Lab 7-8 – MATH 240 – Computational Statistics

Jackson Colby
Colgate University
Mathematics
jcolby@colgate.edu

1 Introduction

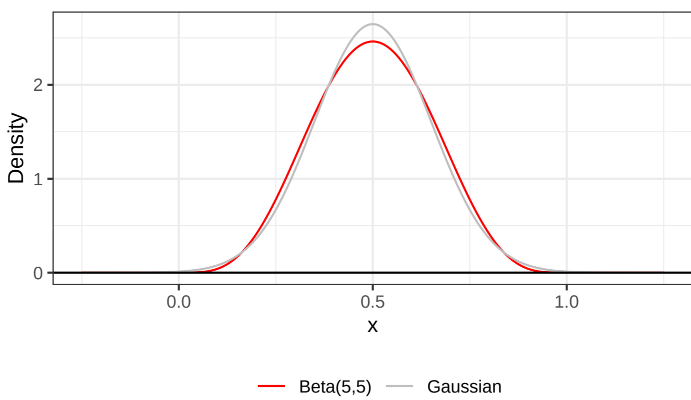
2 Density Functions and Parameters

The Beta distribution with parameters α and β is given by the probability density function:

$$f_X(x|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}, \quad \text{for } x \in [0,1]$$

The probability function for the Beta distribution takes values of 0 everywhere outside of [0,1].

The following plot shows the comparison between a Beta distribution with $\alpha = 5$ and $\beta = 5$ and a Gaussian distribution with the same mean and variance. This figure shows that when alpha and beta are the same or close to the same the beta distribution has a similar density to the normal distribution. If α is greater than β then the distribution will be left skewed and if α is less than β then the distribution will be right skewed.



A table containing the four given cases is below.

alpha	beta	mean	variance	skewness	kurtosis
2.00	5.00	0.29	0.03	0.60	-0.12
5.00	5.00	0.50	0.02	0.00	-0.46
5.00	2.00	0.71	0.03	-0.60	-0.12
0.50	0.50	0.50	0.12	0.00	-1.50

Table 1: Summary of Beta Distribution Statistics

As seen in the table, for larger α and β the variance is lower. All the Beta distributions are platykurtic, the graphs being more platykurtic when α and β are the same or similar.

3 Properties

The population level properties for the Beta Distribution are as follows: The mean is given by:

$$E(X) = \frac{\alpha}{\alpha+\beta}$$

The variance is given by:

$$\text{Var}(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

The skewness is given by:

$$\text{Skew}(X) = \frac{2(\beta-\alpha)}{(\alpha+\beta+1)\sqrt{(\alpha+\beta+2)\alpha\beta}}$$

The excess kurtosis is given by:

$$\text{Kurt}(X) = \frac{6(\alpha-\beta)^2}{(\alpha+\beta+1)\,(\alpha+\beta+2)} - \frac{\alpha\beta(\alpha+\beta+2)}{\alpha\beta(\alpha+\beta+2)(\alpha+\beta+3)}$$

A function called beta.moment was utilized to test both centered and uncentered moments of the population-level characteristics to the approximations calculated.

For the case alpha=2, beta=5: the statistics using the population level were mean=0.285714, var=0.0255102, skew=0.047619, kurt=0.001874. These statistics compared to the table above are very close to the values of the approximation, meaning the characteristics described above are true for the Beta Distribtuion.

4 Estimators

5 Example with Death Rates Data