CS270: LAB #12

More Natural Deduction

You may work in teams of ideally three people (two or four is acceptable in the event of an unscheduled absence).
If your team does not complete the lab during the class period, it can be submitted by 11:59pm on Friday
In order to receive credit, follow these instructions:

[a] Every team member should be discussing simultaneously the same problem – do NOT try to divvy up the labor and assign different problems to different students since the material is cumulative.

[b] Directly edit this lab PDF using Sedja/PDFescape with your answers (extra pages can be added in the rare event you need more than the allotted space)

[c] Each lab, rotate which member has the responsibility of being the Scribe. This is the person that is typing the answers and uploading the final PDF – note that only a single copy of the filled in PDF is turned into Gradescope. Only one lab needs to be submitted for the entire team, and all members receive the same score. Make sure to use a font that your PDF editor is compatible with (otherwise you might find your answers appear as weird shapes/sizes or simply disappear entirely!)

[d] The Gradescope submission must have each answer properly tagged with the appropriate question. Moreover, every member of the team must be listed as a submitter. Although it is the Scribe which executes these actions, it is still the responsibility of the entire team to make certain this is done properly (thus it is highly recommended that the Scribe share their screen so the entire team can witness it). Answers which are improperly tagged cannot be seen by the grader and thus cannot be scored.

[e] **FOR REMOTE ONLY**: Each lab, rotate which member has the responsibility of being the Recorder. This is the person who hits the Zoom Record button (once the technical permission is granted by the TA/RCF/Professor) and ensures that everyone has their camera/microphone on. They are also the member that is responsible to make sure the DrexelStream video is marked as viewable and entered into the https://tinyurl.com/VidLinkForm webform before 11:59pm (they should also email the rest of their team as confirmation.) Note that the video file doesn't get created/processed until after the Recorder has quit Zoom.

[f] Each lab, rotate which member has the responsibility of being the Manager. This is the person that ensures that everyone is participating equally and honestly, keeps the group on task, ensures that all team members understand a solution before going on to the next question, and presses the "hand up" button in Zoom to summon a TA or the professor (but they only do so after surveying the group to make sure everyone has the same question).

Team Name (CS pioneer): Tim Berners-Lee

Scribe name: Brendan Hoag

Recorder name: Jeremy Mathews

Manager name: Jackson Masterson

Other team member (if any): Cole Bardin

Question 1 : 14 points

Match the name of each Basic Deduction Rule with its General Format.

1. Conjunction Elimination
2. Implication Introduction
3. Implication Elimination
4. Disjunction Introduction
5. Conjunction Introduction
6. Disjunction Elimination

Write the number from the above list next to the general form of that Deduction Rule.

(a) (2 points)

1.	$A \implies B$	
2.	A	
3.	B	$\implies$ E 1,2

(a) 3. Implication Elimination

(b) (2 points)

1.	$A \wedge B$	
2.	A	$\wedge$ E 1

(b) 1. Conjunction Elimination

(c) (2 points)

1.	A	
2.	$A \vee B$	$\vee$ I 1

(c) 4. Disjunction Introduction

(d) (2 points)

1.1	A	Assume
1.2	$\ldots$	
1.3	B	
2	$A \implies B$	$\implies$ I1

(d) 2. Implication Introduction

(e) (2 points)

1.	A	
2.	$B \vee A$	$\vee$ I 1

(e) 4. Disjunction Introduction

(f) (2 points)

1.	$A \wedge B$	
2.	B	$\wedge$ E 1

(f) 1. Conjunction Elimination

(g) (2 points)

1.	A	
2.	B	
3.	$A \wedge B$	$\wedge$ I 1,2

(g) 5. Conjunction Introduction

Question 2 : 14 points

Provide Justification for each of the followings Proofs by Deduction.

1	$(P \wedge Q) \implies R$	Premise
2.1	P	Assume
2.2.1	Q	Assume
2.2.2	$P \wedge Q$	?
2.2.3	R	?
2.3	$Q \implies R$	?
3	$P \implies (Q \implies R)$	?

(a) (2 points) What is the justification for line 2.2.2?

(a) ∧ I 2.1, 2.2.1

(b) (2 points) What is the justification for line 2.2.3?

(b) → E 1, 2.2.2

(c) (2 points) What is the justification for line 2.3?

(c) → I 2.2

(d) (2 points) What is the justification for line 3?

(d) → I 2

1.	P	Premise
2.	$P \wedge P$	?
3.	P	?

(e) (2 points) What is the justification for line 2?

(e) ∧ I 1, 1

(f) (2 points) What is the justification for line 3?

(f) ∧ E 2

(g) (2 points) A **Derived Rule** is a rule that can be built from the **Basic Rules**. The following rule is called the repetition rule.

1.	P	
2.	P	R 1

Is the repetition rule a **Basic Rule** or a **Derived Rule**?

This is a derived rule

Question 3 : 10 points

You may have noticed that we have only seen how to create Disjunctions. We will now introduce **Disjunction Elimination**.

To eliminate a Disjunction, we must make two assumptions.

To see why, let's start with the Truth Table for a Disjunction.

A	B	$A \vee B$
T	T	T
T	F	T
F	T	T
F	F	F

(a) (2 points) If we have a premise $A \vee B$, can we conclude that A is true?

- ☒ Yes
- ◯ No

(b) (2 points) If we have a premise $A \vee B$, can we conclude that B is true?

- ☒ Yes
- ◯ No

(c) (2 points) If we have a premise $A \vee B$, can we conclude that *either A is true, B is true or both are true*?

- ☒ Yes
- ◯ No

A **Disjunction Elimination** requires two assumptions. We need to see what happens when A is assumed true, and also what happens when B is assumed true. If both paths lead to the same place, then we can reach a conclusion.

1	$A \vee B$	
2.1	A	Assume
2.2	$\cdots$	
2.3	X	
3.1	B	Assume
3.2	$\cdots$	
3.3	X	
4	X	$\vee$ E 1, 2, 3

(d) (4 points) Why does the Disjunction Elimination's justification require three inputs?

You need to include the original premise as well as the two subproofs that take the assumptions for each symbol.
The two subproofs come from the original premise, and from them you get your conclusion

Question 4 : 12 points

We want to prove the following statement.

$$(A \lor B)\ ;\ A \rightarrow C\ ;\ B \rightarrow C \therefore C$$

(a) (2 points) How many assumptions will be needed in this proof? What are they?

We will need 2 assumptions: that A is true in one case and that B is true in the other.

(b) (10 points) Use Proof Buddy linked on the bbLearn page to prove this argument
Put a screenshot of your proof below once the proof checker confirms it is correct.

Line #	Expression	Rule
1	A∨B	Premise
2	A→C	Premise
3	B→C	Premise
4.1	A	Assumption
4.2	C	→E 2, 4.1
5.1	B	Assumption
5.2	C	→E 3, 5.1
6	C	∨E 1, 4, 5

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Question 5 : 10 points

We only have a few **Basic Rules**. There are a total of 10. We have seen 6 so far.

It is useful to try different strategies when working on a **Proof by Deduction**.

Let's work through some approaches to proving the argument

$$Q \wedge (A \wedge X) \,;\, (Q \wedge X) \rightarrow Z \therefore Z \vee Y$$

(a) (2 points) Conjunction Elimination is one of the easiest rules to use. One of your first steps should be to see if it can give you anything useful.

What are the 4 conclusions we can draw using Conjunction Elimination on the premises?

Our 4 conclusions from the premises are Q, (A∧X), A, and X.

(b) (2 points) None of those get us all the way to the conclusion of our argument.

We might be closer.

We have a premise with an implies in it. It might be nice to use this for something.

Are there any expressions we can make from the answers in part (a) that can allow us use this conditional elimination on this implies?

Since we know Q and we know X, we can construct Q∧X for our premise with the implies.

(c) (2 points) If we use conditional elimination based on (b), what will we get as a result?

We will get Z as the result of the implicative elimination of part b.

(d) (2 points) This is as far as we get get just working forward from the premises. We still haven't reach the conclusion to our argument. What can we do now?

We can look at the conclusion we want to reach for hints. The conclusion is $Z \vee Y$.

Can we make this conclusion from the answer to (c)? What rule do we need to use?

We can make this conclusion. We use the disjunctive introduction rule to go from Z to Z∨Y.

(e) (2 points) Some rules are more difficult to use then others. If we just attack the problem in the order premises are given, we might make it much harder then it needs to be. Start by examining easier rules first, and always keep your conclusion in mind.

If we had the argument $C \vee X; B \Rightarrow X, A \wedge B \therefore X$ which would be the best premise to start work on?

A∧B would be the best premise to start from because it is a simple one that allows us to get both A and B right off the bat.

Question 6 : 10 points

Use Proof Buddy, linke on our bbLearn course page, to prove this argument.

Put a screenshot of your proof below once the proof checker confirms it is correct.

$$(C \wedge D) \vee E \therefore E \vee D$$

Line #	Expression	Rule
1	(C∧D) ∨ E	Premise
2.1	C∧D	Assumption
2.2	D	∧E 2.1
2.3	E ∨ D	∨I 2.2
3.1	E	Assumption
3.2	E ∨ D	∨I 3.1
4	E ∨ D	∨E 1, 2, 3

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Question 7 : 10 points

Use Proof Buddy, linked on our bbLearn course page, to prove this argument.

Put a screenshot of your proof below once the proof checker confirms it is correct.

$$(Z \wedge K) \vee (K \wedge M); K \rightarrow D \quad \therefore D$$

Line #	Expression	Rule
1	(Z ∧ K) ∨ (K ∧ M)	Premise
2	K →D	Premise
3.1	Z ∧ K	Assumption
3.2	K	∧E 3.1
3.3	D	→E 2, 3.2
4.1	K ∧ M	Assumption
4.2	K	∧E 4.1
4.3	D	→E 2, 4.2
5	D	∨E 1,3, 4

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Question 8 : 10 points

Use Proof Buddy, linked on our bbLearn course page, to prove this argument.

Put a screenshot of your proof below once the proof checker confirms it is correct.

$$P \vee (P \wedge Q) \therefore P$$

Line #	Expression	Rule
1	P ∨ (P ∧ Q)	Premise
2.1	P	Assumption
3.1	P ∧ Q	Assumption
3.2	P	∧E 3.1
4	P	∨E 1,2,3

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

f TFL in Proof Buddy
which lets you construct a counter example, if applicable.

Question 9: 10 points [5pts each]

[a] If you believe the following sequent is provable, then give a proof of it in Proof Buddy.
If you believe that it can NOT be proven, provide an assignment of true/false values to the variables that demonstrates that it does not logically follow (in other words, even when all the premises are true, the conclusion isn't).

(H → G) ∨ (Y → G) ; H ∧ Y ; G → Q ∴ Q

Line #	Expression	Rule
1	(H → G) ∨ (Y → G)	Premise
2	H ∧ Y	Premise
3	G → Q	Premise
4.1	H → G	Assumption
4.2	H	∧E 2
4.3	G	→E 4.1, 4.2
5.1	Y → G	Assumption
5.2	Y	∧E 2
5.3	G	→E 5.1, 5.2
6	G	∨E 1, 4, 5
7	Q	→E 3, 6

Check Proof Save

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

[b] If you believe the following sequent is provable, then give a proof of it in Proof Buddy
If you believe that it can NOT be proven, provide an assignment of true/false values to the variables that demonstrates that it does not logically follow (in other words, all the premises are true, but the conclusion isn't).

(H → G) ∨ (Y → G) ; H ∨ Y ; G → Q ∴ Q

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Rules: TFL - Basic & Derived Rules

Premises: (H → G) ∨ (Y → G) ; H ∨ Y ; G → Q

Conclusion: Q

Lemmas allowed

Table Button Info

Line #	Expression	Rule
1	(H → G) ∨ (Y → G)	Premise
2	H ∨ Y	Premise
3	G → Q	Premise

Check Proof Save

Number of Steps:
3

Great job! Don't forget to save your work by clicking "Save"! This will allow you to view this correct proof later!

Result: That is a valid counterexample -- Good Job!

Disproof

H False
G False
Y True
Q False

Check Counter Example

This statement is not provable with the given premises.
A counter example would be: H is true, Y is false.
Even if G is false, Y → G would be true, making the entire expression (H → G) ∨ (Y → G) true.
With G being false, G → Q is also true regardless of Qs value.
This means Q cannot be proven to be true with the given premises.