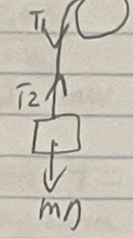
HW 10 I

1. a.



FBD for the problem

First find the tangential acceleration to the Pulleys rim of the stone using the second kinematic equation

starts from rest so 0

$$y = v_0 t + \frac{at^2}{2}$$

$$12.4 = \frac{(4)^2 \times a}{2}$$

$$24.8 = 16a$$

$$a = 1.55 \ m/s^2$$

The tangential acceleration of the stone is equivalent to the angular acceleration of the pulley and its radius:

$$a = R\alpha$$

$$\alpha = \frac{a}{R} = \frac{1.55}{.27} = 5.74 \ rad/s^2$$

The torque exerted by the tension in the wire can be expressed by both equations:

$$\tau = T \times R$$

tangential force

$$\tau = I\alpha$$

since the pulley is a uniform disk, its moment of inertia can be expressed as:

$$I = \frac{mR^2}{2} = \frac{10 \times (.27)^2}{2} = .3645 \ kg \cdot m^2$$

Set it back to other τ form

$$\tau = .3645 \times 5.74 = 2.092 \ m^2 \cdot rad/s^2$$

$$TR = 2.092$$

$$T = \frac{2.092}{.27} = 7.75\ N$$

Newton's second law $\Sigma F = ma$ apply to forces on stone.

$$mg - T = ma$$
$$mg - ma = T$$
$$m(g-a) = T$$
$$m = \frac{T}{g-a} = \frac{7.75}{9.81-1.55} = \boxed{.939\ kg}$$

b. The tension in the wire was found when solving part a = $\boxed{7.75\ N}$

2. a. Since both spheres use the same equation to find rotational kinetic energy:

$$\frac{I\omega^2}{2}$$

The one with the greatest moment of inertia will require a greater magnitude of work to stop.

Moment of inertia for solid sphere:

$$\frac{2MR^2}{5} = \frac{2 \times 5 \times (.12)^2}{5} = .0288\ kg \cdot m^2$$

Moment of inertia for hollow sphere:

$$\frac{2MR^2}{3} = \frac{2 \times 5 \times (.12)^2}{3} = .048\ kg \cdot m^2$$

Since .048 > .0288 the $\boxed{\text{hollow sphere}}$ requires a greater magnitude of work.

b. Both spheres have translational kinetic energy that can be expressed as:

$$\frac{Mv^2}{2}$$

$$\frac{5 \times 4^2}{2} = 40J$$

Now find the angular velocity for the solid sphere: from $V = WR$

$$W = \frac{V}{R} = \frac{4}{.120} = 33.33 \text{ rad/s}$$

Now find rotational kinetic energy with moment of inertia and angular velocity:

$$\frac{.0288 \times 33.33^2}{2} = 16J$$

Total KE solid sphere $= 16 + 40 = \boxed{56.0J}$

C. Rotational velocity and the translational kinetic energy for the hollow sphere is the same as for the solid sphere so substitute moment of inertia:

$$\frac{.048 \times 33.33^2}{2} = 26.7J$$

Total KE hollow sphere $= 26.7 + 40 = \boxed{66.7J}$

3. According to conservation of energy, the hoops initial gravitational potential energy should be converted into translational kinetic energy and rotational kinetic energy as it unwinds:

$$PE_g = KE_{trans} + KE_{rot}$$

$$mgh = \frac{mv^2}{2} + \frac{I\omega^2}{2}$$

moment of inertia for hoop $= MR^2$ and $w = \frac{V}{R}$:

$$mgh = \frac{mv^2}{2} + \frac{MR^2}{2}\left(\frac{V}{R}\right)^2 =$$

$$mgh = \frac{mv^2}{2} + \frac{mv^2}{2} = mv^2$$

$$mgh = mv^2$$

$$v^2 = gh$$

$$v = \sqrt{gh} = 2.53 \text{ m/s}$$

$$\omega = \frac{v}{R} = \frac{2.53}{.08} \approx \boxed{31.5 \text{ rad/s}}$$

b. Since I know angular velocity I can calculate the speed of its center using the equation:

$v = \omega R$ which I used to find ω

$$v = 31.5 \times .08 = \boxed{2.52 \text{ m/s}}$$

4. a. Since the child applies a tangential force the torque can be expressed as:

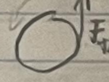
$$\tau = F_{tangential} \times R$$

$$\tau = 24 \times 2.4 = 57.6 \text{ N·m}$$

Using the equivalent expression for torque I can find the angular acceleration since the moment of inertia is given:

$$\tau = I\alpha$$

FBD



$$57.6 = 2400\,\alpha$$

$$\alpha = .024 \text{ rad/s}^2$$

Using rotational kinematic equation I can find the angular velocity:

$\omega_0 = 0$ starts from rest

$$\omega = \omega_0 + \alpha t$$

$$\omega = .024 \times 15 = \boxed{.360 \text{ rad/s}}$$

b. Since $W = \Delta KE$ and there is only rotational KE,

$$W = KE_{rotf} - KE_{roti} = \frac{I\omega^2}{2} - \frac{I\omega_0^2}{2}^{=0} = \frac{2400 \times .36^2}{2} - 0 \approx \boxed{156 \text{ J}}$$

c. Power is merely change in work / change in time, and work was just found and t is provided:

$$P = \frac{\Delta W}{\Delta t} = \frac{156}{15} = \boxed{10.4 \text{ W}}$$

5. Since it takes 2π radians for a clockhand to make a full rotation and its the second hand so I also know it takes 60 seconds I can find the angular velocity:

$$w = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{6} = \frac{\pi}{30} \text{ rad/s}$$

For a slender rod the moment of inertia is given by:

$$I = \frac{Ml^2}{3} = \frac{.006 \times .15^2}{3} = 4.50 \times 10^{-5} \text{ Kg}\cdot\text{m}^2$$

Angular moment is the product of angular velocity and the moment of inertia:

$$L = 4.5 \times 10^{-5} \times \frac{\pi}{30} \approx \boxed{4.71 \times 10^{-6} \text{ Kg}\cdot\text{m}^2/\text{s}}$$

6. First convert the days to seconds:

$$33 \text{ days} \times \frac{24 \text{ hrs}}{1 \text{ day}} \times \frac{3600 \text{ s}}{\text{hr}} = 2.8512 \times 10^6 \text{ s}$$

Since no external torque acts on the star during its collapse its initial and final momentum should be equivalent:

$L = I w$ moment of inertia for solid sphere $= \frac{2 \times M \times R^2}{5}$

$$\frac{2MR_i^2}{5} \times w_0 = \frac{2MR_f^2}{5} \times w_1$$

$$R_i^2 w_0 = R_f^2 w_1$$

$$w_1 = w_0\left(\frac{R_i^2}{R_f^2}\right)$$

$$w_0 = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{2.8512 \times 10^6} = 2.2 \times 10^{-6}$$

$$w_1 = 2.2 \times 10^{-6} \times \left(\frac{9 \times 10^5}{18}\right)^2 = 2.2 \times 10^{-6} \times (5 \times 10^4)^2 =$$

$(2.2 \times 10^{-6}) \times (2.5 \times 10^{9}) = \boxed{5500 \text{ rad/s}}$

7.a, The initial angular momentum comes from only the mud. Since the mud hits the center of the door its distance from the axis of rotation is half the door's width: $r = \frac{w}{2} = \frac{1}{2} = .5$

The linear momentum of the mud $p = mv = .7 \times 13 = 9.10$ kg·m/s

The angular momentum can then be found with the cross product of the linear velocity and the radius:

$L = r \cdot p \cdot \sin(\theta)$ hits perpendicular so theta=90°

initial $L = 9.10 \times .5 = 4.55$ kg·m²/s

To find the final angular momentum I need to find the total moment of inertia times the final angular velocity

$L_{final} = I_{total}\, w$

Moment of inertia for rectangular slab:

$\frac{Mw^2}{3} = \frac{45 \times 1^2}{3} = 15$ kg·m²

Moment of inertia for point mass =

$mr^2 = .7 \times .5^2 = .175$ kg·m²

$I_{total} = 15 + .175 = 15.175$

Because momentum needs to be conserved:

$L_{initial} = I_{total} \times w_f$

$4.55 = 15.175\, w_f$

$\boxed{w_f = .300 \text{ rad/s}}$

b. Since the moment of inertia for the mud is .175 kg·m² which is only 1 percent of the door (15 kg·m²) the mud is $\boxed{\text{insignificant}}$