Lab 7 by Cole Bardin   Section 61

Laplace Workshop

Part A: Definition of the Laplace Transform

1a

```
clc, clear, close all
syms s t; assume(real(s)>0)
f = t;
L = int( exp(-s*t) * f, 0, inf)
```

L =

$$\frac{1}{s^2}$$

```
assume(s,'clear')
```

1b

```
clc, clear, close all
syms s t; assume(real(s)>0)
f = t^2;
L = int( exp(-s*t) * f, 0, inf)
```

L =

$$\frac{2}{s^3}$$

```
assume(s,'clear')
```

1c

```
clc, clear, close all
syms s t; assume(real(s)>5)
f = 3*exp(5*t);
L = simplify(int( exp(-s*t) * f, 0, inf))
```

L =

$$\frac{3}{s-5}$$

```
assume(s,'clear')
```

2a

```
clc, clear, close all
```

```
syms n s t; assume(real(s)>0)
f = t^n;
L = simplify(int( exp(-s*t) * f, 0, inf))
```

L =

$$\frac{\Gamma(n+1)}{s^{n+1}}$$

```
assume(s,'clear')
```

2b

```
clc, clear, close all
syms a s t; assume(s>0) ; assume(a, 'real')
f = sin(a*t);
L = int( exp(-s*t) * f, 0, inf);
L = simplify(L)
```

L =

$$\frac{a}{a^2+s^2}$$

```
assume(s,'clear')
```

2c

```
clc, clear, close all
syms a s t; assume(s>a)
f = exp(a*t);
L = int( exp(-s*t) * f, 0, inf)
```

L =

$$-\frac{1}{a-s}$$

```
assume(s,'clear')
```

3a

```
clc, clear, close all
syms t;
laplace(3*cosh(5*t))
```

ans =

$$\frac{3\,s}{s^2-25}$$

3b

```
clc, clear, close all
```

```
syms t;
u = @ (t) heaviside(t);
laplace((t-3)^2 * u(t-3))
```

ans =

$$\frac{2\,e^{-3\,s}}{s^3}$$

3c

```
clc, clear, close all
syms t;
laplace(t^(0.5))
```

ans =

$$\frac{\sqrt{\pi}}{2\,s^{3/2}}$$

4a

```
clc, clear, close all
syms t;
assume(t, 'real')
ilaplace(sym(1))
```

ans = $\delta(t)$

4b

```
clc, clear, close all
syms s;
ilaplace( (5*s+8)/(16+s^2) )
```

ans = $5\cos(4\,t) + 2\sin(4\,t)$

4c

```
clc, clear, close all
syms s;
ilaplace( 1/(s^(3/2)) )
```

ans =

$$\frac{2\,\sqrt{t}}{\sqrt{\pi}}$$

Part B: Partial fraction expansions

5a

```
clc, clear, close all
syms s;
partfrac( 16/(s^2-8*s))
```

ans =

$$\frac{2}{s-8}-\frac{2}{s}$$

5b

```
clc, clear, close all
syms s;
partfrac( (9*s^2 - 52*s +72)/((s-2)*(s-3)*(s-4)) )
```

ans =

$$\frac{2}{s-2}+\frac{3}{s-3}+\frac{4}{s-4}$$

5c

```
clc, clear, close all
syms s;
partfrac( (3*s^2 - 14*s + 20)/((s-3)^3) )
```

ans =

$$\frac{3}{s-3}+\frac{4}{(s-3)^2}+\frac{5}{(s-3)^3}$$

Part C: Solving a Differential Equation using the Laplace Transform

```
clc, clear, close all
% Question 6
syms y(t);
dy = diff(y,t);
d2y = diff(y,t,t);
DE = d2y + y == 6*sin(2*t);
sol = dsolve(DE, y(0)==0, dy(0)==6)
```

sol = $10\sin(t) - 2\sin(2\,t)$

```
Y = matlabFunction(sol);

% Question 7: Laplace Transform Method for Solution
% a. Define the necessary symbolic variables.
fprintf("Question 7: Solve a DE using the Laplace Transform\n")
```

Question 7: Solve a DE using the Laplace Transform

```
syms s t Y % Now Y(s) denotes the transform of the unknown function y(t).
% b. Find the Laplace transform of y'(t): Y1 = s Y - y(0)
% This is necessary, even though this term does not appear in the LHS
% of the differential equation.
y0 = 0; dy0 = 6; % Initial conditions
f = 6 * sin(2*t) % the forcing function
```

f = $6\sin(2\,t)$

```
disp 'The transform of the derivative is:'
```

The transform of the derivative is:

```
Y1 = s*Y - y0 % Add the initial value y(0)=y0 manually here.
```

Y1 = $Y\,s$

```
% c. Find the Laplace transform of y''(t): Y2 = s Y1 - y'(0)
disp 'The transform of the double derivative is:'
```

The transform of the double derivative is:

```
Y2 = s*Y1 - dy0 % Add the initial value y'(0)=dy0 manually here.
```

Y2 = $Y\,s^2 - 6$

```
% d. Find the Laplace transform F of the forcing term f(t) = 6*sin(2*t)
disp 'The transform F(s) of the forcing term f(t) is:'
```

The transform F(s) of the forcing term f(t) is:

```
F = laplace( f )
```

F =

$$\frac{12}{s^2+4}$$

```
% e. Combine all the terms into the transform of the entire equation,
% which we will name LTofDE for Laplace Transform of DE.
% y'' + y = f(t) with the initial conditions y(0)=y0, y'(0)=dy0
LTofDE = Y2 + Y == F
```

LTofDE =

$$Y\,s^2 + Y - 6 = \frac{12}{s^2+4}$$

```
% f. Use solve to solve this algebraic equation for the unknown Y.
Sol = solve(LTofDE, Y);
Y = matlabFunction(Sol); Y(s)
```

ans =

$$\frac{\frac{12}{s^2+4}+6}{s^2+1}$$

```
Y = partfrac(Y(s)) % express solution in partial fraction form
```

Y =

$$\frac{10}{s^2+1}-\frac{4}{s^2+4}$$

```
% g. Find the inverse Laplace transform of the solution:
sol = ilaplace(Sol,s,t);
y = matlabFunction(sol); y(t) % solution in the time domain
```

ans = $10\sin(t)-2\sin(2\,t)$

Part D: Solve a new DE using the Laplace transform technique

```
clc, clear, close all
% a. Define the necessary symbolic variables.
fprintf("Question 8: Solve a DE using the Laplace Transform\n")
```

Question 8: Solve a DE using the Laplace Transform

```
syms s t Y % Now Y(s) denotes the transform of the unknown function y(t).
% b. Find the Laplace transform of y'(t): Y1 = s Y - y(0)
% This is necessary, even though this term does not appear in the LHS
% of the differential equation.
y0 = 4; dy0 = 2; % Initial conditions
f = 13*exp(-2*t) % the forcing function
```

f = $13\,e^{-2\,t}$

```
disp 'The transform of the derivative is:'
```

The transform of the derivative is:

```
Y1 = s*Y - y0 % Add the initial value y(0)=y0 manually here.
```

Y1 = $Y\,s-4$

```
% c. Find the Laplace transform of y''(t): Y2 = s Y1 - y'(0)
disp 'The transform of the double derivative is:'
```

The transform of the double derivative is:

```
Y2 = s*Y1 - dy0 % Add the initial value y'(0)=dy0 manually here.
```

Y2 = $s\,(Y\,s-4)-2$

```
% d. Find the Laplace transform F of the forcing term f(t) = 6*sin(2*t)
disp 'The transform F(s) of the forcing term f(t) is:'
```

The transform F(s) of the forcing term f(t) is:

```
F = laplace( f )
```

F =

$$\frac{13}{s+2}$$

```
% e. Combine all the terms into the transform of the entire equation,
% which we will name LTofDE for Laplace Transform of DE.
% y'' + y = f(t) with the initial conditions y(0)=y0, y'(0)=dy0
LTofDE = Y2 + Y1 + (5/4)*Y == F
```

LTofDE =

$$\frac{5\,Y}{4} + Y\,s + s\,(Y\,s - 4) - 6 = \frac{13}{s+2}$$

```
% f. Use solve to solve this algebraic equation for the unknown Y.
Sol = solve(LTofDE, Y);
Y = matlabFunction(Sol);
simplifyFraction(Y(s))
```

ans =

$$\frac{4\,(4\,s^2 + 14\,s + 25)}{(s+2)\,(4\,s^2 + 4\,s + 5)}$$

```
Y = partfrac(simplifyFraction(Y(s))) % express solution in partial fraction form
```

Y =

$$\frac{4}{s+2} + \frac{40}{4\,s^2 + 4\,s + 5}$$

```
% g. Find the inverse Laplace transform of the solution:
sol = ilaplace(Sol,s,t);
y = matlabFunction(sol); y(t) % solution in the time domain
```

ans =

$$4\,e^{-2\,t} + 10\,e^{-\frac{t}{2}}\,\sin(t)$$