Chapter 1.3 - Summary
Properties of Graphs and Functions

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September 10, 2024

You can characterize functions based on the following properties:

- Domain and Range
- Zeros and y-intercepts
- Continuity and discontinuity
- Intervals of increase and decrease
- Symmetry
- End behaviour

Lets now define what those are actually.

Interval of increase: The intervals(sections) of the domain where the output is increasing, from left to right.

Interval of decrease: The intervals(sections) of the domain where the output is decreasing, from left to right.

State the intervals of increase and decrease for the function $x \mapsto x^2$

Interval of increase: $(0, \infty)$, Interval of decrease: $(\infty, 0)$

Continuous Function: Any function that has a fully define domain(has no breaks or holes).

Discontinuity: A break in the domain.

End behaviour: The behaviour of a function at end(what is the x and y approaching).

Symmetry: The symmetry of a function, if a function has even symmetry, it's symmetrical over the y axis. If a function has odd symmetry, it's symmetrical rotational around the origin.

If a function is odd then $-f(-x)$ **MUST** equal $f(x)$.
If a function is even then $f(-x)$ **MUST** equal $f(x)$.

For example, lets find the symmetry for $f(x) = \frac{1}{x}$ and $g(x) = x^2$

$$\begin{aligned} -f(-x) &= -1\left(\frac{1}{(-x)}\right) \\ &= \frac{1}{x} \\ -f(-x) &= f(x) \end{aligned}$$

$\therefore$ The function $f(x)$ has odd symmetry.

$$\begin{aligned} f(-x) &= (-x)^2 \\ &= x^2 \\ f(-x) &= f(x) \end{aligned}$$

$\therefore$ The function $f(x)$ has even symmetry.