W3 InLab Activity: **Name:** **Cole** **Bardin**

first *last*

Problem 1: Rabbit Island! Sailors introduced a group of rabbitts on an island with no predators and ample food supply. The rabbit population $N(t)$ increases at a rate proportional to the number of rabbits. The population doubles every two years and after $t = 10$ years the sailors stop by the island and find the population is $N(10) = 384$.

a. Write a differential equation for the number of rabbits $N(t)$ using k for the rate of growth.

$$\frac{dN}{dt} = k * N$$

b. Find the specific solution with an initial population of $N(0)$.

$$\boldsymbol{N(t) = N(0) * e^{kt}}$$

c. Find k given the population doubles every two years. Give an exact expression.

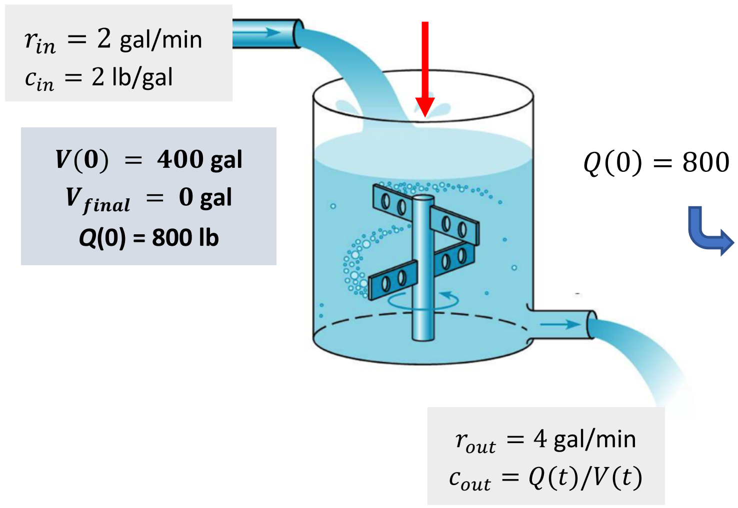
$$k = \frac{\ln(2)}{2}$$

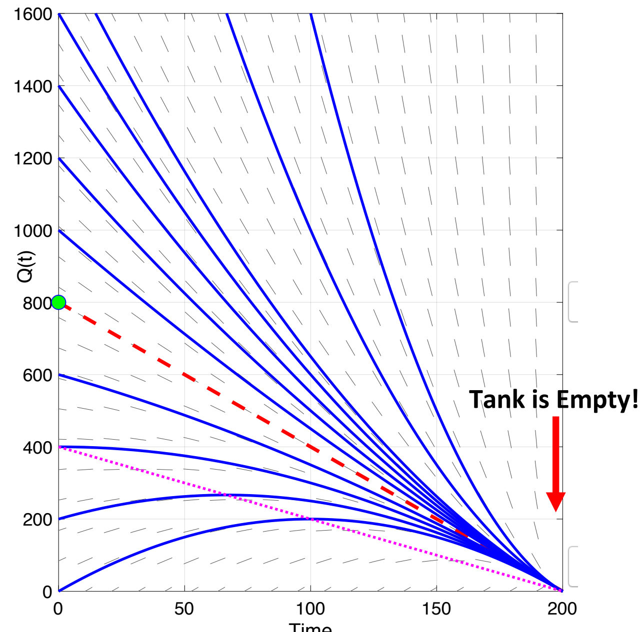
d. Find the initial number of rabbits $N(0)$ the sailors left on the island given $N(10) = 384$.

$$N(0) = 12$$

Problem 2: Tank Problem A 400-gallon tank is initially full, so that $V(0) = 400$ gallons. The tank contains a brine solution and initially the amount of dissolved salt is $Q(0) = 800$ pounds. At time 0, a brine solution with a concentration of 2 pounds/gallon is pumped in at the rate of $r_{in} = 2$ gallons/minute and the well-stirred mixture is pumped out at double that rate, or $r_{out} = 4$ gallons/minute.

Underlying model: $\frac{dQ}{dt} = rate_{in} - rate_{out}$ where $Q(t)$ is the amount of salt in the tank at time t in pounds.





a. The volume is **not constant** but decreases at a constant rate from an initial value of $V(0) = 400$, until the tank is empty at time $T = 200$ minutes. Express $V(t)$ as a linear function from the time $t = 0$ to the moment the tank is empty.

$$V(t) = 400 - 2t \qquad gal$$

b. Salt flows into the tank at the rate: $\quad rate_{in} = 4 \quad \frac{lb}{min}$

c. Salt flows out of the tank at the rate: $\quad rate_{out} = \frac{4Q}{400-2t} \quad \frac{lb}{min}$

d. The differential equation governing the amount of salt $Q(t)$ up until the tank is empty is:

i. $\frac{dQ}{dt} = 6 - \frac{Q}{400}$ **ii.** $\frac{dQ}{dt} = 4 - \frac{Q}{100}$ **iii.** $\frac{dQ}{dt} = 4 + \frac{4Q}{400-2t}$ **iv.** $\frac{dQ}{dt} = 4 - \frac{4Q}{400-2t}$

e. The integrating factor $\mu(t)$ for this DE can be chosen as:

i. $\mu(t) = e^{t/200}$ **ii.** $\mu(t) = \frac{1}{(t-200)^2}$ **iii.** $\mu(t) = (100-t)^2$ **iv.** $\mu(t) = \frac{1}{(t-100)^2}$

Tip: Any multiple of an integrating factor is also an integrating factor. It's not unique.

The general solution can be shown to be: $Q(t) = (800 - 4t) + c \cdot (200 - t)^2$

f. Solve the DE for the quantity of salt $Q(t)$ given that $Q(0) = 800$.
See the small dot and dashed line on the graph. This solution is only valid up until the tank is empty.

i. $Q(t) = (200 - t)^2$ **ii.** $Q(t) = 8 \cdot (100 - t)$
iii. $Q(t) = 200 - t$ **iv.** $Q(t) = 800 - 4t$

g. The equation of the **nullcline** (shown as a dotted line through the local maxima) is:
i. $Q = 200 - t$ **ii.** $Q = 200 - 2t$ **iii.** $Q = 400 - 2t$ **iv.** $Q = 800 - 4t$

h. Using the general solution shown at the top, what is the value of c for this new solution curve that satisfies $Q(0) = 0$? That is the tank starts off filled with **fresh water** instead.
This corresponds to the lowest solution displayed in the previous plot.
i. $c = 0$ **ii.** $c = -\frac{1}{25}$ **iii.** $c = -\frac{1}{50}$ **iv.** $c = -\frac{3}{100}$