CS270: LAB #7

Logic Operations

You may work in teams of three people (2 or 4 is acceptable in the event of an unscheduled absence). Unless stated otherwise, the lab is due to be submitted into Gradescope at the end of the day.
In order to receive credit, follow these instructions:

[a] Every team member should be discussing simultaneously the same problem – do NOT try to divvy up the labor and assign different problems to different students since the material is cumulative.

[b] Directly edit this lab PDF using Sedja/PDFescape with your answers (extra pages can be added in the rare event you need more than the allotted space)

[c] Each lab, rotate which member has the responsibility of being the Scribe. This is the person that is typing the answers and uploading the final PDF – note that only a single copy of the filled in PDF is turned into Gradescope. Only one lab needs to be submitted for the entire team, and all members receive the same score. Make sure to use a font that your PDF editor is compatible with (otherwise you might find your answers appear as weird shapes/sizes or simply disappear entirely!)

[d] The Gradescope submission must have each answer properly tagged with the appropriate question. Moreover, every member of the team must be listed as a submitter. Although it is the Scribe which executes these actions, it is still the responsibility of the entire team to make certain this is done properly (thus it is highly recommended that the Scribe share their screen so the entire team can witness it). Answers which are improperly tagged cannot be seen by the grader and thus cannot be scored.

[e] **FOR REMOTE ONLY**: Each lab, rotate which member has the responsibility of being the Recorder. This is the person who hits the Zoom Record button (once the technical permission is granted by the TA/RCF/Professor) and ensures that everyone has their camera/microphone on. They are also the member that is responsible to make sure the DrexelStream video is marked as viewable and entered into the https://tinyurl.com/VidLinkForm webform before 11:59pm (they should also email the rest of their team as confirmation.) Note that the video file doesn't get created/processed until after the Recorder has quit Zoom.

[f] Each lab, rotate which member has the responsibility of being the Manager. This is the person that ensures that everyone is participating equally and honestly, keeps the group on task, ensures that all team members understand a solution before going on to the next question, and presses the "hand up" button in Zoom to summon a TA or the professor (but they only do so after surveying the group to make sure everyone has the same question).

Team Name (CS pioneer): Tim Berners-Lee

Scribe name: Jackson Masterson

Recorder name: Cole Bardin

Manager name: Jeremy Mathews

Other team member (if any): ____________________

Question 1 : 10 points

Boolean Algebra has operators.

A **Truth Table** is a table that shows the output of an operator for every input.

Our first operator is **AND**. The **AND** operator is called the **Conjunction**. A **Conjunction** is true only when both its inputs are true.

A	B	A and B
T	T	T
T	F	F
F	T	F
F	F	F

If we wrote "*The sky is blue and the sun is cold*" this would look like

$$A = \text{The sky is blue}$$
$$B = \text{The sun is cold}$$
$$A \text{ and } B = False$$

The statement is False. The sky is blue, so A is True. The sun is very hot, so B is False. This matches the second line in the Truth Table, telling us the result is False.

Each of the following symbols means something similar to "A and B", but the most appropriate for us is the first one:

A∧B, A and B, A∩B, A·B, AB, A&B, A&&B

Let x and y be integers (whole numbers $x \geq 0$).

Give numbers for x and y that makes each statement true.

(a) (2 points) $(x > 5) \wedge (y < 9)$ x = 6, y = 8

(b) (2 points) $(x > 20) \wedge (x > 17)$ x = 22

(c) (2 points) $(x \text{ is even}) \wedge (y \text{ is odd})$ x = 4, y = 3

(d) (2 points) $(x \leq y) \wedge (x \geq y)$ x = y

(e) (2 points) $(x + 5 < y) \wedge (x * 2 = y)$ x = 6, y = 12

Question 2 : 12 points

The two other most important operators are **OR** (disjunction) and **NOT** (negation). Their Truth Tables are given below.

For tables with more complicated expressions, it's recommended you put the output directly beneath the operator of the subexpression. See the example below, where Green=inputs, Red=final output

A	B	A or B
T	T	T
T	F	T
F	T	T
F	F	F

A	not A
T	F
F	T

(A	∨	B)	∧	(B	∧	¬	A)
T	T	T	F		F	F	
T	T	F	F		F	F	
F	T	T	T		T	T	
F	F	F	F		F	T	

Each of the following symbols means something similar to "A or B", but the most appropriate for us is the first one A∨B, A∪B, A+B, A|B, A||B

These symbols are similar to saying "not A", but the first is most appropriate for boolean algebra: ¬A, ~A, −A, !A, $\overline{\text{A}}$

You can write a Truth Table for a Boolean Expression by evaluating each operator using its Truth Table.

A	B	(not A) or B
T	T	T
T	F	F
F	T	T
F	F	T

Draw a Truth Table for each of the following statements.

(a) (4 points) $P \wedge \neg P$

P	P	∧	¬	P
0		0	1	
1		0	0	

(b) (4 points) $P \vee \neg P$

P	P	V	¬	P
0		1	1	
1		1	0	

(c) (4 points) $(P \vee Q) \wedge (\neg P \vee \neg Q)$ **Be sure to use the format demonstrated at top, to avoid unnecessary columns**

A	¬	¬	A	⇔	A
0	0	1		1	
1	1	0		1	

Question 3 : 13 points

The Bi-conditional operator $\Leftrightarrow$ is used for equivalence. It is true when both inputs are the same.

A	B	A $\Leftrightarrow$ B
T	T	T
T	F	F
F	T	F
F	F	T

(a) (4 points) Draw the Truth Table for $(A \wedge B) \vee (\neg A \wedge \neg B)$.

(A	∧	B)	∨	(¬	A	∧	¬	B)
0	0	1	0	1		0	0	
0	0	0	1	1		1	1	
1	0	0	0	0		0	1	
1	1	1	1	0		0	0	

(b) (1 point) Was the previous Truth Table the same as $A \Leftrightarrow B$?

Yes

(c) (2 points) Draw the Truth Table for $\neg\neg A \Leftrightarrow A$.

A	¬	¬	A	⇔	A
0	0	1		1	
1	1	0		1	

(d) (2 points) Draw the Truth Table for $A \Leftrightarrow \neg A$.

A	A	⇔	¬	A
0		0	1	
1		0	0	

(e) (4 points) Draw the Truth Table for $(A \vee B) \Leftrightarrow (A \wedge B)$.

A	B	(A	V	B)	⇔	(A	Λ	B)
1	1		1		1		1	
1	0		1		0		0	
0	1		1		0		0	
0	0		0		0		0	

Question 4 : 15 points

A Truth Table is a **Proof**. It shows every possible input and output for an expression. That means it tells you everything there is to know about that expression. A Boolean expression can have 1 of three types of Truth Table.

- **Tautology**: The expression is True for all inputs
- **Contingent**: The expression is True sometimes and False sometimes
- **Contradictory**: The expression is False for all inputs

Draw the Truth Table for each of the following expressions. Note what type of Truth Table each is.

(a) (5 points) Draw the Truth Table for $(\neg A \vee \neg B) \Leftrightarrow (\neg(A \wedge B))$.

4a

A	B	(¬	A	V	¬	B)	⇔	(¬	(A	∧	B))
1	1	0		0	0		1	0		1	
1	0	0		1	1		1	1		0	
0	1	1		1	0		1	1		0	
0	0	1		1	1		1	1		0	

Tautology

(b) (5 points) Draw the Truth Table for $(\neg A \wedge B) \Leftrightarrow (\neg(A \vee B))$.

A	B	(¬	A	^	B)	⇔	(¬	(A	V	B)
1	1	0		0		1	0		1	
1	0	0		0		1	0		1	
0	1	1		1		0	0		1	
0	0	1		0		0	1		0	

Contingent

(c) (5 points) Draw the Truth Table for $((A \wedge \neg B) \wedge \neg A)$.

((A	∧	¬	B)	∧	¬	A)
0	0	1	0	0	1	
0	0	0	1	0	1	
1	1	1	0	0	0	
1	0	0	1	0	0	

Contradictory

Question 5 : 18 points

Another very important operator is the **Implies** (Conditional) operator.

You can think of $P \implies Q$ as being a promise that whenever P is True, Q will also be true.

This is a frequent event in programming.

if x > 10 **then**
  y=5
end if

In Boolean Algebra we would say $(x > 10) \implies (y = 5)$.

A	B	$A \implies B$
T	T	T
T	F	F
F	T	T
F	F	T

this 2nd row of the table, e.g. x=11, y=6 would correspond to it being *False* that the IF line of code was executed

Pretend that the following guarantee is written in the Syllabus (it is not).

If you pass the final exam, **then** you will pass the course.

Let

- E represent the statement *you passed the final exam*
- C represent the statement *you passed the course*

In Boolean Algebra this would be $E \implies C$.

For each of the following situations, fill in the blanks.

(a) (3 points) You pass the final exam and get an A in the class.

In this case, E= 1 and C= 1 therefore $(E \implies C)$= 1.

(b) (3 points) You fail the final and get a C+ in the class.

In this case, E= 0 and C= 1 therefore $(E \implies C)$= 1.

(c) (3 points) You fail the final and get an F in the class.

In this case, E= 0 and C= 0 therefore $(E \implies C)$= 1.

(d) (3 points) You pass the final and get an F in the class.

In this case, E= 1 and C= 0 therefore $(E \implies C)$= 0.

(e) (2 points) In which case did the Syllabus fail to keep its guarantee $((E \implies C) = False)$?

it failed to keep its promise when the student passed the final exam (E=1) but failed the class (C=0), which results in (E=>C)=0

(f) (4 points) Explain why "False→Anything" must always be True

false=>anything is always true because the implies operator can only result to false when the first expression is true AND the second expression is false. if the first expression always false, then the result of the implies operator will always evaluate to true

Question 6 : 4 points

The **NOR** operator is traditionally written as $A \downarrow B$.

The **NOR** operator has the following truth table.

A	B	$A \downarrow B$
T	T	F
T	F	F
F	T	F
F	F	T

Use a Truth Table to prove that $A \downarrow B$ is logically equivalent to $\neg(A \vee B)$.

To show that two expressions are equivalent, you must make TWO separate tables, and then show they both have the same final output column

A	B	(A	↓	B)	⇔	¬	(A	V	B)
1	1		0		1	0		1	
1	0		0		1	0		1	
0	1		0		1	0		1	
0	0		1		1	1		0	

Question 7 : 2 points

Use a Truth Table to prove that $A \downarrow A$ is logically equivalent to $\neg A$.

Q7

A	A	↓	A		A	¬	A
0		1		=	0	1	
1		0			1	0	

Question 8 : 4 points

Use a Truth Table to prove that $(A \downarrow B) \downarrow (A \downarrow B)$ is logically equivalent to $A \vee B$.

((A	↓	B)	↓	(A	↓	B))	↔	(A	v	B)
0	1	0	0		1		1	0	0	0
0	0	1	1		0		1	0	1	1
1	0	0	1		0		1	1	1	0
1	0	1	1		0		1	1	1	1

Question 9 : 4 points

Use a Truth Table to prove that $(A \downarrow A) \downarrow (B \downarrow B)$ is logically equivalent to $A \land B$.

Q9

A	B	(A	↓	A)	↓	(B	↓	B)		A	B	A	∧	B
1	1		0		1		0			1	1		1	
1	0		0		0		1		=	1	0		0	
0	1		1		0		0			0	1		0	
0	0		1		0		1			0	0		0	

Question 10 : 4 points

Use a Truth Table to prove that $A \implies B$ is logically equivalent to $\neg A \lor B$.

A	B	A	⇒	B	⇔	¬	A	V	B
0	0		1		1	1		1	
0	1		1		1	1		1	
1	0		0		1	0		0	
1	1		1		1	0		1	

Question 11 : 9 points

Once we have proven (by Truth Table) that a statement is either a Tautology or Contradictory statement, we can use it as a **Rule**. We can refer to Rules are justifications in proofs.

Here are some rules, may of which you provided proofs for in this lab. Notice that we use ≡ instead of ⇔ to denote that these are rules not just logic expressions.

These Boolean Algebra proofs will use rules that are linked on the bbLearn course page

Normally just starting at a Premise and reaching a Conclusion only proves implication, but these ≡ rules work in both directions. Note that the Ps and Qs represents ANY expression, not merely a single variable

We can use these rules to prove statement without drawing Truth Tables.

Prove: $(P \implies P)$ is equivalent to True.

1.	$P \implies P$	Premise
2.	$\neg P \vee P$	By Rule 20 (Def of Implies)
3.	True	By Rule 16 (Complementation)

(a) (4 points) Use the rules on the linked page to Prove: $\neg(\neg A \vee B)$ is equivalent to $A \wedge \neg B$.

¬(¬A ∨ B) Premise
(¬(¬A)) ∧ ¬B Demorgan 2 (rule 19)
A ∧ ¬B Double negative (17)

(b) (5 points) Use the rules on the linked page to Prove: $\neg(A \Leftrightarrow B)$ is equivalent to $(\neg A \vee \neg B) \wedge (A \vee B)$.

¬(A⇔B) # Premise
¬((A ⇒ B)∧(B ⇒ A)) # By Rule 21 (Def. of Equiv.)
¬((¬A∨B)∧(¬B∨A)) # By Rule 20 (Def. of Implies)
¬((¬A∧¬B)∨(¬A∧A)∨(B∧¬B)∨(B∧A)) # By Rule 5 (Distributivity)
¬((¬A∧¬B)∨(B∧A)) # By Rule 15 (Complementation[And])
¬(¬A∧¬B)∧¬(B∧A)) # By Rule 19 (DeMorgan 2)
(A∨B)∧(¬B∨¬A)) # By Rule 18 (DeMorgan 1)
(A∨B)∧(¬A∨¬B)) # By Rule 4 (Commutativity [Or])

Question 12 : 5 points

RULES FOR NOR↓	Rule #
$\neg(A \vee B) \equiv A \downarrow B$	(23)
$\neg A \equiv A \downarrow A$	(24)
$A \vee B \equiv (A \downarrow B) \downarrow (A \downarrow B)$	(25)
$A \wedge B \equiv (A \downarrow A) \downarrow (B \downarrow B)$	(26)

Using only Rule 20, together with the additional rules for NOR ↓ above, give a proof for the following.

Prove: $A \implies B$ is equivalent to $((A \downarrow A) \downarrow B) \downarrow ((A \downarrow A) \downarrow B)$.

A⇒B Premise
¬A ∨ B Implies Definition
((A ↓ A) ∨ B) Nor rule 24
((A ↓ A) ↓ B) ↓ ((A ↓ A) ↓ B) Nor rule 25