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First *Last*

ENGR 232: Dynamic Engineering Systems – Summer 2022

MATLAB Exam - **Version 60C**

ANSWER TEMPLATE FOR EASY SUBMISSION

Part A: Multiple Choice - Just circle the answer for each question in part A.

Warning: Highlighting may disappear when you print the PDF. Don't use highlighting to show your answers!

1. Find the matrix for which $\lambda = 0$ is a repeated eigenvalue. You can just record the answer in the answer template file.

a. $A_1 = \begin{bmatrix} 6 & -6 & 0 \\ 3 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$ **b.** $A_2 = \begin{bmatrix} -2 & 6 & 9 \\ -2 & 5 & 6 \\ 0 & 0 & 1 \end{bmatrix}$ **c.** $A_3 = \begin{bmatrix} -2 & 6 & 14 \\ -2 & 5 & 8 \\ 0 & 0 & 3 \end{bmatrix}$ **d.** $A_4 = \begin{bmatrix} -4 & 10 & 15 \\ -3 & 7 & 8 \\ 0 & 0 & 3 \end{bmatrix}$

2. Here is a new matrix, for which $\lambda = 0$ is a triple eigenvalue: $A = \begin{bmatrix} -24 & 16 & -16 \\ -40 & 28 & -28 \\ -4 & 4 & -4 \end{bmatrix}$

You can check that it has only one independent eigenvector $\vec{\mathbf{v}} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ for this repeated eigenvalue. Find a generalized eigenvector $\vec{\mathbf{w}}$ so that: $(A - \lambda I)\,\vec{\mathbf{w}} = \vec{\mathbf{v}}$ where $\lambda = 0$ and $\vec{\mathbf{v}}$ is the above eigenvector.

a. $\vec{\mathbf{w}} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ **b.** $\vec{\mathbf{w}} = \begin{bmatrix} 3/4 \\ 1/2 \\ 0 \end{bmatrix}$ **c.** $\vec{\mathbf{w}} = \begin{bmatrix} 1/2 \\ 3/4 \\ 0 \end{bmatrix}$ **d.** $\vec{\mathbf{w}} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

3. The function $y(t) = t \cdot \cos t$ is a solution to one of these differential equations. Using **dsolve** & **simplify**, find the DE. Note the given function implies $y(0) = 0$ and $y'(0) = 1$.

a. $y'' + y = t$ **b.** $y'' + y = -2\sin t$ **c.** $y'' + y = 2\cos t$ **d.** $y'' + y = \sin t$

4. Find the coefficient A_2 in the partial fraction: $F(s) = \frac{3125 \cdot s}{(s-2)^3 \cdot (s+3)^3} = \frac{A_1}{s-2} + \frac{A_2}{(s-2)^2} + \frac{A_3}{(s-2)^3} + \frac{B_1}{s+3} + \frac{B_2}{(s+3)^2} + \frac{B_3}{(s+3)^3}$

Be sure you noticed the s in the numerator!

a. $A_2 = -3$ **b.** $A_2 = -5$ **c.** $A_2 = +50$ **d.** $A_2 = +3$ **e.** $A_2 = 20$ **f.** $A_2 = 75$

5. The function $y = x^2 e^{-x}$ is a solution to one of these differential equations. Note the given function implies $y(0) = 0$ and $y'(0) = 0$. Using **dsolve** and **simplify**, find the DE.

a. $y'' + 3y' + 2y = e^{-x}$ **b.** $y'' + 4y' + 3y = 4x \cdot e^{-x}$

c. $y'' + y' = xe^{-x}$ **d.** $y'' + 3y' + 2y = 2(1 + x) \cdot e^{-x}$

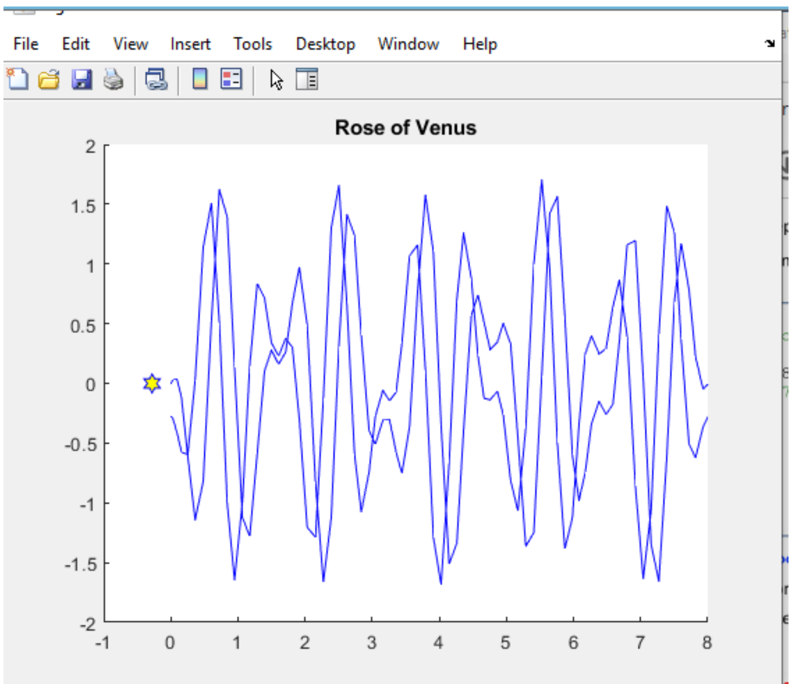
Part B: Numerical Solutions: Earth-Venus Orbital Resonance **(5 points)**

1-2: Complete this MATLAB <u>function</u> to represent the system in matrix form and return **xdot** (i.e. $\frac{d}{dt}\vec{\mathbf{x}}$) using the above equation. Inside the function, define the matrix *A*, the vector $\vec{\mathbf{f}}$ and compute **xdot**.

(2 points)

```
function xdot = diffeq65(t,x)
    % Earth-Venus Orbital Resonance - the Rose of Venus
    Re = 1.00; Rv = 0.72;        % in astronomical units
    we = 2*pi; wv = 2*pi *13/8; % angular frequencies for Earth and Venus
    c = (wv - we) * Rv % 2.8274
    % add A here   ;
    A=[0,-2*pi;2*pi,0];
    % add f here   ;
    f=c*[-1*sin(wv*t);cos(wv*t)];
    % calculate xdot here   ;
    xdot = A*x + f;
end
```

3-5: Paste your filled graph with the Rose of Venus and yellow hexagram here. Replace this sample with your plot!



Part C: Exact Solution for 2nd-Order DE using Laplace Transform **(10 points)**

This new DE features a large limiting circle. That circle passes through the point $(-60,0)$.
Consider this the first initial value.

DE: $y'' + \frac{1}{6} \cdot y' + y = 12 + 12 \cdot \sin t$ **IC:** $y(0) = -60, \quad y'(0) = 0$

Point 1: Find the Laplace transform $F(s)$ of the forcing term $f(t) = 12 + 12\sin(t)$ and record it in the box below.

Point 1: The transform $F(s)$ is: $F(s) = \frac{12}{s} + \frac{12}{s^2+1}$

Point2: Type in the missing numerator. $Y(s) = \frac{\mathbf{-60s^2+12}}{s^3+s}$

Hint, it is a quadratic.

Point 3: Using the **partfrac** command, find the missing coefficient *C*. Hint: It's negative.

$$Y(s) = \frac{12}{s} + \frac{Cs}{s^2+1}$$

Answer: ***C* = -72s** **?**

Point 4: $\lim_{t\to\infty} y(t) = 0$ **? (not true however)**

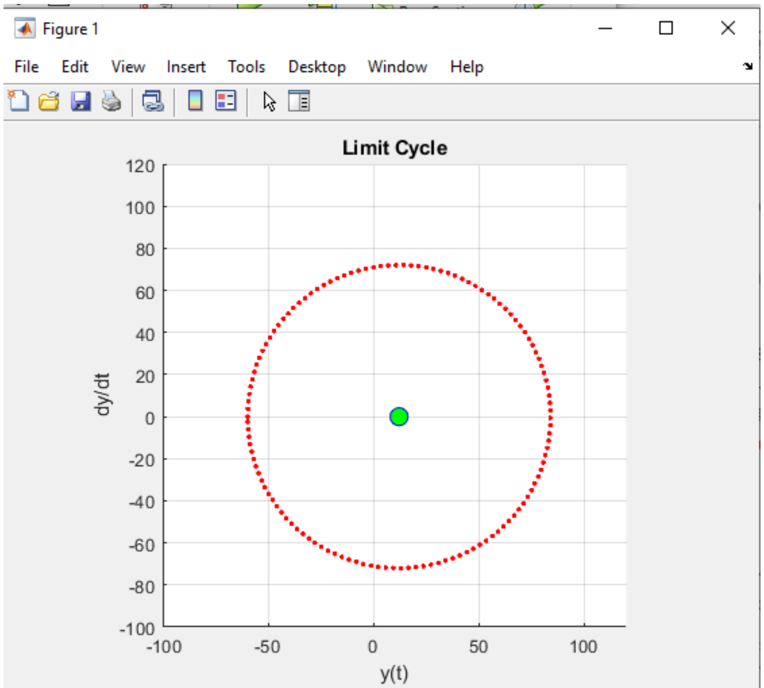
Point 5: Using the `ilaplace` command, and `matlabFunction`, find the exact solution $y(t)$ with the given initial condition $(-60,0)$. There is a constant, and a cosine term. The constant is given for you. Give the missing term.

Point 5: $y(t) = 12 - \mathbf{72cos(t)}$

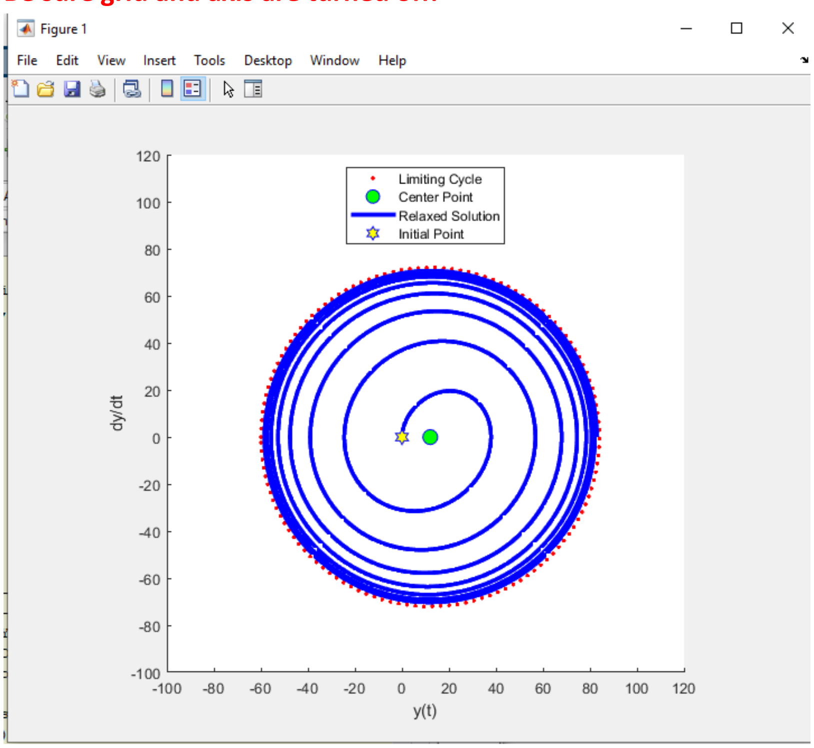
Point 6: Use `matlabFunction`, `simplify` and the `diff` command, to find a symbolic expression for $y'(t)$.

Answer: $y'(t) = -72 * \sin(t)$

Points 7-8: Paste your completed plot with the red limit cirle and yellow hexagram here.



Points 9-10: Paste your combined graph with the black circle and red curve here. Be sure grid and axis are turned off.



Part D: Exact Solution for Rose of Venus using Laplace Transform in Matrix Form **(5 points)**

Point 1: $(sI - A)^{-1} = \frac{1}{s^2+4\pi^2} \cdot \begin{bmatrix} s & -2pi \\ 2pi & s \end{bmatrix}$

Point 2: $\vec{\mathbf{F}}(s) = \frac{c}{s^2+\omega_V{}^2} \cdot \begin{bmatrix} -wv \\ s \end{bmatrix}$

Point 3: $\vec{\mathbf{X}}zin(s) = \frac{1}{s^2+4\pi^2} \cdot \begin{bmatrix} -0.28s \\ -0.56 * pi \end{bmatrix}$

Point 4: $\vec{\mathbf{X}}zs(s) = \frac{c}{(s^2+4\pi^2)(s^2+\omega_V{}^2)} \cdot \begin{bmatrix} -(2\pi + \omega_V) \cdot s \\ s^2 - 2 * pi * wv \end{bmatrix}$

Point 5: $\vec{\mathbf{x}}(4) = \begin{bmatrix} -1.72 \\ 0.00 \end{bmatrix}$ **Give both components.**

Ready to Submit?

Be sure all questions are answered. When your MATLAB Exam is complete, be sure to submit three files:

1. Your **completed Answer Template** as a PDF file
2. A copy of your **MATLAB Live Script**
3. A **PDF** copy of your **MATLAB Live Script** (Save-Export to PDF…)