Lab 9 by Cole Bardin   Section 61

Matrix Laplace Method and Linear Systems

Part A: The Harmonic Oscillator

```
clc, clear, close all
syms s;
A = [0, 1; -1, 0]; x0 = [3;4];
% a
Rs = inv(s*eye(2)-A)
```

Rs =

$$\begin{pmatrix} \frac{s}{s^2+1} & \frac{1}{s^2+1} \\ -\frac{1}{s^2+1} & \frac{s}{s^2+1} \end{pmatrix}$$

```
% b
Ts = ilaplace(Rs)
```

Ts =

$$\begin{pmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{pmatrix}$$

```
% c
x = Ts*x0
```

x =

$$\begin{pmatrix} 3\cos(t) + 4\sin(t) \\ 4\cos(t) - 3\sin(t) \end{pmatrix}$$

```
% d. Plot the solution x2 (vertical axis) versus x1
x1 = matlabFunction(x(1))
```

```
x1 = function_handle with value:
    @(t)cos(t).*3.0+sin(t).*4.0
```

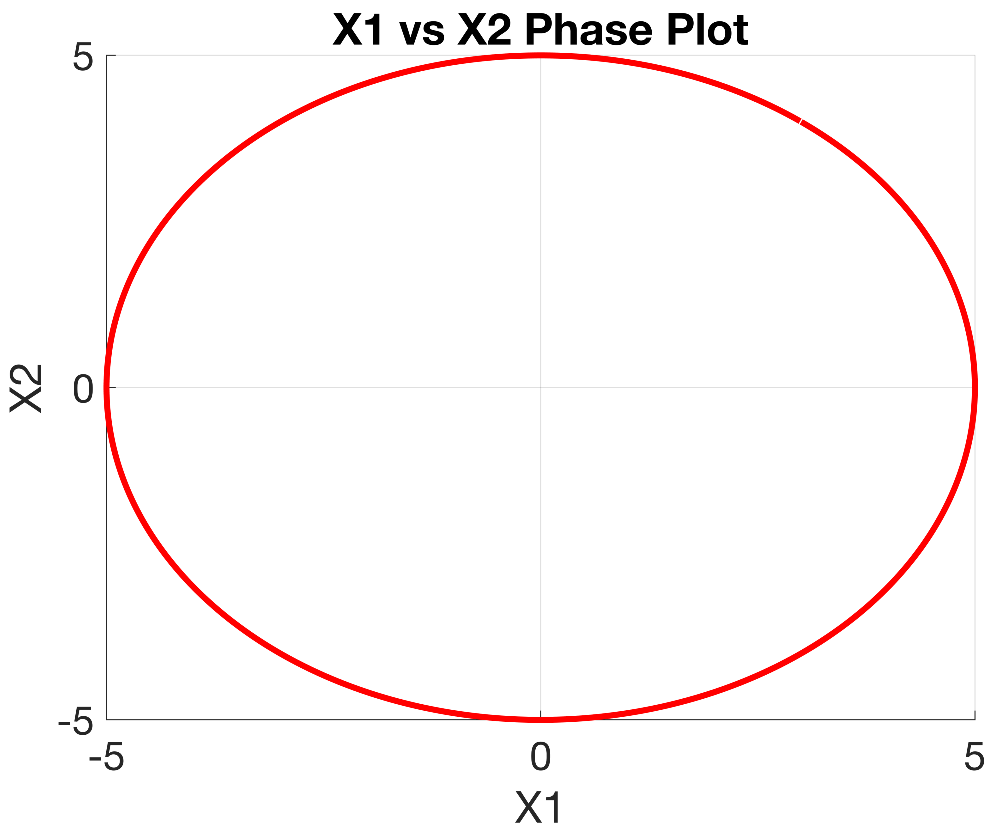
```
x2 = matlabFunction(x(2))
```

```
x2 = function_handle with value:
    @(t)cos(t).*4.0-sin(t).*3.0
```

```
time = 0:0.01: 2*pi;

% e
grid on; hold on
set(gca,'FontSize', 20)
plot(x1(time), x2(time), 'r', "LineWidth", 3)
```

```
xlabel("X1")
ylabel("X2")
title("X1 vs X2 Phase Plot")
```



Part B: Falling Apple, Nonhomogeneous Equation (No friction)

```
clc, clear, close all
syms s h v0 g t;
A = [0,1;0,0]; x0 = [h;v0];
% a
R = inv(s*eye(2)-A)
```

R =

$$\begin{pmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{pmatrix}$$

```
% b
T = ilaplace(R)
```

T =

$$\begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$$

```
% c
```

```
f = sym([0;-g]);
F=laplace(f,t,s)
```

F =

$$\begin{pmatrix} 0 \\ -\frac{g}{s} \end{pmatrix}$$

```
% d
xzi = T*x0
```

xzi =

$$\begin{pmatrix} h + t\, v_0 \\ v_0 \end{pmatrix}$$

```
% e
xzs = ilaplace(R*F)
```

xzs =

$$\begin{pmatrix} -\frac{g\, t^2}{2} \\ -g\, t \end{pmatrix}$$

```
% f
x = xzi + xzs
```

x =

$$\begin{pmatrix} -\frac{g\, t^2}{2} + v_0\, t + h \\ v_0 - g\, t \end{pmatrix}$$

Part C: Two Tanks – Laplace Matrix Method

```
clc, clear, close all
syms s t k;
A = (1/12)*[-4,1;4,-4];
q0=[2;4];
b = [6*k;0];
B = laplace(b, t, s)
```

B =

$$\begin{pmatrix} \frac{6\, k}{s} \\ 0 \end{pmatrix}$$

```


% a
R = inv(s*eye(2)-A)
```

R =

$$\begin{pmatrix} \frac{4\,(3\,s+1)}{12\,s^2+8\,s+1} & \frac{1}{12\,s^2+8\,s+1} \\ \frac{4}{12\,s^2+8\,s+1} & \frac{4\,(3\,s+1)}{12\,s^2+8\,s+1} \end{pmatrix}$$

```
% b
T = ilaplace(R)
```

T =

$$\begin{pmatrix} \frac{e^{-\frac{t}{2}}}{2}+\frac{e^{-\frac{t}{6}}}{2} & \frac{e^{-\frac{t}{6}}}{4}-\frac{e^{-\frac{t}{2}}}{4} \\ e^{-\frac{t}{6}}-e^{-\frac{t}{2}} & \frac{e^{-\frac{t}{2}}}{2}+\frac{e^{-\frac{t}{6}}}{2} \end{pmatrix}$$

```
% c
qzi = T*q0
```

qzi =

$$\begin{pmatrix} 2\,e^{-\frac{t}{6}} \\ 4\,e^{-\frac{t}{6}} \end{pmatrix}$$

```
% d
qzs = ilaplace(R*B)
```

qzs =

$$\begin{pmatrix} 24\,k-6\,k\,e^{-\frac{t}{2}}-18\,k\,e^{-\frac{t}{6}} \\ 24\,k+12\,k\,e^{-\frac{t}{2}}-36\,k\,e^{-\frac{t}{6}} \end{pmatrix}$$

```
% e
x = qzi + qzs
```

x =

$$\begin{pmatrix} 24\,k+2\,e^{-\frac{t}{6}}-6\,k\,e^{-\frac{t}{2}}-18\,k\,e^{-\frac{t}{6}} \\ 24\,k+4\,e^{-\frac{t}{6}}+12\,k\,e^{-\frac{t}{2}}-36\,k\,e^{-\frac{t}{6}} \end{pmatrix}$$

Part D: The Rose of Venus

```
clc, clear, close all
syms s t c wv we;
%we=2*pi; wv=2*pi*(18/3);
Re=1; Rv=0.72;
A = [0, -2*pi;2*pi, 0]; x0=[-0.28;0.00];
%c=(wv-we)*Rv;
f=c*[-sin(wv*t);cos(wv*t)];
xe=[cos(we*t);sin(we*t)];
xv=[cos(wv*t);sin(wv*t)];
z=xv-xe;
% a
R=inv(s*eye(2)-A)
```

R =

$$\begin{pmatrix} \frac{s}{s^2+4\,\pi^2} & -\frac{2\,\pi}{s^2+4\,\pi^2} \\ \frac{2\,\pi}{s^2+4\,\pi^2} & \frac{s}{s^2+4\,\pi^2} \end{pmatrix}$$

```
% b
F = laplace(f,t,s)
```

F =

$$\begin{pmatrix} -\frac{c\,\mathrm{wv}}{s^2+\mathrm{wv}^2} \\ \frac{c\,s}{s^2+\mathrm{wv}^2} \end{pmatrix}$$

```
% c
xzi=R*x0
```

xzi =

$$\begin{pmatrix} -\frac{7\,s}{25\,\left(s^2+4\,\pi^2\right)} \\ -\frac{14\,\pi}{25\,\left(s^2+4\,\pi^2\right)} \end{pmatrix}$$

```
% d
xzs=R*F
```

xzs =

$$\begin{pmatrix} -\frac{2\,\pi\,c\,s}{\sigma_1} - \frac{c\,s\,\mathrm{wv}}{\sigma_1} \\ \frac{c\,s^2}{\sigma_1} - \frac{2\,\pi\,c\,\mathrm{wv}}{\sigma_1} \end{pmatrix}$$

where

$$\sigma_1 = (s^2 + \mathrm{wv}^2)\;(s^2 + 4\,\pi^2)$$

```
% e
x = matlabFunction(simplify(ilaplace(xzi+xzs)))
```

```
x = function_handle with value:
    @(c,t,wv)[(c.*cos(t.*wv))./(wv-pi.*2.0)-(cos(t.*pi.*2.0).*(c.*2.5e+1+wv.*7.0-pi.*1.4e+1))./(wv.*2.5e+1
```

```
x(2.8274,4,2*pi*(18/3))
```

```
ans = 2×1
   -0.2800
   -0.0000
```