Labs 7 and 8 – MATH 240 – Computational Statistics

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Abstract

In labs 7 and 8, we explored the beta distribution and its properties. We compared summary values of beta distributions under different parameter sets, also analyzing cumulative statistics. We then took death rate data and utilized both the method of moments and maximum likelihood estimates to determine approximate values of the parameters that would provide a matching distribution.

Keywords: Beta Distributions, Random Sampling, Estimators

1 Introduction

The beta distribution is a continuous probability distribution with two free parameters, α and β. It models a random variable X with a support of $[0, 1]$. The parameters can be adjusted to allow for flexibility in the distribution. For example, under specific values of α and β, a beta distribution can be skewed in different directions, have different excess kurtosis values, and other properties.

2 Density Functions and Parameters

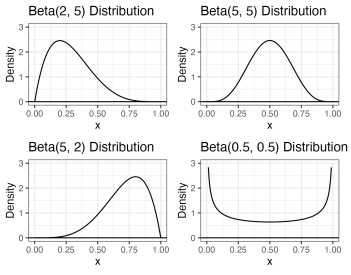

Figure 1: Beta distributions with different values of α and β.

Alpha	Beta	Mean	Variance	Skewness	Excess Kurtosis
2.00	5.00	0.29	0.03	0.60	-0.12
5.00	5.00	0.50	0.02	0.00	-0.46
5.00	2.00	0.71	0.03	-0.60	-0.12
0.50	0.50	0.50	0.12	0.00	-1.50

Table 1: Summary values of beta distributions from different choices of α and β.

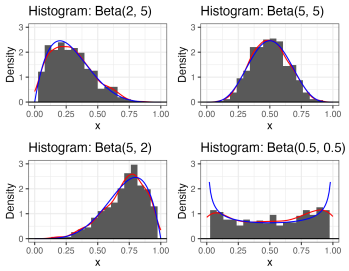

Figure 2: Histograms of random samples from different beta distributions.

3 Properties

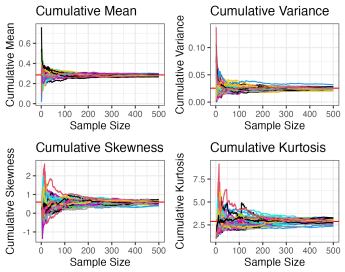

Figure 3: Cumulative statistics compared with sample size.

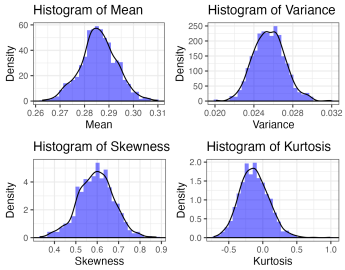

Figure 4: Distributions of summary values under random sampling of the Beta(2,5) distribution. All four appear to be normally distributed.

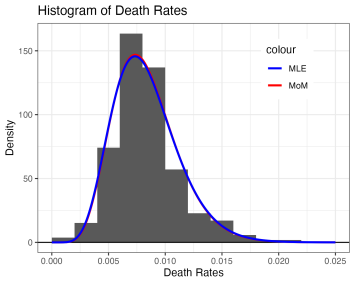

Figure 5: Histogram of 2022 death rates from the world bank, with MLE and MoM estimated distributions superimposed The MoM distribution appears to match slightly better.

4 Estimators

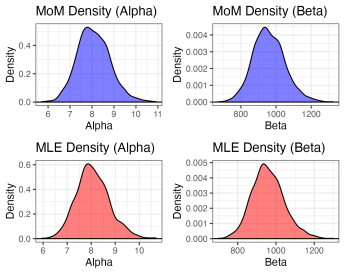

Figure 6: Estimated densities of alpha and beta for the method of moments and maximum likelihood estimates.

Parameter	Method	Bias	Precision	MSE
Alpha	MOM	0.08	1.83	0.55
Alpha	MLE	0.07	2.13	0.48
Beta	MOM	10.29	0.00	8288.46
Beta	MLE	9.11	0.00	7132.70

Table 2: Bias, precision, and mean squared error for the method of moments and maximum likelihood estimates, with $\alpha = 8$ and $\beta = 950$.