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1. Consider the following linear homogenous differential equations with initial conditions.

a) $y" + 4y = 0$ $y(0) = 1,\quad y'(0) = -1$

$$r^2 + 4 = 0$$
$$r = 0 \pm 2i$$
$$y(0) = d_1 = 1$$
$$y'(0) = d_2 = -\frac{1}{2}$$
$$y(t) = \cos(2t) - \frac{1}{2}\sin(2t)$$

b) $y" - 4y' + 13y = 0$ $y(0) = 1,\quad y'(0) = 0$

$$r^2 - 4r + 13 = 0$$
$$r = \frac{4 \pm \sqrt{16 - 4 * 13}}{2}$$
$$r = 2 \pm 3i$$
$$y(0) = d_1 = 1$$
$$y'(0) = 2 * 1 + 3d_2 = 0, \quad d_2 = -\frac{3}{2}$$
$$y(t) = e^{2t}[\cos(3t) - \frac{3}{2}\sin(3t)]$$

For each of the equations given, find the characteristic equation and solve the differential equation to find the solution.

2. For the following 2nd order differential equation, decompose the equation into a system of first order linear differential equations and then write the state equation. That is, express the system of equations in matrix representation of the form $x' = Ax + Bu$:

$$y'' + y' - 2y = 2t,\ \ y(0) = 0,\ \ y'^{(0)} = 1$$
$$x1 = y \qquad x2 = y'$$
$$x1' = y' = x2$$
$$x2' = y'' = 2t + 2y - y'$$
$$x2' = -x2 + 2x1 + 2t$$
$$x' = \begin{bmatrix} x1' \\ x2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2t \end{bmatrix}$$
$$y(0) = \begin{bmatrix} x1(0) \\ x2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$