

1.

$$\begin{bmatrix} -2 & 1 & 4 \\ 5 & 3 & 0 \\ 4 & 6 & -3 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 4 \end{bmatrix}$$

$$R_2 = 5R_1 + R_2 \cdot 2$$

$$R_3 = R_3 + R_1 \cdot 2 \begin{bmatrix} -2 & 1 & 4 \\ 0 & 11 & 20 \\ 0 & 8 & 5 \end{bmatrix} = \begin{bmatrix} 7 \\ 36 \\ 18 \end{bmatrix}$$

$$R_3 \neq 11R_3 - 8R_2$$

$$\begin{bmatrix} -2 & 1 & 4 \\ 0 & 11 & 20 \\ 0 & 0 & -105 \end{bmatrix} = \begin{bmatrix} 7 \\ 36 \\ -90 \end{bmatrix}$$

$$R_3 = R_3 / -105$$

$$\begin{bmatrix} -2 & 1 & 4 \\ 0 & 1 & 20 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 36 \\ 0 \end{bmatrix}$$

$$\therefore z = 76$$

$$\begin{aligned} 11y + 20(76) &= 36 \\ 11y + 20 \cancel{76} &= 36 \\ -2x + y + 9z &= 7 \\ x &= \frac{7 - 4(76) + \frac{1984}{11}}{2} \end{aligned}$$

$$\boxed{\begin{aligned} y &= -\frac{1984}{11} \\ z &= 76 \\ x &= \frac{1783}{22} \end{aligned}}$$

$$2. \quad \begin{array}{l} 7x + 8y + 9z = 6 \\ 4x + 5y + 6z = 3 \\ x + 2y + 3z = 0 \end{array}$$

$$\left( \begin{bmatrix} 7 & 8 & 9 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 0 \end{bmatrix} \right)$$

$$R_1 - (R_1/7)$$

$$\left[ \begin{array}{ccc} 1 & 8/7 & 9/7 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{array} \right] = \begin{bmatrix} 6/7 \\ 3 \\ 0 \end{bmatrix}$$

$$R_2 - R_2 - (R_1/4)$$

$$\left[ \begin{array}{ccc} 1 & 8/7 & 9/7 \\ 0 & 3/7 & 6/7 \\ 1 & 2 & 3 \end{array} \right] = \begin{bmatrix} 6/7 \\ -3/7 \\ 0 \end{bmatrix}$$

$$R_3 - R_2 - R_1$$

$$\left[ \begin{array}{ccc} 1 & 8/7 & 9/7 \\ 0 & 3/7 & 6/7 \\ 0 & 6/7 & 12/7 \end{array} \right] = \begin{bmatrix} 6/7 \\ -3/7 \\ -6/7 \end{bmatrix}$$

$$R_2 - R_2 \cdot (1/3)$$

$$\left[ \begin{array}{ccc} 1 & 8/7 & 9/7 \\ 0 & 1 & 2 \\ 0 & 6/7 & 12/7 \end{array} \right] = \begin{bmatrix} 6/7 \\ -1 \\ -6/7 \end{bmatrix}$$

$$R_3 = R_3 - R_2(6/7)$$

$$\begin{bmatrix} 1 & 8/7 & 9/7 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 6/7 \\ -1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$R_1 = R_1 - R_2(8/7)$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

$$\therefore x - z = 0$$

$$y + 2z = -1$$

$$\therefore R_1 = R_1 \cdot (0.5H)$$

a.

$$\begin{bmatrix} 0.5H & 0 & -0.5H \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_2 = R_2 - R_1$$

$$\begin{bmatrix} 0.5H & 0 & -0.5H \\ -0.5H & 1 & 2.5H \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} 0.5Hx - 0.5Hy = 1 \\ 0.5Hx + 2.5Hy = -1 \\ 0.5Hx + 2.5Hy = -0.429 \end{cases}$$

$$b. \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$$

$$\therefore \begin{array}{l} Y+2Z=-1 \\ X-Z=2 \end{array}$$

\* Still an infinite number of solutions.  
Changing the ordering of the rows without  
changing the contents of the rows shouldn't  
change the result.