Determining Coeficient of Lift, Airfoil Efficiency and Stall Angles of NACA-0012 and NACA-4412 Airfoils Numerically.

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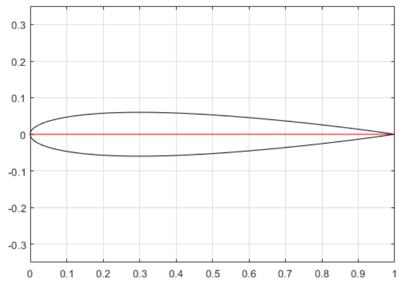
Abstract

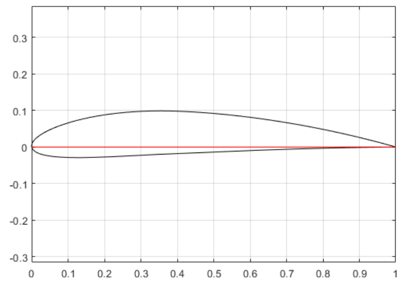
Lift coefficients and aerodynamic efficiencies for NACA-0012 and NACA-4412 airfoils for angles of attack between $-16^\circ < \alpha < 16^\circ$ were determined using numerical results from a 128 vortex panel analysis, and stall angles were determined for both airfoils using thwaite's method and a stall criterion of separation points at $x/c \leq 0.2$. Results were compared to XFOIL data at $R_e = 10^6$ for each airfoil and analytical or experimental values when available.

1 Methodology

NACA-0012 and NACA-4412 airfoils were chosen as subjects of numerical analysis. The two airfoils, pictured below, were modeled by a series of 128 vortex panels to determine the surface velocity of the surrounding air, as well as the local pressure coefficients at the control point of each panel. Results were obtained using the method presented in (*Kuethe & Chow, 1998*) on page 161. Separation points were determined using Thwaites' criterion outlined in the 2022 Engineering Aerodynamics Course Notes on pages 79-80.



(a) NACA-0012 airfoil outline with chord plotted in red



(b) NACA-4412 airfoil outline with chord plotted in red

1.1 Determining the Analytical Lift Coefficient

Thin airfoil theory can be applied to the airfoils in order to determine an analytical value for the lift coefficient. Using the mean camber line of an airfoil, the lift coefficient can be determined by the equation below, derived in chapter 5 of *Kuethe & Chow, 1998*

$$C_L = 2\pi(\alpha + \int_0^\pi \frac{dz}{dx}(\cos(2\theta) - \cos(\theta))\acute{d}\theta \tag{1}$$

where $\frac{dz}{dx}$ is the slope of the mean camber line and θ is the angle of the line an arbitrary point makes with the point $(c/2, 0)$ clockwise from π. The angle of zero lift, α_{L0}, is the result of the integral term of the above equation

$$\alpha_{L0} = \int_0^\pi \frac{dz}{dx}(\cos(2\theta) - \cos(\theta))\acute{d}\theta \tag{2}$$

The values of α_{L0} for common airfoil profiles such as NACA-0012 and NACA-4412 are well known, and some common profiles have their results tabulated on pg 149 of *Kuethe & Chow, 1998*. For NACA-0012, the angle is $\alpha_{L0} = 0^\circ$, and for NACA-4412, the angle is $\alpha_{L0} = -3.9^\circ$.

1.2 Determining the Lift and Drag Coefficients Numerically

The total force F applied to an airfoil can be found by the integral

$$\mathbf{F} = (D, L) = \iint_S -p\mathbf{n}\, dA \tag{3}$$

The pressure, drag force and lift force may be written in terms of their respective coefficients, neglecting frictional drag.

$$\mathbf{F} = \frac{c\rho U_\infty^2}{2}(C_D, C_L) = \iint_S -(C_p\frac{\rho U_\infty^2}{2} + p_\infty)\mathbf{n}\, dA \tag{4}$$

Where $\mathbf{n}$ is the normal vector to the surface of the airfoil. Approximating the integrals as sums and assuming that the airfoil's shape does not change across its span, an equation for the drag and lift coefficients per unit length of the airfoil can be found

$$\frac{c}{s} \cdot (C_D, C_L) = \sum -C_p S\, \mathbf{n} \tag{5}$$

Where S is the panel length for a given control point. Note that the static pressure term disappears, since it is constant over the surface of the airfoil and therefore integrates to zero. The above equation can be applied to the dataset generated via the vortex panel method, however, a clockwise rotation must be applied to the drag and lift forces (and therefore the coefficients) to properly account for the angle of the airfoil. The following rotation matrix can be applied to the drag and lift coefficients to align the forces with the x and y axis

$$\begin{bmatrix} cos(\alpha) & sin(\alpha) \\ -sin(\alpha & cos(\alpha) \end{bmatrix} \begin{bmatrix} C_D \\ C_L \end{bmatrix} = \begin{bmatrix} C_{D,\text{true}} \\ C_{L,\text{true}} \end{bmatrix} \tag{6}$$

2 Results

2.1 Coefficient of Pressure on Upper and Lower Surfaces

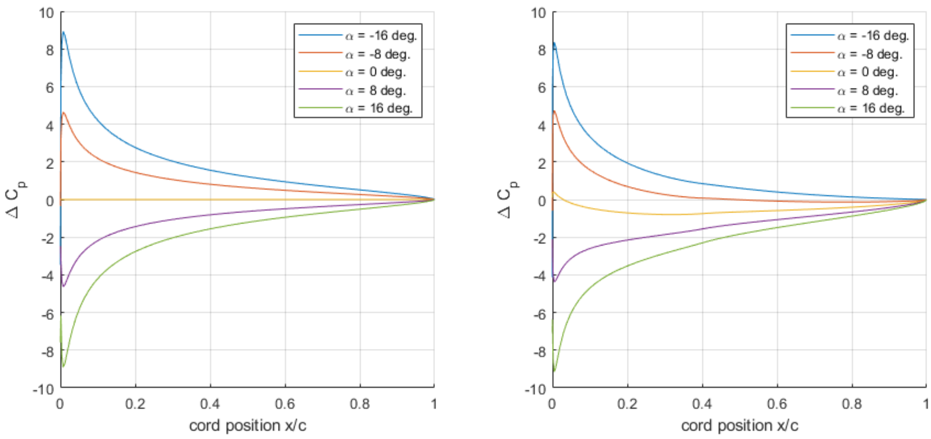


Figure 2: Difference between the pressure coefficients on the upper and lower surface of NACA-0012 (left) and NACA-4412 (right) plotted for angles of attack between $-16°$ and $16°$

As expected, the difference between the pressure coefficients on the upper and lower surfaces for the NACA-0012 airfoil is zero for an angle of attack of zero, and is symmetrical for positive and negative angles of attack. NACA-4412's distribution of ΔC_p is skewed more negative, as the geometry of the airfoil generates higher negative pressure coefficients on the top surface of the airfoil generally, and indicates that the angle of zero lift lies below $0°$. In both cases differences in pressure coefficient are higher near the leading edge where air will be moving the fastest over the side of the surface facing the flow directly, and taper to zero as the geometry of the airfoil encourages becomes less aggressively sloped.

2.2 Lift Coefficients and Aerodynamic Efficiency

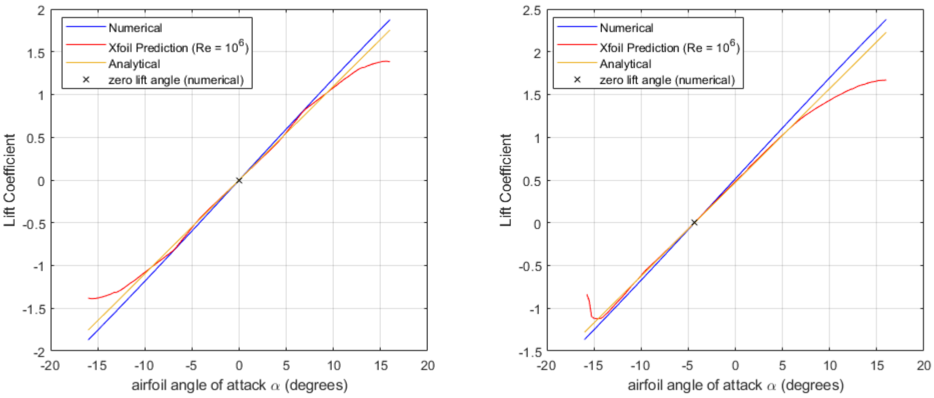


Figure 3: Coefficient of Lift vs Angle of Attack for NACA-0012 (left) and NACA-4412 (right)

For both airfoil geometries, the numerically calculated lift coefficient is accurate to the XFOIL data and thin airfoil analysis surrounding the angle of zero lift (calculated to be $\alpha = 0°$ for NACA-0012 and $\alpha = -4.31°$ for NACA-4412). Accuracy to both data sets degrades as α approaches $-16°$ and $16°$, with the most significant deviation occurring for NACA-4412 as α approaches $16°$.The angle of zero lift for NACA-4412 is given by *Kuethe & Chow, 1998* on page 149 as $-3.9°$.

2.3 Separation Points and Stall Angle

For the purposes of this excercise, stall was assumed to have occurred when the flow separation on the upper surface of the airfoil reached within 20% of the chord from the leading edge of the airfoil.

The numerically predicted stall angle is $\alpha \approx 4.9°$ for NACA-0012 and $\alpha \approx 8.5°$ for NACA-4412. If we take the angle at which a local maximum of the lift coefficient occurs to be the angle that the airfoil stalls, then the XFOIL data seems to suggest a stall angle of $\alpha \approx 15.5°$ for NACA-0012, and $\alpha \approx 15°$ for NACA-4412. From the data presented in appendix IV of *Abbot & Doenhoff, 1959*, the experimentally predicted stall angle for a Reynolds number of 3×10^6 is $\alpha \approx 16°$ for NACA-0012 and $\alpha \approx 13°$ for NACA-4412.

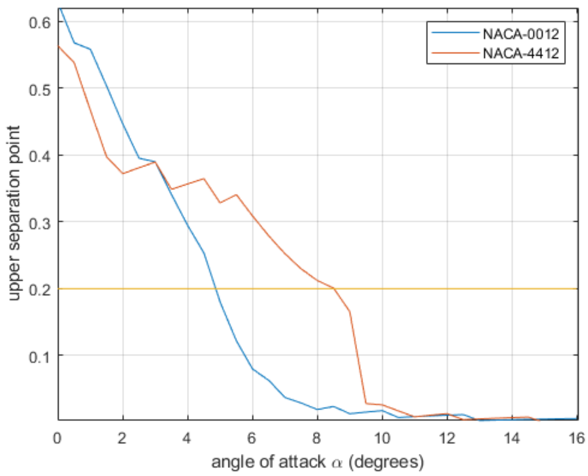


Figure 4: Separation points for upper surfaces of NACA-0012 (colored blue) and NACA-4412 (colored red) plotted against angle of attack. Yellow line at $x_s = 0.2$ is the stall criterion.

There is a large disparity between the numerically obtained values and those obtained experimentally and from XFOIL simulations. The first possible explanation for this discrepancy is the imposition of two separate methods of determining stall angle. Since the vortex panel model cannot take into account turbulence, compressibility and friction on the airfoil in it's current implementation, the lift coefficient behaves differently from the XFOIL and experimental models. It never reaching a local maxima in the range of angles of attack that can be used to determine a stall angle, therefore the 20% chord method must be utilized, which is not always applicable or accurate. Additionally, the accuracy of the pressure coefficients determined by the vortex panel method degrade rapidly as α departs from the angle of zero lift. This can be seen in the coefficients of lift in figure 3, which are directly proportional to the coefficients of pressure. Finally, the relatively low number of panels (128) used in this simulation may contribute to the inaccuracy of Thwaites' method, though convergence testing on the number of panels used would be needed to verify this hypothesis.

3 References

[1] Savaş, Ömer. "Engineering Aerodynamics Course Notes, Fall 2022." University of California, Berkeley, 29 Aug. 2022.
[2] Kuethe, Arnold M., and Chuen-Yen Chow. Foundations of Aerodynamics: Bases of Aerodynamic Design. 5th ed., J. Wiley, 2000.
[3] Abbott, Ira Herbert, and Doenhoff A E Von. Theory of Wing Sections. Dover Pubns., 1959.
XFOIL data from http://airfoiltools.com/search/index?m