W7 InLab Activity: **Name: Cole Bardin**

first *last*

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Problem 1: Find the Laplace Transform $F(s)$ for each $f(t)$

Tip: Consult the Table of Laplace Transforms if necessary

a. Find the transform $F(s)$ if $f(t) = e^t + 2e^{-2t} + 3\cos(3t)$

$$F(s) = \frac{1}{s-1} + \frac{2}{s+2} + \frac{3s}{s^2+9}$$

b. Find the transform $F(s)$ if $f(t) = 1 + t + t^2 + t^3$

$$F(s) = \frac{1}{s} + \frac{1}{s^2} + \frac{2}{s^3} + \frac{6}{s^4}$$

c. Find the transform $F(s)$ if $f(t) = 4e^{5t} + 6\sin(7t) + 8$

$$F(s) = \frac{4}{s-5} + \frac{42}{s^2+49} + \frac{8}{s}$$

Problem 2: Apply Frequency Shifting – Fill in all the boxes below.

See identity **29** in the Table of Laplace Transforms.

Frequency Shifting Property
If the transform of $f(t)$ is $F(s)$, then the transform of $g(t) = e^{ct} \cdot f(t)$ is $G(s) = F(s-c)$.

a. Find the transform $G(s)$ if $g(t) = e^{-2t} \cdot \cos(4t)$

i. Give the function $f(t)$ (which does not include the exponential) and its transform $F(s)$.

We see $f(t) =$ cos(4t) and its Laplace Transform is $F(s) = \dfrac{s}{s^2+16}$

ii. So $G(s) = F(s-c) = F(s+2) = \dfrac{s+2}{(s+2)^2+16}$

b. Find the transform $G(s)$ if $g(t) = e^{-4t} \cdot t^4$

i. Give the function $f(t)$ (which does not include the exponential) and its transform $F(s)$.

We see $f(t) =$ t^4 and its Laplace Transform is $F(s) = \dfrac{24}{s^5}$

ii. So $G(s) = F(s-c) = F(s+4) = \dfrac{24}{(s+4)^5}$

c. Find the **inverse** transform $g(t)$ if $G(s) = \frac{3s+12}{s^2+8s+25}$

i. We can frequency shift G as follows: $G(s) = \frac{3(s+4)}{(s+4)^2+9} = F(s+4)$ where $F(s) = \dfrac{3s}{s^2+9}$

ii. Choosing $c = -4$ and find: $f(t) = 3\cos(3t)$ so that $g(t) = e^{ct} f(t) = e^{-4t} * \dfrac{3s}{s^2+9}$

Problem 3: Solve a DE using the Laplace Transform – Fill in all the boxes below

Use Laplace Transforms to solve this differential equation.

DE: $y'' + 5y' + 6y = 2e^{-t}$ **IC:** $y(0) = 0, \quad y'(0) = 1$

Denote the transform of the unknown $y(t)$ as $Y(s)$ as usual.

a. Find the transform of the middle term on the LHS:

$$\mathcal{L}\{5y'\} = 5 \cdot \big(sY - y(0)\big) = \boxed{5sY}$$

b. Find the transform of the double derivative term on the LHS:

$$\mathcal{L}\{y''\} = s\mathcal{L}\{y'\} - y'(0) = \boxed{s^2Y - 1}$$

c. Find the transform of the RHS forcing function $f(t) = 2e^{-t}$.

$$\mathcal{L}\{2e^{-t}\} = \boxed{\frac{2}{s+1}}$$

d. Combine all four terms from the DE and solve for Y. **DE:** $y'' + 5y' + 6y = 2e^{-t}$

$$[s^2Y - 1] \ + \ 5sY \ + \ 6Y \ = \frac{2}{s+1}$$

Collect all the terms that multiply Y on the LHS and the rest on the RHS.

$$\boxed{Y(s^2 + 5s + 6) = \frac{2}{s+1} + \frac{s+1}{s+1}}$$

Solve for Y:

$$(s+2)(s+3)Y \ = \frac{s+3}{s+1}$$

e. Cancel out the common factor of $(s+3)$ to find Y.

$$Y \ = \boxed{\frac{1}{(s+1)(s+2)}}$$

f. Using partial fractions, you would find: $Y \ = \frac{1}{s+1} - \frac{1}{s+2}$

Give the solution in the time domain.

$$y(t) = \boxed{e^{-t} - e^{-2t}}$$