Lab 5 by Cole Bardin Section 62

Aplications of the Inverse Matrix

Question 1

```
clc, clear, close all

syms x y;
A = @(x,y) [x, y, 15-x-y; 20-2*x-y, 5, 2*x+y-10; x+y-5, 10-y, 10-x];

% Iterate y1 through each odd number 1-9 except 5
for y1 = [1,3,7,9]
    % Iterate x1 through each even number 2-4
    for x1 = 2:2:8
        % Create magic matrix with x1 y1
        M = A(x1,y1);
        % Assume it is a proper magic
        is_magic = 1;
        % Initialize empty cache for each 9 slots
        cache = zeros(1,9);
        % Iterate through each element
        for i = 1:1:9
            % Set is_magic false if any element<1 or >9
            if M(i) < 1 || M(i) > 9
                is_magic = 0;
            else
                % If number is already found in cache, set false
                if cache(M(i)) == 1
                    is_magic = 0;
                % If not, mark number as found in cache
                else
                    cache(M(i)) = 1;
                end
            end
        end
        % If this matrix did not fail any tests display
        if is_magic == 1
            disp("x="+x1+" y="+y1+" makes a magic matrix:")
            disp(M)
        end
    end
end
```

```
x=6 y=1 makes a magic matrix:
     6     1     8
     7     5     3
     2     9     4
x=8 y=1 makes a magic matrix:
```

```
     8     1     6
     3     5     7
     4     9     2
x=4 y=3 makes a magic matrix:
     4     3     8
     9     5     1
     2     7     6
x=8 y=3 makes a magic matrix:
     8     3     4
     1     5     9
     6     7     2
x=2 y=7 makes a magic matrix:
     2     7     6
     9     5     1
     4     3     8
x=6 y=7 makes a magic matrix:
     6     7     2
     1     5     9
     8     3     4
x=2 y=9 makes a magic matrix:
     2     9     4
     7     5     3
     6     1     8
x=4 y=9 makes a magic matrix:
     4     9     2
     3     5     7
     8     1     6
```

Question 2

```
clc, clear, close all

syms x y;
A = @(x,y) [x, y, 15-x-y; 20-2*x-y, 5, 2*x+y-10; x+y-5, 10-y, 10-x];

M3 = A(8,1);
r = ones(3,1);
S = r'*M3*r;
disp("Sum of all elements in M3=")
```

```
Sum of all elements in M3=
```

```
disp(S)
```

```
    45
```

Question 3

```
clc, clear, close all

A = sym(pascal(4));
disp("Pascal(4)")
```

```
Pascal(4)
```

```
disp("Det=")
```

```
Det=
```

```
disp(det(A))
```

$$1$$

```
disp("Adj=")
```

```
Adj=
```

```
disp(adjoint(A))
```

$$\begin{pmatrix} 4 & -6 & 4 & -1 \\ -6 & 14 & -11 & 3 \\ 4 & -11 & 10 & -3 \\ -1 & 3 & -3 & 1 \end{pmatrix}$$

```
disp("Inv=")
```

```
Inv=
```

```
disp(inv(A))
```

$$\begin{pmatrix} 4 & -6 & 4 & -1 \\ -6 & 14 & -11 & 3 \\ 4 & -11 & 10 & -3 \\ -1 & 3 & -3 & 1 \end{pmatrix}$$

```
A = sym(mod(pascal(4),2));
disp("Pascal(4) mod2")
```

```
Pascal(4) mod2
```

```
disp("Det=")
```

```
Det=
```

```
disp(det(A))
```

$$1$$

```
disp("Adj=")
```

```
Adj=
```

```
disp(adjoint(A))
```

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

```
disp("Inv=")
```

```
Inv=
```

```
disp(inv(A))
```

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

```
A = sym(vander(1:4));
disp("vander(1:4)")
```

```
vander(1:4)
```

```
disp("Det=")
```

```
Det=
```

```
disp(det(A))
```

$$12$$

```
disp("Adj=")
```

```
Adj=
```

```
disp(adjoint(A))
```

$$\begin{pmatrix} -2 & 6 & -6 & 2 \\ 18 & -48 & 42 & -12 \\ -52 & 114 & -84 & 22 \\ 48 & -72 & 48 & -12 \end{pmatrix}$$

```
disp("Inv=")
```

```
Inv=
```

```
disp(inv(A))
```

$$\begin{pmatrix} -\frac{1}{6} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{6} \\ \frac{3}{2} & -4 & \frac{7}{2} & -1 \\ -\frac{13}{3} & \frac{19}{2} & -7 & \frac{11}{6} \\ 4 & -6 & 4 & -1 \end{pmatrix}$$

```
A = [1, 2, 3; 4, 5, 6; 7, 8, 9];
```

```
disp("1-9")
```

```
1-9
```

```
disp("Det=")
```

```
Det=
```

```
disp(det(A))
```

```
  -9.5162e-16
```

```
disp("Adj=")
```

```
Adj=
```

```
disp(adjoint(A))
```

```
   -3.0000    6.0000   -3.0000
    6.0000  -12.0000    6.0000
   -3.0000    6.0000   -3.0000
```

```
disp("Inv=")
```

```
Inv=
```

```
disp(inv(A))
```

```
Warning: Matrix is close to singular or badly scaled. Results may be inaccurate. RCOND =  2.202823e-18.
   1.0e+16 *

    0.3153   -0.6305    0.3153
   -0.6305    1.2610   -0.6305
    0.3153   -0.6305    0.3153
```

Question 4

```
clc, clear, close all
A = eye(1000);
whos A
```

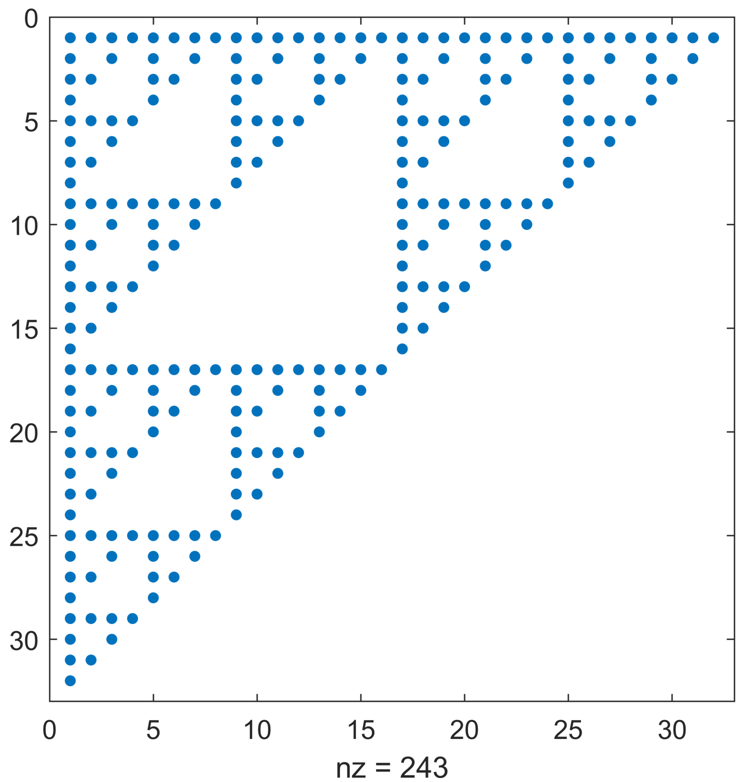
```
  Name         Size                 Bytes  Class     Attributes

  A         1000x1000             8000000  double              
```

```
S = sparse(A);
whos S
```

```
  Name         Size                Bytes  Class     Attributes

  S         1000x1000            24008  double    sparse
```

```

S = sparse(4,3);
A = full(S);

I10 = sparse(eye(10));
```

```
i = [1:10, 1:10];
j = [1:10, 10:-1:1];
k = ones(size(i));
A = sparse(i, j, k);

A = sparse([5, 6], [5, 6], [3, -3],10,10); % two nonzero elements in a 10x10 sparse matrix.

clear
p = pascal(32); % Pascal matrix of dimension 16
s = rem(p,2); % remainder of p mod 2 applied to each component
spy(s) % displays 16x16 Sierpinski gasket
```



Question 5

```
clc, clear, close all

A = mod(pascal(4),2);
A = sym(A);
I = eye(4);
AI = [A, I];
AIR = rref(AI);
disp(AIR(:,5:end))
```

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

Question 6

```
clc, clear, close all

n = 5;
X = -ones(n-1,1);
Y = 2*ones(n,1);
Z = X;

A  = gallery('tridiag',X,Y,Z);
A = sym(full(A));

disp("Det of tridiag A=")
```

```
Det of tridiag A=
```

```
disp(det(A))
```

$$6$$

```
disp("Inverse of tridiag A=")
```

```
Inverse of tridiag A=
```

```
disp(inv(A))
```

$$\begin{pmatrix} \frac{5}{6} & \frac{2}{3} & \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ \frac{2}{3} & \frac{4}{3} & 1 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 1 & \frac{3}{2} & 1 & \frac{1}{2} \\ \frac{1}{3} & \frac{2}{3} & 1 & \frac{4}{3} & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2} & \frac{2}{3} & \frac{5}{6} \end{pmatrix}$$

```
disp("Adjoint of tridiag A=")
```

```
Adjoint of tridiag A=
```

```
disp(adjoint(A))
```

$$\begin{pmatrix} 5 & 4 & 3 & 2 & 1 \\ 4 & 8 & 6 & 4 & 2 \\ 3 & 6 & 9 & 6 & 3 \\ 2 & 4 & 6 & 8 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$

Question 7

```
clc, clear, close all

n = 5;
X = -ones(n-1,1);
Y = 2*ones(n,1);
Z = X;

A  = gallery('tridiag',X,Y,Z);
b = [0; 0; 0; 0; 6];

AI = [A, eye(5)];
AIR = rref(AI);
Ainv = AIR(:,n+1:end);
disp("Inverse of tridiag=")
```

```
Inverse of tridiag=
```

```
disp(Ainv)
```

```
    0.8333    0.6667    0.5000    0.3333    0.1667
    0.6667    1.3333    1.0000    0.6667    0.3333
    0.5000    1.0000    1.5000    1.0000    0.5000
    0.3333    0.6667    1.0000    1.3333    0.6667
    0.1667    0.3333    0.5000    0.6667    0.8333
```

```

x = Ainv*b;
x = x(:,end);
disp("Solution set=")
```

```
Solution set=
```

```
disp(x)
```

```
     1
     2
     3
     4
     5
```

```

disp("Ax-b =")
```

```
Ax-b =
```

```
disp(A*x - b)
```

```
     0
     0
     0
     0
     0
```

Question 8

```
clc, clear, close all

A11 = pascal(3);
A12 = pascal(3);
A12 = A12(:, 1:2);
A21 = zeros(2,3);
A22 = pascal(2);
A = [A11 A12; A21 A22];

B11 = inv(A11)
```

```
B11 = 3×3
    3.0000   -3.0000    1.0000
   -3.0000    5.0000   -2.0000
    1.0000   -2.0000    1.0000
```

```
B22 = inv(A22)
```

```
B22 = 2×2
     2    -1
    -1     1
```

```
B12 = -1*B11*A12*B22
```

```
B12 = 3×2
   -2.0000    1.0000
    1.0000   -1.0000
         0         0
```

```
B = inv(A)
```

```
B = 5×5
     3    -3     1    -2     1
    -3     5    -2     1    -1
     1    -2     1     0     0
     0     0     0     2    -1
     0     0     0    -1     1
```

Questions 9-10

```
clc, clear, close all
```

```
message1 = "Beware the Jabberwock, my son!\nThe jaws that bite, the claws that catch!\n";
message2 = "Beware the Jubjub bird,\nand shun The frumious Bandersnatch!\n";
message = message1 + message2; % combine the two strings
message = char(message); % convert to character array
%fprintf(message) % display the full message

A = [1, -2, 2; 2, -1, 2; 2, -2, 3];

% We better encode it, so the Jabberwock is not warned in advance!
while mod(length(message),3) ~= 0
    message = [message ' ']; % Add another space.
end
M = reshape( message, 3, []); % Reshape message into a 3xn matrix
A = [1 -2 2; 2 -1 2; 2 -2 3]; % Matrix to encrypt the message.
coded_message = A*M;

% The son will now decode his father's message using the matrix inverse.
decoder = round( inv(A) );
message_in_a_matrix = decoder*coded_message;
original_message = char( reshape( message_in_a_matrix, 1, [] ) );
fprintf(original_message)
```

```
Beware the Jabberwock, my son!
The jaws that bite, the claws that catch!
Beware the Jubjub bird,
and shun The frumious Bandersnatch!
```

```
coded_message1 = [68 258 278 -34 88 -59 95 121 111 -49 -61 257 109 77 -20 110 240 -53 249 115 1

message_in_a_matrix = decoder*coded_message1;
original_message = char( reshape( message_in_a_matrix, 1, [] ) );
fprintf(original_message)
```

```
Deep in the human unconscious is a pervasive need for a logical universe that makes sense.
But the real universe is always one step beyond logic. - Frank Herbert, Dune
```