Lab 10 – MATH 240 – Computational Statistics

Avery Johnson
Colgate University
Department of Mathematics
aqjohnson@colgate.edu

Abstract

This lab investigates Gallup's (Gallup, 2010) claim that increasing the sample size reduces the margin of error, specifically comparing the results of 1,000-sample poll (±4%) with a 2,000-sample poll (±2%). Using simulations and resampling from the Gallup data, we confirm that larger sample sizes reduce the margin of error, but the population proportion also plays a significant role. Resampling results were consistent with simulations, reinforcing the reliability of this method when the true population is unknown. Additionally, theoretical estimates from the Wilson formula aligned with the simualted results. Overall, this study shows that Gallup's claims oversimplify the relationship between the margin of error, sample size, and population proportion.

Keywords: Simulation, Resampling, Margin of Error

1 Introduction

Understanding how confident we can be in a sample-based proportion is fundamental to survey statistics. Gallup (Gallup, 2010), for example, claims their margin of error is within ±2% for a sample size of 2,000 and ±4% for a sample size of 1,000. However, these claims are often presented without a detailed explanation. This lab aims to break down these claims using simulation, resampling, and theoretical formulas. We begin by simulating from a known population to observe how sample proportions vary. Next, we use resampling methods to estimate variability from real survey data. Finally, we expand the analysis by systematically varying sample sizes and population proportions, comparing the resulting variability to a theoretical margin of error formula. Together, these approaches offer a deeper understanding of how sample size and population proportion influence uncertainty, and provide insight into the accuracy of the claims made by Gallup.

2 Methods

2.1 Basic Simulation

We simulated 10,000 samples using the binomial distribution with a known success probability of $p = 0.39$. We examined sample sizes of 1004 and 2008- values used in recent Gallup polls. For each sample, we calculated the proportion of successes and estimated the middle 95% interval of these proportions, which we used to compute a margin of error.

2.2 Resampling from Real Data

In most real-world situations, the true population proportion p is typically unknown. When this is the case, we can not simulate directly from a known distribution, so we use resampling. Resampling involves repeatedly drawing samples from the observed data with replacement, allowing us to mimic the process of sampling from the population without assuming the underlying distribution. We constructed a data frame representing Gallup's survey of 1,004 Americans, based on the reported breakdowns: 39% were satisfied, 59% were dissatisfied, and 2% had no opinion. We then performed 1,000 resamples, each with a sample size of 1,004, drawn with replacement from the original data. For each resample, we calculated the sample proportion $\hat{p}$ of respondents who were satisfied. The resample proportions were plotted on a histogram with a superimposed density curve to visually approximate the sampling distribution for the sample proportion. Additionally, we calculated the range of the middle 95% along with the margin of error to compare with the simulation data.

2.3 Simulation Over n and p

Next, we explored how the margin of error behaves across a range of sample sizes (from 100 to 3000) and population proportions (from 0.01 to 0.99). We created a double `for()` loop to iterate over each n,p pair. For each of these pairs, we simulated 10,000 sample proportions using `rbinom()` and calculated the 95% interval and margin of error. The margin of error results were stored and then plotted in a `geom_raster()` plot.

2.4 Actual Margin of Error

Using the Wilson margin of error formula, we computed the theoretical margin of errors for the same grid of n,p values. Once again, we used a double `for()` loop to iterate over the pairs of n,p values, stored the Wilson margin of error at each iteration, and plotted the results in a `geom_raster()` plot.

3 Results

3.1 Basic Simulation

Figure 1 presents the histograms of the sample proportions for the two sample sizes, $n = 1004$ and $n = 2008$. Each plot includes a superimposed density curve, providing a visual approximation of the sampling distribution of the sample proportion. For both sample sizes, the histogram exhibits a roughly bell-shaped distribution, which is characteristic of normality if the sample size is large enough. As can be found in Table 1, for the sample size of $n = 1004$, the middle 95% of the sample proportions lies within a range approximately from 0.36 to 0.42, yielding a margin of error of about 0.03. When the sample size is doubled to $n = 2008$, the range of the middle 95% decreases to approximately 0.40 to 0.41, resulting in a margin of error of about 0.02. This reduction in the margin of error as the sample size increases is consistent with the behavior expected by Gallup, though the margin of error of 0.030 for $n = 1004$ issmaller than the stated ±4%.

3.2 Resampling

Figure 2 shows the histogram of the sample proportions obtained from the 1,000 resamples of Gallup's survey data, with a superimposed density curve to approximate the sampling distribution for the sampling proportion $\hat{p}$. The distribution appears roughly bell-shaped, similar to the results from the simulations, indicating that the sampling distribution of the sample proportion is roughly normal. The middle 95% of the sample proportions lies within a range approximately from 0.36 to 0.42, giving a margin of error of about 0.03. This margin of error is very similar to the result from the simulation with $n = 1004$. Table 1 summarizes the comparison between the simulation results for $n = 1004$, $n = 2008$, and the resampling results. While we cannot increase the sample size with resampling, the margin of error closely mirrors that observed in the simulation for $n = 1004$, which is smaller than the ±4% margin of error reported by Gallup.

Sample Size	Lower (95%)	Upper (95%)	MOE
Simulation (n=1004)	0.36	0.42	0.03
Simulation (n=2008)	0.37	0.41	0.02
Resample	0.36	0.42	0.03

Table 1: Margin of Error Results

3.3 Simulation Over n and p

The simulation results, shown in Figure 3, reveal the relationship between sample size n and probability proportion p in determining the margin of error. This demonstrates that the margin of error story is not as simple as Gallup described. While it is true that increasing n reduces the margin of error, the value of p also plays a significant role. Specifically, when p is close to 0 or 1, the margin of error becomes smaller due to the constrained parameter space. This highlights that the margin of error does not solely depend on sample size but also on the extremity of the population proportion.

3.4 Actual Margin of Error

As can be seen in Figure 4, the heat map of the Wilson-based margins of error looks nearly identical to the one from the simulation. This indicates that the Wilson formula is a reliable and efficient way to approximate sampling variability. Both plots that the variability peaks at $p = 0.5$, that increasing n leads to tighter estimates, and that when p is extreme, the margin of error is smaller.

4 Discussion

This lab reinforced key ideas about statistical uncertainty in estimating proportions. Simulations in a known p showed how sampling variability plays out as sample size increases. Resampling showed that even when we don't know the truth, we can still estimate variability using just the observed data, and the results tend to look the same. Extending the simulations over a grid of values for n and p showed that increasing the sample size alone does not guarantee a small margin of error; the value of p matters too. When p is near the extreme, the margin of error shrinks because the proportions can't go below 0 or above 1. Finally, comparing simulated margins to those derived from the Wilson formula showed similarity. Overall, these analyses complicate the excerpts in the Gallup document, informing us that the relationship between sample size and margin of error is more nuanced than it may initially appear. While larger sample sizes certainly help reduce variability, the true precision of an estimate also depends on the underlying population propotion.

References

Gallup (2010). How are polls conducted? PDF available at https://news.gallup.com/poll/101872/how-does-gallup-polling-work.aspx. Accessed: 2025-04-07.

5 Appendix

5.1 Basic Simulation

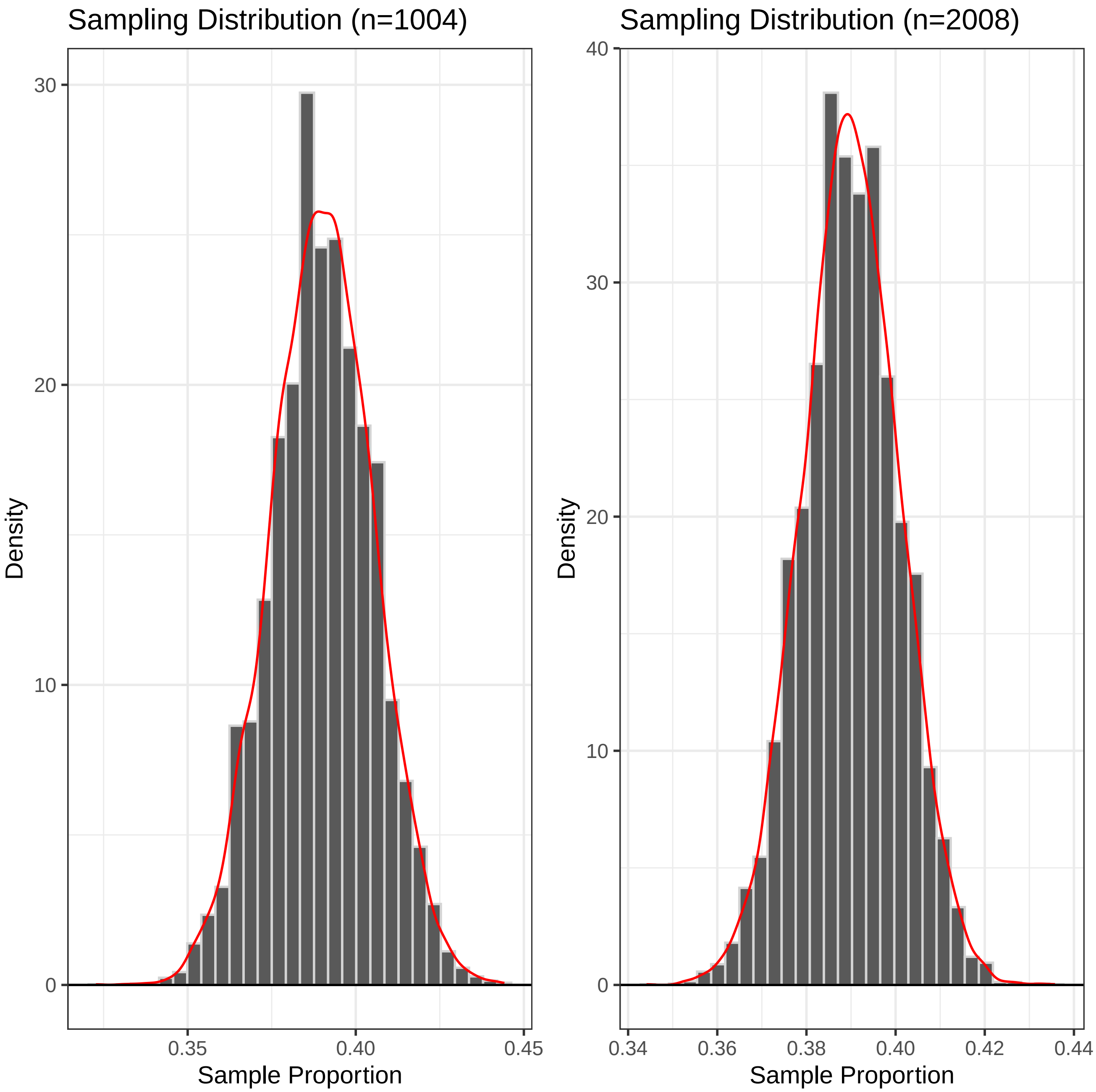

Figure 1: Simulations for $n = 1004$ and $n = 2008$

5.2 Resampling

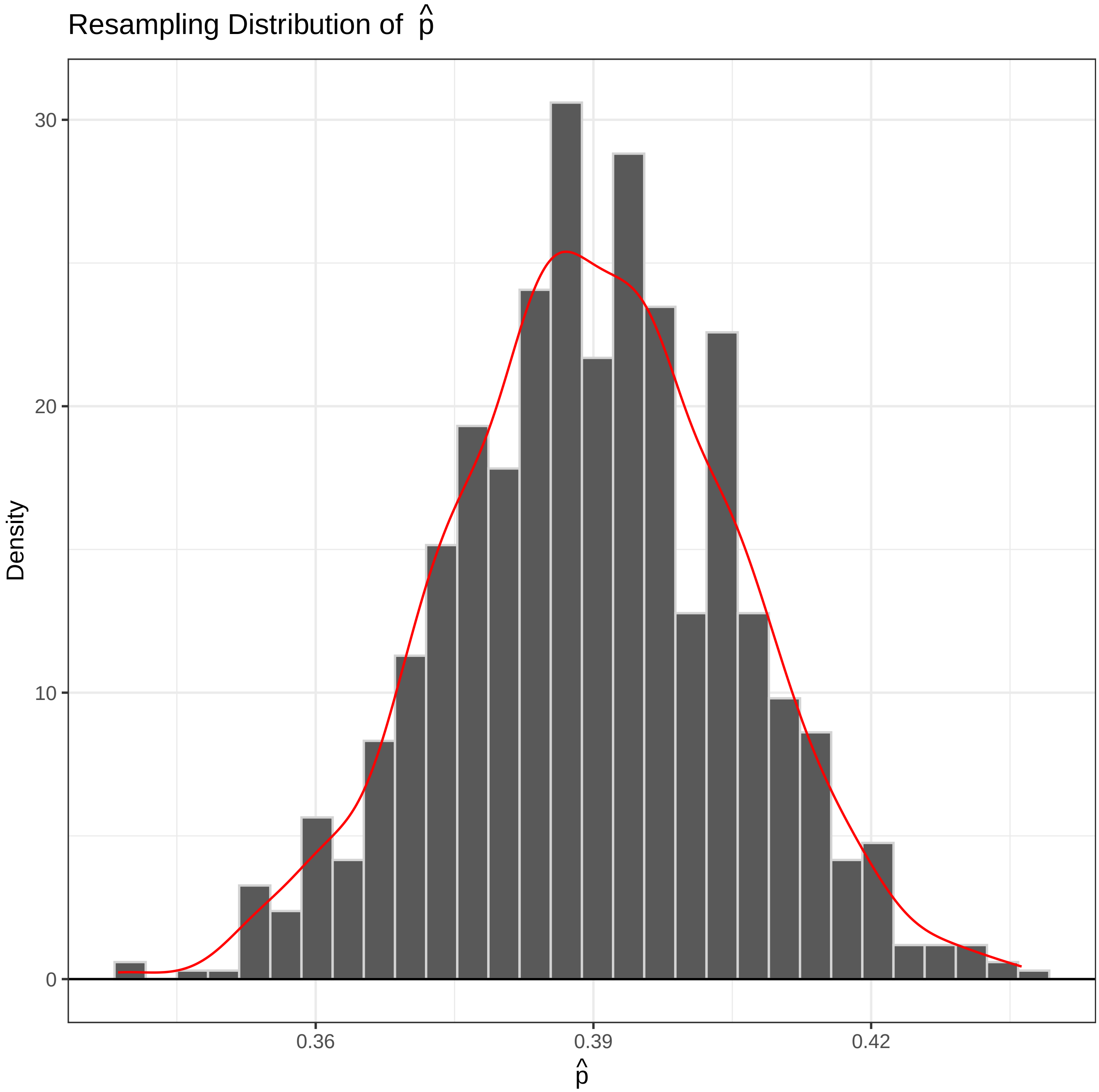

Figure 2: Resampling Distribution of $\hat{p}$

5.3 Simulated Margin of Error

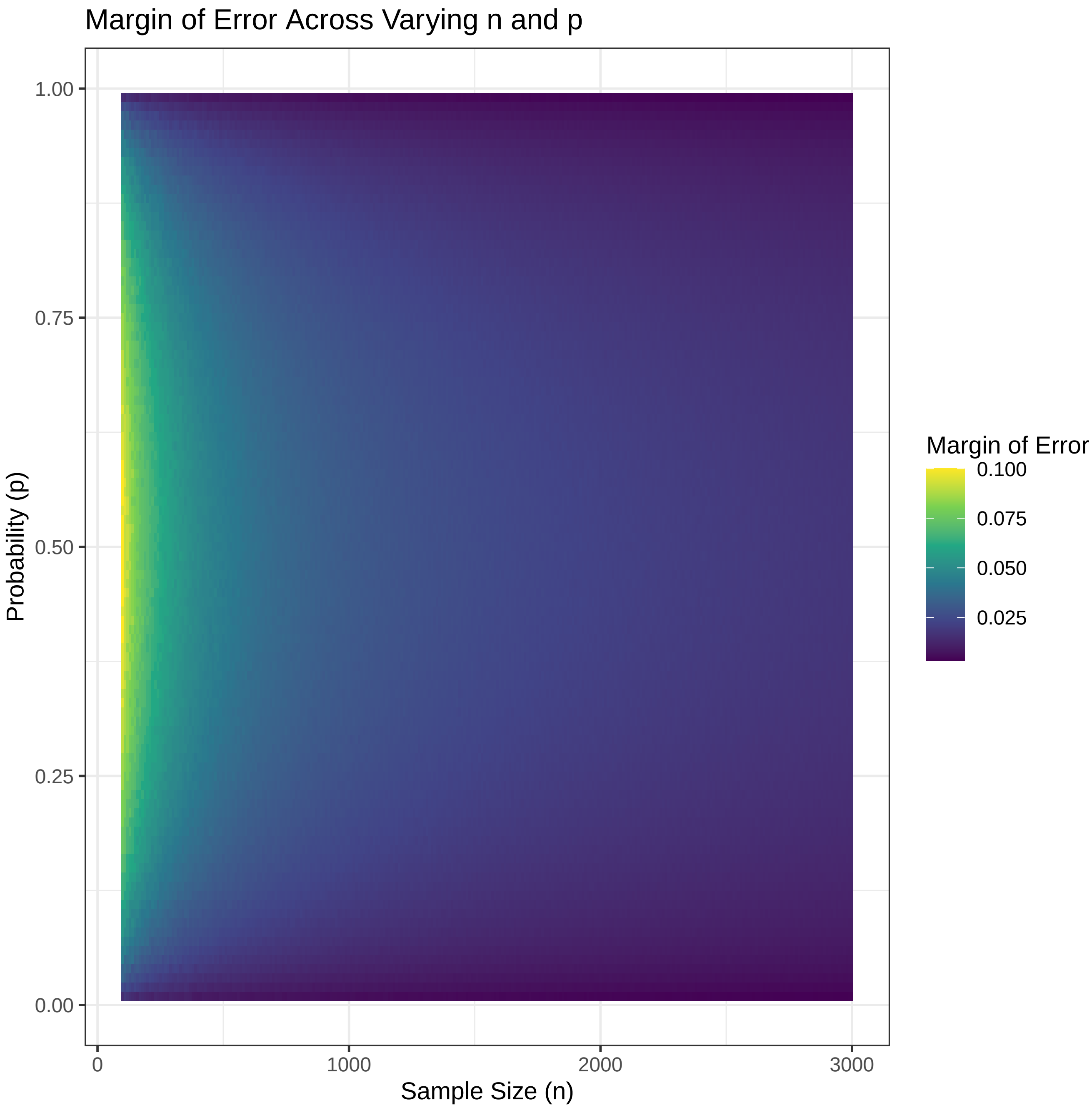

Figure 3: Margin of Error Across n and p

5.4 Wilson Margin of Error

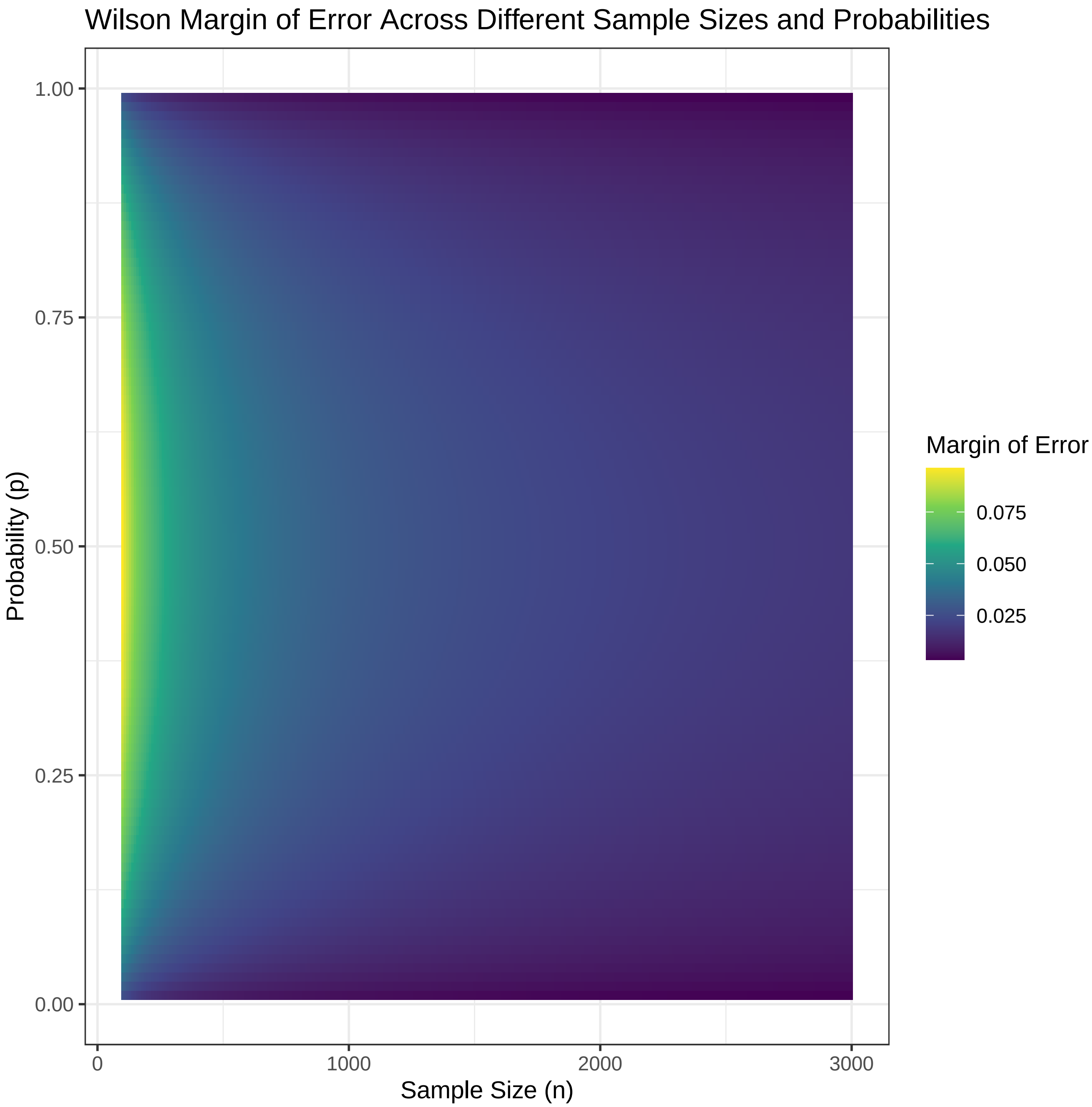

Figure 4: Wilson Margin of Error