MECH 539: Computational Aerodynamics
Department of Mechanical Engineering, McGill University

Project #1: Numerical Methods for the Linear-Advection Equation
Due 4th. February, 2020

Solve the one-dimensional wave equation (linear-advection equation)

$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial u}{\partial x} = 0.$$

Use the following schemes upwind, Lax, Lax–Wendroff, Leap-Frog, and MacCormack, for the initial condition

$$u = \frac{1}{2}\left(1 + \tanh[250(x-20)]\right), \qquad 0 \leq x \leq 40$$

and exact Dirichlet boundary conditions. Choose initially 41 grid point mesh with $\Delta x = 1$, and compute to $t = 10$.

1. Solve this problem for all three methods for $\Delta t = 1.0$ and $\Delta t = 0.5$, and compare graphically with the exact stationary solution. Discuss the differences between the various solutions. [10 points]
2. Perform a grid study in space and time with two of the schemes of your choice. [Note: Use grids of successive refinement starting with the 41 grid point mesh as the coarsest grid. [2 points]
3. Demonstrate that the order of accuracy of the scheme on a plot where the y-axis is the log of the error and the x-axis is Δx. [1 point]
4. Select two of the schemes and derive its stability condition. Demonstrate through numerical experiments for both schemes that the derived stability condition confirms what is observed. Note any discrepancies. [4 point]
5. Select one of the schemes and demonstrate that its consistent. [3 points]