Vortex Panel and Thwaite's Method Models for a Cylinder

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Abstract

In this project, vortex panel modeling and Thwaite's method are conducted to approximate the coefficient of pressure distribution and flow separation points for two cylindrical airfoils with kutta conditions of $\alpha = 180^{\circ} and - 150^{\circ}$. Distributions of coefficients of pressure are found to match analytical solutions and separation points are found to converge is both cases, validating the results and proving the validity of the simulation method for future use in more complex airfoils.

1 Introduction

Flow modeling and separation point determination are important aspects of determining the characteristics of airfoils. Modeling of the flow surrounding an airfoil and determining the coefficient of pressure is important in determining the lift and drag forces that airfoils are subject to. Determining the points at which flow separates from the surface of an airfoil at a given angle of attack is vital in knowing the maximum speeds and angles airfoils may be used at before the airfoil begins to stall. Unfortunately, for complex geometries analytical solutions for determining both of these metrics are difficult to derive, and therefore we turn to numerical models. To approximate both the coefficient of pressure distribution and the flow separation points we can use the vortex panel method and Thwaite's Method respectively.

1.1 Vortex Panel Modeling

One objective of the following exercise is to determine the local coefficient of pressure and tangential of velocity surrounding an object. We can determine these values numerically by modeling the shape as a series of linear vortex panels.

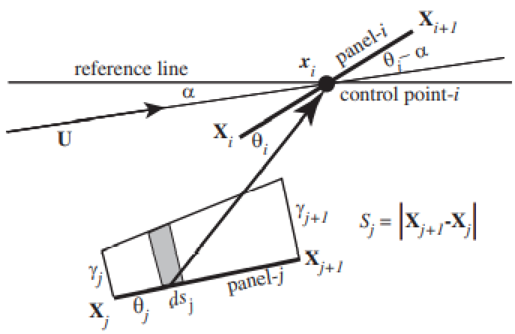

Figure 1: Symbol definition for vortex panel method, image from ME 263 course notes [1]

The velocity potential function at position (x_i, y_i) for a system of n vortex panels immersed in a uniform flow can be written as

$$\phi(x_i, y_i) = U(x_i cos(\alpha), y_i sin(\alpha)) - \sum_{j=1}^{n} \int_j \frac{\gamma(s_j)}{2\pi} \arctan(\frac{y_i - y_j}{x_i - x_j}) ds_j \tag{1}$$

We take the circulation along each vortex panel to be varying linearly

$$\gamma(s_j) = \gamma_j + (\gamma_{j+1} - \gamma_j)\frac{s_j}{S_j} \tag{2}$$

Where s_j is the position along the panel. The normal component of the velocity at each control point vanishes, therefore we can write two systems of equations, one concerning the normal component and one concerning the tangential component of the velocity at each panel.

$$\sum_{j=1}^{n}(C_{n1_{ij}}\gamma_j + C_{n2_{ij}}\gamma_{j+1}) = 2\pi U \sin(\theta_i - \alpha) \tag{3}$$

$$U_{t_i} = \sum_{j=1}^{n}(C_{t1_{ij}}\gamma_j + C_{t2_{ij}}\gamma_{j+1}) + 2(\theta_i - \alpha) \tag{4}$$

solving the first system of equations for γ_i allows us to substitute those values of γ_i into the tangential equation and solve for the tangential velocity at each panel. These velocities can then be used to find the coefficient of pressure.

$$C_{p_i} = 1 - U_{t_i}^2 \tag{5}$$

The coefficients C_{n1}, C_{n2}, C_{t1} and C_{t2} are found for each combination of two panels i and j, one acting upon the other, based on each panel's position and angle made with the reference line. Equations for these coefficients as well as general implementation of the panel method can be found on pages 156-163 of [2]

1.2 Thwaite's Method

The point at which flow separates from the surface of a given airfoil can be found as a function of the momentum thickness. This thickness can be found through integration of the momentum equation as a function of the surface velocity distribution U_e (see pg 68-72 of [1])

$$\theta(x)^2 = \frac{0.45\nu}{U_e(x)^6}\int_0^x U_e(\xi)^5 d\xi \tag{6}$$

And a function K can be defined such that when the distance along the surface of the airfoil from a stagnation point $x = x_s$, $K(x_s) = -0.09$

$$K(x_s) = \frac{\theta^2}{\nu}\frac{dU_e}{dx} = \frac{dU_e}{dx}(x_s)\frac{0.45}{U_e(x_s)^6}\int_0^{x_s} U_e(\xi)^5 d\xi = -0.09 \tag{7}$$

2 Implementation

2.1 Vortex Panel Analysis

Points corresponding to the points at the ends of each vortex panel were read from a file, and from those points the control points at the center of each panel and the angle each panel made with the reference line was determined. From there coefficients $A-G, P$ and Q were determined for each combination of panels, which are then used to find C_{n1}, C_{n2}, C_{t1} and C_{t2}. The system of linear equations from equations 3 may be written as

$$\gamma \mathbf{A_n} = \sin(\theta_\mathbf{i} - \alpha) \tag{8}$$

Where γ is a vector of $n+1$ unknown elements, the right hand side is an n-element array with α being the kutta condition angle and A_n is an $n \times n+1$ matrix comprised of the following elements

$$\begin{array}{ll} A_{n_{i,j}} = C_{n1_{i,j}} & \text{if } j = 1 \\ A_{n_{i,j}} = C_{n1_{i,j}} + C_{n1_{i,j-1}} & \text{if } 2 \le j \le n \\ A_{n_{i,j}} = C_{n2_{i,j-1}} & \text{if } j = n+1 \end{array} \tag{9}$$

Therefore Cramer's Rule can be applied to solve for γ of each panel. Once γ is found, the velocity can be solved for using equation 4, and the coefficient of pressure may be found using equation 5.

2.2 Thwaite's Method

In order to find the separation points of the flow around the cylinder, the stagnation points also must be found. Since a tangential velocity of exactly zero is unlikely, a script was programmed which finds the control points between which the sign of the velocity changes, and the stagnation point is set to be the midpoint of the two panels.

Once the location of stagnation points is determined, two arrays Su and Sl are defined that approximate the arc distance between each control point and the leading edge stagnation point by using the lengths of the panels going both clockwise and counterclockwise. The integral term of equation 7 is then approximated using the MATLAB cumulative trapezoidal integration function for each control point between the stagnation points both the upper and lower side, and $\frac{\Delta U_{t_{i,i+1}}}{\Delta S_{i,i+1}}$ approximates the differential term of equation 7 for the control point i. This introduces error that approaches zero as the number of control points increases. A value for an approximate K function is then found for each control point, and the code finds the two control points which -0.09 lies between, and the angle of the separation point is linearly interpolated using the angle of the straddling control points and the K values.

3 Results

3.1 Coefficients of Pressure

Coefficients of pressure are shown below plotted against the angle θ made from the line between each control point on the surface of the cylinder and the origin. It can be seen that the coefficient of pressure converges to the analytic solution as the number of panels increases.

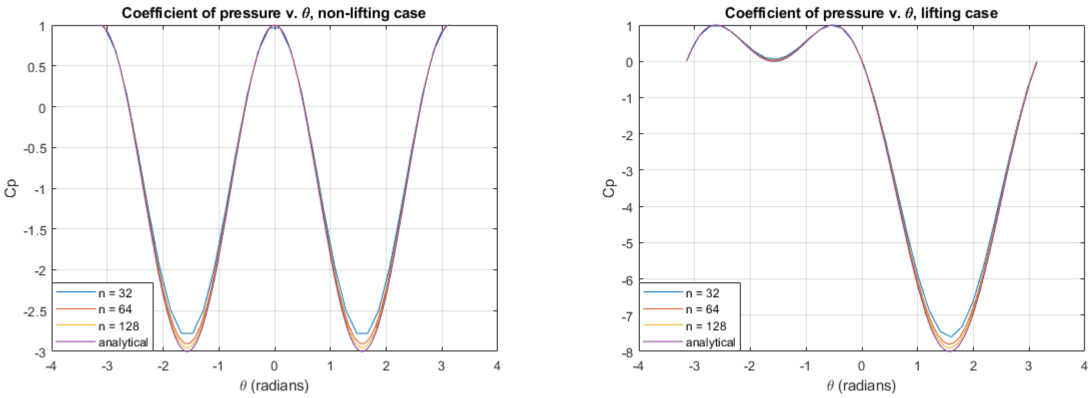

Figure 2: Coefficients of Pressure for lifting and non-lifting case, where n is the number of panels

For the non-lifting case (kutta condition set at $\alpha = 180°$), the coefficient of pressure is symmetric about $\theta = 0$, with peaks at $\theta = \pm\frac{\pi}{2}$ of around -3. Since the pressure distribution is symmetric, we can be certain that there are no net pressure forces on the object and that there is in fact, no lift

For the lifting case with the kutta condition placed at $\alpha = -150°$, there is an extreme asymmetry across the X axis, with C_p approaching -8 at its greatest magnitude and the other peak at $\theta = -\frac{\pi}{2}$ approaching zero. This imbalance of pressure indicates that there is a lifting force on the cylinder pushing directly upwards.

3.2 Separation Points

As seen by the spacing of the points in the figure below, the separation points rapidly converge as the number of panels increases. The leading edge for each figure is the leftmost stagnation point, and the trailing edge is the rightmost in both figures. In both cases the flow separates on the side of the trailing edge, and detach in similar regions. In the lifting case, flow separation points are seemingly rotated clockwise on the surface.

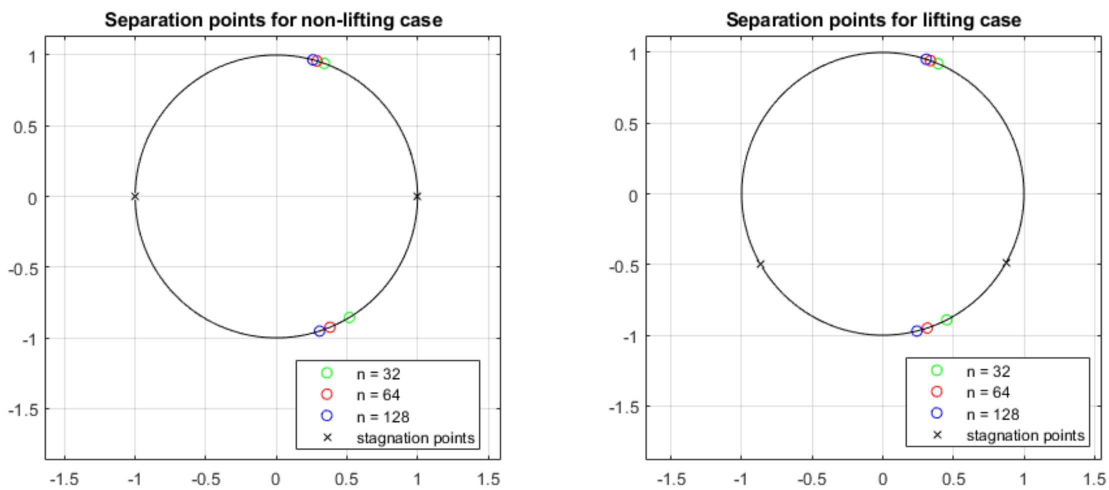

Figure 3: Separation points for non-lifting (left) and lifting (right) cases

4 References

[1] Savaş, Ömer. “Engineering Aerodynamics Course Notes, Fall 2022.” University of California, Berkeley, 29 Aug. 2022.

[2] Kuethe, Arnold M., and Chuen-Yen Chow. Foundations of Aerodynamics: Bases of Aerodynamic Design. 5th ed., J. Wiley, 2000.