In lecture 16, we looked at precipitation amounts in Madison County (at Morrisville station). We found that the Weibull distribution had a good fit to the monthly precipitation amounts.

We found that the MLEs for the Weibull distribution were

$$\hat{a} = 2.1871$$
$$\hat{\sigma} = 3.9683$$

and

$$-\mathcal{L}(\{\hat{a}, \hat{\sigma}\}|\mathbf{x}) = 2166.496$$

is the realized negative log-likelihood. Note this means that the log-likelihood is

$$\mathcal{L}(\{\hat{a}, \hat{\sigma}\}|\mathbf{x}) = -2166.496,$$

and the usual likelihood is

$$L(\{\hat{a}, \hat{\sigma}\}|\mathbf{x}) = e^{[\mathcal{L}(\{\hat{a}, \hat{\sigma}\}|\mathbf{x})]} \approx= e^{-2166.496},$$

which `R` cannot differentiate from 0.

1. Someone asked "why Weibull?" in class. That is, why wouldn't we use another right-skewed distribution like the Gamma (see Lecture 15), or the Log-Normal (see Lecture 17).

   (a) Compute the MLEs for these data using a Gamma distribution.

```
llgamma <- function(data, par, neg=F){
  alpha <- par[1]
  beta <- par[2]

  loglik <- sum(log(dgamma(x=data, shape=alpha, rate=beta)))

  return(ifelse(neg, -loglik, loglik))
  }

(mles <- optim(par = c(1,1),     ##how do I know what to guess here?
         fn = llgamma,
         data=dat.wind$speed,      #### WHAT DATA SHOULD I PUT IN????
         neg=T))

## Error:  object 'dat.wind' not found

alpha.hat.mle <- mles$par[1]

## Error:  object 'mles' not found

beta.hat.mle <- mles$par[2]

## Error:  object 'mles' not found
```

   (b) Compute the MLEs for these data using the Log-Normal distribution.

```
ll_lognorm <- function(data, par, neg=F){
  mu <- par[1]
  sigma <- par[2]

  loglik <- sum(dlnorm(x=data, meanlog=mu, sdlog=sigma, log=TRUE))

  return(ifelse(neg, -loglik, loglik))
  }
```

```r
(mles <- optim(par = c(0,1),  ##how do I know what to guess here?
               fn = ll_lognorm,
               data = dat.wind$speed,  #### WHAT DATA SHOULD I PUT IN????
               neg = TRUE))

## Error:  object 'dat.wind' not found

mu.hat.mle <- mles$par[1]

## Error:  object 'mles' not found

sigma.hat.mle <- mles$par[2]

## Error:  object 'mles' not found
```

(c) Compute the likelihood ratio to compare the Weibull and the Gamma distribution. Which has a better fit according to the likelhiood ratio?

$$Q = \frac{L(\{\hat{a}, \hat{\sigma}\}|\mathbf{x})}{L(\{\hat{\alpha}, \hat{\beta}\}|\mathbf{x})} = e^{\left[\mathcal{L}(\{\hat{a},\hat{\sigma}\}|\mathbf{x}) - \mathcal{L}(\{\hat{\alpha},\hat{\beta}\}|\mathbf{x})\right]}$$

(d) Compute the likelihood ratio to compare the Weibull and the Log-Normal distribution. Which has a better fit according to the likelihood ratio?

$$Q = \frac{L(\{\hat{a}, \hat{\sigma}\}|\mathbf{x})}{L(\{\hat{\mu}, \hat{\sigma}\}|\mathbf{x})} = e^{\left[\mathcal{L}(\{\hat{a},\hat{\sigma}\}|\mathbf{x}) - \mathcal{L}(\{\hat{\mu},\hat{\sigma}\}|\mathbf{x})\right]}$$

(e) Compute the likelihood ratio to compare the Gamma and the Log-Normal distribution. Which has a better fit according to the likelhiood ratio?

$$Q = \frac{L(\{\hat{\alpha}, \hat{\beta}\}|\mathbf{x})}{L(\{\hat{\mu}, \hat{\sigma}\}|\mathbf{x})} = e^{\left[\mathcal{L}(\{\hat{\alpha},\hat{\beta}\}|\mathbf{x}) - \mathcal{L}(\{\hat{\mu},\hat{\sigma}\}|\mathbf{x})\right]}$$

2. Optional Coding Challenge. Choose the "best" distribution and refit the model by season.

   (a) Fit the Distribution for Winter (December-February).

   (b) Fit the Distribution for Spring (March-May).

   (c) Fit the Distribution for Summer (June-August).

   (d) Fit the Distribution for Fall (September-November).

   (e) Plot the four distributions in one plot using `cyan3` for Winter, `chartreuse3` for Spring, `red3` for Summer, and `chocolate3` for Fall. Note any similarities/differences you observe across the seasons.