

Homework 4 Calculations

$P(e)$
0.25

e	$P(f e)$
true	0.5
false	0.1

f	$P(a f)$
true	0.8750
false	0.2500

f	$P(t f)$
true	0.7500
false	0.0625

a. What is the marginal probability that a random student is enrolled in 383?

Marginal Probability is the degree of belief that an event is true, regardless of all other events. In the Primer assignment, we found that 25% of the students were enrolled in 383. Because E is not dependent on any other factor (A , F or T), we can assume that the probability that the student is enrolled, regardless of all the other events, is also 25%.

b. Calculate the marginal probability of a random student testing positive or 383 Fever.

Tested is dependent on Fever (Fever is dependent on Enrolled). We can use the law of total probability:

$$P(A) = P(A \cap B) + P(A \cap B^c)$$

and using the definition of conditional probability, $P(A \cap B) = P(A|B)P(B)$, we can write

$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c).$$

From <https://www.probabilitycourse.com/>, a textbook used in ECE 213 Probability and Statistics.

$$P(T) = P(T = \text{True} | F) P(F) + P(T = \text{False} | F) P(F')$$

But F is also dependent on E, so we must use this law of total probability to find P(F) first:

$$P(F) = P(F = \text{True} | E) P(E) + P(F = \text{False} | E) P(E')$$

$$= (0.5 \times 0.25) + (0.1 \times 0.75) = 0.15$$

Now we can apply this to the previous equation:

$$P(T) = (0.75 \times 0.15) + (0.065 \times 0.85) = 0.16775 \approx 0.17$$

c. Calculate the marginal probability of a random student being excessively awesome

We will take a similar approach to this that we used in the previous problem by using the law of total probability again.

$$P(A) = P(A = \text{True} | F)P(F) + P(A = \text{False} | F) P(F')$$

We already found $P(F) = 0.15$:

$$P(A) = (0.875 \times 0.15) + (0.25 \times 0.85) = 0.34375$$

d. Calculate the joint probability of a random student being excessively awesome *and* testing positive for 383 Fever.

Awesome and Tested are not connected and therefore independent:

$$P(A \wedge T) = P(A) * P(T) = 0.375 * 0.2 = 0.075$$

e. Calculate the probability of a random student being excessively awesome *or* testing positive for 383 Fever.

Awesome and Tested are not connected and therefore independent:

$$P(A \vee T) = P(A) + P(T) = 0.375 + 0.2 = 0.575$$

f. Calculate the conditional probability of a random student being enrolled in 383 given that they are excessively awesome.

$$P(E|A) = \frac{P(E,A)}{P(A)} = \frac{0.158333}{0.375} = 0.422.$$

Note that the values for $P(E, A)$ and $P(A)$ were found in the Primer assignment using an Excel sheet.

g. Calculate the conditional probability of a random student being enrolled in 383 given that they are excessively awesome and tested positive.

$$P(E | A,T) = \frac{P(E,A,T)}{P(A,T)} = \frac{0.108333}{0.175} = 0.62$$

