Progress Report Part 1a: Check Final Answers

Find : Required CT Jacobians needed to obtain linearized CT model parameters

```
In[1]:= x = {X, Xdot, Y, Ydot};
        x // MatrixForm
```

Out[2]//MatrixForm=

$$\begin{pmatrix} X \\ Xdot \\ Y \\ Ydot \end{pmatrix}$$

```
In[3]:= rrx = {X → x1, Xdot → x2, Y → x3, Ydot → x4, r → Sqrt[X^2 + Y^2]};
```

```
In[4]:= ((x /. rrx) // MatrixForm) == (x // MatrixForm)
```

Out[4]=

$$\begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \end{pmatrix} == \begin{pmatrix} X \\ Xdot \\ Y \\ Ydot \end{pmatrix}$$

```
In[5]:= u = {u1, u2};
        u // MatrixForm
```

Out[6]//MatrixForm=

$$\begin{pmatrix} u1 \\ u2 \end{pmatrix}$$

```
In[7]:= w̃ = {w̃1, w̃2};
        w̃ // MatrixForm
```

Out[8]//MatrixForm=

$$\begin{pmatrix} \tilde{w}1 \\ \tilde{w}2 \end{pmatrix}$$

System Dynamics Jacobians

In[9]:=

$$Xddot = \frac{-\mu X}{r^3} + u1 + \tilde{w}1;$$

$$Yddot = \frac{-\mu Y}{r^3} + u2 + \tilde{w}2;$$

```
In[11]:= f = {Xdot, Xddot, Ydot, Yddot};
         f // MatrixForm
```

Out[12]//MatrixForm=

$$\begin{pmatrix} Xdot \\ u1 - \frac{X\mu}{r^3} + \tilde{w}1 \\ Ydot \\ u2 - \frac{Y\mu}{r^3} + \tilde{w}2 \end{pmatrix}$$

In[13]:=
```
f //. rrx // MatrixForm
```

Out[13]//MatrixForm=

$$\begin{pmatrix} \mathrm{u1} - \dfrac{\mathrm{x2}}{\frac{\mathrm{x1}\,\mu}{\left(\mathrm{x1}^2+\mathrm{x3}^2\right)^{3/2}}} + \tilde{\mathrm{w}}1 \\ \mathrm{u2} - \dfrac{\mathrm{x4}}{\frac{\mathrm{x3}\,\mu}{\left(\mathrm{x1}^2+\mathrm{x3}^2\right)^{3/2}}} + \tilde{\mathrm{w}}2 \end{pmatrix}$$

In[14]:=
```
CTJacobian[f_, x_] := Table[Table[D[f[[i]], x[[j]]], {j, 1, Length[x]}], {i, 1, Length[f]}]
```

In[15]:=
```
dFdx = CTJacobian[(f //. rrx), (x //. rrx)];
dFdx // MatrixForm // FullSimplify
```

Out[16]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{\left(2\,\mathrm{x1}^2-\mathrm{x3}^2\right)\mu}{\left(\mathrm{x1}^2+\mathrm{x3}^2\right)^{5/2}} & 0 & \frac{3\,\mathrm{x1}\,\mathrm{x3}\,\mu}{\left(\mathrm{x1}^2+\mathrm{x3}^2\right)^{5/2}} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{3\,\mathrm{x1}\,\mathrm{x3}\,\mu}{\left(\mathrm{x1}^2+\mathrm{x3}^2\right)^{5/2}} & 0 & -\frac{\left(\mathrm{x1}^2-2\,\mathrm{x3}^2\right)\mu}{\left(\mathrm{x1}^2+\mathrm{x3}^2\right)^{5/2}} & 0 \end{pmatrix}$$

In[17]:=
```
dFdu = CTJacobian[f //. rrx, u];
dFdu // MatrixForm
```

Out[18]//MatrixForm=

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$

In[19]:=
```
dFdw = CTJacobian[f //. rrx, w̃];
dFdw // MatrixForm
```

Out[20]//MatrixForm=

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$

System Measurement Jacobians

In[21]:=
```
ρ = Sqrt[(X - Xs)^2 + (Y - Ys)^2]
```

Out[21]= $\sqrt{(\mathrm{X}-\mathrm{Xs})^2+(\mathrm{Y}-\mathrm{Ys})^2}$

In[22]:=
```
ρdot = ((X - Xs) (Xdot - Xsdot) + (Y - Ys) (Ydot - Ysdot)) / ρ
```

Out[22]=

$$\frac{(\mathrm{X}-\mathrm{Xs})\,(\mathrm{Xdot}-\mathrm{Xsdot})+(\mathrm{Y}-\mathrm{Ys})\,(\mathrm{Ydot}-\mathrm{Ysdot})}{\sqrt{(\mathrm{X}-\mathrm{Xs})^2+(\mathrm{Y}-\mathrm{Ys})^2}}$$

In[23]:=
```
ϕ = ArcTan[(Y - Ys)/(X - Xs)];
```

In[24]:=
```
h = {ρ, ρdot, ϕ};
h //. rrx // MatrixForm
```

Out[25]//MatrixForm=

$$\begin{pmatrix} \sqrt{(x1-Xs)^2+(x3-Ys)^2} \\ \frac{(x1-Xs)\,(x2-Xsdot)+(x3-Ys)\,(x4-Ysdot)}{\sqrt{(x1-Xs)^2+(x3-Ys)^2}} \\ \mathrm{ArcTan}\left[\frac{x3-Ys}{x1-Xs}\right] \end{pmatrix}$$

In[26]:=
```
dHdx = CTJacobian[h //. rrx, (x //. rrx)];
dHdx // MatrixForm
```

Out[27]//MatrixForm=

$$\begin{pmatrix} \frac{x1-Xs}{\sqrt{(x1-Xs)^2+(x3-Ys)^2}} & 0 & \cdots \\ \frac{x2-Xsdot}{\sqrt{(x1-Xs)^2+(x3-Ys)^2}}-\frac{(x1-Xs)\,((x1-Xs)\,(x2-Xsdot)+(x3-Ys)\,(x4-Ysdot))}{((x1-Xs)^2+(x3-Ys)^2)^{3/2}} & \frac{x1-Xs}{\sqrt{(x1-Xs)^2+(x3-Ys)^2}} & -\frac{(x3-Ys)\,((x1-Xs)\,(x2-Xsdot)+\cdots}{((x1-Xs)^2+(x3-\cdots} \\ -\frac{x3-Ys}{(x1-Xs)^2\left(1+\frac{(x3-Ys)^2}{(x1-Xs)^2}\right)} & 0 & \cdots \end{pmatrix}$$

In[28]:=
```
dHdu = CTJacobian[h //. rrx, u];
dHdu // MatrixForm
```

Out[29]//MatrixForm=

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$