LAB 6: LaPlace Work Shop

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061

Part 1:

(1.1) $Y(s) = \frac{s-18}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$

$s-18 = As - 3A + Bs + 2B$ $\quad A+B=1$

$s-18 = (A+B)s - 3A + 2B$ $\quad -3A+2B=-18$

$B = 1-A$ $\quad -3A + 2(1-A) = -18$

$-3A + 2 - 2A = -18$ $\quad 5A = 20$

$A = 4 \quad B = -3$

$Y(s) = \frac{4}{s+2} - \frac{3}{s-3}$ $\quad y(t) = L^{-1}\left(\frac{4}{s+2} - \frac{3}{s-3}\right)$

$$\boxed{y(t) = 4e^{-2t} - 3e^{3t}}$$

(1.2) $Y(s) = \frac{-3s^2 - 14s + 32}{(s+4)(s^2+4)} = \frac{A}{s+4} + \frac{Bs+C}{s^2+4}$

$-3 \cdot s^2 - 14s + 32 = As^2 + 4A + Bs^2 + Cs + 4Bs + 4C$

$-3 \cdot s^2 - 14 \cdot s + 32 = (A+B)s^2 + (4B+C)s + 4A + 4C$

$A+B=-3 \quad 4B+C=-14 \quad 4A+4C=32$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & -3 \\ 0 & 4 & 1 & -14 \\ 4 & 0 & 4 & 32 \end{array}\right] \left[\begin{matrix} A \\ B \\ C \end{matrix}\right] = \left[\begin{matrix} 2 \\ -5 \\ 6 \end{matrix}\right]$$

solved in MATLAB

$Y(s) = \frac{2}{s+4} + \frac{-5 \cdot s}{s^2+2^2} + \frac{3 \cdot 2}{s^2+2^2}$

$$\boxed{y(t) = 2e^{-4t} - 5\cos(2t) + 3\sin(2t)}$$

(1.3) $Y(s) = \frac{2s-3}{s^2+2s+10} = \frac{2\cdot s - 3}{(s+1)^2+9}$

$Y(s) = 2\cdot\frac{s+1}{(s+1)^2+9} - 5\cdot\frac{3\cdot\frac{1}{3}}{(s+1)^2+9}$

$$\boxed{y(t) = 2\cdot e^{-t}\cdot\cos(3t) - \frac{5}{3}e^{-t}\cdot\sin(3t)}$$

(1.4) $Y(s) = \frac{90}{(s+5)(s+2)^2} = \frac{A}{s+5} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$

$s^2+7s+10 \quad s^2+4s+4$

$90 = As^2 + 4As + 4A + Bs^2 + 7Bs + 10B + Cs + 5C$

$90 = (A+B)s^2 + (4A+7B+C)\cdot s + 4A + 10B + 5C$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 4 & 7 & 1 & 0 \\ 4 & 10 & 5 & 90 \end{array}\right] = \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 10 \\ -10 \\ 30 \end{bmatrix}$$

Solved in MatLab

$n=1$
$a=-2$

$\frac{1!}{(s+2)^2}\cdot 30$

$Y(s) = \frac{10}{s+5} + \frac{-10}{s+2} - \frac{30}{(s+2)^2}$

$$\boxed{Y(t) = 10e^{-5t} - 10e^{-2t} + 30\cdot t\,e^{-2t}}$$

Part 2:

(1.5) $y'' + 6y' + 8y = 0 \quad y(0) = 1 \quad y'(0) = -4$

$s^2Y(s) - s\cdot 1 + 4 + 6sY(s) - 6 + 8Y(s) = 0$

$(s^2+6s+8)Y(s) = s+2$

$Y(s) = \frac{s+2}{(s+2)(s+4)} = \frac{1}{s+4} \quad y(t) = L^{-1}\left\{\frac{1}{s+4}\right\}$

$\boxed{y(t) = e^{-4t}}$

(1.6) $y'' + 6y' + 8y = 0 \quad y(0) = 1 \quad y'(0) = 1$

$s^2Y(s) - s\cdot 1 - 1 + 6sY(s) - 6 + 8Y(s) = 0$

$(s^2+6s+8)Y(s) = s+7$

$Y(s) = \frac{s+7}{(s+2)(s+4)} = \frac{A}{s+2} + \frac{B}{s+4}$

$s+7 = A\cdot s + 4A + Bs + 2B \quad A+B=1 \quad B = 1-A$

$s+7 = (A+B)s + 4A + 2B \quad 4A+2B = 7$

$4A + 2(1-A) = 7 \quad 4A + 2 - 2A = 7 \quad 2A = 5 \quad A = \frac{5}{2}$

$B = -\frac{3}{2}$

$Y(s) = \frac{5/2}{s+2} - \frac{3/2}{s+4} \quad \boxed{y(t) = \frac{5}{2}e^{-2t} - \frac{3}{2}e^{-4t}}$

(1.7) $y'' + 6y' + 8y = 5 \quad y(0) = 1 \quad y'(0) = 1$

$s^2Y(s) - s\cdot 1 - 1 + 6sY(s) - 6 + 8Y(s) = \frac{5}{s}$

$(s^2+6s+8)Y(t) = \frac{5}{s} + s + 7$

$Y(t) = \frac{s^2+7s+5}{s(s+4)(s+2)} = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+2} \qquad s^2+2s \quad s^2+8s$

$s^2+7s+5 = As^2 + 6As + 8A + Bs^2 + 2Bs + Cs^2 + 4Cs$

$s^2+7s+5 = (A+B+C)s^2 + (6A+2B+4C)\cdot s + 8A$

$\begin{bmatrix} 1 & 1 & 1 \\ 6 & 2 & 4 \\ 8 & 0 & 0 \end{bmatrix}\begin{pmatrix} 1 \\ 7 \\ 5 \end{pmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$

$A = 5/8, \quad B = -7/8, \quad C = 5/4$

$Y(s) = \frac{5/8}{s} + \frac{-7/8}{s+4} + \frac{5/4}{s+2}$

$\boxed{y(t) = \frac{5}{8} - \frac{7}{8}\cdot e^{-4t} + \frac{5}{4}\cdot e^{-2t}}$

(1.8) $y'' + 5y' + 6y = 5e^{-5t}$ $\quad y(0)=0,\ y'(0)=0$

$s^2Y(s) - s\cdot 0 - 0 + 5sY(s) - 0 + 6Y(s) = \frac{5}{s+5}$ $\quad s^2+5s+6$

$(s^2+5s+6)Y(s) = \frac{5}{s+5}$

$Y(s) = \frac{5}{(s+5)(s+3)(s+2)} = \frac{A}{s+5} + \frac{B}{s+3} + \frac{C}{s+2}$ $\quad s^2+7s+10$ $\quad s^3+8s+15$

$5 = As^2 + 5As + 6A + Bs^2 + 7Bs + 10B + Cs^2 + 8Cs + 15C$

$5 = (A+B+C)s^2 + (5A+7B+8C)\cdot s + 6A+10B+15C$

$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 5 & 7 & 8 & 0 \\ 6 & 10 & 15 & 5 \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 5/6 \\ -5/2 \\ 5/3 \end{bmatrix}$ solved in Matlab

$Y(s) = \frac{5/6}{s+5} + \frac{-5/2}{s+3} + \frac{5/3}{s+2}$

$\boxed{y(t) = \frac{5}{6}e^{-5t} - \frac{5}{2}e^{-3t} + \frac{5}{3}e^{-2t}}$

(1.9) $y' + 6y = t \quad y(0)=1$

$s\cdot Y(s) - 1 + 6Y(s) = \frac{1}{s^2}$ $\quad Y(s) = \frac{s^2+1}{s^2(s+6)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+6}$

$(s+6)Y(s) = \frac{s^2+1}{s^2}$

$s^2+1 = As^2 + 6As + Bs + 6B + Cs^2$

$s^2+1 = (A+C)s^2 + (6A+B)s + 6B$

$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 6 & 1 & 0 & 0 \\ 0 & 6 & 0 & 1 \end{array}\right] = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$ solved in Matlab

$A = -1/36, \quad B = 1/6, \quad C = 37/36$

$Y(s) = \frac{-1/36}{s} + \frac{1/6}{s^2} + \frac{37/36}{s+6}$

$\boxed{y(t) = -\frac{1}{36} + \frac{1}{6}t + \frac{37}{36}e^{-6t}}$