Lab 5 by Cole Bardin Section: 61

Method of Undetermined Coefficients

Part A: Homogeneous DE

```
clc, clear, close all
% Question 2
syms y(t)
Dy = diff(y,t); D2y = diff(y,t, t);

DE = 4*D2y +4*Dy + 17*y == 0;
sol = dsolve(DE, y(0)==4, Dy(0)==0)
```

```
sol =
```

$$e^{-\frac{t}{2}} (4\cos(2t) + \sin(2t))$$

```
% Question 3

Y = matlabFunction(sol);
Dy = matlabFunction(diff(Y, t));

Y(1)
```

```
ans = -0.4581
```

```
Dy(1)
```

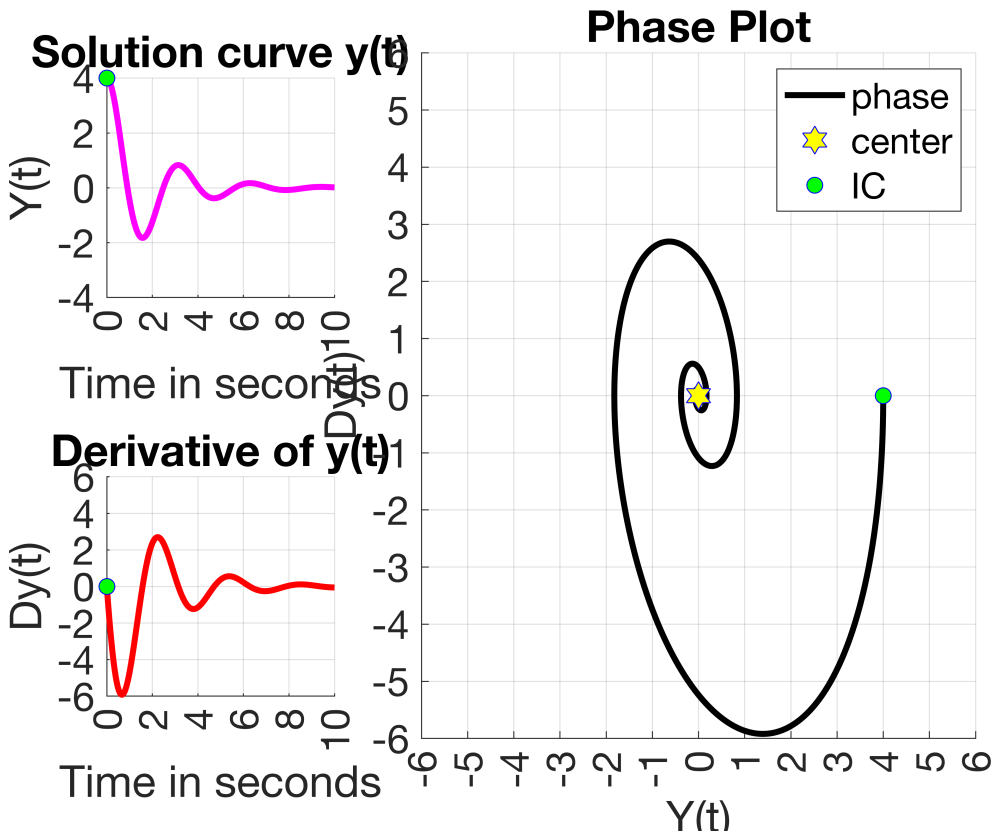
```
ans = -4.6879
```

```
% Questions 4-5
figure
time = 0: 0.01 : 10;

subplot(2,3,1) % tile 1
set(gca,"FontSize",20)
hold on
grid on
plot(time, Y(time), 'm', 'LineWidth',3)
plot(0,4, 'bo', 'MarkerSize',8, 'MarkerFaceColor','g')
title("Solution curve y(t)")
xlabel("Time in seconds")
ylabel("Y(t)")
axis([0, 10, -4, 4])
xticks(0:2:10)
yticks(-4:2:4)
```

```
subplot(2, 3, 4) % tile 4
set(gca,"FontSize",20)
hold on
grid on
plot(time, Dy(time), 'r', 'LineWidth',3)
plot(0,0, 'bo', 'MarkerSize',8, 'MarkerFaceColor','g')
title("Derivative of y(t)")
xlabel("Time in seconds")
ylabel("Dy(t)")
axis([0, 10, -6, 6])
xticks(0:2:10)
yticks(-6:2:6)

subplot(2, 3, [2, 3, 5, 6]) % 2-3 and 5-6
set(gca,"FontSize",20)
hold on
grid on
phase = plot(Y(time), Dy(time), 'k', 'LineWidth',3);
center = plot(0, 0, 'bh', 'MarkerSize', 14, 'MarkerFaceColor', 'yellow'); % Center Poi
ic = plot(4,0, 'bo', 'MarkerSize',8, 'MarkerFaceColor','g');
title("Phase Plot")
xlabel("Y(t)")
ylabel("Dy(t)")
axis([-6, 6, -6, 6])
xticks(-6:6)
yticks(-6:6)
legend([phase, center, ic], ["phase", "center", "IC"])
```



Part B: Non-Homogeneous DE: Method of Undetermined Coefficients

```
clc, clear, close all
% Questions 6-7
syms y(t)
Dy = diff(y,t); D2y = diff(y,t, t);

% a)
DE = 4*D2y +4*Dy + 17*y == 17 + 289*t;
sol = dsolve(DE, y(0)==4, Dy(0)==0);
disp("a)")
```

```
a)
```

```
sol = simplify(sol)
```

```
sol =
```

$$17\,t + 7\cos(2\,t)\,\mathrm{e}^{-\frac{t}{2}} - \frac{27\sin(2\,t)\,\mathrm{e}^{-\frac{t}{2}}}{4} - 3$$

```
% b)
DE = 4*D2y +4*Dy + 17*y == 100*exp(-2*t);
sol = dsolve(DE, y(0)==4, Dy(0)==0);
disp("b)")
```

b)

```
sol = simplify(sol)
```

sol =

$$4\,e^{-2\,t} + 4\,\sin(2\,t)\,e^{-\frac{t}{2}}$$

```
% c)
DE = 4*D2y +4*Dy + 17*y == 260*cos(2*t);
sol = dsolve(DE, y(0)==4, Dy(0)==0);
disp("c)")
```

c)

```
sol = simplify(sol)
```

sol =

$$4\,\cos(2\,t) + 32\,\sin(2\,t) - 32\,\sin(2\,t)\,e^{-\frac{t}{2}}$$

```
% d)
DE = 4*D2y +4*Dy + 17*y == 16*exp(-t/2)*cos(2*t) + 32*exp(-t/2)*sin(2*t);
sol = dsolve(DE, y(0)==4, Dy(0)==0);
disp("d)")
```

d)

```
sol = simplify(sol)
```

sol =

$$e^{-\frac{t}{2}}\,(4\,\cos(2\,t) + 2\,\sin(2\,t) - 2\,t\,\cos(2\,t) + t\,\sin(2\,t))$$

Questions 8-10: The Method of Undetermined Coefficients from SCRATCH

```
clc, clear, close all
% Question 8
syms A B t C1 C2
syms y(t) a b
Dy = diff(y,t); D2y = diff(y,t, t);
f = 260*cos(2*t)
```

f = $260\,\cos(2\,t)$

```
DE = 4*D2y + 4*Dy + 17*y - f == 0 % non-homogeneous differential equation
```

DE(t) =

$$4\,\frac{\partial^2}{\partial t^2}\,y(t) + 4\,\frac{\partial}{\partial t}\,y(t) - 260\,\cos(2\,t) + 17\,y(t) = 0$$

```

```

```
Y = A * cos(2*t) + B * sin(2*t) % our guess
```

Y = $A\cos(2t) + B\sin(2t)$

```
plug_it_in = subs(DE, y, Y) % Plug our guess into the DE
```

plug_it_in(t) = $A\cos(2t) - 260\cos(2t) + 8B\cos(2t) - 8A\sin(2t) + B\sin(2t) = 0$

```
eqn = collect(plug_it_in, [cos(2*t), sin(2*t)]) % arrange by similar terms
```

eqn(t) = $(A + 8B - 260)\cos(2t) + (B - 8A)\sin(2t) = 0$

```
% MATLAB can find the undetermined coefficients for us
equations = coeffs( lhs(eqn), [cos(2*t), sin(2*t)] )
```

equations(t) = $(B - 8A \quad A + 8B - 260)$

```
variables = [A, B]
```

variables = $(A \quad B)$

```
% find and display the undetermined coefficients
[A, B] = solve(equations, variables)
```

A = 4

B = 32

```
% Questions 9-10
Yp = A * cos(2*t) + B * sin(2*t); % our guess;

% Solve for homogenous solution
DE = 4*D2y + 4*Dy + 17*y == 0;
Yh = simplify(dsolve(DE));

% Get general form
Y = simplify(Yh + Yp);
Dy = diff(Y, t);

% Solve for C1 and C2 in general form
Y = matlabFunction(Y);
Dy = matlabFunction(diff(Y,t));
eqns = [Y(sym(C1), sym(C2), 0)==4, Dy(sym(C1), sym(C2), 0)==0];
C = solve(eqns, C1, C2);

% Sub in C1 and C2 values
Y = simplify(Yh + Yp);
Y = subs(Y, C1, C.C1);
Y = subs(Y, C2, C.C2);
```

```
% Make new matlab functions for new solutions
Y = matlabFunction(Y)
```

```
Y = function_handle with value:
    @(t)cos(t.*2.0).*4.0+sin(t.*2.0).*3.2e+1-sin(t.*2.0).*exp(t.*(-1.0./2.0)).*3.2e+1
```

```
Dy = matlabFunction(diff(Y,t))
```

```
Dy = function_handle with value:
    @(t)cos(t.*2.0).*6.4e+1-sin(t.*2.0).*8.0-cos(t.*2.0).*exp(t.*(-1.0./2.0)).*6.4e+1+sin(t.*2.0).*exp(t.*
```

```
Yp = matlabFunction(Yp)
```

```
Yp = function_handle with value:
    @(t)cos(t.*2.0).*4.0+sin(t.*2.0).*3.2e+1
```

```
Ypp = matlabFunction(diff(Yp, t))
```

```
Ypp = function_handle with value:
    @(t)cos(t.*2.0).*6.4e+1-sin(t.*2.0).*8.0
```

```
fig3 = figure(3);
time = 0: 0.01 : 10;

subplot(2,3,1) % tile 1
set(gca,"FontSize",20)
hold on
grid on
plot(time, Y(time), 'm', 'LineWidth',3)
plot(0,4, 'bo', 'MarkerSize',8, 'MarkerFaceColor','g')
title("Solution curve y(t)")
xlabel("Time in seconds")
ylabel("y(t)")
axis([0, 10, -40, 40])
xticks(0:2:10)
yticks(-40:20:40)

subplot(2, 3, 4) % tile 4
set(gca,"FontSize",20)
hold on
grid on
plot(time, Dy(time), 'r', 'LineWidth',3)
plot(0,0, 'bo', 'MarkerSize',8, 'MarkerFaceColor','g')
title("Derivative of y(t)")
xlabel("Time in seconds")
ylabel("Dy(t)")
axis([0, 10, -75, 75])
xticks(0:2:10)
yticks(-50:50:50)

subplot(2, 3, [2, 3, 5, 6]) % 2-3 and 5-6
set(gca,"FontSize",20)
```

```
hold on
grid on
phase = plot(Y(time), Dy(time), 'k', 'LineWidth',3);
ic = plot(4,0, 'bo', 'MarkerSize',8, 'MarkerFaceColor','g');
limit = plot(Yp(time), Ypp(time), 'r', 'LineWidth',3);
title("Phase Plot")
xlabel("y(t)")
ylabel("dy/dt")
axis([-40, 40, -80, 80])
xticks(-40:20:40)
yticks(-60:20:60)
legend([phase, limit, ic], ["phase", "limit cycle", "IC"])
```