

Part A

Question 1

```
clc, clear, close all
a1=[6,-6,0;3,-3,0;1,-1,0]
```

```
a1 = 3x3
     6    -6     0
     3    -3     0
     1    -1     0
```

```
a2=[-2,6,9;-2,5,6;0,0,1]
```

```
a2 = 3x3
    -2     6     9
    -2     5     6
     0     0     1
```

```
a3=[-2,6,14;-2,5,8;0,0,3]
```

```
a3 = 3x3
    -2     6    14
    -2     5     8
     0     0     3
```

```
a4=[-4,10,15;-3,7,8;0,0,3]
```

```
a4 = 3x3
    -4    10    15
    -3     7     8
     0     0     3
```

```
eig(a1)
```

```
ans = 3x1
     0
     0
     3
```

```
eig(a2)
```

```
ans = 3x1
  1.0000
  2.0000
  1.0000
```

```
eig(a3)
```

```
ans = 3x1
  1.0000
  2.0000
  3.0000
```

```
eig(a4)
```

```
ans = 3×1
    1.0000
    2.0000
    3.0000
```

Question 2

```
clc, clear, close all
A=[-24,16,-16;-40,28,-28;-4,4,-4]
```

```
A = 3×3
   -24    16   -16
   -40    28   -28
    -4     4    -4
```

```
eig(A)
```

```
ans = 3×1 complex
10-4 ×
    0.3608 + 0.6249i
    0.3608 - 0.6249i
   -0.7216 + 0.0000i
```

```
rref([A,[0;0;0]])
```

```
ans = 3×4
     1     0     0     0
     0     1    -1     0
     0     0     0     0
```

```
v1=[0;1;1]
```

```
v1 = 3×1
     0
     1
     1
```

```
rref([A,v1])
```

```
ans = 3×4
    1.0000     0     0    0.5000
         0    1.0000   -1.0000    0.7500
         0     0     0         0
```

Question 3

```
clc, clear, close all
syms y(t);
y0=0;yp0=1;
dy=diff(y,t); d2y=diff(y,t,t);
DE = d2y+y
```

```
DE(t) =
```

$$\frac{\partial^2}{\partial t^2} y(t) + y(t)$$

$$F_a = t$$

$$F_a = t$$

$$F_b = -2*\sin(t)$$

$$F_b = -2 \sin(t)$$

$$F_c = 2*\cos(t)$$

$$F_c = 2 \cos(t)$$

$$F_d = \sin(t)$$

$$F_d = \sin(t)$$

$$\text{simplify}(\text{dsolve}(\text{DE}==F_a, y(0)==y_0, dy(0)==y_{p0}))$$

$$\text{ans} = t$$

$$\text{simplify}(\text{dsolve}(\text{DE}==F_b, y(0)==y_0, dy(0)==y_{p0}))$$

$$\text{ans} = t \cos(t)$$

$$\text{simplify}(\text{dsolve}(\text{DE}==F_c, y(0)==y_0, dy(0)==y_{p0}))$$

$$\text{ans} = \sin(t) (t + 1)$$

$$\text{simplify}(\text{dsolve}(\text{DE}==F_d, y(0)==y_0, dy(0)==y_{p0}))$$

$$\text{ans} =$$

$$\frac{3 \sin(t)}{2} - \frac{t \cos(t)}{2}$$

Question 4

```
clc, clear, close all
syms s;
Fs = (3125*s)/((s-2)*(s-2)*(s-2)*(s+3)*(s+3)*(s+3))
```

$$F_s =$$

$$\frac{3125 s}{(s-2)^3 (s+3)^3}$$

$$PF = \text{partfrac}(F_s)$$

$$PF =$$

$$\frac{3}{s+3} - \frac{5}{(s-2)^2} - \frac{3}{s-2} + \frac{50}{(s-2)^3} + \frac{20}{(s+3)^2} + \frac{75}{(s+3)^3}$$

Question 5

```
clc, clear, close all
syms y(x);
y0=0;dy0=0;
dy=diff(y,x); d2y=diff(y,x,x);
DEa = d2y + 3*dy + 2*y == exp(-1*x)
```

DEa(x) =

$$\frac{\partial^2}{\partial x^2} y(x) + 3 \frac{\partial}{\partial x} y(x) + 2 y(x) = e^{-x}$$

```
DEb = d2y + 4*dy + 3*y == 4*x*exp(-1*x)
```

DEb(x) =

$$\frac{\partial^2}{\partial x^2} y(x) + 4 \frac{\partial}{\partial x} y(x) + 3 y(x) = 4 x e^{-x}$$

```
DEc = d2y + dy == x*exp(-1*x)
```

DEc(x) =

$$\frac{\partial^2}{\partial x^2} y(x) + \frac{\partial}{\partial x} y(x) = x e^{-x}$$

```
DEd = d2y + 3*dy + 2*y == 2*(1+x)*exp(-1*x)
```

DEd(x) =

$$\frac{\partial^2}{\partial x^2} y(x) + 3 \frac{\partial}{\partial x} y(x) + 2 y(x) = e^{-x} (2 x + 2)$$

```
simplify(dsolve(DEa,y(0)==y0,dy(0)==dy0))
```

$$\text{ans} = e^{-2 x} (x e^x - e^x + 1)$$

```
simplify(dsolve(DEb,y(0)==y0,dy(0)==dy0))
```

ans =

$$x^2 e^{-x} - \frac{e^{-3 x}}{2} - \frac{e^{-x} (2 x - 1)}{2}$$

```
simplify(dsolve(DEc,y(0)==y0,dy(0)==dy0))
```

ans =

$$-\frac{e^{-x} (2 x - 2 e^x + x^2 + 2)}{2}$$

```
simplify(dsolve(DEd,y(0)==y0,dy(0)==dy0))
```

$$\text{ans} = x^2 e^{-x}$$

Part B

Questions 3-5

```
clc, clear, close all
syms x(t) y(t);
xm=[x;y];
x0=[-0.28;0.00];
time = 00;001;8;
```

Part C

Points 1-10

```
clc, clear, close all
syms Y t s;
y0=-60;dy0=0;
f=12+12*sin(t)
```

$$f = 12 \sin(t) + 12$$

```
% Point 1
F=laplace(f)
```

$$F =$$

$$\frac{12}{s^2 + 1} + \frac{12}{s}$$

$$d2Y = s*s*Y - s*y0 - dy0$$

$$d2Y = Y s^2 + 60 s$$

$$dY = (1/6)*(s*Y - y0)$$

$$dY =$$

$$\frac{Y s}{6} + 10$$

$$LTofDE = d2Y + dY + Y == F$$

$$LTofDE =$$

$$Y + 60 s + \frac{Y s}{6} + Y s^2 + 10 = \frac{12}{s^2 + 1} + \frac{12}{s}$$

$$Sol = \text{solve}(LTofDE, Y)$$

$$Sol =$$

$$-\frac{60 s^2 - 12}{s^3 + s}$$

$$Y = \text{matlabFunction}(Sol);$$

```
% Point 2
Y(s);
% Point 3
PF = partfrac(Y(s))
```

```
PF =


$$\frac{12}{s} - \frac{72s}{s^2 + 1}$$

```

```
% Point 4
limit(Y(s),s,inf)
```

```
ans = 0
```

```
% Point 5
y=ilaplace(Y(s))
```

```
y = 12 - 72 cos(t)
```

```
% Point 6
dy = matlabFunction(simplify(diff(y,t)))
```

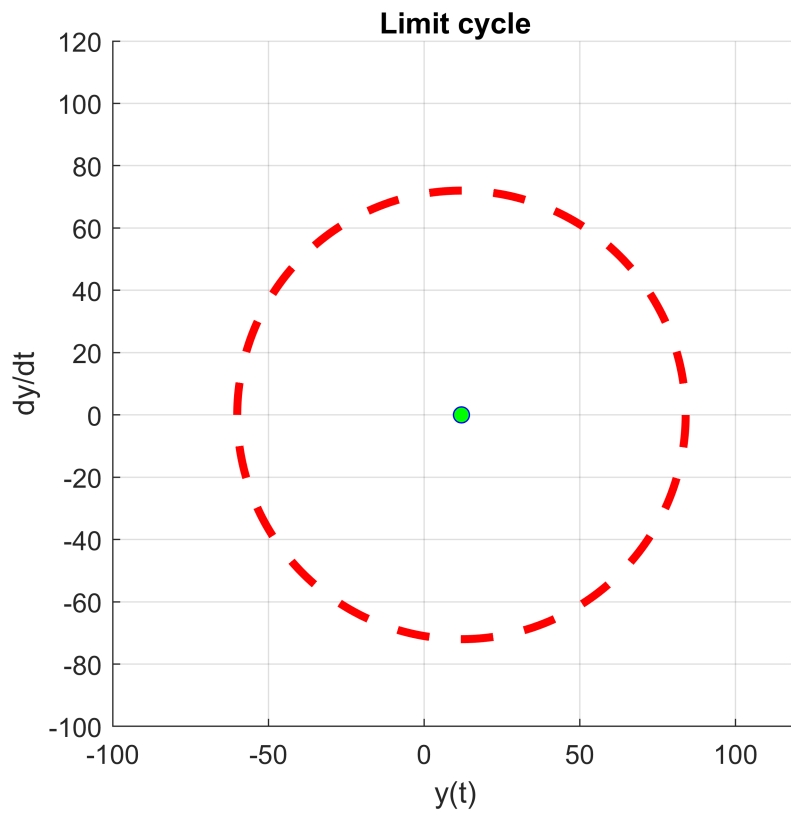
```
dy = function_handle with value:
@(t)sin(t).*7.2e+1
```

```
y=matlabFunction(y)
```

```
y = function_handle with value:
@(t)cos(t).*-7.2e+1+1.2e+1
```

```
% Points 7-8
time=[0:0.01:2*pi];

grid on
hold on
limit = plot(y(time), dy(time), 'r--', "LineWidth",3);
center = plot(12,0, 'bo', "MarkerFaceColor",'g');
axis equal
axis([-100, 120, -100, 120])
xlabel("y(t)");
ylabel("dy/dt");
title("Limit cycle");
```



Points 9-10

```
clc, clear, close all
syms x(t) y(t);
xm=[x;y];
x0=[-0.28;0.00];
time = [0,8];
sol = ode45(@diffeq65,time,x0)
```

```
c = 2.8274
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```

[illegible]

[illegible]

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[illegible]

[illegible]

```

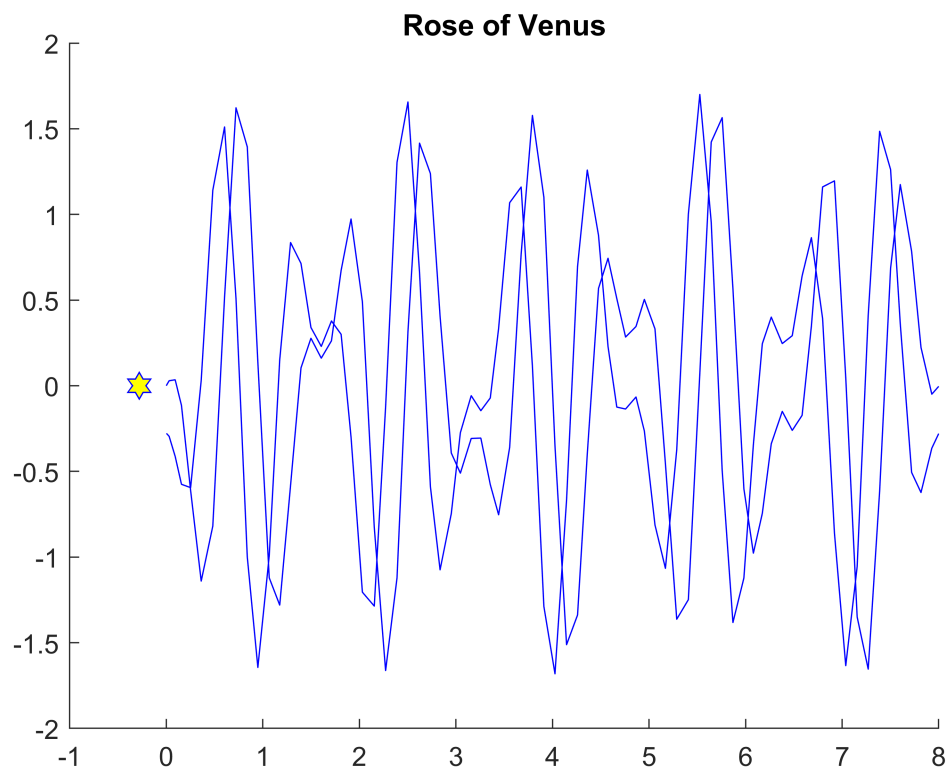
c = 2.8274
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c = 2.8274
c = 2.8274
c = 2.8274
sol = struct with fields:
    solver: 'ode45'
    extdata: [1x1 struct]
        x: [1x79 double]
        y: [2x79 double]
    stats: [1x1 struct]
    idata: [1x1 struct]

```

```

hold on
plot(sol.x, sol.y, 'b')
plot(-0.28, 0, "bh", "MarkerSize",10,"MarkerFaceColor",'y');
title("Rose of Venus")

```



Part C

Points 1-10

```
clc, clear, close all
syms Y t s;
y0=-60;dy0=0;
f=12+12*sin(t)
```

$$f = 12 \sin(t) + 12$$

```
% Point 1
F=laplace(f)
```

$$F = \frac{12}{s^2 + 1} + \frac{12}{s}$$

$$d2Y = s*s*Y - s*y0 - dy0$$

$$d2Y = Y s^2 + 60 s$$

$$dY = (1/6)*(s*Y - y0)$$

$$dY = \frac{Y s}{6} + 10$$

$$LTofDE = d2Y + dY + Y == F$$

$$LTofDE = Y + 60 s + \frac{Y s}{6} + Y s^2 + 10 = \frac{12}{s^2 + 1} + \frac{12}{s}$$

$$Sol = \text{solve}(LTofDE, Y)$$

$$Sol = -\frac{60 s^2 - 12}{s^3 + s}$$

```
Y = matlabFunction(Sol);
% Point 2
Y(s);
% Point 3
PF = partfrac(Y(s))
```

$$PF = \frac{12}{s} - \frac{72 s}{s^2 + 1}$$

```
% Point 4
limit(Y(s),s,inf)
```

$$ans = 0$$


```
% Point 5
y=ilaplace(Y(s))
```

$$y = 12 - 72 \cos(t)$$

```
% Point 6
dy = matlabFunction(simplify(diff(y,t)))
```

```
dy = function_handle with value:
@(t)sin(t).*7.2e+1
```

```
y=matlabFunction(y)
```

```
y = function_handle with value:
@(t)cos(t).*-7.2e+1+1.2e+1
```

```
% Points 7-8
time=[0:0.05:2*pi];
grid on
hold on
limit = plot(y(time), dy(time), 'r.', "LineWidth",3);
center = plot(12,0, 'bo', "MarkerFaceColor",'g', "MarkerSize",10);
axis equal
axis([-100, 120, -100, 120])
xlabel("y(t)");
ylabel("dy/dt");
title("Limit Cycle")
```

```
% Points 9-10
clear
grid off
syms Y t s;
y0=0;dy0=0;
f=12+12*sin(t)
```

$$f = 12 \sin(t) + 12$$

```
F=laplace(f)
```

$$F = \frac{12}{s^2 + 1} + \frac{12}{s}$$

$$d2Y = s*s*Y - s*y0 - dy0$$

$$d2Y = Y s^2$$

$$dY = (1/6)*(s*Y - y0)$$

$$dY = \frac{Y s}{6}$$

```
LTofDE = d2Y + dY + Y == F
```

```
LTofDE =
```

$$Y s^2 + \frac{Y s}{6} + Y = \frac{12}{s^2 + 1} + \frac{12}{s}$$

```
Sol = solve(LTofDE, Y)
```

```
Sol =
```

$$\frac{\frac{12}{s^2 + 1} + \frac{12}{s}}{s^2 + \frac{s}{6} + 1}$$

```
Y = matlabFunction(Sol);  
Y(s);  
PF = partfrac(Y(s))
```

```
PF =
```

$$\frac{12}{s} - \frac{72 s}{s^2 + 1} + \frac{360 s + 60}{6 s^2 + s + 6}$$

```
limit(Y(s),s,inf)
```

```
ans = 0
```

```
y=ilaplace(Y(s))
```

```
y =
```

$$60 e^{-\frac{t}{12}} \left(\cos\left(\frac{\sqrt{143} t}{12}\right) + \frac{\sqrt{143} \sin\left(\frac{\sqrt{143} t}{12}\right)}{143} \right) - 72 \cos(t) + 12$$

```
dy = matlabFunction(simplify(diff(y,t)))
```

```
dy = function_handle with value:
```

```
@(t)sin(t).*7.2e+1-sqrt(1.43e+2).*exp(t.*(-1.0./1.2e+1)).*sin((sqrt(1.43e+2).*t)./1.2e+1).*(7.2e+2./1.43e+2)
```

```
y=matlabFunction(y)
```

```
y = function_handle with value:
```

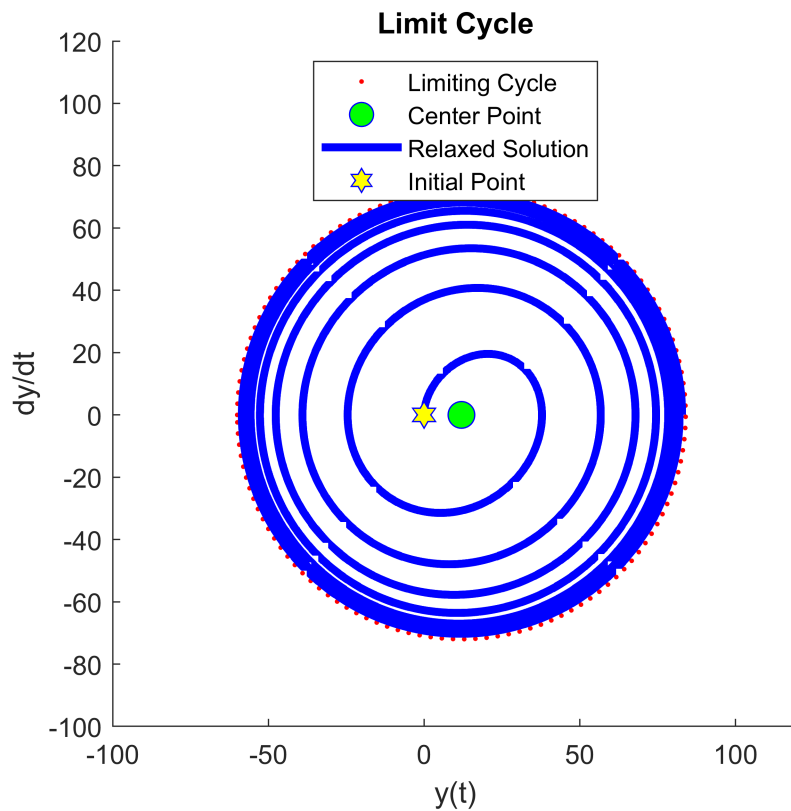
```
@(t)cos(t).*-7.2e+1+exp(t.*(-1.0./1.2e+1)).*(cos((sqrt(1.43e+2).*t)./1.2e+1)+(sqrt(1.43e+2).*sin((sqrt(1.43e+2).
```

```
time=[0:0.01:15*pi];
```

```
rel = plot(y(time), dy(time), 'b', "LineWidth",3);
```

```
ip = plot(0,0, 'bh', "MarkerFaceColor",'y', "MarkerSize",10);
```

```
legend(["Limiting Cycle","Center Point","Relaxed Solution","Initial Point"],"Location","north",
```



Part D

```
% Points 1-5
clc, clear, close all
syms s t;
Re = 1.00; Rv = 0.72;           % in astronomical units
we = 2*pi; wv = 2*pi *13/8; % angular frequencies for Earth and Venus
c = (wv - we) * Rv % 2.8274
```

```
c = 2.8274
```

```
A=[0,-2*pi;2*pi,0];
f=c*[-1*sin(wv*t);cos(wv*t)];
x0=[-0.28;0.00];
```

```
% Point 1
R = inv(s*eye(2)-A)
```

```
R =
```

$$\begin{pmatrix} \frac{s}{s^2 + 4\pi^2} & -\frac{2\pi}{s^2 + 4\pi^2} \\ \frac{2\pi}{s^2 + 4\pi^2} & \frac{s}{s^2 + 4\pi^2} \end{pmatrix}$$

```
% Point 2
F = laplace(f)
```

F =

$$\begin{pmatrix} -\frac{117 \pi^2}{40 \left(s^2 + \frac{169 \pi^2}{16} \right)} \\ \frac{9 \pi s}{10 \left(s^2 + \frac{169 \pi^2}{16} \right)} \end{pmatrix}$$

% Point 3

```
xzi = R*x0
```

xzi =

$$\begin{pmatrix} -\frac{7 s}{25 (s^2 + 4 \pi^2)} \\ -\frac{14 \pi}{25 (s^2 + 4 \pi^2)} \end{pmatrix}$$

% Point 4

```
xzs = simplify(R*F)
```

xzs =

$$\begin{pmatrix} -\frac{189 s \pi^2}{40 (s^2 + 4 \pi^2) \left(s^2 + \frac{169 \pi^2}{16} \right)} \\ -\frac{36 \pi (13 \pi^2 - 2 s^2)}{5 (16 s^4 + 233 \pi^2 s^2 + 676 \pi^4)} \end{pmatrix}$$

% Point 5

```
X=xzi+xzs
```

X =

$$\begin{pmatrix} -\frac{7 s}{25 \sigma_1} - \frac{189 s \pi^2}{40 \sigma_1 \left(s^2 + \frac{169 \pi^2}{16} \right)} \\ -\frac{14 \pi}{25 \sigma_1} - \frac{36 \pi (13 \pi^2 - 2 s^2)}{5 (16 s^4 + 233 \pi^2 s^2 + 676 \pi^4)} \end{pmatrix}$$

where

$$\sigma_1 = s^2 + 4 \pi^2$$

```
x=matlabFunction(ilaplace(X))
```

x = function_handle with value:

```
@(t)[-cos(t.*pi.*2.0)+cos(t.*pi.*(1.3e+1./4.0)).*(1.8e+1./2.5e+1);-sin(t.*pi.*2.0)+sin(t.*pi.*(1.3e+1./4.0)).*(1.8e+1./2.5e+1)]
```

```
x(4)
```

```
ans = 2x1  
-1.7200
```

-0.0000