Name: ___________________ _______________________________ **Grade: ______ / 25**

ENGR 232: Dynamic Engineering Systems – Summer 2022

MATLAB Exam - **Version 60C**

Instructions: You have approximately **1 hour and 40 min** to finish and **upload** your MATLAB EXAM to BBLearn. Use the provided ANSWER TEMPLATE for your convenience. You are required to use MATLAB to complete this exam and no other software.

ALL WORK MUST BE COMPLETED INDEPENDENTLY. Absolutely NO collaboration.

Tip: Save your work at least every 10 minutes! Maximum time granted for a computer crash is 10 minutes.

Be sure to upload your answers as a PDF before the submission window closes. Remember to also submit your Live Script and a PDF copy of your Live Script!!

Part A: Multiple Choice: **(5 points)**

Answer each of the following quick-answer questions using single MATLAB commands such as `eig, det, simplify, expand, dsolve and partfrac`.

1. Find the matrix for which $\lambda = 0$ is a repeated eigenvalue. You can just record the answer in the answer template file.

a. $A_1 = \begin{bmatrix} 6 & -6 & 0 \\ 3 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$ **b.** $A_2 = \begin{bmatrix} -2 & 6 & 9 \\ -2 & 5 & 6 \\ 0 & 0 & 1 \end{bmatrix}$ **c.** $A_3 = \begin{bmatrix} -2 & 6 & 14 \\ -2 & 5 & 8 \\ 0 & 0 & 3 \end{bmatrix}$ **d.** $A_4 = \begin{bmatrix} -4 & 10 & 15 \\ -3 & 7 & 8 \\ 0 & 0 & 3 \end{bmatrix}$

2. Here is a new matrix, for which $\lambda = 0$ is a triple eigenvalue: $A = \begin{bmatrix} -24 & 16 & -16 \\ -40 & 28 & -28 \\ -4 & 4 & -4 \end{bmatrix}$

You can check that it has only one independent eigenvector $\vec{\mathbf{v}} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ for this repeated eigenvalue. Find a generalized eigenvector $\vec{\mathbf{w}}$ so that: $(A - \lambda I)\,\vec{\mathbf{w}} = \vec{\mathbf{v}}$ where $\lambda = 0$ and $\vec{\mathbf{v}}$ is the above eigenvector.

a. $\vec{\mathbf{w}} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ **b.** $\vec{\mathbf{w}} = \begin{bmatrix} 3/4 \\ 1/2 \\ 0 \end{bmatrix}$ **c.** $\vec{\mathbf{w}} = \begin{bmatrix} 1/2 \\ 3/4 \\ 0 \end{bmatrix}$ **d.** $\vec{\mathbf{w}} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

3. The function $y(t) = t \cdot \cos t$ is a solution to one of these differential equations. Using **dsolve** & **simplify**, find the DE. Note the given function implies $y(0) = 0$ and $y'(0) = 1$.

a. $y'' + y = t$ **b.** $y'' + y = -2\sin t$ **c.** $y'' + y = 2\cos t$ **d.** $y'' + y = \sin t$

4. Find the coefficient A_2 in the partial fraction: $F(s) = \frac{3125 \cdot s}{(s-2)^3 \cdot (s+3)^3} = \frac{A_1}{s-2} + \frac{A_2}{(s-2)^2} + \frac{A_3}{(s-2)^3} + \frac{B_1}{s+3} + \frac{B_2}{(s+3)^2} + \frac{B_3}{(s+3)^3}$

Be sure you noticed the s in the numerator!

a. $A_2 = -3$ **b.** $A_2 = -5$ **c.** $A_2 = +50$ **d.** $A_2 = +3$ **e.** $A_2 = 20$ **f.** $A_2 = 75$

5. The function $y = x^2 e^{-x}$ is a solution to one of these differential equations.
Note the given function implies $y(0) = 0$ and $y'(0) = 0$. Using **dsolve** and **simplify**, find the DE.

a. $y'' + 3y' + 2y = e^{-x}$ **b.** $y'' + 4y' + 3y = 4x \cdot e^{-x}$
c. $y'' + y' = xe^{-x}$ **d.** $y'' + 3y' + 2y = 2(1 + x) \cdot e^{-x}$

Part B: Numerical Solutions: Earth-Venus Orbital Resonance **(5 points)**

Consider the following coupled-system of first-order differential equations with unknowns $x(t)$ and $y(t)$ which describes the Earth-Venus Orbital Resonance. Here, $R_E = 1.00$ and $R_V = 0.72$ are the radius of the orbits for Earth and Venus respectively in astronomical units. The periods of the orbits are such that Venus completes 13 orbits in the time it takes Earth to complete 8 orbits. For Earth, the angular frequency in radians per year is $\omega_E = 2\pi$ while for Venus the angular frequency is: $\omega_V = 2\pi \cdot \frac{13}{8}$ All these values are given inside the function below. We defined $c = (\omega_V - \omega_E) \cdot R_V = 2.8274$

$$\textbf{DE: } \frac{dx}{dt} = -2\pi \cdot y - c \cdot \sin \omega_V t \quad \frac{dy}{dt} = +2\pi \cdot x + c \cdot \cos \omega_V t \qquad \textbf{IC: } \ x(0) = -0.28, \ y(0) = 0$$

Defining the state vector $\vec{\mathbf{x}} = \begin{bmatrix} x \\ y \end{bmatrix}$ and the initial condition $\vec{\mathbf{x}}(0) = \begin{bmatrix} -0.28 \\ 0.00 \end{bmatrix}$ this system can be represented in the matrix form:

$$\frac{d}{dt}\vec{\mathbf{x}} = A\vec{\mathbf{x}} + \vec{\mathbf{f}}$$ where $A = \begin{bmatrix} 0 & -2\pi \\ 2\pi & 0 \end{bmatrix}$ and $\vec{\mathbf{f}} = c \cdot \begin{bmatrix} -\sin(\omega_V t) \\ +\cos(\omega_V t) \end{bmatrix}$ represents the <u>forcing</u> term.

1-2: Complete this MATLAB <u>function</u> to represent the system in matrix form and return **xdot** (i.e. $\frac{d}{dt}\vec{\mathbf{x}}$) using the above equation. Inside the function, define the matrix *A*, the vector $\vec{\mathbf{f}}$ and compute **xdot**. Save your function file as **diffeq65.m** and paste your code below. **(2 points)**

```
function xdot = diffeq65(t,x)
    % Earth-Venus Orbital Resonance - the Rose of Venus
    Re = 1.00; Rv = 0.72;         % in astronomical units
    we = 2*pi; wv = 2*pi *13/8; % angular frequencies for Earth and Venus
    c = (wv - we) * Rv % 2.8274

    % add A here   ;

    % add f here   ;

    % calculate xdot here   ;
end
```

3-5: Solve this system <u>**numerically**</u> using **ode45** over the time interval from 0 to 8 years with a step size of 0.001 for better accuracy. Don't forget the initial conditions (in vector form) are $x0 = \begin{bmatrix} -0.28 \\ 0.00 \end{bmatrix}$. Plot y (vertical) against x in **blue**. There is no need to use subplot here, since we just want the one plot. Add the title **'Rose of Venus'**. Add a large <u>yellow hexagram</u> at the initial point.

Sample: Your Rose of Venus will resemble this ornament but will be slightly rotated. Paste your filled graph with the Rose of Venus and yellow hexagram in the Answer Template.

Note to Grader: Assign one point each.
1. The **blue** Rose of Venus curve is correct.
2. The yellow hexagram marks the initial point.
3. The title says Rose of Venus.

Rose of Venus

Part C: Exact Solution for 2nd-Order DE using Laplace Transform **(10 points)**

This new DE features a large limiting circle. That circle passes through the point $(-60,0)$. Consider this as our first Initial Value.

DE: $y'' + \frac{1}{6} \cdot y' + y = 12 + 12 \cdot \sin t$ **IC:** $y(0) = -60, \quad y'(0) = 0$

Point 1: Find the Laplace transform $F(s)$ of the forcing term $f(t) = 12 + 12\sin(t)$ and record it in the box below.

Point 1: The transform $F(s)$ is: $F(s) = \ ?$

Point 2: Let $Y(s)$ denote the Laplace transform of the solution $y(t)$ for these initial conditions. Find Y by taking the transform of the above DE. Be sure to plug in both initial values $(-60,0)$. If you apply the **`simplify`** or **`simplifyFraction`** command, you should see the transform Y in the form $Y(s) = N(s)/D(s)$. Provide the missing numerator. The denominator has been provided for you.

Point2: Type in the missing numerator. $Y(s) = \frac{?}{s^3+s}$

Hint, it is a quadratic.

Point 3: Using the **partfrac** command, find the missing coefficient C. Hint: It's negative.

$$Y(s) = \frac{12}{s} + \frac{Cs}{s^2+1}$$

Point 3: ***C*** **= _ _ _ ?**

Point 4: Ack! The final value theorem does not apply in this situation! The solution just keeps oscillating. Nevertheless, apply the **FVT** and calculate what it predicts. Using the **limit** command, and the final value theorem, find the predicted value for $y(t)$ as $t \to \infty$. (Even though this is incorrect.)

Point 4: $\lim_{t\to\infty} y(t) = \_\_\_\_\_$ **? (not true however)**

Point 5: Using the **`ilaplace`** command, and **`matlabFunction`**, find the exact solution $y(t)$ with the given initial condition $(-60,0)$. There is a constant, and a cosine term. The constant is given for you. Give the missing term.

Point 5: $y(t) = 12 - \_\_\_\_$ **?**

Point 6: Use **`matlabFunction`**, **`simplify`** and the **`diff`** command, to find a symbolic expression for $y'(t)$.

Point 6: $y'(t) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$ **?**

Points 7-8: Create just a phase plot showing the exact solution to the DE through $(-60,0)$ by plotting $y'(t)$ versus $y(t)$. Use a **red dotted line** with a thickness of 3. Set the **axis to equal** so you can see the true shape of this circle.

Add a **green-filled circle** at the center point which is $\vec{\mathbf{x}}_{center} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}$. Do not show the starting point here.

Paste your completed phase plot in the Answer Template for credit.

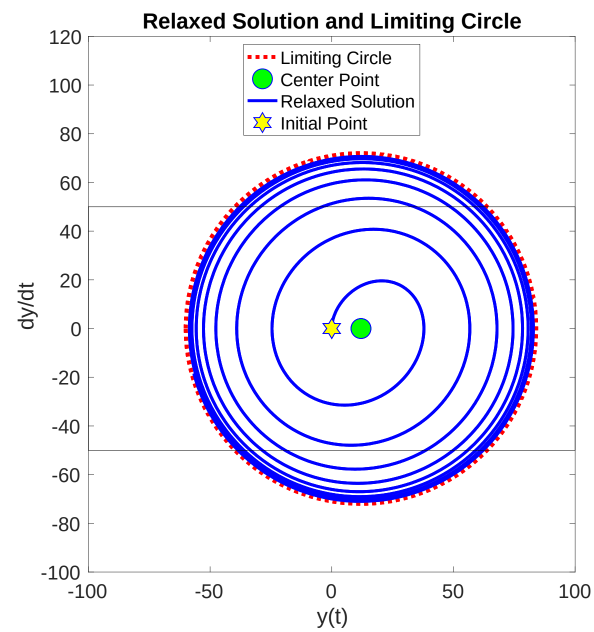
Point 9 and 10: Now add the **relaxed solution**. Everything is the same as before, except the solution now starts off at the origin so that the new initial condition is $y(0) = 0, \quad y'(0) = 0$

Tip: You can clone most of your previous code, but be sure to comment out any `figure` command, so you don't erase the limiting circle. Draw the relaxed solution as a **blue** curve with thickness 3. You will need a large time interval now, say from 0 to 50. Place a **yellow** **hexagram** at the new initial point (0,0).

Combine your plots and add a **legend**. Place both your phase plots on the same figure. You may only need to remove the **figure** command in your code, reset the initial conditions and then rerun your code. Show a legend similar to the sample.

Paste your combined phase plots in the Answer Template.

Sample: This sample actually shows everything that is wanted, but your answer must not include the red rectangle which partially masks the desired graph.



Part D: Exact Solution for Rose of Venus using Laplace Transform in Matrix Form **(5 points)**

Consider once again our Rose of Venus equation in the matrix form:

$\frac{d}{dt}\vec{\mathbf{x}} = A\vec{\mathbf{x}} + \vec{\mathbf{f}}$ where $A = \begin{bmatrix} 0 & -2\pi \\ 2\pi & 0 \end{bmatrix}$ and $\vec{\mathbf{f}} = c \cdot \begin{bmatrix} -\sin(\omega_V t) \\ +\cos(\omega_V t) \end{bmatrix}$ represents the forcing term.

We defined $c = (\omega_V - \omega_E) \cdot R_V = 2.8274$ and $\omega_V = 2\pi \cdot \frac{13}{8}$ and the initial condition $\vec{\mathbf{x}}(0) = \begin{bmatrix} -0.28 \\ 0.00 \end{bmatrix}$

We have seen the solution can be obtained all at once using: $\vec{\mathbf{X}}(s) = (sI - A)^{-1}\,[\vec{\mathbf{x}}(0) + \vec{\mathbf{F}}(s)]$

where $\vec{\mathbf{F}}(s)$ is the laplace transform of the vector $\vec{\mathbf{f}}(t) = c \cdot \begin{bmatrix} -\sin(\omega_V t) \\ +\cos(\omega_V t) \end{bmatrix}$.

Point 1: Find the inverse of the matrix $(sI - A)$. A common factor has been pulled outside the matrix. Enter π as **pi**. Declare s to be a symbol. Two of the missing four elements are multiples of π. Express answers here using s and π.

Point 1: $(sI - A)^{-1} = \frac{1}{s^2+4\pi^2} \cdot \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$

Point 2: Find the Laplace transform $\vec{\mathbf{F}}(s)$ of the forcing vector $\vec{\mathbf{f}}(t) = c \cdot \begin{bmatrix} -\sin(\omega_V t) \\ +\cos(\omega_V t) \end{bmatrix}$

For now, treat the constants c and ω_V as symbols. >> syms c wv

Note a common factor has been given for free. Express answers here using s and ω_V.

Point 2: $\vec{\mathbf{F}}(s) = \frac{c}{s^2+\omega_V{}^2} \cdot \begin{bmatrix} ? \\ ? \end{bmatrix}$

Point 3: Find the **zero-input** solution $\vec{\mathbf{X}}zin(s)$ (in the s domain) using $\vec{\mathbf{X}}zin(s) = (sI - A)^{-1}\,\vec{\mathbf{x}}(0)$ where $\vec{\mathbf{x}}(0) = \begin{bmatrix} -0.28 \\ 0.00 \end{bmatrix}$

Note a common factor has been given for free. Note MATLAB will express –0.28 as –7/25.

Point 3: $\vec{\mathbf{X}}zin(s) = \frac{1}{s^2+4\pi^2} \cdot \begin{bmatrix} ? \\ ? \end{bmatrix}$

Point 4: Find the **zero-state** solution $\vec{\mathbf{X}}zs(s)$ (in the s domain) using $\vec{\mathbf{X}}zs(s) = (sI - A)^{-1}\,\vec{\mathbf{F}}(s)$

The top component was given for free. Use **simplify** or **simplifyFraction**, to see the form asked for.

As before, leave the constants c and ω_V as symbols. >> syms c wv

Point 4: $\vec{\mathbf{X}}zs(s) = \frac{c}{(s^2+4\pi^2)(s^2+\omega_V{}^2)} \cdot \begin{bmatrix} -(2\pi + \omega_V) \cdot s \\ ? \end{bmatrix}$

Point 5: Combine the zero-state solution and the zero-input solution to find the **total** solution $\vec{\mathbf{X}}(s)$.

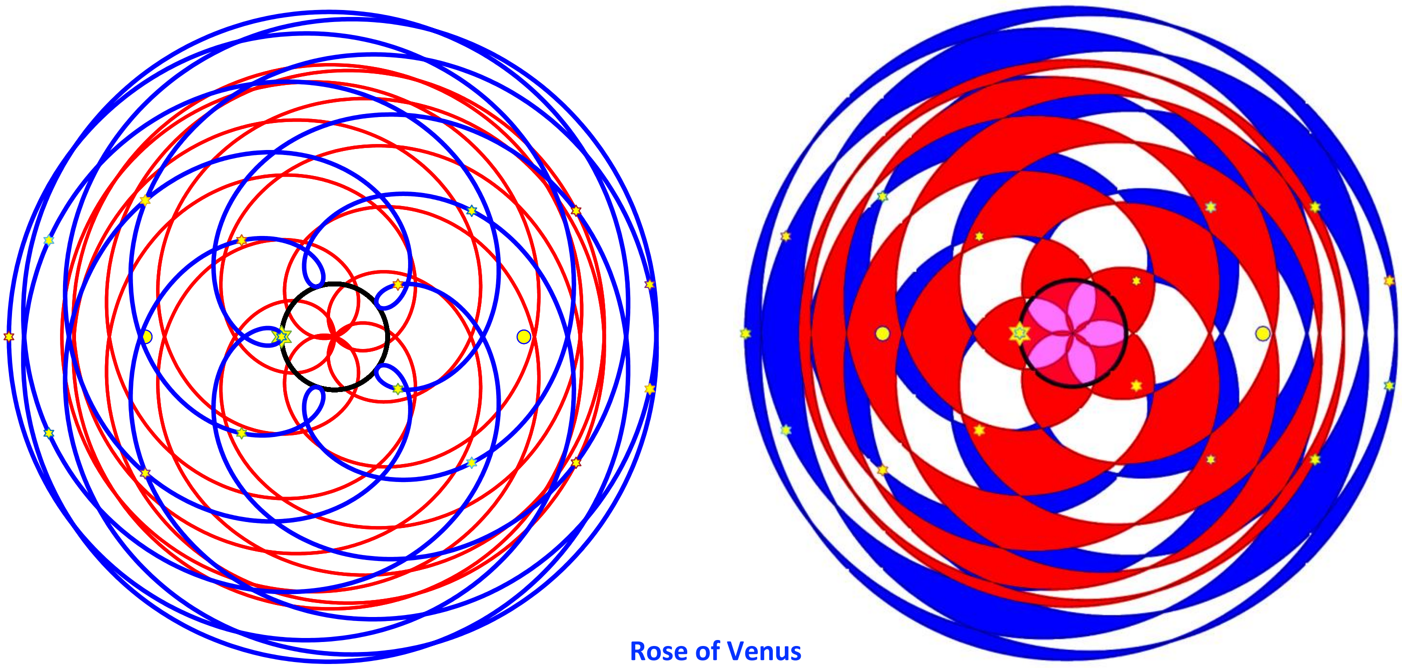
Then take the inverse transform to find the solution in the time domain $\vec{\mathbf{x}}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$.

The **total solution** in the time domain has the form shown below. Find the value of $\vec{\mathbf{x}}(4)$, half-way thru the rose.
Hint: Both entrees are real numbers.

$$\vec{\mathbf{x}}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \frac{18}{25} \cdot \cos\left(\frac{13}{4}\pi\right) t \; - \cos 2\pi t \\ \frac{18}{25} \cdot \sin\left(\frac{13}{4}\pi\right) t \; - \; \sin 2\pi t \end{bmatrix}$$

Point 5: $\vec{\mathbf{x}}(4) = [\;\;]$ **Give both components.**

This figure is not required! Here are two views of the Rose of Venus. The **black** circle is the **zero-input solution**. The **red** flower inside with five-fold symmetry is the **zero-state solution**. The **blue** curve is the **total solution** which you should have produced in section **B**. The view on the right uses **fill** to create a more colorful rose. The yellow stars show the relative location of Venus once every six months. You can see these trace out two circles. There is hidden beauty inside many differential equations. Enjoy!



Rose of Venus

Ready to Submit?

Be sure all questions are answered. When your MATLAB Exam is complete, be sure to submit three files:

1. Your **completed Answer Template** as a PDF file
2. A copy of your **MATLAB Live Script**
3. A **PDF** copy of your **MATLAB Live Script** (Save-Export to PDF…)