Drexel University
Office of the Dean of the College of Engineering
ENGR 232 – Dynamic Engineering Systems

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First *Last*

Lab 9 Answer Template: Matrix Laplace Method and Linear Systems **Winter 2022**

Part A: Undamped Harmonic Oscillator **2/2**

TA will randomly pick two of these to grade.

a. Find the resolvent $R(s) = (sI - A)^{-1}$ for the harmonic oscillator. One component has been given for you.

$$R(s) = \frac{1}{s^2+1} \cdot \begin{bmatrix} s & 1 \\ -1 & s \end{bmatrix}$$

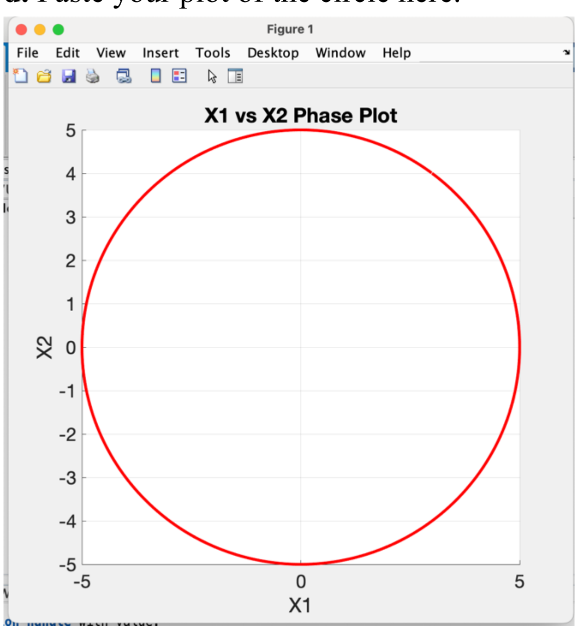
b. Find the state-transition matrix $\Phi(t) = \mathcal{L}^{-1}((sI - A)^{-1}) = \mathcal{L}^{-1}(R)$. One component has been given for you.

$$\Phi(t) = \begin{bmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{bmatrix}$$

c. Find the solution $\vec{\mathbf{x}}(t)$ at any time t using the state-transition matrix $\Phi(t)$ and the initial condition $\vec{\mathbf{x}}(0) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$. The top component has been given for you.

$$\vec{\mathbf{x}}(t) = \Phi(t)\,\vec{\mathbf{x}}(0) = \begin{bmatrix} 3\cos(t) + 4\sin(t) \\ 4\cos(t) - 3\sin(t) \end{bmatrix}$$

d. Paste your plot of the circle here.



Part B: Falling Apple, Nonhomogeneous Equation (No friction)

3/3

TA will randomly pick <u>three</u> of these to grade.

a. Find the **resolvent** $R(s) = (sI - A)^{-1}$ for the falling apple. One component has been given for you.

$$R(s) = \begin{bmatrix} 1/s & 1/s^2 \\ 0 & 1/s \end{bmatrix}$$

b. Find the **state-transition matrix** $\Phi(t) = \mathcal{L}^{-1}((sI - A)^{-1}) = \mathcal{L}^{-1}(R)$ for the falling apple. One component is free!

$$\Phi(t) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

c. Find the transform $\vec{\mathbf{F}}(s)$ of the forcing vector $\vec{\boldsymbol{f}}(t) = \begin{bmatrix} 0 \\ -g \end{bmatrix}$.

$$\vec{\mathbf{F}}(s) = \begin{bmatrix} 0 \\ -g/s \end{bmatrix}$$

d. Give the **zero-input solution** in the <u>**time**</u> domain only.
Leave h, g and v_0 as symbolic quantities. One component given for free.

$$\vec{\mathbf{x}}_{zi}(t) = \mathcal{L}^{-1}\left(\frac{1}{s^2} \cdot \begin{bmatrix} sh + v_0 \\ sv_0 \end{bmatrix}\right) = \begin{bmatrix} h + tv_0 \\ v_0 \end{bmatrix}$$

e. Give the **zero-state solution** in the <u>**time**</u> domain only.
Leave h, g and v_0 as symbolic quantities. One component given for free and g has been factored outside.

$$\vec{\mathbf{x}}_{zs}(t) = \mathcal{L}^{-1}\left(-\frac{g}{s^3} \cdot \begin{bmatrix} 1 \\ s \end{bmatrix}\right) = -g\begin{bmatrix} t^2/2 \\ t \end{bmatrix}$$

f. Combine the zero-input and zero-state solution to obtain the **total solution** in the time domain.
Leave h, g and v_0 as symbolic quantities. One component given for free.

$$\vec{\mathbf{x}}_{\mathbf{total}}(t) = \vec{\mathbf{x}}_{zi}(t) + \vec{\mathbf{x}}_{zs}(t) = \begin{bmatrix} h + tv_0 - 0.5gt^2 \\ v_0 - gt \end{bmatrix}$$

Part C: Two Tanks – Laplace Matrix Method 2/2

TA will randomly pick two of these to grade.

a. Find the **resolvent** $R(s) = (sI - A)^{-1}$ for the two-tank system. A common denominator and one component have been given for you.

$$R(s) = \frac{1}{12s^2 + 8s + 1} \cdot \begin{bmatrix} 4 + 12s & 1 \\ 4 & 4 + 12s \end{bmatrix}$$

b Give the **state-transition matrix** $\Phi(t) = \mathcal{L}^{-1}((sI - A)^{-1})$ for the two-tank system. The first column has been given for you.

$$\Phi(t) = \begin{bmatrix} \frac{1}{2} \cdot e^{-\frac{t}{2}} + \frac{1}{2} \cdot e^{-t/6} & e^{-t/6} * \frac{1}{4} - e^{-\frac{t}{2}} * \frac{1}{4} \\ -e^{-\frac{t}{2}} + e^{-t/6} & e^{-t/2} * \frac{1}{2} + e^{-\frac{t}{6}} * \frac{1}{2} \end{bmatrix}$$

c. Give the **zero-input solution** in the **time** domain only. One component given for free.

$$\vec{\mathbf{q}}_{\mathbf{zi}}(t) = 2e^{-t/6} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

d. Give the **zero-state solution** in the **time** domain only. One component given for free and k has been factored outside.

$$\vec{\mathbf{q}}_{\mathbf{zs}}(t) = k \cdot \begin{bmatrix} 24 - 6e^{-t/2} - 18e^{-t/6} \\ 24 + 12e^{-t/2} - 38e^{-t/6} \end{bmatrix}$$

e. Give the **total solution** in the **time** domain only. Fill in all the missing components.

$$\vec{\mathbf{q}}_{\text{total}}(t) = \vec{\mathbf{q}}_{\mathbf{zi}}(t) + \vec{\mathbf{q}}_{\mathbf{zs}}(t) = 2e^{-\frac{t}{6}} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} + k \cdot \begin{bmatrix} 24 - 6e^{-t/2} - 18e^{-t/6} \\ 24 + 12e^{-t/2} - 38e^{-t/6} \end{bmatrix}$$

Part D: The Rose of Venus **3/3**

TA will randomly pick three of these to grade.

a. Find the **resolvent** $R(s) = (sI - A)^{-1}$ for the Rose of Venus. A common factor and the first row has been given for you. Express answers here using *s* and π.

$$R(s) = (sI - A)^{-1} = \frac{1}{s^2+4\pi^2} \cdot \begin{bmatrix} s & -2\pi \\ 2\pi & s \end{bmatrix}$$

b. Find the Laplace transform $\vec{\mathbf{F}}(s)$ of the forcing vector $\vec{\mathbf{f}}(t) = c \cdot \begin{bmatrix} -\sin(\omega_V t) \\ +\cos(\omega_V t) \end{bmatrix}$

$$\vec{\mathbf{F}}(s) = \frac{c}{s^2 + {\omega_V}^2} \cdot \begin{bmatrix} -\omega_V \\ s \end{bmatrix}$$

c. Find the **zero-input** solution $\vec{\mathbf{X}}zin(s)$ (in the *s* domain) using $\vec{\mathbf{X}}zin(s) = (sI - A)^{-1}\,\vec{\mathbf{x}}(0)$

$$\vec{\mathbf{X}}zin(s) = \frac{1}{s^2 + 4\pi^2} \cdot \begin{bmatrix} -\frac{7s}{25} \\ -\frac{14\pi}{25} \end{bmatrix} = \frac{1}{s^2 + 4\pi^2} \cdot \begin{bmatrix} -0.28\,s \\ -1.76 \end{bmatrix}$$

d. Find the **zero-state** solution $\vec{\mathbf{X}}zs(s)$ (in the s domain) using $\vec{\mathbf{X}}zs(s) = (sI - A)^{-1}\,\vec{\mathbf{F}}(s)$

$$\vec{\mathbf{X}}zs(s) = \frac{c}{(s^2 + 4\pi^2)(s^2 + {\omega_V}^2)} \cdot \begin{bmatrix} -(2\pi + \omega_V) \cdot s \\ s^2 - 2\pi\omega_V \end{bmatrix}$$

e. Find the value of $\vec{\mathbf{x}}(4)$, half-way thru the rose.

$$\vec{\mathbf{x}}(4) = \begin{bmatrix} -0.28 \\ 0.00 \end{bmatrix}$$

You do not need to plot the **Rose of Venus**!

Ready to Submit?

Be sure all ten questions are answered. When your lab is complete, be sure to submit three files:

1. Your **completed Answer Template** as a PDF file
2. A copy of your **MATLAB Live Script**
3. A **PDF** copy of your **MATLAB Live Script** (Save-Export to PDF…)

The due date is the day after your lab section by **11:59pm** to receive full credit. You have one more day, to submit the lab (but with a small penalty), and then the window closes for good and your grade will be zero.