Homework 2

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Question 1

i. (a)

$$G(s) = \frac{4s - 8}{s^2 + 2s - 9}$$

$$\text{DC GAIN} = G(0) = \frac{-8}{-9}$$

(b)

$$G(s) = \frac{10}{s^2 + 2s + 10}$$

$$\text{DC GAIN} = G(0) = \frac{10}{10} = 1$$

ii. (a)

$$N(0) = s^2 + 2s - 9 = 0$$

$$\text{Poles:} s = -1 \pm \sqrt{10}$$

$$Real(s) = -1 + \sqrt{10} > 0, \text{so this system is unstable.}$$

(b)

$$N(0) = s^2 + 2s + 10 = 0$$

$$\text{Poles:} s = -1 \pm 3i$$

$$Real(s) = -1 < 0, \text{so this system is stable.}$$

iii. (a)

From TF, we get the ODE:

$$\ddot{y(t)} + 2\dot{y(t)} - 9y(t) = 4\dot{u(t)} - 8u(t)$$

Guess that: $y(t) = Ce^{st}, \dot{y(t)} = Cse^{st}, \ddot{y(t)} = Cs^2e^{st}$ So we get:

$$Cs^2e^{st} + 2Cse^{st} - 9Ce^{st} = 0$$

$$Ce^{st}(s^2 + 2s - 9) = 0$$

$$s = -1 \pm \sqrt{10}$$

$$y(t) = c_1e^{(-1+\sqrt{10})t} + c_2e^{(-1-\sqrt{10})t}$$

(b)

From TF, we get the ODE:

$$\ddot{y(t)} + 2\dot{y(t)} + 10y(t) = 10u(t)$$

Guess that: $y(t) = Ce^{st}, \dot{y(t)} = Cse^{st}, \ddot{y(t)} = Cs^2e^{st}$ So we get:

$$Cs^2e^{st} + 2Cse^{st} + 10Ce^{st} = 0$$

$$Ce^{st}(s^2 + 2s + 10) = 0$$

$$s = -1 \pm 3i$$

$$y(t) = c_1e^{-t}cos(3t) + c_2e^{-t}sin(3t)$$

iv. (a) Roots: $s = -1 \pm \sqrt{10}$

Homogeneous Solution:

$$y_h(t) = c_1e^{(-1+\sqrt{10})t} + c_2e^{(-1-\sqrt{10})t}$$

Input: $u(t) \equiv 2$

$$\ddot{y_p(t)} + 2\dot{y_p(t)} - 9y_p(t) = -16$$

Guess $y_p(t) = C$, need: $-9C = -16$,and then get: $C = \frac{16}{9}$

So general form of Input response:

$$y(t) = y_h(t) + y_p(t) = c_1e^{(-1+\sqrt{10})t} + c_2e^{(-1-\sqrt{10})t} + \frac{16}{9}$$

(b) Roots: $s = -1 \pm 3i$

Homogeneous Solution:

$$y(t) = c_1e^{-t}cos(3t) + c_2e^{-t}sin(3t)$$

Input: $u(t) \equiv 2$

$$\ddot{y(t)} + 2\dot{y(t)} + 10y(t) = 20$$

Guess $y_p(t) = C$, need: $10C = 20$,and then get: $C = 2$

So general form of Input response:

$$y(t) = y_h(t) + y_p(t) = c_1e^{-t}cos(3t) + c_2e^{-t}sin(3t) + 2$$

v. (a) For the zero initial conditions: $y(0) = 0, \dot{y(0)} = 0$

We get:

$$y(0) = c_1 + c_2 + \frac{16}{9} = 0$$

$$\dot{y(0)} = (-1 + \sqrt{10})c_1 + (-1 - \sqrt{10}c_2) = 0$$

So:

$$c_1 = -1.17, c_2 = -0.6078$$

So input response:

$$y(t) = -1.17e^{(-1+\sqrt{10})t} - 0.6078e^{(-1-\sqrt{10})t} + \frac{16}{9}$$

(b) For the zero initial conditions: $y(0) = 0, \dot{y(0)} = 0$

We get:

$$y(0) = c_1 + 2 = 0$$

$$\dot{y(0)} = -c_1 + 3c_2 = 0$$

So:

$$c_1 = -2, c_2 = -2/3$$

So input response:

$$y(t) = -2e^{-t}cos(3t) - \frac{2}{3}e^{-t}sin(3t) + 2$$

Question 2

a TF:

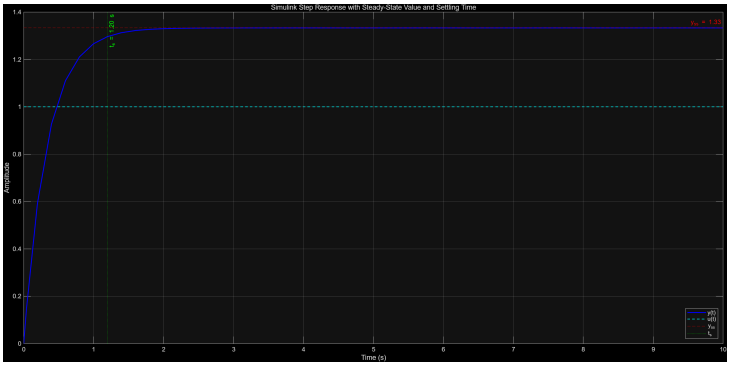
$$G(s) = \frac{4}{s+3}$$

Roots: $s = -3 < 0$So this system is stable.

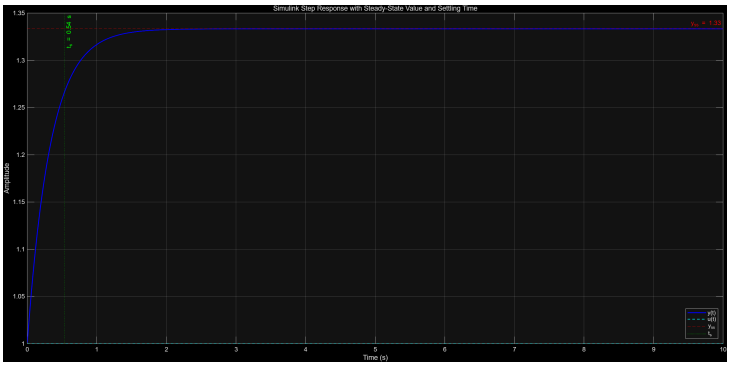
b Poles: $s = -3$

$$\text{Time constant: } \tau = \frac{1}{|Re(p)|} = \frac{1}{3}s$$

c y(0) = 0:



d $y(0) = 1$:


Simulink Step Response with Steady-State Value and Settling Time
Amplitude
Time (s)