

Part A

Question 1

```
clc, clear
A = [10;-20;10;20;-40;20];
B = [50;40;30;20;10;-150];
C = [10;10;10;20;30;30];
D = [10;10;10;10;6;4];
```

```
ANorm = norm(A)
```

```
ANorm = 54.7723
```

```
BNorm = norm(B)
```

```
BNorm = 167.3320
```

```
CNorm = norm(C)
```

```
CNorm = 50
```

```
DNorm = norm(D)
```

```
DNorm = 21.2603
```

Quesion 2

```
clc, clear
A = [6;0;6;0];
B = [12;0;0;0];
C = [6;6;6;6];
D = [-12;12;-12;12];
```

```
Anorms = [norm(A,1);norm(A);norm(A,Inf)]
```

```
Anorms = 3x1
12.0000
8.4853
6.0000
```

```
Bnorms = [norm(B,1);norm(B);norm(B,Inf)]
```

```
Bnorms = 3x1
12
12
12
```

```
Cnorms = [norm(C,1);norm(C);norm(C,Inf)]
```

```
Cnorms = 3x1
```

24
12
6

```
Dnorms = [norm(D,1);norm(D);norm(D,Inf)]
```

```
Dnorms = 3×1  
48  
24  
12
```

Question 3

```
clc, clear  
A = pascal(5);  
B = pascal(5,1);  
C = pascal(5,2);
```

```
A2 = A*A
```

```
A2 = 5×5  
    5    15    35    70    126  
   15    55   140   294   546  
   35   140   371   798  1506  
   70   294   798  1742  3321  
  126   546  1506  3321  6376
```

```
B2 = B*B
```

```
B2 = 5×5  
    1    0    0    0    0  
    0    1    0    0    0  
    0    0    1    0    0  
    0    0    0    1    0  
    0    0    0    0    1
```

```
C2 = C*C
```

```
C2 = 5×5  
    0    0    0    0    1  
    0    0    0   -1   -4  
    0    0    1    3    6  
    0   -1   -2   -3   -4  
    1    1    1    1    1
```

Question 4

```
clc, clear  
A = [4,3,2,1;5,4,3,2];  
AI = [A, eye(2)]
```

```
AI = 2×6  
    4    3    2    1    1    0  
    5    4    3    2    0    1
```

```
AIR = rref(AI)
```

```
AIR = 2x6
      1      0     -1     -2      4     -3
      0      1      2      3     -5      4
```

```
R = AIR(:,5:end)
```

```
R = 2x2
      4     -3
     -5      4
```

```
B = AIR(:,1:4)
```

```
B = 2x4
      1      0     -1     -2
      0      1      2      3
```

```
if isequal(B, R*A)
    fprintf("Verified that B = R*A\n")
end
```

```
Verified that B = R*A
```

Question 5

```
clc, clear
disp("Question 5:")
```

Question 5:

```
C = [-1,2,2;2,2,-1;2,-1,2];
A = [C;C];
```

```
V1 = [1;1;1;1;1;1];
V2 = [1;2;3;1;2;3];
V3 = [6;6;6;0;0;0];
V4 = [1;2;3;-1;-2;-3];
```

```
V1_res = A'*V1
```

```
V1_res = 3x1
      6
      6
      6
```

```
V2_res = A'*V2
```

```
V2_res = 3x1
     18
      6
     12
```

```
V3_res = A'*V3
```

```
V3_res = 3×1
    18
    18
    18
```

```
V4_res = A'*V4
```

```
V4_res = 3×1
     0
     0
     0
```

```
% Question 6
disp("Question 6:")
```

Question 6:

```
b = [14;5;11;14;5;11];
AM = [A,b];
RAM = rref(AM)
```

```
RAM = 6×4
     1     0     0     2
     0     1     0     3
     0     0     1     5
     0     0     0     0
     0     0     0     0
     0     0     0     0
```

```
% Question 7
disp("Question 7:")
```

Question 7:

```
x = inv(A'*A)*A'*b
```

```
x = 3×1
    2.0000
    3.0000
    5.0000
```

Question 8

```
clc, clear
A = toeplitz( [1 0 0 0 0]', [1 2 3 4 5]);
V = [1;1;1;1;1];
```

```
VtAtAV = V'*A'*A*V
```

```
VtAtAV = 371
```

Questions 9-10

```
clc, clear
A = toeplitz( [1 0 0 0 0]', [1 2 3 4 5]);

AI = [A, eye(5)]
```

```
AI = 5x10
    1     2     3     4     5     1     0     0     0     0
    0     1     2     3     4     0     1     0     0     0
    0     0     1     2     3     0     0     1     0     0
    0     0     0     1     2     0     0     0     1     0
    0     0     0     0     1     0     0     0     0     1
```

```
AIR = rref(AI)
```

```
AIR = 5x10
    1     0     0     0     0     1    -2     1     0     0
    0     1     0     0     0     0     1    -2     1     0
    0     0     1     0     0     0     0     1    -2     1
    0     0     0     1     0     0     0     0     1    -2
    0     0     0     0     1     0     0     0     0     1
```

```
Ainv = AIR(:,6:end)
```

```
Ainv = 5x5
    1    -2     1     0     0
    0     1    -2     1     0
    0     0     1    -2     1
    0     0     0     1    -2
    0     0     0     0     1
```

Part B

Question 1

```
clc, clear
A = sym(hadamard(8));
b = [20;-4;-8;0;0;0;0;0];
```

```
disp("Question 1:")
```

Question 1:

```
detA = det(A)
```

```
detA = 4096
```

```
% Question 2
disp("Question 2:")
```

Question 2:

```
A8 = A;
```

```
A8(:,8) = b
```

```
A8 =
```

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 20 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -4 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -8 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 0 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & 0 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 0 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 0 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & 0 \end{pmatrix}$$

```
% Question 3  
disp("Question 3:")
```

Question 3:

```
x8 = det(A8)/det(A)
```

```
x8 = 4
```

```
% Questions 4-5  
disp("Question 4:")
```

Question 4:

```
N = size(A,2);  
x = zeros(N,1);  
for i= 1:N  
    Ai = A;  
    Ai(:,i) = b;  
    x(i) = det(Ai)/det(A);  
end  
x
```

```
x = 8x1  
    1  
    2  
    3  
    4  
    1  
    2  
    3  
    4
```

Part C

Question 1

```
clc, clear
```

```
M = [1,0,3,0;0,2,0,4;1,2,3,4;-1,2,-3,4];
A = [M, M*M];
disp("Question 1:")
```

Question 1:

```
Ared = rref(A)
```

```
Ared = 4x8
    1     0     3     0     4     6    12    12
    0     1     0     2    -2     6    -6    12
    0     0     0     0     0     0     0     0
    0     0     0     0     0     0     0     0
```

```
% Question 2
disp("Question 2:")
```

Question 2:

```
R = Ared(1:2,:)
```

```
R = 2x8
    1     0     3     0     4     6    12    12
    0     1     0     2    -2     6    -6    12
```

```
% Question 3
disp("Question 3:")
```

Question 3:

```
N = null(A, 'r')
```

```
N = 8x6
   -3     0    -4    -6   -12   -12
    0    -2     2    -6     6   -12
    1     0     0     0     0     0
    0     1     0     0     0     0
    0     0     1     0     0     0
    0     0     0     1     0     0
    0     0     0     0     1     0
    0     0     0     0     0     1
```

```
% Question 4
disp("Question 4:")
```

Question 4:

```
RN = R*N
```

```
RN = 2x6
    0     0     0     0     0     0
    0     0     0     0     0     0
```

```
% Question 5
disp("Question 5:")
```

Question 5:

```
AtA = null(A'*A, 'r')
```

AtA = 8×6

-3	0	-4	-6	-12	-12
0	-2	2	-6	6	-12
1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1

```
AAAt = null(A*A', 'r')
```

AAAt = 4×2

-1	1
-1	-1
1	0
0	1

```
AAAtA = null(A*A'*A, 'r')
```

AAAtA = 8×6

-3	0	-4	-6	-12	-12
0	-2	2	-6	6	-12
1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1

```
AtAAAtA = null(A'*A*A'*A, 'r')
```

AtAAAtA = 8×6

-3	0	-4	-6	-12	-12
0	-2	2	-6	6	-12
1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1

Part D

Question 1

```
clc, clear, close all
format short
clear, clc, close all
% The years the Olympics were held.
x = [1900 1904 1908 1912 1920 1924 1928 1932 1936 1948 1952 1956 1960 1964 1968 1972 1976 1980]
% Winning time in seconds.
y = [246.2 245.4 243.4 236.8 241.8 233.6 233.2 231.2 227.8 229.8 225.2 221.2 215.6 218.1 214.9
```



```
N = numel(x);
D = [ones(N,1), x];

disp("Question 1:")
```

Question 1:

```
DtD = D'*D
```

```
DtD = 2x2
      23      44900
      44900   87674608
```

```
% Question 2
disp("Question 2:")
```

Question 2:

```
b = inv(DtD)*D'*y
```

```
b = 2x1
      874.4631
     -0.3323
```

```
% Question 3
disp("Question 3:")
```

Question 3:

```
yest = D*b;
e = y - yest;
RMSE = sqrt( (e'*e)/N )
```

```
RMSE = 3.6711
```

```
% Questions 4-5
hold on
grid on
plot(x,y,'bo','MarkerFaceColor','y')
plot(x, yest, 'b--')
title("Olympic 1500-Meter Records: Men")
xlabel("Year")
ylabel("Time in Seconds")
axis([1900,2000,200,260])

for i=1:N
    plot([x(i), x(i)], [y(i), yest(i)], 'k')
end

legend(["Raw Data Points", "Best-Fit Line"])
```

