Lab 7 and 8 – MATH 240 – Computational Statistics

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Abstract

This document provides a basic template for the 2-page labs we will complete each week. Here, briefly summarize what you did and why it might be helpful. Provide all the top-line conclusions, but avoid providing *all* the details. Results should be limited to "we show X, Y, and Z."

Keywords: What topics does the lab cover concerning class? List 3-4 key terms here, separated by semicolons.

1 Introduction

2 Density Functions and Parameters

ADD PDF of beta ADD parameters

To explore the beta distribution, we focused on four cases: Beta($\alpha = 2, \beta = 5$), Beta($\alpha = 5, \beta = 5$), Beta($\alpha = 5, \beta = 2$), and Beta($\alpha = 0.5, \beta = 0.5$). We calculated the population-level characteristics(mean, variance, skewness, and excess kurtosis) for all four cases by deriving the formulas and numerically analyzing them for each case (Summarized in Table 1).

Alpha	Beta	mean	variance	skewness	kurtosis
2.00	5.00	0.29	0.03	0.60	-0.12
5.00	5.00	0.50	0.02	0.00	-0.46
5.00	2.00	0.71	0.03	-0.60	-0.12
0.50	0.50	0.50	0.12	0.00	-1.50

Table 1: Population-Level Summary by Case

To visually see the distribution, all four cases are plotted in Figure 1. The Beta($\alpha = 2, \beta = 5$) distribution is right-skewed, the Beta($\alpha = 5, \beta = 5$) is symmetric, the Beta($\alpha = 5, \beta = 2$) is left-skewed, and the Beta($\alpha = 0.5, \beta = 0.5$) is u-shaped indicating majority of the distribution is close to 0 or 1. We can see from Figure 1 that when $\alpha < \beta$, it is right-skewed; when $\alpha > \beta$, it is left-skewed; when $\alpha = \beta$, it is symmetric.

3 Properties

The beta distribution includes many important properties such as mean, variance, skewness, and excess kurtosis. These values were calculated for all four cases using provided formulas from beta distribution (Table 1). To compare the results, the moments of the beta distribution were calculated directly using numerical integration. Our function, `beta.moment()`, calculated both the centered and uncentered moments, summarizing the same characteristics as in Table 1. This process resulted in values that match the theoretical values from Table 1 and can be seen in Table 2 in Appendix.

Since the goal of summarizing this data is to approximate what the population distribution might be, we further analyze how different sample sizes effect the sample estimates. Initially randomly selecting sample sizes of `n` = 500 from the beta distribution, histograms for each sample with estimated density and true PDF were plotted (Figure 2) accompanied by a numerical summary (Table 3). Comparing numerical summaries and plots from sample to Table 1, confirms that the sample estimates the approximate theoretical values.

4 Estimators

5 Example

6 Discussion

Bibliography: Note that when you add citations to your bib.bib file *and* you cite them in your document, the bibliography section will automatically populate here.

7 Appendix

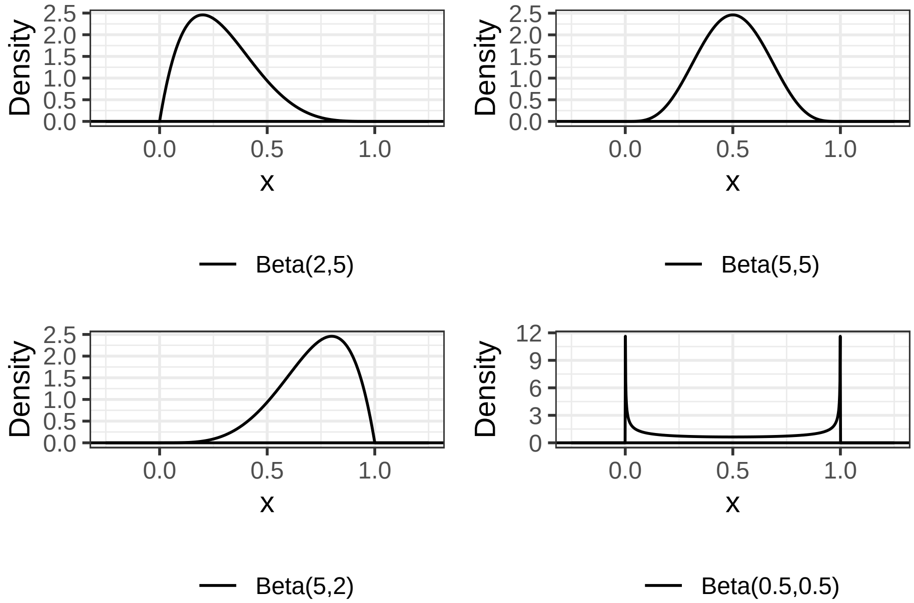

Figure 1: Population Distribution Plot by Case

Alpha	Beta	mean	variance	skewness	excess.kurtosis
2.00	5.00	0.29	0.03	0.60	-0.12
5.00	5.00	0.50	0.02	-0.00	-0.46
5.00	2.00	0.71	0.03	-0.60	-0.12
0.50	0.50	0.50	0.12	-0.00	-1.50

Table 2: Moments Summary Table

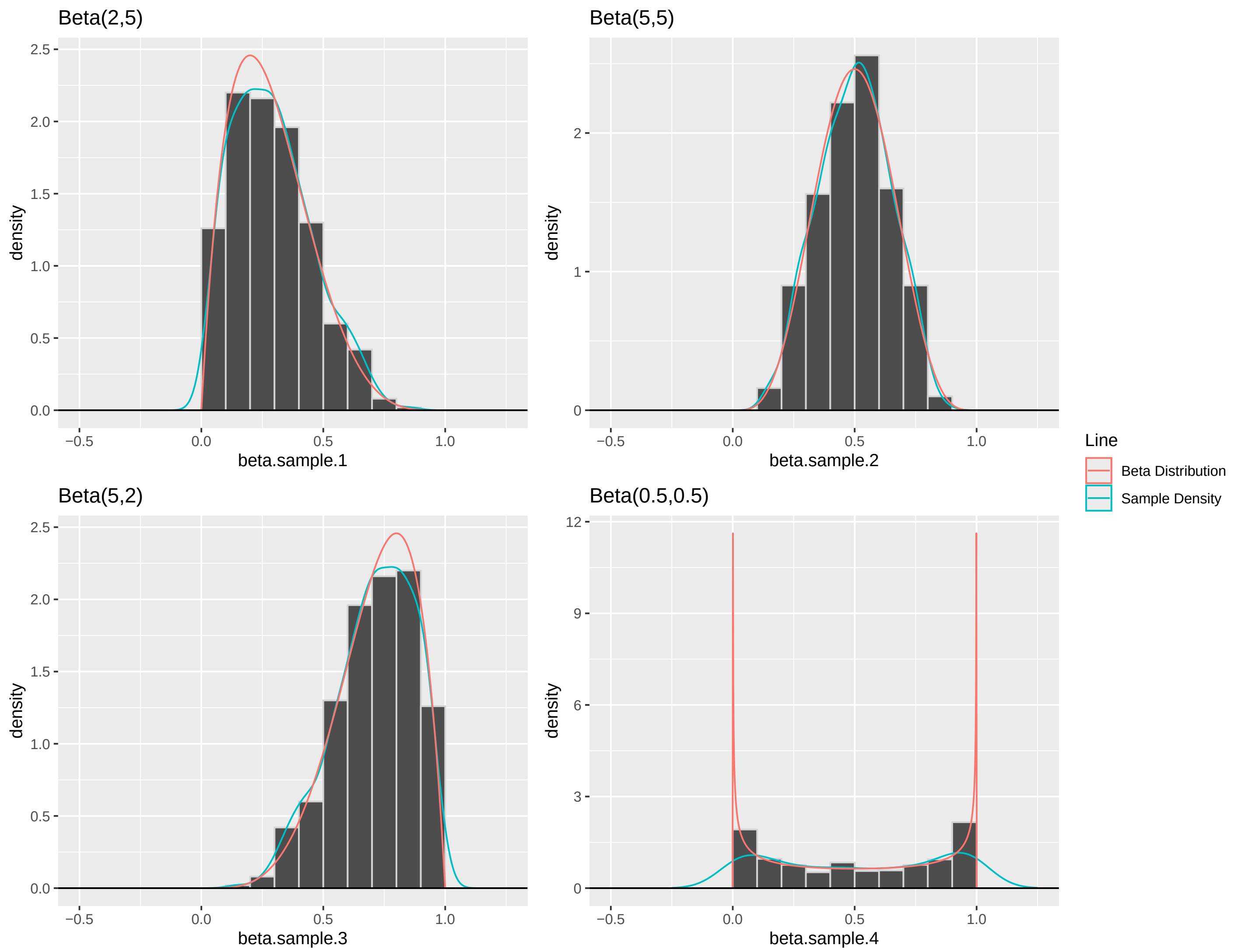

Figure 2: Sample Plot by Four Beta Distributions

Alpha	Beta	mean	variance	skewness	excess.kurtosis
2.00	5.00	0.29	0.03	0.60	-0.12
5.00	5.00	0.50	0.02	-0.00	-0.46
5.00	2.00	0.71	0.03	-0.60	-0.12
0.50	0.50	0.50	0.12	-0.00	-1.50

Table 3: Sample Summary Table