

Lab 08 – MATH 240 – Computational Statistics

Cristian Palmer
Student
Mathematics
cpalmer@colgate.edu

1 Introduction

Lab 8 is a continuation of the work which we began during lab 7. In lab 7, we were tasked with computing the population moments for four distinct cases of the beta distribution and graphically comparing the different cases. In lab 8 we continued to build on our understanding of the beta distribution by modeling country death rates worldwide with the beta distribution. Our end goal with this lab was to be able to describe the beta distribution. Particularly, this write up aims to provide answers to questions such as: What is the beta distribution? What does it look like? What is it used for? What are its properties? And, what additional information do we gain from the simulations and real data analysis?

2 Density Functions and Parameters

To begin, the beta distribution's probability density function (PDF) is given by:

$$f_X(x | \alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1} I(x \in [0, 1])$$

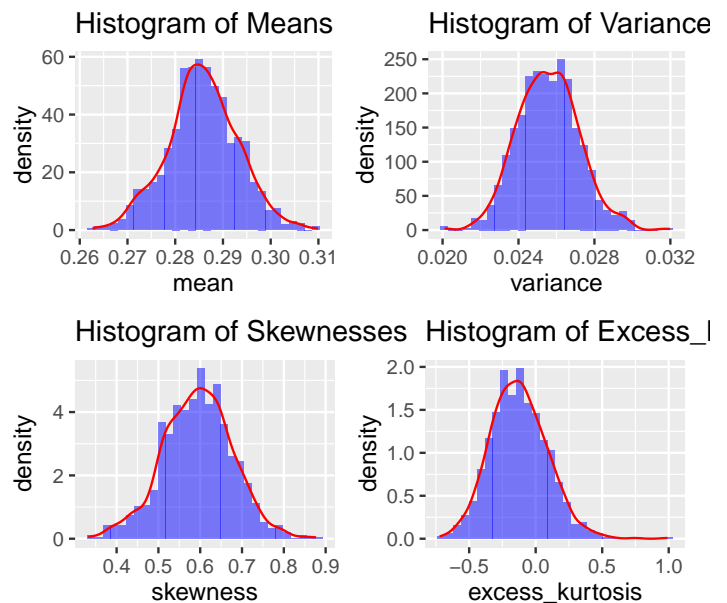
This PDF is expressed using the gamma function and involves the random variable x along with parameters α and β . The domain of the beta distribution is restricted to the interval $[0, 1]$, meaning $0 \leq x \leq 1$. Additionally, both shape parameters α and β must be strictly positive for the distribution to be properly defined.

3 Properties

During lab 7 multiple of our tasks saw us calculating certain properties of the beta distribution for our alternate α and β values. Since the distribution's shape is defined by its parameters, the distributions properties, notably its mean, variance, skewness, and excess kurtosis are also described by the α and β parameters.

Pictured below is a 2x2 grid of histograms depicting the respective distributions and density curves for

the beta distribution with an α of 2, and a β of 5. We iterated over this specific beta distribution 1000 times, each time generating 500 values for each property which we saved to a tibble and graphed below. We utilized the `patchwork` (Pedersen, 2024) library in order to orient the four graphs in this 2x2 grid fashion.



4 Estimators

5 Example

References

- Pedersen, T. L. (2024). *patchwork: The Composer of Plots*. R package version 1.3.0.
- Wickham, H., Averick, M., Bryan, J., Chang, W., McGowan, L. D., François, R., Grolemond, G., Hayes, A., Henry, L., Hester, J., Kuhn, M., Pedersen, T. L., Miller, E., Bache, S. M., Müller, K., Ooms, J., Robinson, D., Seidel, D. P., Spinu, V., Takahashi, K., Vaughan, D., Wilke, C., Woo, K., and Yutani, H. (2019). Welcome to the tidyverse. *Journal of Open Source Software*, 4(43):1686.

6 Appendix

If you have anything extra, you can add it here in the appendix. This can include images or tables that don't work well in the two-page setup, code snippets you might want to share, etc.