Homework 2

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Question 1

i. (a)

$$G(s) = \frac{4s-8}{s^2+2s-9}$$

$$\text{DC GAIN} = G(0) = \frac{-8}{-9}$$

(b)

$$G(s) = \frac{10}{s^2+2s+10}$$

$$\text{DC GAIN} = G(0) = \frac{10}{10} = 1$$

ii. (a)

$$N(0) = s^2 + 2s - 9 = 0$$

$$\text{Poles:} s = -1 \pm \sqrt{10}$$

$$Real(s) = -1 + \sqrt{10} > 0, \text{so this system is unstable.}$$

(b)

$$N(0) = s^2 + 2s + 10 = 0$$

$$\text{Poles:} s = -1 \pm 3i$$

$$Real(s) = -1 < 0, \text{so this system is stable.}$$

iii. (a)

From TF, we get the ODE:

$$y\ddot{(t)} + 2y\dot{(t)} - 9y(t) = 4u\dot{(t)} - 8u(t)$$

Guess that: $y(t) = Ce^{st}, y\dot{(t)} = Cse^{st}, y\ddot{(t)} = Cs^2e^{st}$ So we get:

$$Cs^2e^{st} + 2Cse^{st} - 9Ce^{st} = 0$$

$$Ce^{st}(s^2 + 2s - 9) = 0$$

$$s = -1 \pm \sqrt{10}$$

$$y(t) = c_1e^{(-1+\sqrt{10})t} + c_2e^{(-1-\sqrt{10})t}$$

(b)

From TF, we get the ODE:

$$\ddot{y(t)} + 2\dot{y(t)} + 10y(t) = 10u(t)$$

Guess that: $y(t) = Ce^{st}, \dot{y(t)} = Cse^{st}, \ddot{y(t)} = Cs^2e^{st}$ So we get:

$$Cs^2e^{st} + 2Cse^{st} + 10Ce^{st} = 0$$

$$Ce^{st}(s^2 + 2s + 10) = 0$$

$$s = -1 \pm 3i$$

$$y(t) = c_1e^{-t}cos(3t) + c_2e^{-t}sin(3t)$$

iv. (a) Roots: $s = -1 \pm \sqrt{10}$

Homogeneous Solution:

$$y_h(t) = c_1e^{(-1+\sqrt{10})t} + c_2e^{(-1-\sqrt{10})t}$$

Input: $u(t) \equiv 2$

$$\ddot{y_p(t)} + 2\dot{y_p(t)} - 9y_p(t) = -16$$

Guess $y_p(t) = C$, need: $-9C = -16$,and then get: $C = \frac{16}{9}$

So general form of Input response:

$$y(t) = y_h(t) + y_p(t) = c_1e^{(-1+\sqrt{10})t} + c_2e^{(-1-\sqrt{10})t} + \frac{16}{9}$$

(b) Roots: $s = -1 \pm 3i$

Homogeneous Solution:

$$y(t) = c_1e^{-t}cos(3t) + c_2e^{-t}sin(3t)$$

Input: $u(t) \equiv 2$

$$\ddot{y(t)} + 2\dot{y(t)} + 10y(t) = 20$$

Guess $y_p(t) = C$, need: $10C = 20$,and then get: $C = 2$

So general form of Input response:

$$y(t) = y_h(t) + y_p(t) = c_1e^{-t}cos(3t) + c_2e^{-t}sin(3t) + 2$$

v. (a) For the zero initial conditions: $y(0) = 0, y(\dot{0}) = 0$

We get:

$$y(0) = c_1 + c_2 + \frac{16}{9} = 0$$

$$y(\dot{0}) = (-1 + \sqrt{10})c_1 + (-1 - \sqrt{10}c_2) = 0$$

So:

$$c_1 = -1.17, c_2 = -0.6078$$

So input response:

$$y(t) = -1.17e^{(-1+\sqrt{10})t} - 0.6078e^{(-1-\sqrt{10})t} + \frac{16}{9}$$

(b) For the zero initial conditions: $y(0) = 0, y(\dot{0}) = 0$

We get:

$$y(0) = c_1 + 2 = 0$$

$$y(\dot{0}) = -c_1 + 3c_2 = 0$$

So:

$$c_1 = -2, c_2 = -2/3$$

So input response:

$$y(t) = -2e^{-t}cos(3t) - \frac{2}{3}e^{-t}sin(3t) + 2$$

Question 2

a TF:

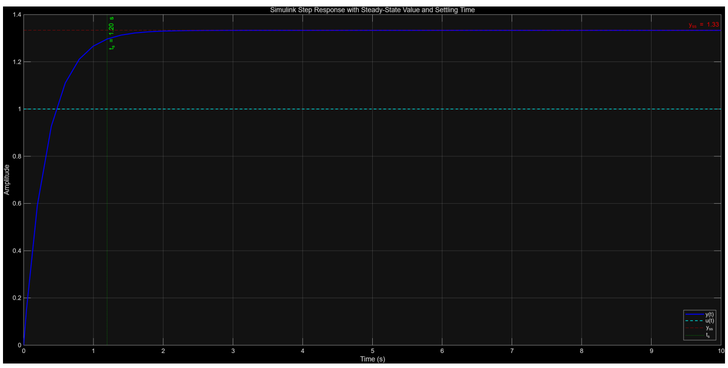
$$G(s) = \frac{4}{s+3}$$

Roots: $s = -3 < 0$So this system is stable.

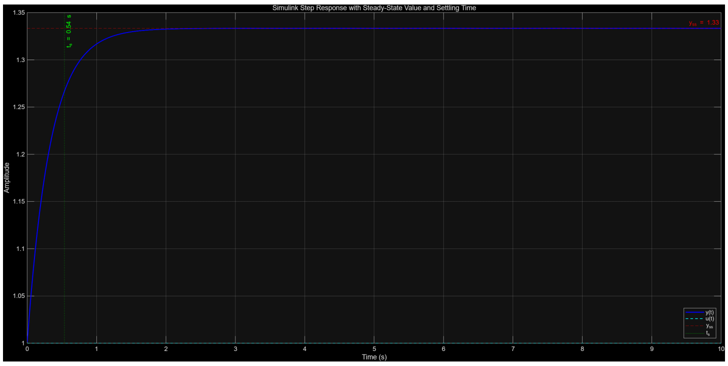
b Poles: $s = -3$

$$\text{Time constant: } \tau = \frac{1}{|Re(p)|} = \frac{1}{3}s$$

c y(0) = 0:



d y(0) = 1:



Question 3

i. (a) TF:

$$G(s) = \frac{12}{s^2 + 2s + 10}$$

$$2\zeta\omega_n = 2, \omega_n^2 = 10$$

$$\omega_n = \sqrt{10}, \zeta = \frac{1}{\sqrt{10}} \approx 0.3162 < 1$$

So the system is under-damped. (b) TF:

$$G(s) = \frac{8}{s^2 + 20s + 4}$$

$$2\zeta\omega_n = 20, \omega_n^2 = 4$$

$$\omega_n = 2, \zeta = 5 > 1$$

So the system is over-damped.

ii. (a)

$$T_s \approx \frac{3}{\zeta\omega_n} = 3s$$

(b) This is a over-damped system, and usually the settling time of a over-damped system can be obtain from simulation which is told by professor on piazza and recommended book (Control system: An introduction). But I found a way to estimate the settling time of a over-damped system on the internet called Dominant pole method, from which we can get an approximate value without simulation:

$$T_s \approx \frac{3}{|Re(P_d)|} = \frac{3}{0.202} = 14.8515s$$

Also, for verification, result from simulation:

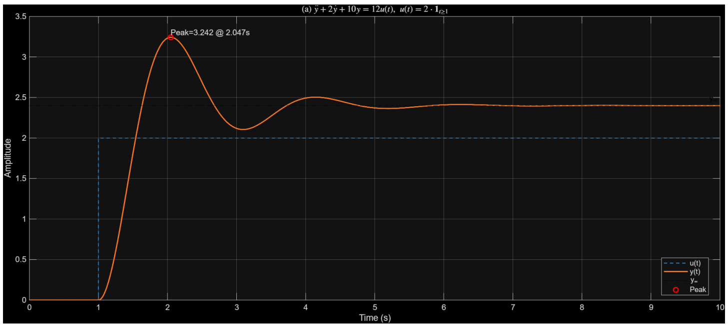
$$T_s = 14.8792$$

iii. (a)

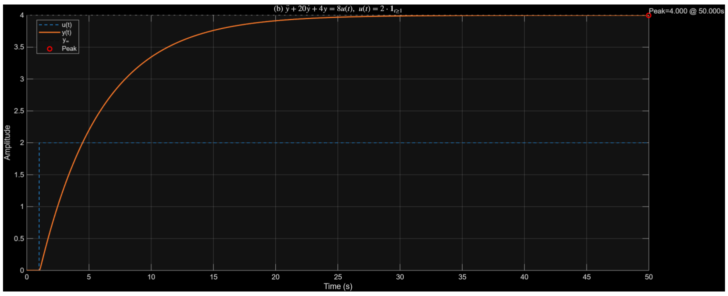
$$M = e^{\frac{-\zeta}{\sqrt{1-\zeta^2}}\pi} = 0.3509 = 35.09\%$$

(b) For it is over-damped system, there is nop overshoot and $M = 0$

iv. (a) From iii, $M = 35.09$, step value $= 2$, DC Gain $= 1.2$, $y_{ss} = 2.4$ So Peak Value $= 2.4 * (1 + 0.3509) = 3.2422$ And from picture, Peak Value $= 3.242$ which is corresponding to the result we get from iii.



(b) From iii, $M = 0$, step value $= 2$, DC Gain $= 2$, $y_{ss} = 4$ So Peak Value $= 4 * (1 + 0) = 4$ And from picture, Peak Value $= 4$ which is also corresponding to the result we get from iii.



Question 4

a

$$G(s) = \frac{8000}{s^5 + 77s^4 + 2051s^3 + 21325s^2 + 80000s + 406250}$$

$$\text{Poles:} s_1 = s_2 = s_3 = -25, s_4 = -1 + 5i, s_5 = -1 - 5i$$

b

$$\tau_1 = \tau_2 = \tau_3 = 0.04, \tau_4 = \tau_5 = 1$$

c A second-order approximation because the poles i choose is complex.

d

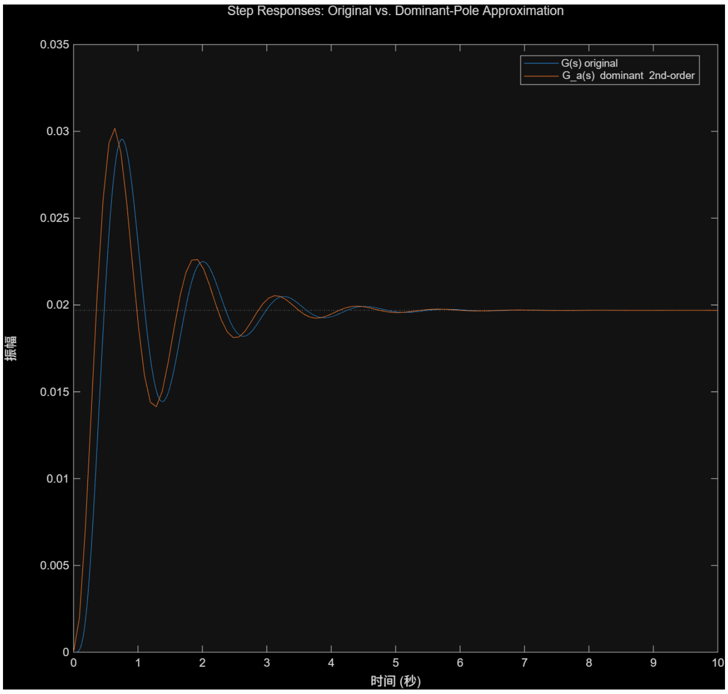
$$G_a(s) = \frac{b_0}{s^2 + 2s + 26}$$

$$G_a(0) = \frac{b_0}{26} = \frac{8000}{406250}$$

$$b_0 = 0.512$$

$$G_a(0) = \frac{0.512}{s^2 + 2s + 26}$$

e The step response:



Question 5

a

$$H(s) = \frac{8000}{s^5 + 637s^4 + 7571s^3 + 44685s^2 + 194400s + 406250}$$
$$\text{Poles:} s_1 = -625, s_2 = 1 + 5i, s_3 = 1 - 5i, s_4 = s_5 = -5$$

b

$$\tau_1 = 0.0016, \tau_2 = \tau_3 = 1, \tau_4 = \tau_5 = 0.2$$

c

$$H_a(s) = \frac{b_0}{s^2 + 2s + 26}$$
$$H_a(0) = \frac{b_0}{26} = \frac{8000}{406250}$$
$$b_0 = 0.512$$
$$H_a(0) = \frac{0.512}{s^2 + 2s + 26}$$

Question 6

When they have the same poles 1, $G_a(s)$ is better because the other poles are 0.04, which is smaller enough than $H_a(s)$ which is 0.2.

Question 7

$$DCGAIN = J(0) = \frac{a_0}{10}$$

Consider the step value is 1 as conventional: $\frac{a_0}{10} = 5$

$$a_0 = 50$$

Then from simulation on MATLAB, tuning the value of a_1, get $a_1 = -51$