Design of a Single Plane Coaxial Rotor.

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Abstract

In this exercise, a co-planar nested rotor is designed to meet requirements for the chord and blade tip Reynolds numbers, net torque between the inner and outer blades, rotor stability and difference in bending moment at the root of the inner and outer blades. Through the application of rotor dynamics theory with a set blade configuration, thrust ratio, rotor speed and the radius of the boundary between the inner and outer blades was varied to determine a final design which met all design criteria.

1 Introduction and Methodology

1.1 Rotor craft and Coaxial Blade Configurations

From wikipedia, the definition of a rotorcraft is "a heavier-than-air aircraft with rotary wings or rotor blades, which generate lift by rotating around a vertical mast" [2]. Common rotorcraft include helicopters and quadrotor drones, and such aircraft provide essential capabilities in areas ranging from emergency services to videography. The use of rotors, however, provides an issue which must be accounted for; torque. The rapid rotation of a single rotor requires a large torque to be applied, one which cannot be counterbalanced entirely by the inertia of the craft, therefore without some sort of stabilization a single-rotor aircraft will begin to rotate undesirably. One way that engineers have solved this issue with some helicopters is the use of a coaxial blade configuration. In this configuration, two counter rotating rotors move around a shared axis. Done correctly, this configuration can get rid of undesired rotation of the craft, however stacked coaxial rotors have increased power requirements for the bottom rotor due to the increased energy of the working fluid as it enters the second set of propellers [1]. This project aims to provide a design for a single-plane coaxial rotor, which may eliminate this increased power requirement.

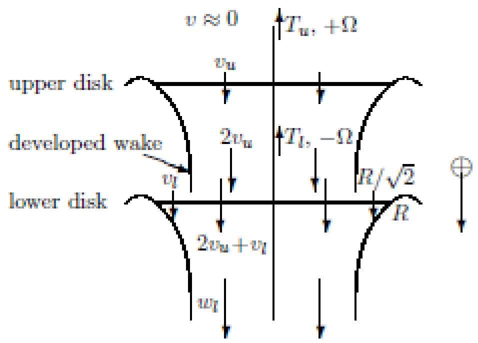

Figure 1: Visualization of a stacked coaxial rotor [1]

1.2 Rotor Dynamics and Blade Element Theory

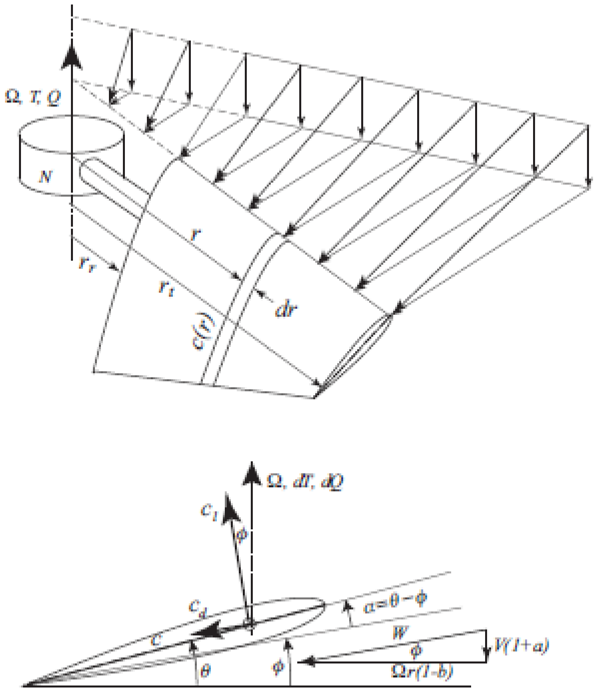

Figure 2: Visualization of rotor blade with design parameters labeled [1]

A discrete form of Blade element theory can be used to approximate the lift and drag generated by each blade using the sectional lift and drag coefficients. Integration across the span of the sectional lift and drag produces the total lift and drag on the airfoil. Breaking the airfoil into even sections across its span and summing the product of the sectional lift and the length of each section gives a decent approximation of the forces experienced by the airfoil.

$$L_b = \int_{b_i}^{b_o} q(r)c_l(r)c(r)dr \approx \sum_{i=1}^{m} q(r_i)c_l(r_i)c(r_i)\Delta r \tag{1}$$

$$D_b = \int_{b_i}^{b_o} q(r)c_d(r)c(r)dr \approx \sum_{i=1}^{m} q(r_i)c_d(r_i)c(r_i)\Delta r \tag{2}$$

Using thin airfoil theory, the sectional lift and drag coefficients for a circular airfoil such as the one selected can be found via the equations below.

$$c_l = 2\pi(\alpha + \frac{2z_m}{c}) \tag{3}$$

$$c_d = -\alpha c_l \tag{4}$$

Derivations for these equations can be found in section 9 of the course notes [1]. Lift and Drag are not directly useful metrics for rotorcraft however, more useful values being the torque applied to the blades by the air and the upward thrust contributed by each blade. Since the rotor forces air downward, each blade must be angled even further to achieve a given angle of attack. The air velocity required to achieve a given thrust value is given by the equation

$$v_h = \sqrt{T/2\rho A} \tag{5}$$

where A is the projected disc area of the rotor, ρ is the density of the fluid and T is the thrust. The angle of the velocity vector aligned with α is given by

$$\phi = \arctan(v_h/\Omega r) \tag{6}$$

and the dynamic pressure q is given by

$$q = \frac{1}{2}\rho(v_h^2 + \Omega^2 r^2) \tag{7}$$

Due to the relatively small angle of attack α, for the purposes of this exercise the drag component of the induced force on each blade will be ignored. Therefore the total upward thrust provided by each blade may be written as

$$T_b = L_b \cos(\phi) \approx \sum_{i=1}^{m} q(r_i)c_l(r_i)\cos(\phi(r_i))c(r_i)\Delta r \tag{8}$$

The total torque imparted on each blade can be found as a product of the component of the force on the blade and the distance from the center of the rotor

$$Q_b = L_b \sin(\phi) \approx \sum_{i=1}^{m} q(r_i)c_l(r_i)\sin(\phi(r_i))c(r_i)r_i\Delta r \tag{9}$$

Derivations of these equations can be found in section 16.8 of the course notes [1]. Another metric of interest is the bending moment at the root of each blade. This moment is given by the integral

$$\int_{b_i}^{b_o} (rD(r), rL(r), 0)dr \approx (\sum_{i=1}^{n} q(r_i)c_l(r_i)r_i\Delta r, \sum_{i=1}^{n} q(r_i)c_l(r_i)r_i\Delta r, 0) \tag{10}$$

Where $L(r)$ and $D(r)$ are the sectional lift and drag forces, in units of (N/m).

1.3 Desired Characteristics and Restrictions

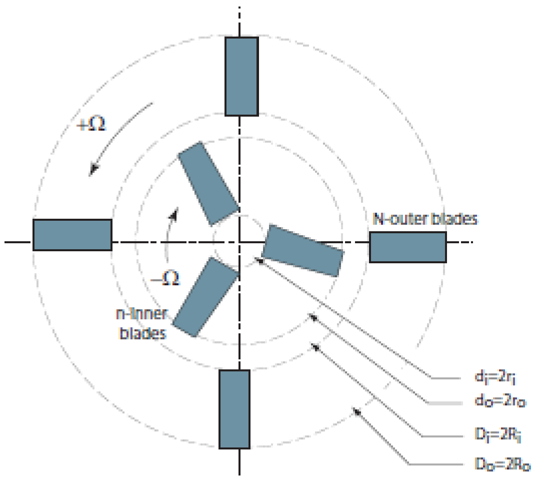

Figure 3: Aerial view of blade configuration [1]. Note that there are four outer blades, however this exercises dictates the use of three outer blades.

The general design consists of two concentric sets of blades with wingtips on one plane. The inner radius r_i is required to be a quarter of the outer radius R_o, and the distance between R_i and r_o must be $0.05R_o$. The midline refers to the radius at the center of r_o and R_i. The rotor must be able to provide 100N of thrust while satisfying several design requirements. These requirements constrain the values of the radii of the propellers, as well as the rotational speed and the thrust ratio.

Rotational velocity is constrained by two major parameters; the rotor tip mach number and the chord Reynolds number. The rotor tip mach number, given by $\frac{R_o\Omega}{a}$, is to be kept below 0.5 in order to not deal with compressibility effects and shocks at the airfoil tip. Keeping the chord Reynolds number ($R_{e,c} = c(r)\Omega r/\nu$) below 250,000 is meant to ensure that the flow stays mostly laminar. Since all variables except for the rotational speed are known in the rotor tip mach number equation once an outer radius is selected, an initial maximum value can be set for Ω.

Additionally, the torques applied to the inner versus outer blades were to be kept within five percent of each other to fully capitalize on the benefits of using a coaxial rotor. Since the number of blades was equal, the percent difference between the total inner and outer torques was equal to the percent difference between one of the blades on the inside and outside. The difference in torques was primarily a function of the Thrust ratio and the relative length of each set of blades.

Rotor solidity is a measure of the proportion of the rotor's projected disc area that is occupied by the propellers. A rotor stability of $\sigma \leq 0.10$ is required for this exercise.

$$\sigma = \frac{NA_o + nA_i}{A} \tag{11}$$

Where N and n are the number of inner and outer blades respectively. This restriction forces a lower limit on the outer radius, as a wider propeller is necessary to produce the same amount of thrust with a shorter blade.

Finally, the bending moment at the root of each blade is to be kept within five percent of the others. This is primarily impacted by the thrust ratio.

1.4 Design Methodology

An outer radius of $R_o = 0.5m$ was selected, which satisfied rotor stability requirements for a large set of initial conditions. A uniform thrust distribution was decided upon, so each blade element produces the same amount of thrust. Three parameters were swept based on reasoning discussed in section 1.3 in order to determine a blade geometry that met all requirements; rotational speed, midline radius and T_R, the proportion of the thrust the inner blades will provide. For the purposes of this exercise, each individual segment of the propeller blades is modeled as a circular arc of radius $R_c = 2R_o$. The angle of attack is designed to be tangent to the each blade at the leading edge $\alpha(r) = \arcsin(c/(2R_c)$. Additionally, the maximum height of the circular airfoil can be written as $z_m = c(r)^2/16R_o$. This allows for a sectional thrust equation to be written which has the chord as the only unknown.

$$\Delta T_b(c, r) = T_b/m = 2c(r)\pi[\arcsin(\frac{c(r)}{(4R_o)}) + \frac{c(r)}{8R_o}]cos(\phi)q\Delta r \tag{12}$$

This relationship indicates a correlation between radius and chord of roughly $c \propto 1/r$ if the small angle approximation is made. Since a desired thrust is known and a uniform thrust distribution across the blade is chosen, the chord is solved for either using an iterative method such as Newton-Raphson or by applying the small angle approximation and solving for $c(r)$ algebraically. The total thrust is to be distributed between 3 inner and 3 outer blades, therefore the amount of thrust each individual blade will provide is selected as $T_{bi} = T(T_R)/3$ for the inner blades and $T_{bo} = T(1 - T_R)/3$ for the outer blades

Once the chord is obtained for both the inner and outer blades given a set of initial parameters, the torque, root bending moments, rotor solidity, rotor tip mach number and maximum chord reynolds numbers can be found from previously mentioned equations. From there, a set of initial parameters which satisfies all of the aforementioned requirements is selected.

2 Final Design and Specifications

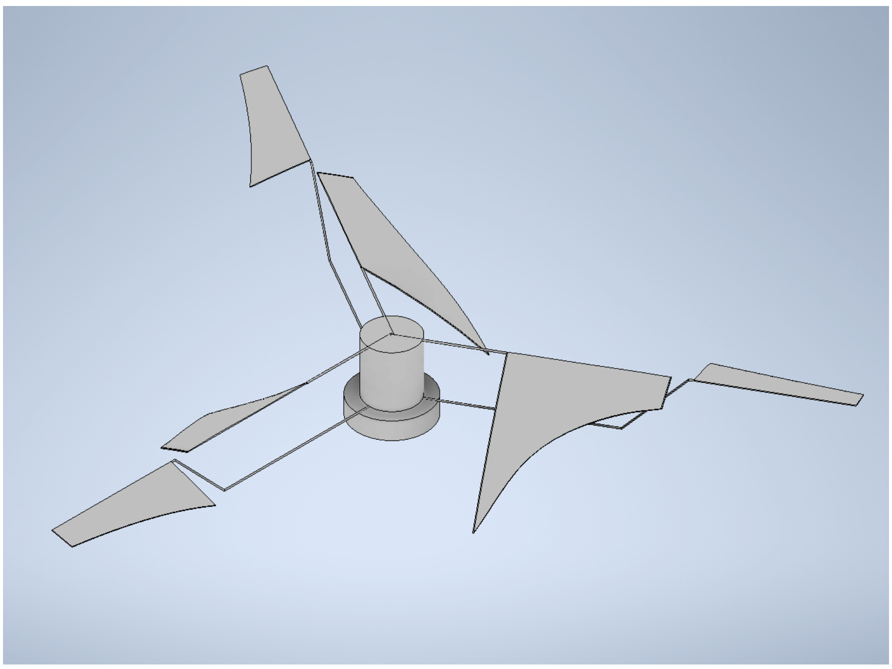

Figure 4: Final Rotor design isometric view

Below are two tables which hold the values of all considered design parameters and requirements respectively.

Table 1: Final Design Parameters and Geometry

Outer Radius (m)	Inner Radius (m)	Mid-line Location (m)	Inner Thrust Proportion (N/N)	Rotor Speed (rad/s)	Hover Power (W)	Total Torque (N-m)
0.50	0.125	0.3126	0.3378	75.51	771.47	0.6220

Table 2: Values of all Constraint Parameters

Torque % Difference	Root Bending Moment % Difference	Rotor Solidity	Max Chord Reynolds No.	Rotor Tip Mach No.
0.428	4.96	0.09133	1.2465×10^5	0.1109

Total torque required to drive the rotor is found by

$$Q = |Q_i| + |Q_o| = 3Q_{bi} + 3Q_{bo} \tag{13}$$

Power is found by

$$P = P_i + P_o = T(T_R)\sqrt{\frac{T(T_R)}{(2\rho\pi(r_o^2 - r_i^2))}} + T(1 - T_R)\sqrt{\frac{T(1 - T_R)}{(2\rho\pi(R_o^2 - R_i^2))}} \tag{14}$$

The final design was not selected out of the final batch of viable candidates for any particular reason, as all candidates had roughly similar performance and design characteristics This is likely due to the set outer radius, althoug additional investigation would be required to reveal such claims. a mid-line of 0.3126m gives $r_o = 0.300$m and $R_i = 0.325$m. The total power required to get the rotor to hover is 771.47 Watts. The torque required to drive the rotor was 0.6220 N-m. The net torque vanished to a surprising extent, especially in comparison to the net moment. The difference between the inner and outer torques was able to be taken down to 0.43%, while the moment was barely under 5%. The upper bound of the rotor speed was set to be under the required rotor tip mach number, and the maximum chord Reynolds number was well within allowances at 1.25×10^5, so restrictions on the rotor speed were well under their required values.

The chord lengths as a function of radius are portrayed below.

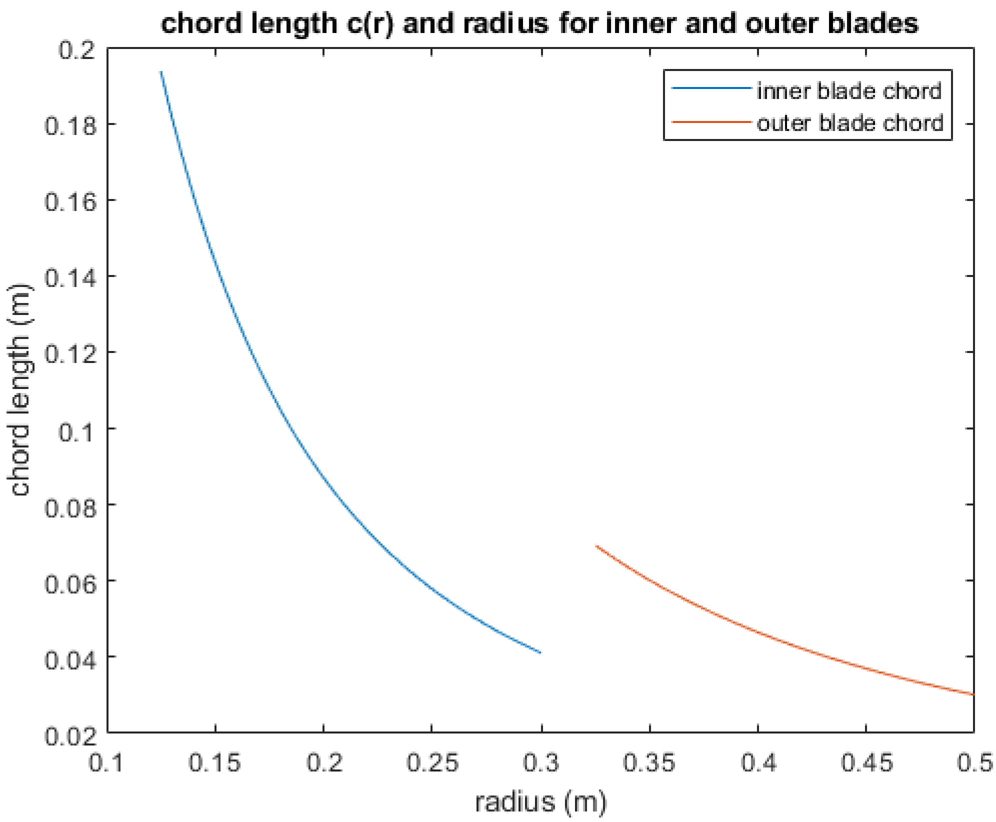

Figure 5: Chord Length and Radius

As can be seen in the figure above, the chord of the propellers was found to vary inversely with r, which is expected for an equal thrust distribution. Since the dynamic pressure increases drastically with radius, the chord length required to produce the same amount of thrust between blade elements goes down as radius increases. Discontinuity of c(r) between the inner and outer propellers is due to the difference in thrust between the two sets of propellers, as well as the placement of the midline.

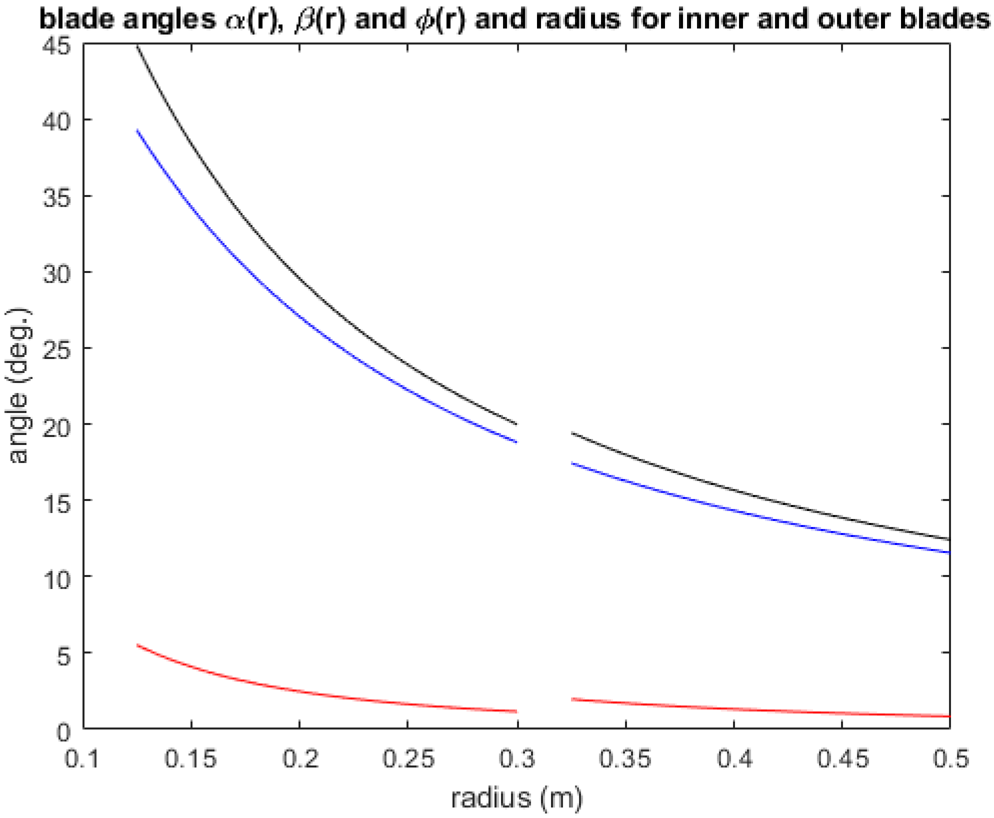

Figure 6: Blade angles $\alpha(r)$, $\beta(r)$ and $\phi(r)$ in red, black and blue respectively plotted against radius

From the figure above, it is seen that angle of attack is relatively small in comparison to ϕ. The decrease in ϕ is expected, as the planar component of the velocity at the leading edge of the blade increases linearly with radius, which is roughly inversely proportional to ϕ.

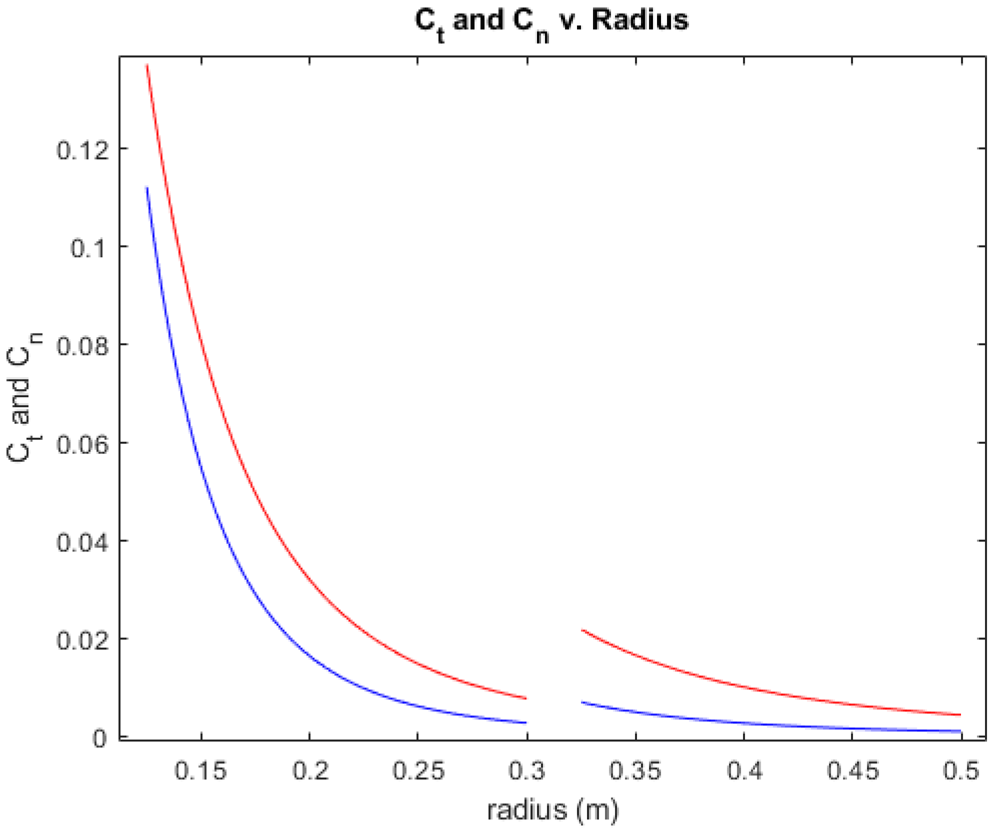

Figure 7: C_n (red) and C_t (blue) vs radius

C_n and C_t when drag is neglected end up being $C_n = cos(\phi)c_l$ and $C_t = sin(\phi)$. Therefore distribution of C_n and C_t roughly mirror the chord distribution along the radius. This is expected for the same reasons that the chord and airfoil angles decrease with radius. The neglecting of the drag component when calculating the thrust and torque is likely the cause of some of the discrepancy between the two coefficients, however even if drag had been included it would have only adjusted the values by about a factor of α, and would not have closed the gap by a long shot.

3 References

[1] Savaş, Ömer. "Engineering Aerodynamics Course Notes, Fall 2022." University of California, Berkeley, 29 Aug. 2022.
[2] https://en.wikipedia.org/wiki/Rotorcraft