In-Lab Group Activity for Week 5: Row, Column and Null Space

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Problem 1: Row, Column and Null Space

The coefficient matrix A and its reduced form B are shown below.

$$A = \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 7 & 7 \end{bmatrix} \leftrightarrow \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \begin{bmatrix} \mathbf{1} & 1 & 0 & 0 \\ 0 & 0 & \mathbf{1} & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = B$$

a. The transformation $\vec{\mathbf{x}} \mapsto A\,\vec{\mathbf{x}}$ maps the vector $\vec{\mathbf{x}} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ in $\mathbb{R}^4$ to the vector: ii.

i. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ **ii.** $\begin{bmatrix} x_3 + x_4 \\ x_1 + x_2 \\ 7x_3 + 7x_4 \end{bmatrix}$ **iii.** $\begin{bmatrix} x_1 + x_2 \\ x_3 \\ x_4 \end{bmatrix}$ **iv.** $\begin{bmatrix} x_3 + x_4 \\ x_1 + x_2 \\ x_2 + x_3 \end{bmatrix}$

b. Fill out the following table of properties of the matrix A. **Rank** is always the number of pivots.

Rank	Dimension of Row Space	Dimension of Column Space	Dimension of Null Space
2	**R4**	**R3**	**2**

c. Find a basis for the **column space** of A. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ It's the ramp or plane $x_3 = 7x_1$.

Required Answer Method: Choose the column(s) from A that have pivots in B.

d. Do the pivot columns of B also form a basis for the column space of A? **Yes** **No**

e. Find a basis for the **row space** of A. Must give rows!

$$\{ [1 \quad 1 \quad 0 \quad 0], \ [0 \quad 0 \quad 1 \quad 1] \}$$

Answer Method: Choose the non-zero rows from B.

f. Write out the **null space** for the homogenous equation $A\vec{\mathbf{x}} = \vec{\mathbf{0}}$ in vector parametric form:

$$\vec{\mathbf{x}} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -x2 \\ x2 \\ -x3 \\ x3 \end{bmatrix} = x_2 \cdot \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \cdot \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

Problem 2: Row, Column and Null Space

The coefficient matrix A and its reduced form B are shown below. Below, R combines all the row-reducing operations into a single, square invertible matrix.

$$A = \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ 2 & 2 & -4 \end{bmatrix} \end{array} \leftrightarrow \begin{array}{c} \begin{array}{ccc} x_1 & \boldsymbol{x_2} & \boldsymbol{x_3} \end{array} \\ \begin{bmatrix} \mathbf{1} & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} = B = RA$$

a. One of these is **not** the same for the matrices A and B.

i. Null space **ii.** Column space **iii.** Row space

b. Fill out the following table of properties of the matrix A. **Rank** is always the number of pivots.

Rank	**Dimension of Row Space**	**Dimension of Column Space**	**Dimension of Null Space**
1	**3**	**3**	**2**

c. Find a basis for the **column space** of A. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \right\}$ It's the **line**: $x_1 = x_2 = \frac{x_3}{2}$.

Required Answer Method: Choose the column(s) from A that have pivots in B.

d. Do the pivot columns of B also form a basis for the column space of A? **Yes** **No**

e. Find a basis for the **row space** of A. Must give row(s)!

$$\{ \begin{bmatrix} 1 & 1 & -2 \end{bmatrix} \}$$

Answer Method: Choose the non-zero rows from B.

f. Write out the null space for the homogenous equation $A\vec{\mathbf{x}} = \vec{\mathbf{0}}$ in vector parametric form:

$$\vec{\mathbf{x}} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x3 - x2 \\ x_2\ (free) \\ x_3\ (free) \end{bmatrix} = x_2 \cdot \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + x_3 \cdot \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

g. Find a **particular** solution to the nonhomogeneous equation $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$ where $\vec{\mathbf{b}} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$.

i. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ **ii.** $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ **iii.** $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ **iv.** $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ **v.** $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

Problem 3: Match each matrix with its determinant!

Hints: Subtract R1 from R2 & R4 for D. Use Cofactor expansion for E.

Do not use MATLAB except to check your hand calculations.

$A = \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 3 & 5 \\ 2 & 2 & 2 \end{bmatrix}$ $D = \begin{bmatrix} 4 & 1 & 1 & 1 \\ 4 & 5 & 1 & 1 \\ 0 & 0 & 1 & 7 \\ 4 & 1 & 1 & 2 \end{bmatrix}$ $E = \begin{bmatrix} 0 & 1 & 1 \\ 3 & 9 & 5 \\ 0 & 1 & 4 \end{bmatrix}$

Two-arrow rule	Triangular matrix rule!	Identical rows!	Use the EROs suggested above.	Cofactor expansion!

a. Each of these determinants is a perfect square or the negative of a square.

i.	This matrix has determinant equal to +4.	*A*	*B*	*C*	*D*	*E*
ii.	This matrix has determinant equal to 0.	*A*	*B*	*C*	*D*	*E*
iii.	This matrix has determinant equal to –9.	*A*	*B*	*C*	*D*	*E*
iv.	This matrix has determinant equal to +16.	*A*	*B*	*C*	*D*	*E*
v.	This matrix has determinant equal to +25.	*A*	*B*	*C*	*D*	*E*

b. Find the determinant $|A|$ of the 4×4 matrix A. We are given that A can be row-reduced to the matrix

$B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$ using the row operations itemized below. Starting with A:

Fill in the missing constant after each step.

i. Swap the first two rows and name the result A_1. $|A_1| =$ **-1** $\cdot |A|$

ii. Subtract row 1 of A_1 from its second row and name the result A_2. $|A_2| =$ **-1** $\cdot |A|$

iii. Subtract twice row 1 of A_2 from its last row and name the result A_3. $|A_3| =$ **-1** $\cdot |A|$

iv. Divide row 5 of A_3 by 5 and name the result B. $|B| =$ **-1/5** $\cdot |A|$

v. Give the determinant of B above. $|B| =$ **5**

vi. Give the determinant of A. $|A| =$ **-25**

If you want to double check, the mystery matrix A is revealed on the next page as well as the row operations that result in B.

$$\text{Mystery Matrix } A = \begin{bmatrix} 1 & 2 & 3 & 3 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 3 \\ 2 & 2 & 2 & 27 \end{bmatrix} \longrightarrow A_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 3 \\ 0 & 0 & 1 & 3 \\ 2 & 2 & 2 & 27 \end{bmatrix} \longrightarrow A_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 3 \\ 2 & 2 & 2 & 27 \end{bmatrix}$$

$$\longrightarrow A_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 25 \end{bmatrix} \longrightarrow B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$