Unemployment, Factor Substitution, and Capital Formation

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Abstract

We incorporate a wage bargaining structure in a dynamic general equilibrium model and show how this feature changes short and long-run properties of equilibria compared with a perfectly competitive setting. We discuss how employment, capital, and income shares respond to wage setting shocks and show that adjustment dynamics depend decisively on the magnitude of the elasticity of substitution between labour and capital. Values of the elasticity below unity add persistence, tend to preserve stability, and lead to empirically plausible adjustment patterns. By contrast, values above unity introduce additional volatility, thereby making steady states potentially unstable.

Keywords: Wage bargaining, Unemployment, Overlapping generations

JEL classification: D51, E24, E25

Zusammenfassung

Wir betrachten ein intertemporales Gleichgewichtsmodell mit Lohnverhandlungen am Arbeitsmarkt und diskutieren die Eigenschaften von kurz- und langfristigen Gleichgewichten im Vergleich zu einer vollständig kompetitiven Ökonomie. Wir analysieren die Reaktion von Beschäftigung, Kapital und Einkommensverteilung auf Lohnschocks und diskutieren dabei insbesondere, wie sich Annahmen bezüglich der Substitutionselastizität zwischen Arbeit und Kapital auf die dynamischen Anpassungsprozesse auswirken. Werte der Elastizität, die kleiner als eins sind, erhöhen die Persistenz, wirken stabilisierend und ergeben empirisch plausible Anpassungspfade. Unterstellt man hingegen Werte für die Elastizität, die größer als eins sind, erhöht dies die Volatilität aller Variablen, und langfristige Gleichgewichte werden dadurch potenziell instabil.

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1 Introduction

This paper explores effects of insider-dominated wage setting in an intertemporal equilibrium framework. While, in static treatments of insider-outsider relationships, unemployment results by and large from wages which are set too high to be consistent with full employment, the issue becomes more complicated from a dynamic perspective. Attempts by labour to raise wages and thereby to capture rents which are available at a given capital stock are likely to trigger (possibly slow) reactions of capital which in the long run is the more elastic factor.[1] In particular, capital accumulation may slow down in order to restore the initial level of profitability, and, as a by-product, employment may well fall even if current wages are seemingly not out of line. Moreover, as discussed by Blanchard (1997, 1998) in his analysis of recent developments in continental European countries, income shares are particularly susceptible to strong shifts if the interaction between labour and capital takes place in not fully competitive settings. More specifically, the findings of Blanchard indicate that adverse wage setting shocks tend to raise the labour share on impact, while in the long run this process is likely to be reversed and labour may even be worse off than in the initial situation.[2]
To address the interaction between wage setting, employment and capital formation, we consider an overlapping generations model with production in which we augment an otherwise perfectly competitive set-up with wage bargaining in the labour market. Owing to this feature, unemployment in our model is not a matter of misperceptions but an equilibrium phenomenon.[3] Reflecting the general equilibrium character of the model, the evolution of employment, capital and income shares depends solely on parameters describing (i) the economy's technology, (ii) characteristics of the

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[1] For a detailed discussion of this interaction, see Caballero and Hammour (1997).

[2] We return to a more detailed discussion of these findings in Section 6.

[3] Thus, our work follows the agenda set out in Phelps (1994) on 'structural slumps': "Behind the equilibrium path of unemployment are nonmonetary factors working through nonmonetary mechanisms: the propensity to quit or shirk, hysteresis effects of idleness, insider-outsider relationships, welfare-state subsidies, rent-seeking unions, balance sheet factors in financial markets, and the institutional substructure." (Phelps, 1994, p. vii.)

labour market, such as the bargaining power of the negotiating sides, benefit levels and turnover rates, and (iii) the preferences of agents.

We consider a right–to–manage set-up in which sector–specific unions and employers' federations bargain on behalf of a large amount of firms and workers over wages which are mandatory for all employment contracts.[4] In every period, taking the sector–wide capital stock as given, the two sides decide on the wage, and individual firms then have full discretion over the level of employment. With all firms being identical, employment in the representative sector will depend on the bargaining power of the two sides, the value of the reservation wage maintained during the bargain by the union, and the location of the labour demand curve which shifts with changes in the capital stock. To ensure outcomes which are at all stages consistent with equilibrium unemployment, we assume that the labour market is subject to a certain degree of (exogenously imposed) mismatch. In every period some jobs cannot be filled with workers from the same sector so that unemployed persons always have a chance of obtaining a job at the going wage. This in turn feeds back into the reservation wage maintained by the union in the bargaining process. As a result, some equilibrium unemployment will always emerge, irrespective of the level of the capital stock, in order to restrain the wage demand of the representative union.

Since firms act as price-takers in the labour market, the right–to–manage approach allows for a convenient dichotomy between short and long-run developments. In particular, assuming that in general equilibrium all unemployment benefits are funded through taxes on labour, the long-run stationary capital intensity will be identical to the one resulting under perfect competition, i.e. shocks affecting long-run employment translate into equiproportionate changes in the capital stock. Thus, in our context long-run equilibria naturally relate to steady states of competitive overlapping generations models with production as discussed, for example, in Azariadis (1993).

Using a standard aggregate production function, we leave the elasticity of substitution σ between labour and capital a priori unrestricted. However, it turns out that, depending on this elasticity, short-run properties of equilibria and off-steady state dynamics are much richer than in the fully competitive framework. If $\sigma < 1$ there is under plausible conditions a unique short-run equilibrium, parametrized by arbitrary levels of the capital stock. In this case, shocks affecting the labour market (such as a shift in relative bargaining power or changes in the replacement rate) create persistent employment dynamics. Moreover, long-run steady states will be stable whenever they are stable in the benchmark case of full employment dynamics. By contrast, if $\sigma > 1$ uniqueness of short-run equilibria is no longer ensured, and due to strong substitution effects all series become more volatile. Moreover, this volatil-

[4]For detailed treatments of this and alternative bargaining concepts in static frameworks, see McDonald and Solow (1981) and Oswald (1985).

ity breaks the stability correspondence between full employment and unemployment dynamics and creates the possibility of unstable steady states and endogenous cycles. Regarding the evolution of income shares, $\sigma < 1$ leads to empirically plausible adjustment processes. By contrast, $\sigma > 1$ would imply that labour loses out on impact - a finding which seems to be inconsistent with the empirical evidence.

The fact that the steady state level of employment is under bargaining no longer exogenously given leads us to a modification of the golden rule criterion prevailing under "first best" conditions. In particular, consider a steady state with gross complementarity between inputs ($\sigma < 1$) which is dynamically inefficient. Then, taking the static inefficiency resulting from wage bargaining as given, capital investment should be reduced by less than in a competitive setting, since the positive effects of a higher return rate on capital need now to be weighted against a lower level of employment, reducing the steady state level of income.

In related work, Benassy (1997) investigates adjustment dynamics around full employment steady states in a stochastic OLG economy with monopolistic wage setting and an aggregate production function with $\sigma < 1$. The degree of persistence of adjustment dynamics depends crucially on whether wages set by the monopoly union become binding or not, which in turn depends on the magnitude of random technology shocks shifting the labour demand curve. For small shocks, union wages remain non-binding (in the sense that full employment prevails in the adjustment process) and dynamics show little persistence. Only for large shocks union wages become binding and the corresponding unemployment dynamics, though only transitory, show pronounced persistence.[5]

In line with our results derived for short-run equilibria, Rowthorn (1999) shows within a rich right-to-manage framework that for the CES case of $\sigma < 1$ the unemployment rate depends negatively (positively) on the capital stock (labour force), quite in contrast to the limiting case of a Cobb-Douglas function with $\sigma = 1$. Yet, using essentially a static framework, Rowthorn falls short of a dynamic (long-run) analysis which should derive the process of capital accumulation from within the system.

Addressing differences in the evolutions of unemployment and income shares in Anglo-Saxon and continental European countries over the past decades, Blanchard (1997, 1998) uses a framework in which, much as in our approach, the long-run capital intensity is independent of parameters describing a non-competitive labour market. However, his analysis differs from ours in several ways. First, Blanchard considers a constant, exogenously given interest rate. Second, firms are subject to adjustment costs when choosing their inputs.[6] Third, the labour supply of the econ-

[5] In Section 4, we discuss how our stability results relate to the model of Benassy.

[6] Since adjustment costs cause only transitory dynamics, the capital supply is in the long run infinitely elastic, similar to the case of a small open economy. For a similar specification, see Burda (1988).

omy is approximated at the aggregate level by a wage-setting relationship which asserts a negative relationship between the real wage and the unemployment rate. Fourth, Blanchard also considers imperfections in the output market which we specify as being perfectly competitive.

Finally, Hansen (1999) takes a general equilibrium perspective on monopolistic wage setting and considers a Ramsey-type economy in which the long-run capital intensity is uniquely pinned down by the modified golden rule. Addressing the effects of a permanent and unanticipated wage shock, Hansen shows that saddlepath stability prevails as in the prototype Ramsey model with full employment. Yet, dynamics under unemployment are shown to imply an overshooting of employment, i.e. triggered by consumption smoothing, employment falls on impact more than required by the new steady state, which has the same capital intensity as the initial one. Moreover, considering a two-stage production process with sector-specific labour, Hansen assumes rather special functional forms. In particular, he assumes a unit elasticity of substitution between capital and labour.

The remainder of the paper proceeds as follows. In Section 2, we describe the details of the model. In Section 3, we characterize properties of the short-run equilibrium in which the capital stock is fixed. In Section 4, we turn to the dynamic behaviour of the model. Section 5 analyzes effects resulting from increased wage pressure, while Section 6 discusses our results in the context of the existing literature. The final section concludes with a summary and a brief outline of future research.

2 The model

Consider an overlapping generations economy which consists of a large number M of sectors, a continuum $[0, M]$ of consumers/workers living for two periods and a continuum $[0, M]$ of firms. Consumers supply labour when they are young and consume in both lifetime periods. Firms produce a composite consumption/investment good from inputs of capital and labour. The government pays an unemployment benefit which is financed by a wage income tax. The capital and goods markets are perfectly competitive, but wages are the outcome of a bargain between trade unions and employers' federations at the sector level. In each sector a single trade union representing a mass 1 of workers and a single employers' federation representing a mass 1 of firms bargain over the wage. Since the number of sectors is large, negotiation partners ignore the effect of their decision on the unemployment rate and on the level of unemployment benefits.[7] After wages are negotiated, employment

[7] Compared with centralized bargaining, in which a single union and a single employers' federation internalize the impact of their decisions on aggregate outcomes, this assumption tends to increase the unemployment rate (see, for example, Calmfors and Drifill (1988)).

is decided at the level of the firm ("right–to–manage model"). Since each employers' federation represents a large number of firms, the capital investment decision of any single firm does not affect the outcome of the wage bargain and therefore all firms behave in a perfectly competitive fashion. We further assume some turnover of workers between sectors. This implies that wages paid in other sectors and the aggregate unemployment rate matter for wage formation in each sector. As a consequence, any equilibrium involves some positive rate of unemployment. In detail, the economy is described as follows.

Consumers

Each consumer born at date t supplies one unit of indivisible labour when young and wishes to consume in periods t and $t+1$. Consumers save part of their labour income for retirement by holding capital shares which pay a gross real rate of return R_{t+1}. An employed worker receives a (real) net wage $w_t(1-\tau_t)$ whereas an unemployed worker receives the (real) unemployment benefit b_t. Workers are randomly allocated to jobs. Their von Neumann–Morgenstern utility function $u(c_t, c_{t+1})$ is assumed to be linearly homogenous, strictly quasi–concave and differentiable. In particular, workers are risk–neutral. Thus, each worker's savings behaviour is described by a savings function $s(R_{t+1})I_t$ where $I_t \in \{w_t(1-\tau_t), b_t\}$ denotes the first–period income, and the consumer's indirect utility is $v(R_{t+1})I_t$ where $v(R_{t+1}) \equiv u(1 - s(R_{t+1}), s(R_{t+1})R_{t+1})$. We assume $s' \geq 0$, i.e. savings are non–decreasing in the interest rate.

Trade unions

Each trade union represents a continuum of mass 1 of workers in its sector. For the sake of simplicity we assume that all workers are union members. However, workers need not be employed in their home sector for the whole period, but there is some turnover of workers between sectors.[8] A fraction $0 < \pi < 1$ of the initially created work relationships turns out to be unproductive, and these relationships are separated immediately. The resulting vacancies are then filled with (unemployed) workers from some other sector. If $L_t \leq 1$ jobs are created in some sector, a fraction πL_t of these jobs is eventually not matched with workers of the home sector, but is filled with workers of some other sector. Thus $(1-\pi)L_t$ workers receive the negotiated sector net wage $w_t(1-\tau_t)$, and $1-(1-\pi)L_t$ workers are either employed in another sector or stay unemployed. Their expected income, denoted w_t^*, will be determined below. The trade union's objective is to maximize the expected utility of a representative member which is $V_t \equiv v(R_{t+1})\left(w_t(1-\tau_t)(1-\pi)L_t + w_t^*(1-(1-\pi)L_t)\right)$. When negotiations break down, the union's fallback payoff is $\overline{V}_t \equiv v(R_{t+1})w_t^*$, and

[8] Other, more ad–hoc specifications of turnover in wage bargaining models can be found in Layard, Nickel, and Jackman (1991) and in Rowthorn (1999).

thus the union surplus of a successful negotiation is

$$V_t - \overline{V}_t = v(R_{t+1})(1-\pi)(w_t(1-\tau_t) - w_t^*)L_t. \tag{1}$$

Firms and employers' federations

Firms produce the output good from capital and labour using the constant returns production technology $Y_t = F(K_t, L_t) = L_t f(k_t)$ where $k_t = K_t/L_t$ is the capital intensity. The intensive–form production function f is assumed to be increasing and strictly concave. Capital investment is decided a period in advance and thus before wages are negotiated. However, since the investment decision of any single firm has a negligible effect on the total capital stock and thus on labour demand in its sector, firms ignore the impact of their investment decision on the outcome of wage negotiations. Thus firms take the (perfectly foreseen) wage and the interest rate as given, which implies the usual marginality conditions

$$\begin{aligned} w_t &= w(k_t) \equiv f(k_t) - k_t f'(k_t) \text{ and} \\ R_t &= R(k_t) \equiv 1 - \delta + f'(k_t) \ , \end{aligned}$$

where δ is the depreciation rate. Firms of each sector are organized in an employers' federation whose objective is the profit of each of its members,

$$\Pi_t \equiv F(K_t, L_t) - w_t L_t = (f(k_t) - w(k_t))L_t = k_t f'(k_t) L_t \ . \tag{2}$$

The fallback payoff of the employers' federation is assumed to be $\overline{\Pi}_t = 0$.

The wage bargain

Given a sector capital stock K_t, the trade union and the employers' federation negotiate the sector wage taking into account that employment is decided by firms. The large number of sectors implies that negotiation partners ignore the impact of their decision on capital formation and thereby on the capital return in the next period. In particular, unions take R_{t+1} in their objective function as given. The outcome of the wage bargain is determined by the Nash bargaining solution which maximizes the Nash product $(\Pi_t - \overline{\Pi}_t)^{\beta}(V_t - \overline{V}_t)^{1-\beta}$ subject to $w_t = w(K_t/L_t)$. $\beta \in [0,1]$ denotes the bargaining power of the employers' federation. Using (1), (2), $L_t = K_t/k_t$ and cancelling out constants, the Nash program can be reformulated in terms of the sector's capital intensity:

$$\max_{k_t \geq K_t} \left(k_t f'(k_t)\right)^{\beta} \left(w(k_t)(1-\tau_t) - w_t^*\right)^{1-\beta} k_t^{-1} \ .$$

The Nash product is zero at k^* such that $w(k^*)(1-\tau_t) = w_t^*$ and it is zero at $k = \infty$ since $w(k)/k$ tends to zero as $k \to \infty$. We assume that the Nash product is strictly quasi–concave (which is satisfied, for example, for the CES cases considered below). Hence a maximum necessarily exists and an interior (or unemployment) solution of the Nash program is characterized by the first-order condition[9]

$$\beta \frac{f''(k_t)k_t}{f'(k_t)} + (1-\beta)\left(\frac{k_t w'(k_t)(1-\tau_t)}{w(k_t)(1-\tau_t)-w_t^*} - 1\right) = 0 \ . \tag{3}$$

Note that (3) implicitly balances three effects against each other, the first one with weight β, the other two with weight $1-\beta$: First, for given K_t higher wages reduce profits by the factor price frontier, i.e. $f'(k_t)$ and thus profits decline. Second, employed workers benefit from higher wages, i.e. $w_t(1-\tau_t)-w_t^*$ rises. Third, higher wages reduce employment and thus the surplus of the union.

The general equilibrium

Since the workers' outside option w_t^* is the same in all sectors and since (3) has only one solution when the Nash product is strictly quasi–concave, employment and wages in all sectors coincide and every general equilibrium is symmetric. When L_t is employment in each sector, $M(1-(1-\pi)L_t)$ workers in the economy are eventually not employed in their home sector. With some probability φ_t each of them finds a job in some other sector. Given that there are $M\pi L_t$ vacancies left, this probability is

$$\varphi_t = \frac{\pi L_t}{1-(1-\pi)L_t} \ . \tag{4}$$

The government maintains a constant replacement rate $\lambda < 1$, which implies that the unemployment benefit is $b_t = \lambda w_t(1-\tau_t)$. Thus the expected income of a worker who is not employed in his home sector is

$$w_t^* = \varphi_t w_t(1-\tau_t) + (1-\varphi_t)b_t = (\varphi_t + \lambda(1-\varphi_t))w_t(1-\tau_t) \ . \tag{5}$$

Note that there cannot be full employment in general equilibrium. If $L_t = 1$, all workers are employed with probability one, $\varphi_t = 1$, and therefore the union's fall-back payoff coincides with the net wage of the other sectors. Since any outcome of the wage bargain must pay some markup over the fallback wage whenever unions have bargaining power ($\beta < 1$), this wage cannot be set in equilibrium. Some unemployment is required to keep union wage demands in check.

Inserting (4) into (5) shows that the workers' outside wage is increasing in aggregate employment: the higher employment is, the more likely it is for an unemployed worker to find a job in some other sector. Combining this with the bargaining

[9] Below it will be shown that there can only be an unemployment outcome in general equilibrium, and that it is therefore sufficient to consider an interior solution.

solution (3) and solving for employment yields a relation between (sector) employment and the capital intensity that has to hold in the short-run (or temporary) equilibrium:

$$L_t = L(k_t) \equiv \frac{1-\mu(k_t)}{1-(1-\pi)\mu(k_t)} \ , \tag{6}$$

where

$$\mu(k) \equiv \frac{(1-\beta)\frac{w'(k)k}{w(k)}}{(1-\lambda)\left(1-\beta\left(1+\frac{f''(k)k}{f'(k)}\right)\right)} \ . \tag{7}$$

The other equation that (trivially) holds in any equilibrium is

$$L_t = K_t/k_t \ . \tag{8}$$

For a given capital stock equations (6) and (8) determine sector employment L_t and the economy's capital intensity in period t. Equation (6) captures equilibrium in the labour market, whereas only (8) includes the capital stock. Finally, the government balances its budget by adjusting the tax rate so that $\tau_t w_t L_t = \lambda(1-\tau_t)w_t(1-L_t)$. This can always be achieved by a unique tax rate τ_t which, however, does not affect employment and the gross wage in our model. This result, which has also been demonstrated in a similar model by Pissarides (1998), depends on the assumption that the government fixes the replacement rate, i.e. the ratio of unemployment benefits to net wages. If, instead, the government did not adjust benefits to wages but fixed the level of (real) unemployment benefits, a higher income tax rate unambiguously reduces employment.

3 The short-run equilibrium

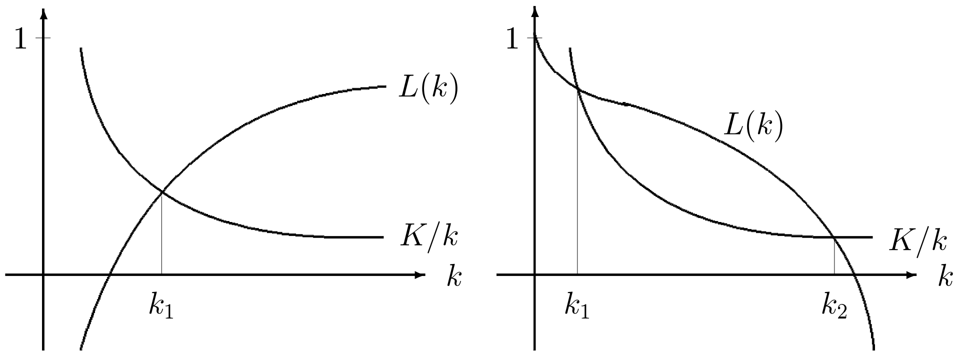
We concentrate in this section on CES production functions given by

$$F(K,L) = [\alpha(A_K K)^{(\sigma-1)/\sigma} + (1-\alpha)(A_L L)^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)} \ , \ 1 \neq \sigma > 0,$$

which, in the boundary case $\sigma = 1$, becomes a Cobb–Douglas production function $F(K,L) = AK^\alpha L^{1-\alpha}$. Existence, uniqueness and comparative statics properties of a short-run equilibrium depend crucially on the elasticity of substitution σ. The function (7) turns out to be

$$\mu(k) = \frac{(1-\beta)\alpha}{(1-\lambda)\left(\sigma\alpha(1-\beta) + (\sigma(1-\beta)+\beta)(1-\alpha)(kA_k/A_L)\right)^{-(\sigma-1)/\sigma})} \ .$$

In the Cobb–Douglas case $\mu(.)$ is a constant, $\mu = (1-\beta)\alpha/((1-\lambda)(1-\alpha\beta))$, which is less than one (and thus $L(k) > 0$) iff $\lambda < (1-\alpha)/(1-\alpha\beta)$. Thus only if the



$(a) : 0 < \sigma < 1$ $(b) : \sigma > 1$

Figure 1: Short-run equilibrium

replacement rate does not exceed this upper bound, there exists a short-run equilibrium with positive employment. The employment level in this case is independent of the capital stock, which is quite consistent with the findings of Layard, Nickell, and Jackman (1991). Note, however, that this result again depends on our assumption that the government fixes the replacement rate instead of real unemployment benefits.

More interesting results are obtained if the elasticity of substitution σ is different from 1. Consider first the case in which capital and labour are gross complements, $\sigma < 1$. Then $\mu(k)$ is strictly decreasing with $\mu(0) = 1/(\sigma(1-\lambda)) > 1$ and $\mu(\infty) = 0$. Thus there exists a unique $\widetilde{k}$ so that $\mu(\widetilde{k}) = 1$, and $L(k)$ is strictly increasing with $L(\widetilde{k}) = 0$ and $L(\infty) = 1$. Figure 1.a shows the two equilibrium curves (6) and (8) and reveals that there exists a unique short-run equilibrium k_1 for any level of the capital stock K. If the capital stock is increased, (8) shifts outwards and employment and the capital intensity (and thereby wages) increase.

Consider next gross substitutes, $\sigma > 1$. Then $\mu(k)$ is strictly increasing with $\mu(0) = 0$ and $\mu(\infty) = 1/(\sigma(1-\lambda))$. Suppose first that $\sigma < 1/(1-\lambda)$, i.e. substitutability between capital and labour is not too strong. Then $\mu(\infty) > 1$ and there exists a unique $\widetilde{k}$ such that $\mu(\widetilde{k}) = 1$. $L(k)$ is strictly decreasing with $L(0) = 1$ and $L(\widetilde{k}) = 0$. Figure 1.b shows the two equilibrium curves (6) and (8) in this case. It can be seen that there are (generically) two short-run equilibrium solutions, denoted here k_1 and k_2, whenever the capital stock is not too large.[10] However, as will be shown in Proposition 2 below, only the left-hand equilibrium can occur in a stable steady state of the dynamic model, i.e. only the left short-run equilibrium can be a stable

[10] In principle, there could be more than two solutions if the slope of the downward sloping function $L(k)$ was sufficiently non–monotone. However, this turns out not to be the case for CES production functions.

long-run equilibrium. A steady state at which the right equilibrium occurs would be unstable. We therefore concentrate below only on the equilibrium k_1 at which (8) cuts (6) from above. If the elasticity of substitution is larger, $\sigma \geq 1/(1-\lambda)$, $L(k)$ is strictly positive for all capital intensities and tends to $(\sigma(1-\lambda)-1)/(\sigma(1-\lambda)-1+\pi)$ as k tends to infinity. In this case there is only one short-run equilibrium k_1 at which the curve (8) cuts (6) from above. In both cases, when the capital stock is increased, k_1 increases (and thus wages) and employment $L(k_1)$ falls. Therefore, capital crowds out labour as a production factor, as can be expected if capital and labour are substitutes.

Evidently, the behaviour of the labour share in response to a rise in K depends on the elasticity of substitution. As can be easily checked, the labour share, $w(k)/f(k)$ is decreasing (increasing) in the capital intensity if and only if $\sigma > 1$ ($\sigma < 1$). Thus, when capital and labour are substitutes, the fall in employment after a rise in K is strong enough to overcompensate the rise in wages: the labour share falls.

The findings of this section are summarized in the following proposition:

Proposition 1 *Assume that the production function has constant elasticity of substitution σ. If $\sigma < 1$, there exists a unique short-run equilibrium for any level of the capital stock. Employment and the labour share increase if the capital stock is increased. If $\sigma > 1$, there are at most two short-run equilibria when the capital stock is not too large. At the equilibrium with the lower capital intensity, employment and the labour share are falling if the capital stock is increased.*

We note in passing that the proposition can easily be generalized to cover situations in which both the capital stock and the labour endowment are subject to changes in the representative sector. Similar findings are also derived in Rowthorn (1999). Assume for a moment that in each sector workers are of mass N_t. This leaves (3) and (5) unaltered, while (4) becomes $\varphi_t = \pi L_t/[N_t - (1-\pi)L_t]$. Combining this with (3) and (5) yields the modified (short-run) equilibrium condition:

$$\frac{K_t}{N_t k_t} = L(k_t) \tag{9}$$

where now $L(k_t)$ denotes the employment rate L_t/N_t. From (9) and Figure 1 one readily infers the effects of a change in relative factor endowments K_t/N_t: (only) for the special case of a Cobb-Douglas technology a rise in K_t/N_t leaves both the unemployment rate and income shares unaffected. By contrast, if $\sigma < 1$ ($\sigma > 1$) a rise in K_t/N_t reduces (increases) the unemployment rate $u_t = 1 - L(k_t)$ and raises (decreases) the labour share.[11]

[11] Note that irrespective of the magnitude of σ the wage (profit rate) rises (falls) if capital becomes relatively more abundant than labour.

Finally, we find it worthwhile to point out that, regardless of the magnitude of σ, full employment equilibria could well emerge in the special case of no turnover, i.e. if one were to admit $\pi = 0$. Assuming $\pi = 0$, $L(k)$ becomes vertical at the value of k which solves (3)-(5) (or equivalently $\mu(k) = 1$), and for K being larger than this value, $L = 1$ would turn into a binding restriction. In other words, given a fixed labour supply, full employment can emerge for $\pi = 0$ if the outward shift of the labour-demand schedule induced by a rise in K is sufficiently strong.

4 Dynamics

The total income of young consumers (employed and unemployed workers) equals

$$M(1-\tau_t)w(k_t)L(k_t) + M\lambda(1-\tau_t)w(k_t)(1-L(k_t)) = Mw(k_t)L(k_t) \ ,$$

since the budget of the government is balanced. A fraction $s(R_{t+1})$ of this income is saved as capital holdings giving the capital stock MK_{t+1} of the next period. Using (8) and cancelling out M, we obtain the following dynamic equation describing the evolution of the capital intensity:

$$k_{t+1}L(k_{t+1}) = s(R(k_{t+1}))w(k_t)L(k_t) \ . \tag{10}$$

It can be immediately seen that the steady state capital intensity is independent of labour market conditions and coincides with the stationary capital intensity of the full employment model with a competitive labour market ($L = 1$), in which case the dynamics follows the equation

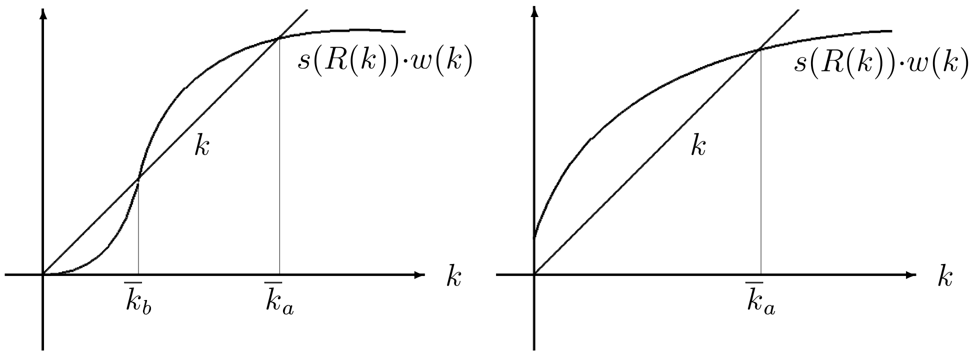
$$k_{t+1} = s(R(k_{t+1}))w(k_t) \ . \tag{11}$$

In both models the steady state capital intensity is determined by[12]

$$\overline{k} = s(R(\overline{k}))w(\overline{k}).$$

This result has the important consequence that the gross wage and the interest rate in the steady state do not depend on labour market parameters. In the long run, changes in labour market conditions translate completely into changes of employment and of the capital stock, both in the same proportions. In terms of Figure 1, employment is determined at the intersection of (6) with the (not necessarily unique) stationary capital intensity $\overline{k}$, and dynamic adjustments to the steady state are described by shifts of (8). In the gross complements case (Figure 1.a), capital and labour move in the same direction along an adjustment path to the steady state,

[12] Note that this result crucially depends on our assumption of homothetic utility which implies that savings are a linear function of income.



$(a) : 0 < \sigma < 1$ $(b) : \sigma > 1$

Figure 2: Long-run equilibrium

whereas for gross substitutes (k_1 in Figure 1.b), employment falls when the capital stock increases, and vice versa.
It is well known that there can be multiple positive steady states. For instance, suppose that savings do not depend on the interest rate (which is the case if the intertemporal utility function is Cobb–Douglas) or, more generally, that the interest elasticity of savings is not too large. If the production function has CES $\sigma \geq 1$, there is only one positive steady state, as shown in Figure 2.b. This steady state is globally stable in the full employment dynamics (11). But if $\sigma < 1$, the equation has typically no or two positive solutions $\overline{k}_a$ and $\overline{k}_b$, as shown in Figure 2.a, of which the larger one is locally stable, whereas the other one is unstable in the full employment dynamics (11).
To examine stability in more generality, denote by $\eta_h(x) \equiv h'(x)x/h(x)$ the elasticity of some function h at x. Implicit differentiation of (10) at a steady state $\overline{k}$ yields

$$d(\overline{k}) \equiv \frac{dk_{t+1}}{dk_t}(\overline{k}) = \frac{\eta_w(\overline{k}) + \eta_L(\overline{k})}{1 + \eta_L(\overline{k}) - \eta_s(R(\overline{k}))\eta_R(\overline{k})} \ . \tag{13}$$

From our assumptions with respect to the production function and the savings function we have

$$\eta_w(\overline{k}) > 0 \ , \ \eta_s(R(\overline{k})) \geq 0 \ , \ \eta_R(\overline{k}) < 0 \ .$$

Hence, it is clear that the dynamics with full employment ($\eta_L(\overline{k}) = 0$) is monotonic, $d(\overline{k}) > 0$, and that a steady state is locally stable iff

$$\eta_w(\overline{k}) < 1 - \eta_s(R(\overline{k}))\eta_R(\overline{k}) \ . \tag{14}$$

In particular, for a CES production function this condition is satisfied at the unique positive steady state when $\sigma \geq 1$ and it is satisfied at the larger of the two positive steady states if $\sigma < 1$.

In the dynamics with unemployment (10), stability depends crucially on the elasticity of the employment function L. For a CES production function, Figure 1.a tells us that $\eta_L(\overline{k}) > 0$ if $\sigma < 1$. Proposition 2 below shows that such a steady state is stable if and only if it is stable in the competitive model, i.e. iff the stability condition (14) is satisfied. Thus, the larger of the two steady states ($\overline{k}_a$ in Figure 2.a) is locally stable also in the unemployment dynamics. When $\sigma > 1$, there exists a unique steady state which can be of two different types, as indicated in Figure 1.b. Either it is as k_1 at which the curve (8) cuts (6) from above and consequently we have $\eta_L(k_1) > -1$, or (8) cuts (6) from below as at k_2 which implies $\eta_L(k_2) < -1$. Both such situations are possible, depending on technology, utility and labour market parameters. However, whenever the interest elasticity of savings is less than one, it emerges that a steady state at k_2 cannot be stable, whereas a steady state at k_1 may be stable or unstable. Denoting by $\sigma(k)$ the elasticity of substitution at k, we have

Proposition 2 *Suppose that $\eta_s(R(\overline{k})) < 1$.*
(i) If $\eta_L(\overline{k}) < -1$ and $\sigma(\overline{k}) > 1$, then $\overline{k}$ is unstable.
(ii) If $\eta_L(\overline{k}) > -1/2$, then $\overline{k}$ is stable if and only if (14) is satisfied.
(iii) In particular, if the production function has CES $\sigma \geq 1$, then the unique positive steady state $\overline{k}_a$ is locally stable if $\eta_L(\overline{k}_a) > -1/2$ and the steady state is unstable if $\eta_L(\overline{k}_a) < -1$. If $\sigma < 1$, then $\overline{k}_a$ is stable and $\overline{k}_b$ is unstable, as they are in the competitive model.

Proof: Appendix.

Three remarks are worth making. First, the proposition does not answer whether in the CES case $\sigma > 1$ a steady state with $-1 < \eta_L(\overline{k}) < -1/2$ is stable or not (which may be the case at k_1 in Figure 1). Suppose that savings are independent of the interest rate ($\eta_s(R(\overline{k})) = 0$). Stability of the steady state then requires that $d(\overline{k}) > -1$, which means that $\eta_w(\overline{k}) + 2\eta_L(\overline{k}) + 1 > 0$. Since $\eta_w(\overline{k}) < 1$ because of (16) and $\sigma > 1$, this condition is fulfilled if $\eta_L(\overline{k})$ is close to $-1/2$, but it is certainly not fulfilled if $\eta_L(\overline{k})$ is close to -1. Furthermore, even if the steady state is locally stable, convergence to the steady state can be cyclical (this is also possible if $\eta_L(\overline{k}) > -1/2$). Moreover, endogenous cycles can exist.[13] Thus, in our OLG model with unemployment, steady states may be unstable if capital and labour are gross substitutes (and, moreover, consumption goods in the two lifetime periods are gross substitutes in the intertemporal utility function). This contrasts

[13] By the flip bifurcation theorem, a cycle of order 2 exists close to a bifurcation point at which $d(\overline{k}) = -1$ (a set of parameters close to a bifurcation point is $\sigma = 1.5$, $A_K = A_L = 100$, $\alpha = .3$, $\lambda = .6$, $\beta = .3$, $\pi = .1$, $s = .2$) Simulation studies suggest, however, that such bifurcations are subcritical, i.e. an unstable cycle coexists with a stable steady state (corridor stability).

with other results in the literature who find bifurcations and endogenous cycles in overlapping generations economies only if there is either complementarity in utility (e.g. Grandmont (1985)) or in production (e.g. Reichlin (1986) and Grandmont, Pintus, and de Vilder (1998)).

Second, the stability analysis and, in particular, the dichotomy between the substitutes and complements cases is not only confined to the overlapping generations (or Diamond) model with two period lives. In the Solow growth model and in the OLG model with stochastic lives of Benassy (1997), capital accumulates according to $K_{t+1} = (1-\delta)K_t + sF(K_t, L_t)$ with some $0 < \delta, s < 1$.[14] If our labour market equilibrium is embedded in this model the dynamic equation is, instead of (10), given by

$$k_{t+1}L(k_{t+1}) = L(k_t)((1-\delta)k_t + sf(k_t)) \ .$$

It is easy to see that the steady state capital intensity is again independent of labour market conditions, and that (13) is replaced by

$$d(\overline{k}) = \frac{dk_{t+1}}{dk_t}(\overline{k}) = \frac{1-\delta+\delta\eta_f(\overline{k})+\eta_L(\overline{k})}{1+\eta_L(\overline{k})} \ .$$

It can again be easily checked that the steady state is stable if $\eta_L(\overline{k}) > -1/2$, and that the steady is unstable if $\eta_L(\overline{k}) < -1$. Hence the results of Proposition 2 apply also to this model, but additional assumptions about the savings elasticity or about the elasticity of $w(k)$ are not required, since the first is zero by assumption and since savings are a constant fraction of total income, and not just of labour income as in the Diamond model.

Third, not only stability but also optimality of steady states depends crucially on the technological substitutability between capital and labour. Suppose that there is a central planner who is incapable of changing labour market institutions but who can influence capital accumulation (for instance, by a lump–sum redistribution between the young and the old generation). If the central planner is far–sighted, he would be interested in maximizing steady state consumption which is given by $C = F(K,L) - \delta K = L(k)(f(k) - \delta k)$. Maximizing this expression reveals that only if employment is independent of the capital intensity (i.e. if the production function is Cobb–Douglas) the optimal steady state will be characterized by the usual golden rule condition $f'(k) = \delta$. But if capital and labour are complements, a higher capital intensity (and a lower interest rate) are required for optimality since a higher capital stock triggers a higher employment level, and vice versa if capital and labour are substitutes.

[14] In the model of Benassy (1997) (which is based on Huffman (1993)), households work only in the first period of their lives and receive capital income in all future periods. A constant probability of death in each period and logarithmic preferences imply that savings are a constant fraction of aggregate income, as in the Solow model. Our labour market equilibrium can be embedded in this model in the same way as in the standard Diamond model with two period lives.

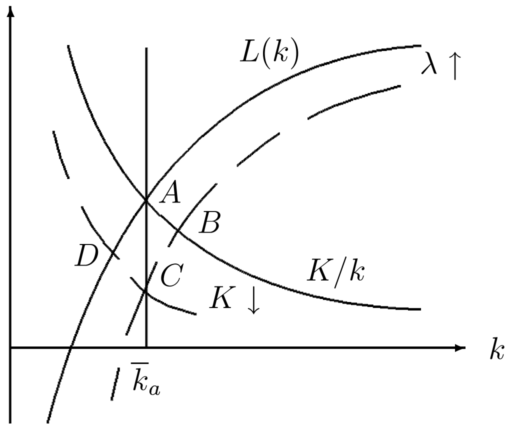


Figure 3: Dynamics ($0 < \sigma < 1$)

5 Effects of wage pressure

To illustrate how sensitively the dynamic behaviour of our economy depends on the magnitude of σ, we return now to the CES specification introduced in Section 3. Specifically, suppose that the economy is initially in a steady state which is stable under full employment dynamics ($\bar{k}_a$ in Figure 2). Moreover, consider an adverse shock to wage setting which is, for instance, induced by stronger power of unions in wage negotiations (a fall in β) or by more generous payments of unemployment benefits (a higher replacement rate λ). As can be seen from (7), both such shocks increase μ and, since $L(k)$ is decreasing in μ, they shift the curve (6) downwards. Figures 1.a and 1.b reveal that, irrespective of the magnitude of σ, such shocks lead to a decline in employment and to a higher capital intensity (and wages). Thus, the impact effect of an adverse shock to wage setting on employment is the same in both cases.

Suppose first that the shock is *permanent*. From Proposition 2, we know that for $\sigma < 1$ the dynamics will always be (locally) stable. As shown in Figure 3, gross complementarity of inputs ensures that the initial decline in employment (move from A to B) will be reinforced over time as capital decumulates on the way to the new steady state C. Moreover, the labour share increases in this case on impact (move from A to B) and returns gradually over time to its initial level, as does the capital intensity k. However, the levels of employment and capital in the new steady state are permanently lower.

As indicated by Proposition 2, for $\sigma > 1$ it is indeed possible to find parametric constellations in which the same experiment turns out to be unstable. Yet, assume dynamics are stable. Again, employment will be lower in the long run than in the initial situation. However, in contrast to the case $\sigma < 1$, employment now

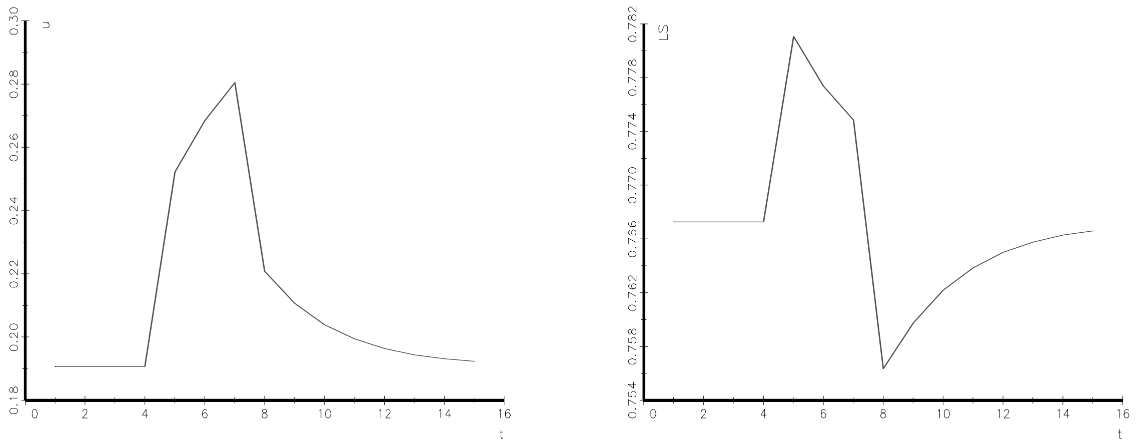


Figure 4: Temporary wage pressure: unemployment rate and labour share ($\sigma = 0.5$)

overshoots on impact, since the ensuing process of reduced capital formation leads to a crowding-in of employment under gross substitutability. Moreover, the labour share falls as a response to the shock and returns slowly to its long-run equilibrium level from below.

Differences between the two regimes become even more pronounced if one considers a *temporary* shock to wage setting. The qualitative effects of such temporary shocks can be read off from Figure 3 if one considers movements from A over B to D back to A. To illustrate this in more detail, we present results from a stylized simulation exercise. Technology parameters are $A_K = A_L = 20$, $\alpha = 0.5$, labour market parameters are $\lambda = 0.6$, $\beta = 0.3$, $\pi = 0.1$, and the savings rate is $s = 0.14$. When $\sigma = 0.5$, there is a stable steady state at about $k = 3.297$ with unemployment rate 0.19 and labour share 0.77. The simulation starts in period 1 at the steady state, and in period 5 there is a 10 percent increase in the replacement ratio to $\lambda = 0.66$, lasting three periods. From period 8 on, the replacement rate is back at 0.6. Figure 4 shows the adjustment paths of the unemployment rate and of the labour share. The unemployment dynamics exhibits some persistence, whereas the labour share increases first, but falls below its steady state value when the wage pressure is over, before it gradually climbs back to its original value.

Figure 5 illustrates the same simulation exercise for $\sigma = 2$. Note that unemployment overshoots first and falls temporarily even below its original value when the wage pressure is over, since now labour substitutes partly for the lower capital stock. On the other hand, the labour share falls first (due to the strong rise in employment), but exceeds its original value after the wage pressure is over.

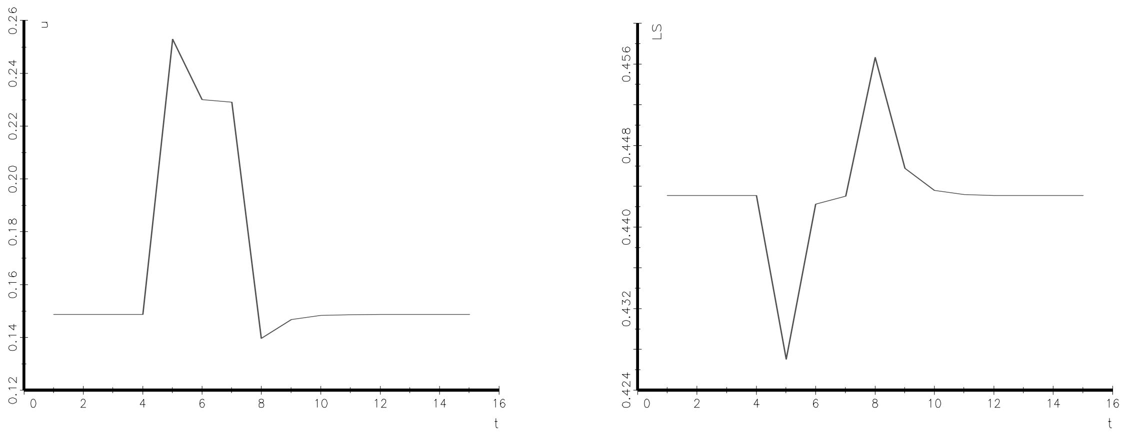


Figure 5: Temporary wage pressure: unemployment rate and labour share ($\sigma = 2$)

6 Discussion

This section discusses from three perspectives how our model and the results fit into the existing literature.

(i) Technology
The dependence of virtually all our results on the elasticity of substitution between labour and capital invariably raises the question of how to appropriately specify the technological set-up in an aggregate framework. The concept of a production function with undated inputs suggests that the entire capital stock can be freely combined with all available units of labour. Yet, as discussed at large in the literature on putty-clay aspects of technology, this notion is misleading since the embodiment of technology in the capital stock limits ex-post substitution possibilities once investments have been made. Addressing this 'putty-clay' feature of capital in a particularly clear-cut way, Caballero and Hammour (1997), for example, present a framework with high ex-ante substitution elasticities for new units of still uncommitted capital. However, once investment decisions have been carried out, capital intensities become fixed and vintage-specific. As a result, the productive structure is characterized by the entire distribution of vintages with associated capital intensities.[15]
If, despite this compelling criticism, one adheres to the concept of an aggregate production function with undated inputs, assumptions made with respect to the elasticity of substitution are of key importance. However, the literature disagrees on the magnitude by which high ex-ante values of this elasticity for the current vintage

[15] For a similar discussion, see Bean (1989).

of capital goods should be discounted in order to obtain a single ex-post value for the entire capital stock already in place.[16] In a recent study on various OECD countries, Andersen et al. (1999) address thoroughly the misspecification problem if estimates for the elasticity are directly obtained from a standard 'putty-putty' framework. Allowing for long adjustment lags in an error correction specification, the study recovers for all countries values of the elasticity well below unity. Findings of this type are also reported in Rowthorn (1999), who gives a broad review of the empirical literature on this issue. However, deviating evidence for France and Germany is given by Berthold, Fehn, and Thode (1999), who use an error-correction model and estimate values of σ significantly above unity.

Similarly, the assumptions made in the theoretical literature are also inconclusive: Benassy (1997), Rowthorn (1999), and Acemoglu (2000), for example, confine themselves to values below unity. Daveri and Tabellini (2000), Hansen (1999) and the widely cited textbook by Layard, Nickell, and Jackman (1991) maintain the assumption of a unit elasticity. Blanchard (1997, 1998) considers scenarios for $\sigma = 1$ and $\sigma = 2$, although the implications of this assumption are different from ours due to the explicit introduction of adjustment costs.

Taken at face value, an overlapping generations economy with two period lived agents seems not to be the ideal framework to address this debate. However, to make the story underlying our model more compelling in the putty-clay context, one could simply assume that firms learn the realization of some shock describing the state of technology only after investments in unspecific capital have been made. More importantly, recall from Section 4 that our results apply also, for example, in the context of a standard Solow growth model. Thus, we think that our results give some guidance for this debate.

In particular, given the pronounced persistence in observed time series and the evolution of income shares, the predictions of our model for $\sigma > 1$ are not particularly plausible. Thus, complementing empirical studies on putty-clay aspects of technology, our analytical findings suggest that values of $\sigma < 1$ do indeed have more reasonable properties if one uses a standard production function with undated inputs.

(ii) Evolution of income shares

As argued in detail by Blanchard (1997, 1998) in his analysis of continental European unemployment over the past decades, adverse labour market shocks may in isolation potentially explain the rise in unemployment, but not in combination with the observed evolution of income shares. The key finding reported by Blanchard is as follows: The early phase of (rising) unemployment in continental Europe begin-

[16] Caballero and Hammour (1997) set the ex-ante elasticity at a value of $\sigma = 6$ and broadly conlude "that this value is empirically consistent with much lower short-run substitutability in the aggregate production function" (p. 14).

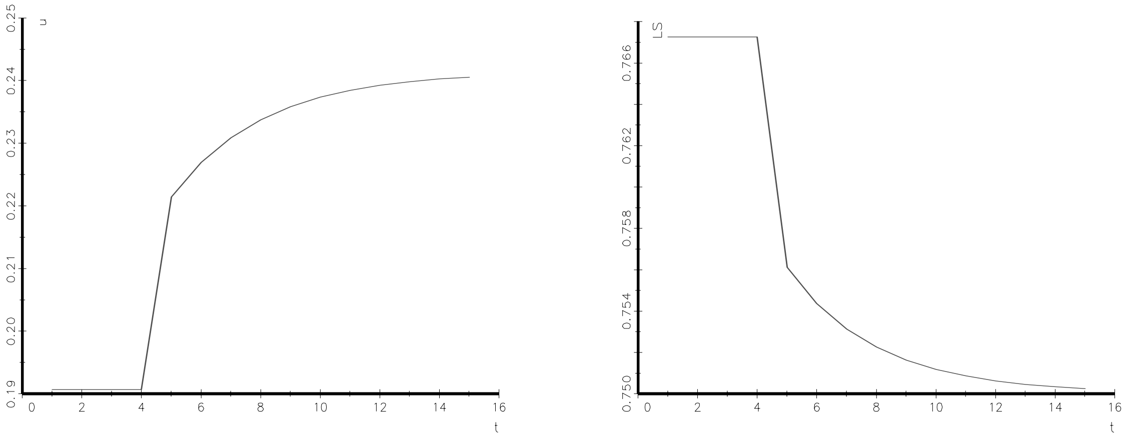


Figure 6: Technological bias towards capital: unemployment rate and labour share ($\sigma = 0.5$)

ning in the 1970s went hand in hand with a rise in the labour share. Yet, since the mid 1980s the share of capital has improved again and reached levels even beyond those prevailing in the early 1970s, while unemployment kept rising.[17] To account for this puzzling reversal of income shares, Blanchard considers the possibility of a gradually emerging technological bias against labour, i.e. a shift of the production function towards less labour-intensive production.[18] Our discussion of the effects of wage pressure, if considered in isolation, shows that our set-up runs into the same difficulties in explaining the puzzling joint behaviour of income shares and unemployment - irrespective of the magnitude of σ. Moreover, we can confirm that technological changes can indeed be an explanation for this puzzle if we reproduce a shock as assumed by Blanchard in the context of our model. To see this, consider the effect of a permanent technological bias from labour to capital, described by a 5 percentage increase of the capital coefficient α in the CES production function from period 5 onwards. All other parameters are as in the simulation exercise in Section 5, and we set again $\sigma = 0.5$. Figure 6 shows the resulting evolution of the unemployment rate and of the labour share. If the same exercise is repeated for

[17] By contrast, in Blanchard's classification the Anglo-Saxon countries lack "medium-run" evolutions of this type. For further empirical evidence on shifts in income shares, see Bentolila and Saint-Paul (1999). However, de Serres, Scarpetta, and de la Maisonneuve (2000) argue that the recent fall in the labour share may be partly due to measurement errors.

[18] To a certain extent, the adoption of such technologies seems to be just another type of a long-run response of capital to earlier wage shocks. Ideally, this should be explained within the model. Drawing on Blanchard's findings, Acemoglu (2000) presents a model in which the direction of technological change becomes endogenous.

$\sigma > 1$, both employment and income shares eventually move in the same direction, but they overshoot initially.
Nevertheless, there are quite different types of shocks which may be considered as well. For the special case of Germany, for example, the exceptional event of unification may have contributed to the puzzling behaviour of income shares. Surely, unification in Germany has prompted a significant change in relative factor endowments. In particular, given the poor quality of the pre-unification east German capital stock, one might argue that in unified Germany labour has become relatively more abundant. And, quite in accordance with the empirical evidence, our brief discussion of short-run effects resulting from changes in relative factor endowments would predict (assuming $\sigma < 1$) that such a shock would lead on impact to a rise in both the unemployment rate and the income share of capital.[19]

(iii) Strategic interaction/Time horizons
To keep things transparent at a general equilibrium level, our model describes strategic aspects of wage setting in a deliberately simple manner. In particular, negotiation partners take as given the sector-wide capital stock which enters the bargain. Thus, we abstract from the typical hold-up problem of firm-specific bargaining as discussed by Grout (1984), Van der Ploeg (1987), Devereux and Lockwood (1991) and Coloma (1999). As a common feature, in these studies the size of the surplus that is relevant to the bargain depends on the investment decision of firms. Depending on the exact specification of the moves of the players, this leads to outcomes which are biased compared with a competitive outcome.
Moreover, the overlapping generations specification addresses the conceptually difficult question of how to specify the attitude of institutions such as unions and employers' federations towards members of different generations in a particularly transparent way. Since individuals work only when they are young, both unions and employers' federations bargain in every period only on behalf of agents belonging to a single generation, and they do not take intertemporal considerations into account.

7 Conclusion

This paper presents a dynamic general equilibrium model to study the effects of insider-dominated wage setting on employment and income shares. As a general feature, we show that policies which seek to exploit a trade-off between employment and real wages as described in the short run by the aggregate labour demand schedule

[19] Certainly, a thorough analysis tied to the specifics of the German unification process would have to address additional issues such as the effects of post-unification subsidies of capital on relative factor prices and the effects of expansionary fiscal policies on capital formation in general.

may well turn out to be self-defeating when the long-run response of capital is taken into account as well.

More specifically, we augment an otherwise standard overlapping generations economy of the Diamond-type with a wage bargaining structure and obtain explicit solutions for short and long-run properties of equilibria. Using fully competitive outcomes as a benchmark, we show that under wage bargaining the existence, uniqueness, and stability of equilibria as well as (second-best) welfare properties depend sensitively on the magnitude of the elasticity of substitution between labour and capital. Compared with a perfectly competitive setting, values of the elasticity below unity tend to preserve stability and lead to empirically plausible responses of employment and income shares to wage setting shocks. By contrast, for values of the elasticity above unity the response pattern to such shocks is less plausible, and, more fundamentally, long-run stationary equilibria may well become unstable.

We demonstrate that our results are not confined to the special structure of overlapping generations economies, but apply also, for example, in the context of a standard Solow model when augmented correspondingly by wage bargaining. Thus, given the broad range of estimates of the substitution elasticity presented in the empirical literature, we think that our results give some guidance for this debate from a theoretical perspective.

We conclude by pointing out two aspects which we find promising for future work. First, the fact that steady state capital intensities are identical under full employment and unemployment dynamics depends strongly on our assumption that the budget of the unemployment insurance is always balanced by appropriate taxation of wage income. In a companion paper (Kaas and von Thadden (2000)), we relax this assumption and discuss how the possibility of bond financing leads to additional steady states and much richer (two-dimensional) dynamics. Second, the stylized picture of income shares depends sensitively on how one accounts for shifts in the composition of the workforce between employees and self–employed workers. Yet, beyond this measurement problem, differences in union membership between these groups (and across sectors) make it desirable to endogenize the decision to join a union. Atkinson (1999), for example, presents a bargaining model in which workers with different skills enter the union contingent on receiving an expected wage which exceeds their outside productivity. While our model treats the bargaining parameter as exogenous, further work along the lines of Atkinson (1999) offers a promising way to determine the (time-varying) strength of insiders from within the system.

Appendix

Proof of Proposition 2:
(i) Notice that the elasticity of substitution is

$$\sigma(k) = -\left(\frac{d\left(f'(k)/w(k)\right)}{dk}\right)^{-1} \frac{f'(k)/w(k)}{k} = \frac{\eta_f(k) - 1}{\eta_{f'}(k)} \;, \tag{15}$$

and that

$$\eta_w(k) = \frac{\eta_f(k)}{\sigma(k)} \;. \tag{16}$$

Now $\eta_w(\overline{k}) < 1$ since $\eta_f(\overline{k}) < 1$ and $\sigma(\overline{k}) > 1$. Hence the numerator in (13) is negative. If the denominator is also negative, $\overline{k}$ is unstable since $d(\overline{k}) > 1$ because of $\eta_w(\overline{k}) < 1 - \eta_s(R(\overline{k}))\eta_R(\overline{k})$. If the denominator is positive, suppose that $\overline{k}$ is stable, so that $d(\overline{k}) > -1$ which means

$$\eta_w(\overline{k}) + 2\eta_L(\overline{k}) + 1 > \eta_s(R(\overline{k}))\eta_R(\overline{k}) \;. \tag{17}$$

Since $\eta_L(\overline{k}) < -1$ and since $\eta_R(\overline{k}) = \eta_{f'}(\overline{k})f'(\overline{k})/(f'(\overline{k})+1-\delta) \geq \eta_{f'}(\overline{k})$, this implies

$$\eta_s(R(\overline{k})) > \eta_{f'}(\overline{k})^{-1}(\eta_w(\overline{k}) - 1) \;.$$

Using (15) and (16) yields

$$\eta_s(R(\overline{k})) > \frac{\sigma(\overline{k})}{\eta_f(\overline{k}) - 1} \cdot \frac{\eta_f(\overline{k}) - \sigma(\overline{k})}{\sigma(\overline{k})} = \frac{\sigma(\overline{k}) - \eta_f(\overline{k})}{1 - \eta_f(\overline{k})} > 1 \;,$$

a contradiction to the assumption that the interest elasticity of savings does not exceed one.
(ii) Under the assumptions, the denominator of (13) is clearly positive. Suppose that $\overline{k}$ is locally stable. Then $d(\overline{k}) < 1$ implies (14). Now suppose that (14) is satisfied. Then, if the numerator of (13) is non–negative, the steady state is locally stable since $0 \leq d(\overline{k}) < 1$. If the numerator is negative, $\overline{k}$ is locally stable if $d(\overline{k}) > -1$ which is equivalent to (17). But since the right hand side of this inequality is non–positive, and since $\eta_L(\overline{k}) > -1/2$, this condition is fulfilled. Hence $\overline{k}$ is stable if and only if (14) is satisfied.
(iii) If $\sigma \geq 1$, (14) is satisfied at the unique steady state. Hence, according to (ii), this steady state is stable if $\eta_L(\overline{k}) > -1/2$ and according to (i) it is unstable if $\eta_L(\overline{k}) < -1$. If $\sigma < 1$, $\eta_L(\overline{k}) > 0$, and according to (ii) a steady state is stable iff (14) is satisfied, which is the case at $\overline{k}_a$ but not at $\overline{k}_b$. □

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