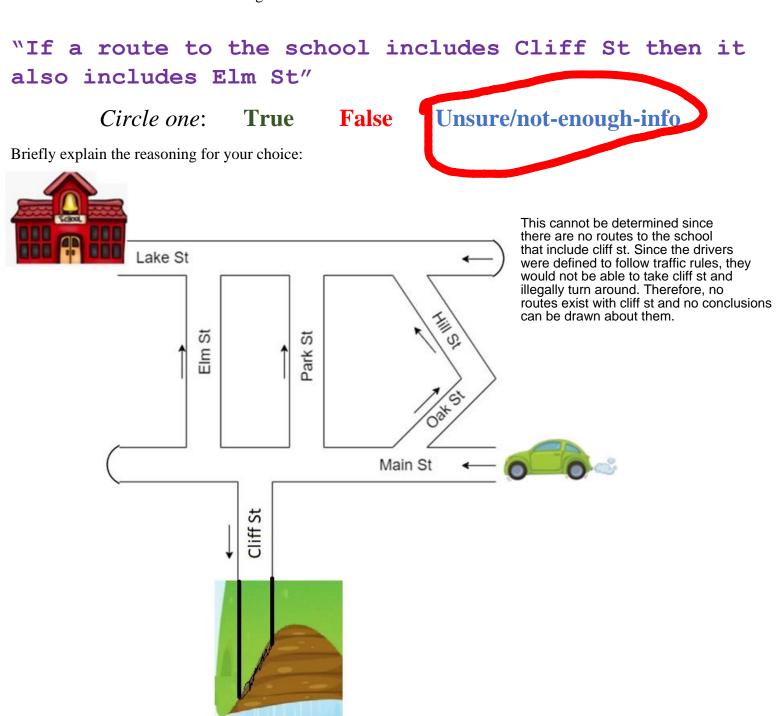
Where the domain of g (i.e. all and only legal inputs) has been defined as the set of all x's such $x^5 \neq 34 - x$

PersonC claims that the function g always outputs the number 0, although PersonD says this isn't true. Whom do you think is correct? Be sure to fully explain your answer in a few sentences of **plain English** (do not use logic symbols, Racket, ProofBuddy, Natural Deduction rules, Boolean Algebra, or Equational Reasoning). Imagine you have to convince a skeptic who is smart, but does not know much math/cs.

This function will always output zero. It would output 1 if x=2. But we need to check if 2 is in the domain. The domain specifies that x^5 must not equal 34-x. for x=2, $2^5=32=34-2$. Thus the one condition that would make this function not zero, is not in the domain. Therefore, the function is always 0.

#3. It is known that the driver of the green car takes the same route every day from their starting location on Main St, to their school on Lake St (see map at bottom of page). We also know they follow all the rules of the road (i.e. don't go the wrong way on the one-way streets marked by the arrows, nor drive off the streets cutting across grass etc). A complete map of the town is given below (there are no other hidden roads).

What is the truth value of the following sentence:



#4. Consider the following two recursively defined numerical sequences (and be sure to read the examples carefully to make sure you fully understand how each term is generated from the previous one):

X = starts off with a 0, and then if the preceding number is x then the next number would be $\frac{3x-x^2+2}{2}$

So, the next number in the Xsequence after 0 would be $\frac{3*0-0^2+2}{2} = \frac{2}{2} = 1$. After the 1 would come the number 2, since $\frac{3*1-1^2+2}{2} = \frac{4}{2} = 2$, etc etc.

There's also a different sequence Y which starts off with a 0, and if the proceeding number is y then the next number would be $\frac{y^2-5y+18}{6}$. So, the next number in the Ysequence after 0 would be $\frac{0^2-5*0+18}{6}=\frac{18}{6}=3$. After the 3 would be 2, since $\frac{3^2-5*3+18}{6}=\frac{12}{6}=2$

PersonA claims that the despite the two sequences having some values different at the start with $X=\{0,1,...\}$ and $Y=\{0,3,...\}$ and although their generating formulas are different, that at some point the values in the sequence X will be always be the same as the values in the sequence Y. PersonB disagrees and says that while it might happen by "coincidence" that sometimes the Xvalue at a spot in the sequence might be the same as the Yvalue at the same spot (e.g. the third number for both sequences happens to be 2), that it doesn't happen very often.

Whom do you think is correct? Be sure to fully explain your answer in a few sentences of **plain English** (do not use logic symbols, Racket, ProofBuddy, Natural Deduction rules, Boolean Algebra, or Equational Reasoning). Imagine you have to convince a skeptic who is smart, but does not know much math/cs.

Even though some of the sequence numbers at the start are the same, it is not guaranteed that these sequences will eventually always be the same. I agree with PersonB because the two sequences are functions that diverge from one another. They are both parabolas; however, X sequence is a decreasing parabola and Y sequence is an increasing one. Meaning as their indices get larger and larger, their differences will become more apparent.