Lab 7-8 – MATH 240 – Computational Statistics

Jackson Colby
Colgate University
Mathematics
jcolby@colgate.edu

1 Introduction

2 Density Functions and Parameters

The Beta distribution with parameters α and β is given by the probability density function:

$$f_X(x|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}, \quad \text{for } x \in [0,1]$$

The probability function for the Beta distribution takes values of 0 everywhere outside of [0,1].

The following plot shows the comparison between a Beta distribution with $\alpha = 5$ and $\beta = 5$ and a Gaussian distribution with the same mean and variance. This figure shows that when alpha and beta are the same or close to the same the beta distribution has a similar density to the normal distribution. If α is greater than β then the distribution will be left skewed and if α is less than β then the distribution will be right skewed.
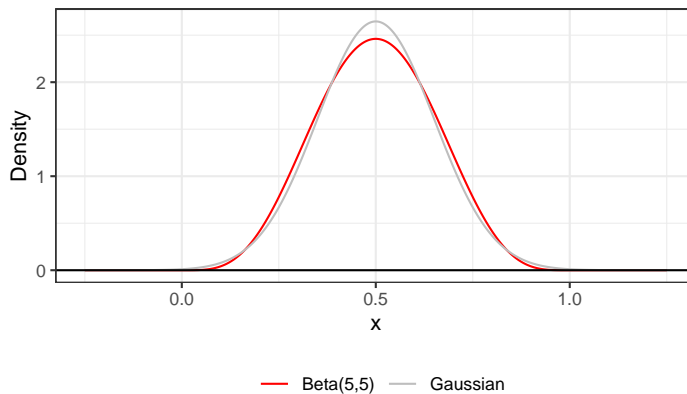
A table containing the four given cases is below.

alpha	beta	mean	variance	skewness	kurtosis	case
2.00	5.00	0.29	0.03	0.60	-0.12	Beta(2,5)
5.00	5.00	0.50	0.02	0.00	-0.46	Beta(5,5)
5.00	2.00	0.71	0.03	-0.60	-0.12	Beta(5,2)
0.50	0.50	0.50	0.12	0.00	-1.50	Beta(0.5,0.5)

Table 1: Summary of Beta Distribution Statistics

As seen in the table, for larger α and β the variance is lower. All the Beta distributions are platykurtic, the graphs being more platykurtic when α and β are the same or similar.

3 Properties

4 Estimators

5 Example with Death Rates Data