Impact of Social Media Information Spread Analysis on X

Progress Report

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I. Abstract

This project investigates the flow of information on X (formerly Twitter) by analyzing the platform's network structure and user interactions. The primary objective of this project is to identify subgraphs and their properties to

gain deeper insight into how information flows on X. Characterizing subgraphs and their properties reveals patterns of influence, the roles of gatekeepers, and the mechanisms by which information reaches distant parts of the network. These findings allow for a better understanding of the structural and behavioral factors of subgraphs that ultimately reveal how information flows on X.

II. Project Description

2.1 Purpose

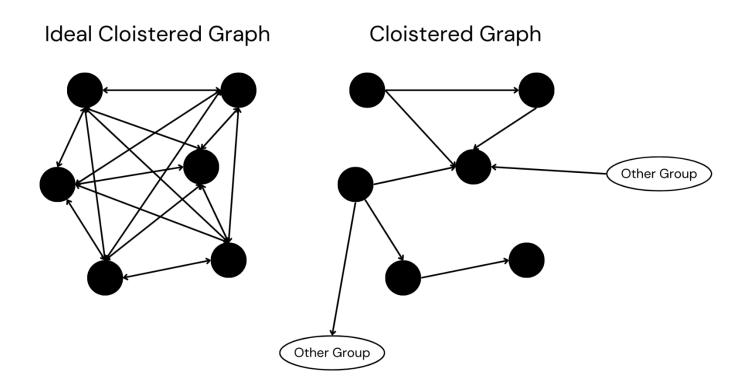
The purpose of this project is to gain a better understanding of how information flows on social media platforms primarily on X. Information flow is crucial for understanding how ideas, news, and opinions spread across networks, influence public perspectives, and affect people's decision-making. By studying these processes, we can reveal patterns of influence, understand who is influenced, and calculate how quickly information reaches different parts of the network. This knowledge is key to addressing challenges such as preventing an increase in news of misinformation, optimizing the spread of accurate information, and improving communication strategies.

Analyzing communities within social media networks plays a critical role in achieving said goals. Communities, which represent smaller subsets of the network, allow us to focus on specific areas of the network. This localized analysis is valuable for understanding the behavior and dynamics of clustered communities and their impact on the network. Community analysis helps identify significant nodes—such as influencers and gatekeepers—that control the flow of information. Studying communities also makes network analysis more manageable. By focusing on smaller areas of a network, we can gain understanding that apply to the entire network without being overwhelmed by its complexity.

2.2 Description

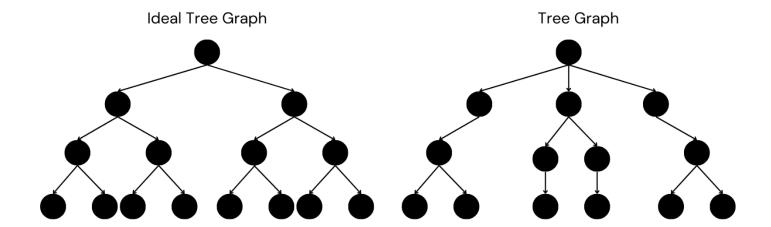
In this project, the primary focus was to analyze the flow of information on X by exploring how different graph structures influence the network. Three distinct graph types were studied: cloistered graph, tree graph, and line branch graph. Each structure represents a different analysis and understanding of how information flows through a network. I will provide a summary of each type of subgraph

The cloistered graph simulates tightly knit communities, also referred to as groups, within a network, where each user has many connections with others in the same community. An ideal cloistered graph would be a disconnected complete graph. However, since such groups are difficult to see in a real-world setting, a more practical definition for a cloistered graph would be a group that has few to none edges connected to other groups within a network. This structure would be analogous to closed groups on X, such as private discussions or niche communities where information tends to flow among active users.



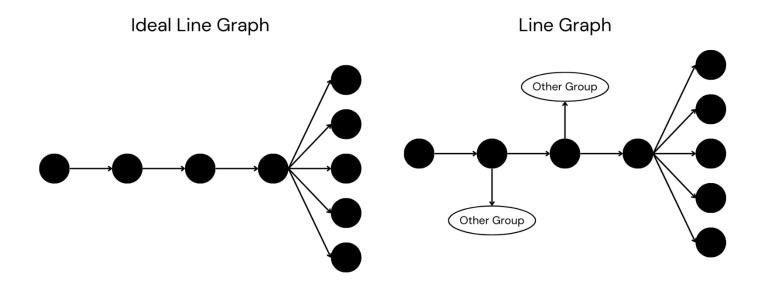
Tree Graph:

The tree graph is a hierarchical data structure where an ideal tree graph is a noncyclic graph that develops from a single node (root node) and each node splits into 2 child nodes. This model reflects scenarios on X where a central figure (an influencer or organization) posts information to their followers, which then spreads through subsequent layers of users.



Line Branch Graph:

The line branch graph is a line of nodes where the final node in the line is connected to several nodes. An ideal line branch graph would be a group where the line of nodes only has connections to the node before and after itself, and the final node in the line has several connections to other nodes. This sequence of nodes would be a group where information passes sequentially from one user to the next, representing the retweet chain seen in niche topics or during initial phases of information propagation.



Analyzing these three graph types revealed how certain community configurations affect the speed, reach, and flow of information in networks on X. These findings contribute to understanding the properties of real-world social networks and provide a baseline for further exploration of how information flows.

2.3 What was built

To implement the analysis of information flow on X, the project focused on graphing the network and analyzing the network through the lens of three distinct graph structures: a cloistered graph, tree graph, and line branch graph. These graph types were chosen to represent different community configurations commonly observed in networks.

The implementation was carried out using R, leveraging its robust libraries for graph representation and analysis. The program written in R is structured to organize the data, graph the network, and analyze the communities in networks by categorizing them into subgraphs.

The program starts with loading the necessary packages for the program.

```
1 * # startup
2 * ```{r}
3 library(tidyverse)
4 library(igraph)
5 library(visNetwork)
6 library(MASS)
7
```

The program then reads from a CSV file that contains the data representing the network. The following lines in the program then organize data by splitting the data into two lists: nodes that point to other nodes and nodes that are pointed to.

```
7
8 # read file
9 nd <- as.data.frame(read_csv("./[INSERT_FILE_NAME].csv"))
10 # users sending
11 from_id <- nd[,1]
12 # users receiving
13 to_id <- nd[,2]
14 ^ ```</pre>
```

After organizing the data, the program then adds one to every node id in the nodes pointing to other nodes and nodes that are pointed to lists. This is due to R not being able to graph nodes that start with 0 or search through lists using an index of 0.

```
16 - # add 1 to from_id list and to_id list
17 + ```{r}
18 # create new from_id list where we add 1 to each node id (this is because you can't have a node 0, must start at node 1)
19 # initialize from_id_plus list
20 from_id_plus <- c()
21 # iterate through nodes in from_id list
22 - for (i in from_id) {
    # add node id plus 1 to from_id_plus list
from_id_plus <- c(from_id_plus, i+1)</pre>
23
24
25 4 }
26 # create new to_id list where we add 1 to each node id (this is because you can't have a node 0, must start at node 1)
27 # initialize to_id_plus list
28 to_id_plus <- c()
29 # iterate through nodes in to_id list
30 - for (i in to_id) {
31  # add node id plus 1 to to_id_plus list
32  to_id_plus <- c(to_id_plus, i+1)</pre>
33 * }
34 -
```

The program then creates a directed list that connects the nodes using the two lists created above.

```
35
36 - # create directed list connecting from and to nodes
37 + ```{r}
38 # list with nodes
39 from_to <- c()
40 # index
41 j <- 1
42 # for loop to generate nodes with directed edges
43 - for (i in 1:dim(nd)) {
44
     # get users sending (need to add 1 as can't have nodes starting with 0 in R)
45
    from_to[j] <- nd[i,1]+1
    j <- j + 1
46
47
     # get users receiving
48
    from_to[j] <- nd[i,2]+1
49
    j <- j + 1
50 4 }
51 -
```

A graph object is created (from the *igraph* package) using the directed list. The program then applies the Louvain algorithm to identify communities in the network. This algorithm is a built-in function (from the *igraph* package) that detects communities in a network by optimizing a parameter called modularity, which quantifies how well a

network can be divided into distinct groups. It separates the network into groups of nodes that are more densely connected to each other than to other nodes. Since my objective was to sort communities into subgraphs, the Louvain algorithm was an excellent choice. Its efficiency on large datasets made it perfect for analyzing my data, which included networks with over 1 million connections. The following lines apply the Louvain algorithm to the graph object and manipulate the data to prepare a visual representation of the network.

```
55 → ### GRAPHING
57 + # create graph objects
58 + ```{r}
59 # create graph object
60 g <- graph(from_to)
61 # apply Louvain algorithm on graph object
62 new_g <- cluster_louvain(as.undirected(g, mode = "collapse"), weights = NULL, resolution = 1)</p>
65 - # create variables used to create node/edge data frame
66 + ```{r}
67 # initialize list to get number of elements in each group
68 group_lengths <- c()
69 # get all unique node ids
70 unique_node_ids <- c()
71 # iterate through nodes in new_g
72 - for(i in 1:length(new_g)){
73 # add group of nodes to unique_node_ids list
74 unique_node_ids <- c(unique_node_ids, c(new_g[[i]]))
75
    # add group length to group_lengths list
     group_lengths <- c(group_lengths, length(new_g[[i]]))</pre>
77 - }
78 - ` ` `
```

The program creates two data frames that contain information about the nodes and edges in the network. These two data frames are used to assist with graphing the network.

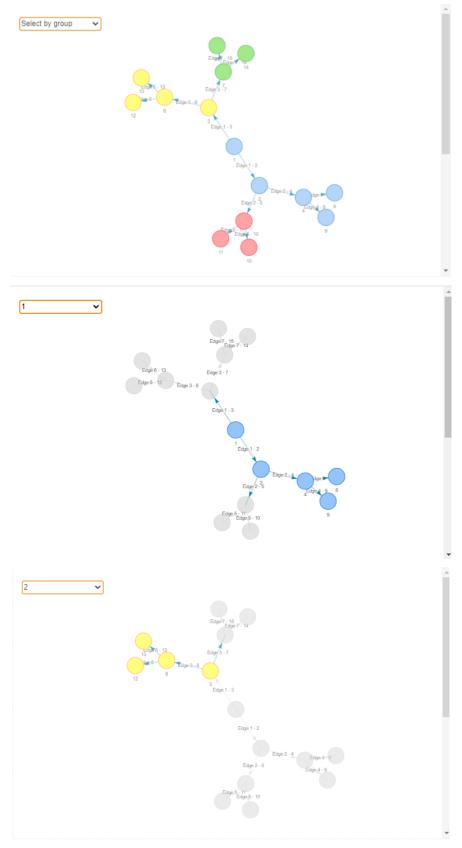
```
80 - # create node data frame variables (for visNetwork graph)
 81 + '''{r}
 82 # create node labels
 83 # initialize node labels list
 84 node_labels <- c()
 85 # iterate through unique node ids
 86 - for (i in unique_node_ids) {
      # add node to node_labels
 87
 88
      node_labels <- c(node_labels, as.character(i))</pre>
 89 4 }
 90 # create node groups
 91 # initialize node groups list
 92 node_groups <- c()
 93 # temp counter
 94 j <- 1
 95 # iterate through group_lengths list
 96 - for (i in group_lengths)
 97
      # repeat group id by value of i and add to node_groups list
     node_groups <- c(node_groups, rep(j, i))
 98
     # increase j
100
     j <- j + 1
101 4 }
102 # create node titles
103 # initialize node titles list
104 node_titles <- c()
105 # iterate through unique node ids
106 - for (i in unique_node_ids)
107
      # create node title and add to list
108
      node_titles <- c(node_titles, paste("Node", as.character(i), "info"))</pre>
109 4 }
110 # create nodes data frame for Louvain algorithm applied graph
111 new_g_nodes <- data.frame(</pre>
112
      id = unique_node_ids,
       label = node_labels,
113
115
      title = node_titles
116 )
117 ^ ` · · ·
119 - # create edge data frame variables (for visNetwork graph)
```

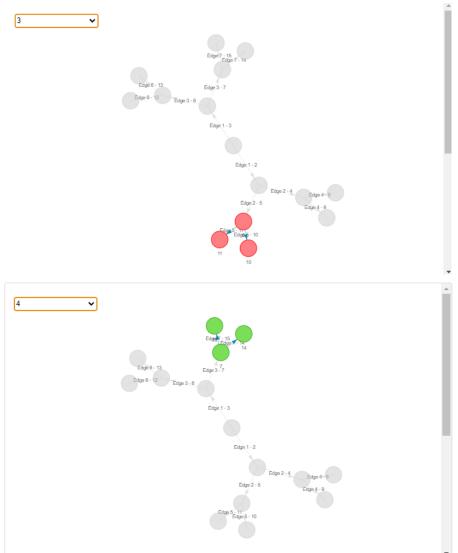
```
120 + ```{r}
121 # create edge labels
122 # initialize edge labels list
123 edge_labels <- c()
124 # iterate with length of from_id_plus list
125 - for (i in 1:length(from_id_plus)) {
# create edge label and add to list
      edge_labels <- c(edge_labels, paste("Edge", from_id_plus[i], "-", to_id_plus[i]))</pre>
127
128 - }
129 # create edge titles
130 # initialize edge titles list
131 edge_titles <- c()
132 # iterate with length of from_id_plus list
133 - for (i in 1:length(from_id_plus)) {
     # create edge title and add to list
135
      edge_titles <- c(edge_titles, paste("Edge from", from_id_plus[i], "to", to_id_plus[i]))</pre>
136 - }
137
    # create edges data frame for Louvain algorithm applied graph
138 new_g_edges <- data.frame(
139
     from = from_id_plus,
140
     to = to_id_plus,
      label = edge_labels,
141
      arrows = "to"
142
143
      title = edge_titles
144 )
```

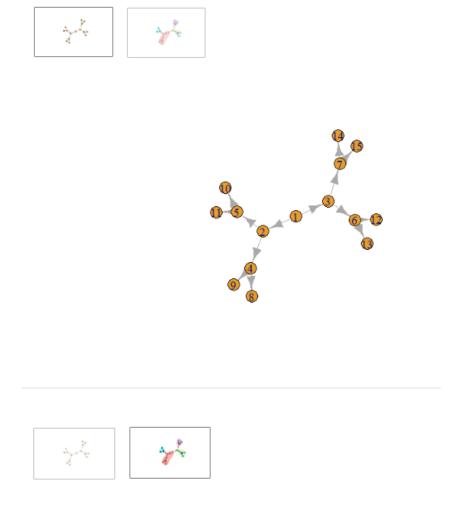
After preparing the data, the program graphs the network using *visNetwork* and *igraph*. While both graphs represent the same network, they offer slightly different perspectives and insights. *visNetwork* provides an

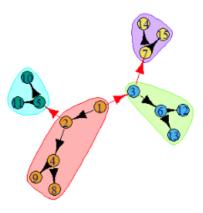
interactive representation of the network as well as an option to export the representation as an HTML file. *igraph* provides a static image of the network and focuses on network analysis.

```
147 → # graph using visNetwork
148 + ```{r}
149 #set.seed(11) (seed for similar looking graph)
150 set.seed(24)
151 visNetwork(new_g_nodes, new_g_edges, height = "850px", width = "100%") %>%
152 visIgraphLayout() %>%
153 visNodes(
154
       shape = "dot",
155
       color = list(
         background = "#0085AF",
156
         border = "#013848",
157
          highlight = "<mark>#FF80</mark>00"
158
      ),
shadow = list(enabled = FALSE, size = 35)
159
160
161 ) %>%
162 visEdges(
      shadow = FALSE,
color = list(color = "#0085AF", highlight = "#C62F4B")
163
164
165
      ) %>%
visOptions(highlightNearest = list(enabled = T, degree = 1, hover = T),
                  selectedBy = "group")
167
168 - * * * *
169
170 - # graph using igraph
171 → ```{r}
172 set.seed(2)
173 plot(g)
174 set.seed(2)
175 plot(new_g, g)
176 -
```









After dividing the network into communities and graphing it, the program then analyzes the communities and applies the subgraph analysis.

To identify all communities that resemble cloistered graphs, the program compares the number of edges in the group to the maximum number of possible edges in the group.

I created a few helper functions to assist with the analysis. The first helper function finds every connection between nodes in the data. The second helper function finds every edge that connects to a different community and every node that belong to the same community. The third helper function finds the MPNE (maximum possible number of edges) of the group.

```
180 - ### ANALYSIS
182 + # CLOISTERED GRAPH
184 - # get all connections between nodes
185 + ```{r}
186 # node connections list
187 node_connections <- c()
188 # iterate through all nodes in from_to list
189 - for (i in seq(1, length(from_to), 2)){
# append each to node to respective from node
       node\_connections[[as.character(from\_to[i])]] = c(node\_connections[[as.character(from\_to[i])]], from\_to[i+1]) \\
191
192 - }
193 -
195 - # find edges that belong to same group or different group and find all node connections in same group
197 # nodes in same group list
198 group_node_connections <- list()
199 # edges that belong to a group
200 in_edge_groups <- list()
201 # edges that leave a group
202 out_edge_groups_f <- list()</pre>
203 # edges that enter a group
204 out_edge_groups_s <- list()
205
206 # iterate through all edges
207 - for (i in 1:nrow(new_g_edges)){
     # get from node
208
209
      fnum <- new_g_edges$from[i]
210
      # get to node
      snum <- new_g_edges$to[i]
211
212
       # find which group fnum belongs to
213
      fnum_group <- new_g_nodes$group[which(new_g_nodes$id == fnum)]</pre>
214
       # find which group snum belongs to
215
      snum_group <- new_g_nodes$group[which(new_g_nodes$id == snum)]</pre>
216
       # if fnum and snum belong to same group
217
218 -
      if(fnum_group == snum_group){
219
        # add edge to in_edge_group
220
        in_edge_groups[[as.character(fnum_group)]] <- c(in_edge_groups[[as.character(fnum_group)]], sprintf('(%i, %i)', fnum, snum))
221
         # add nodes to group_node_connections
222
         group_node_connections[[as.character(fnum)]] <- c(group_node_connections[[as.character(fnum)]], snum)</pre>
223
       # fnum and snum don't belong to same group
224 -
       # add edge to out_edge_group_f
225
        out_edge_groups_f[[as.character(fnum_group)]] <- c(out_edge_groups_f[[as.character(fnum_group)]], sprintf('(%i, %i)', fnum, snum))
226
227
         # add edges to out_edge_group_s
         out\_edge\_groups\_s[[as.character(snum\_group)]] <- c(out\_edge\_groups\_s[[as.character(snum\_group)]], sprintf('(\%i, \%i)', fnum, snum)) \\
228
229 -
230 4 }
231 4
```

```
233 * # find MPNE for each group
234 * ```{r}
235  # max possible number of edges in a group
236  MPNE <- c()
237  # iterate through all groups
238 * for (i in 1:length(new_g)) {
239  # max possible number of edges calculated as n * (n-1) / 2 where n = number of nodes
240  # append max possible number of edges to MPNE
241  MNPE <- c(MPNE, (length(new_g[[i]])*(length(new_g[[i]])-1))/2)
242 * }
243 * ```
```

Using these three helper functions, the program then performs the cloistered graph identification analysis on every group within the network.

```
245 → # Cloistered Graph Analysis
       print("------ Cloistered Graph Analysis -----")
249 # iterate over different gr
250 * for (i in 1:length(MPNE)) {
251 # Group i analysis
251 # Group i analysis
252 print(sprintf("Group %i:", i))
          # find total number of edges entering and leaving edges_le <- length(out_edge_groups_f[[as.character(i)]]) + length(out_edge_groups_s[[as.character(i)]])
         edges_l_e <- length(out_edge_groups_f[[as.character(1)]]) + leng
# how many edges are in each group
len_egroup <- length(in_edge_groups[[as.character(i)]])
# how close group is to that of a complete graph as a percentage
percent <- len_egroup / (MPNE[[i]]) * 100
# how many edges are missing from that of a complete graph
missing_e <- MPNE[[i]] - len_egroup
# group is cloistered graph or not
cloistered <- FALSE
255
257 # now c.o. 258
258 percent <- len_c
259 # how many edges
260
261
262
263
# if number of edges leaving/entering group is 2 percent of max possible number of edges
# then consider it a cloistered group
if (edges_l_e < MPNE[[i]] * 0.02) {
268
269
                           (sprintf(" Group %i is a cloistered graph with %i edges leaving/entering the group", i, edges_l_e)) cloistered to TRUE
270
                cloistered <- TRUE
270 clostered <- INCE
271 # if more than 2 percent then don't consider a cloistered graph
272 * } else {
273 # print statement
274 print(sprintf(" Group %i is not a cloistered graph with %i (
                                               Group %i is not a cloistered graph with %i edges leaving/entering the group", i, edges_l_e))
275 ^ }
276
277 #
# if percentage = 100% and graph is a cloistered graph then group is an ideal cloistered graph 278 - if (percent == 100 & cloistered & length(edges_l_e) == 0) {
278 • if (percent == 100 & cloistered & length(logs-1-),
279 # print statement
280 print(sprintf(" Group %i is an ideal cloistered graph", i))
281 # if percentage = 100% but not cloistered graph then only complete graph
282 • } else if (percent == 100 & !cloistered){
                # print statement
printf(sprintf(" Group %i is a complete graph", i))
          # if percentage != 100% and not a cloistered graph 
| else {
| # print statement
285
                print(sprintf("
                                               Group %i is %f%% (%i/%i) of a complete graph with %i missing edges compared to that of a complete graph", i, percent, len_egroup, MPNE[[i]], missing_e))
```

Next, the program determines which communities are tree graphs by comparing every community against the properties of an ideal tree graph.

Once again, I created several helper functions to assist with the analysis. The first helper function constructs a tree structure using the nodes within a group. The second helper function checks for cycles within a group. The third helper function examines the group in its tree structure and calculates specific statistics, which will be discussed in Section 3.1. The fourth helper function evaluates these statistics to determine how similar the tree structure is to an ideal tree graph.

```
295 + # TREE GRAPH
296
297 + # function to create tree
298 + ```{r}
299 - create_tree <- function(tree = list(), level = 1, node) {
300 # if no nodes in current level of tree
301 - if (is.null(tree[[as.character(level)]])) {
       # add node to tree at level
302
303
        tree[[as.character(level)]] <- node</pre>
     # nodes exist in current level of tree
304
305 → } else {
      # add node to tree at level
306
        tree[[as.character(level)]] <- c(tree[[as.character(level)]], node)</pre>
307
308 -
309
310
    # get children of node
     children <- group_node_connections[[as.character(node)]]</pre>
311
312
313 # if node has children
314 → if (!is.null(children)) {
       # iterate through children
315
       for (child in children) {
316 +
       # recursive call
317
318
          tree <- create_tree(tree, level + 1, child)
      }
319 -
320 4 }
321
322
      # return tree
323
      return(tree)
324 4 }
325 -
```

```
327 ≠ # check for cycle function
328 + ```{r}
329 - has_cycle <- function(node_list, group_node_connections) {
      # helper function for DFS
331 +
     dfs <- function(node) {</pre>
332
333
        # if a node has not been visited
334 →
         if (!visited[[node]]) {
335
           # mark node as visited
336
           visited[node] <- TRUE
337
          # add node to recursion stack
338
          rec_stack[node] <- TRUE</pre>
339
340
          # if the node is connected to other nodes in group
341 -
          if (!is.null(group_node_connections[[as.character(node)]])) {
342
           # iterate through other nodes in group
343 -
            for (near_node in group_node_connections[[as.character(node)]]) {
344
              # if the node has not been visited and recursion call on near_node
345 -
               if (!visited[[near_node]] && dfs(near_node)) {
346
                 # return TRUE
347
                return (TRUE)
348
              # otherwise if the node near is on the recursion stack
349 →
              } else if (rec_stack[[near_node]]) {
350
                # return TRUE
351
                 return (TRUE)
              3
352 -
353
354 -
          }
355 -
356 -
        # remove the node from recursion stack
35.7
358
        rec_stack[node] <- FALSE
359
        # return FALSE
360
        return (FALSE)
361 -
362
363
    # list for visited nodes
     visited <- list()</pre>
364
365
    # list for recursion stack
366 rec_stack <- list()</pre>
367
      # initialize visited and recursion stack
368 - for (node in node_list){
369
       visited[node] <- FALSE</pre>
370
       rec_stack[[node]] <- FALSE</pre>
371 -
372
373
      # check each node in the group
374 → for (node in node_list) {
375
       # if dfs(node is true then there is a cycle)
        if (dfs(node)){
376 +
377
          # return TRUE
378
           return (TRUE)
379 -
380 -
       # there is no cycle so return FALSE
381
      return (FALSE)
382
383 - }
384 -
```

```
386 + # function to find statistics
387 + ```{r}
388 - tree_statistics <- function(tree, group_node_connections){
       # set level error to 0
389
       level_error <- 0
390
       # set extra nodes to 0
392
      extra_nodes <- 0
       # set missing nodes to 0
393
       missing_nodes <- 0
       # set height balance to number of levels
height_balance <- length(names(tree))</pre>
395
396
       # set nodes relative to ideal nodes to 0
397
       nrin <- 0
398
       # set total nodes to 1 (root node)
399
       total nodes <- 1
400
        # set largest number of nodes in a level to 1 (root node)
401
402
       lnol <- 1
403
       # set number of nodes in a level to 1 (root node)
404
405
406
        # iterate through all levels in tree (skip level 0)
407 -
       for (level in names(tree)[-1]){
         # increase total nodes by number of nodes on level
408
         total_nodes <- total_nodes + length(tree[[as.character(level)]])</pre>
409
         # get level number of previous level
         prev_level <- as.character(as.numeric(level)-1)</pre>
411
412
         \mbox{\# if current number of nodes is greater than 2 $\wedge$ number of nodes from previous level}
413
414 +
         if (length(tree[[as.character(level)]]) > 2^length(tree[[prev_level]])){
           # increase extra nodes by the difference
415
           extra_nodes <- extra_nodes + length(tree[[as.character(level)]]) - 2^length(tree[[prev_level]])
416
            # increase level error by 1
417
418
           level_error <- level_error + 1
         # if current number of nodes is less than 2 ^ number of nodes from previous level
419
         } else if (length(tree[[as.character(level)]]) < 2^length(tree[[prev_level]])){
420 -
            # increase missing nodes by differ
          missing_nodes <- missing_nodes + 2^length(tree[[prev_level]]) - length(tree[[as.character(level)]])
# increase level error by 1</pre>
422
423
424
           level_error <- level_error + 1
425 ^
426
         # nol equal to number of nodes on current level
427
         nol <- length(tree[[as.character(level)]])</pre>
428
          # if nol less than or equal to <u>lnol</u>
429
         if (nol <= lnol){
430 +
           # decrease height_balance by 1
431
         height_balance <- height_balance - 1
433
         # if nol greater than lnol
434 -
         } else {
           # set lnol to nol
435
         # set mos
lnol <- nol
436
437 ^
438
439 ^
```

```
# set left_nodes to left half of children of root node
441
         # set total left_nodes <- head(tree[['1']], floor(length(tree[['1']])/2))
# set total left nodes to length of left_nodes
total_left_nodes <- length(left_nodes)
443
444
         # set right_nodes to right half of children of root node right_nodes <- tail(tree[['1']], ceiling(length(tree[['1']])/2)) # set total right nodes to length of right_nodes
446
448
         total\_right\_nodes <- \ length(right\_nodes)
449
        # iterate through all nodes of left_nodes
while (length(left_nodes) != 0){
    # if first node in left_nodes has children
450
451 -
         if (!is.null(group_node_connections[[as.character(left_nodes[1])]])){
    # increase total_left_nodes by number of children
453 +
454
              total_left_nodes <- total_left_nodes + length(group_node_connections[[as.character(left_nodes[1])]])
456
                 append children to
457
              left_nodes <- c(left_nodes, group_node_connections[[as.character(left_nodes[1])]])</pre>
458 ^
              pop off first node in left_nodes
459
           left_nodes <- left_nodes[-1]</pre>
460
461 -
462
         while (length(right_nodes) != 0) {
    # if first node of right_nodes has children
463 -
464
           if \ (!is.null(group\_node\_connections[[as.character(right\_nodes[1])]])) \\ \{
466
                 increase total_right_nodes by number of
467
            total_right_nodes <- total_right_nodes + length(group_node_connections[[as.character(right_nodes[1])]])
468
              # append children to right nodes
             right_nodes <- c(right_nodes, group_node_connections[[as.character(right_nodes[1])]])
469
470 -
            # pop off first node in right_nodes
471
           right_nodes <- right_nodes[-1]
473 4 }
474
        nrin <- sprintf("%i/%i", total nodes, 2^(length(names(tree)))-1)
476
478
         # return statistics
479
         return (c(extra_nodes, missing_nodes, level_error, total_left_nodes, total_right_nodes, nrin, height_balance))
```

```
483 - # function to find value of tree based on analysis
484 + ```{r}
485 - tree_analysis <- function(temp_tree_statistic){
486
       left_nodes <- as.numeric(temp_tree_statistic[4])</pre>
       right_nodes <- as.numeric(temp_tree_statistic[5])
       # if left_nodes < right_nodes
488
489 - if (left_nodes < right_nodes) {
       # side_balance is ratio of left_nodes to right_nodes
490
491
         side_balance <- left_nodes / right_nodes
       # left_nodes > right_nodes
492
493 → } else{
        # side_balance is ratio of right_nodes to left_nodes
494
495
        side_balance <- right_nodes / left_nodes
496 -
497
      # split nrin into numerator and denominator by /
498
      fraction <- strsplit(temp_tree_statistic[6], "/")[[1]]</pre>
499
500
     # get numerator of fraction
      numerator <- as.numeric(fraction[1])</pre>
5.01
      # get denominator of fraction
502
503
      denominator <- as.numeric(fraction[2])</pre>
504
       # calculate nrin
      nrin <- numerator / denominator
505
506
507
       node_error <- as.numeric(temp_tree_statistic[1]) + as.numeric(temp_tree_statistic[2])</pre>
508
      # node_error has weight of 0.35
509
      node_error_weight <- (1 - (node_error / denominator)) * 0.35</pre>
510
       # level_error has weight of 0.25
511
       level_error_weight <- (1 - (as.numeric(temp_tree_statistic[3]) / log((denominator+1), 2))) * 0.25</pre>
512
      # nrin has weight of 0.15
513
514
      nrin_weight <- nrin * 0.15
       # side_balance has weight of 0.15
515
516
       side_balance_weight <- side_balance * 0.15
       # height_balance has weight of 0.1
517
518
       height_balance_weight <- (1 - (as.numeric(temp_tree_statistic[7]) / log((denominator+1), 2))) * 0.1
519
       # sum up weights to calculate temp_tree_statistic and return max(0, weights)
520
521
      return (max(0, (node_error_weight + level_error_weight + side_balance_weight + height_balance_weight + nrin_weight)))
522 4 }
523 A
```

Using these four helper functions, the program then performs the tree graph identification analysis on every group within the network.

```
525 + # Tree Graph Analysis
526 + ```{r}
 527 print("------ Tree Graph Analysis -----")
528 # create new from_to list for finding cycle
530 tree_cycle_graph_list <- list()
531 # iterate through nodes in same group
532 * for (group in unique(new_g_nodesSgroup)){
533 # add nodes in same group to tree_cycle_gr
               # add nodes in same group to tree_cycle_graph_list
tree_cycle_graph_list[[as.character(group)]] <- new_g[[as.character(group)]]
535 - }
536 -
537 # create potential root nodes list
538 potential_root_nodes <- list()
 540 # get each group in tree_cycle_graph_list
541 * for (group in names(tree_cycle_graph_list)){
542 # group must have more than 4
 542 # group must nave more than 1 node
543 * if (length(tree_cycle_graph_list[[group]]) > 1) {
544 # if there is a cycle
545 * if (has_cycle(tree_cycle_graph_list[[group]], group_node_connections)) {
546 # mrint tathement
                 # print statement
print(sprintf(" Group %s has a cycle so not a tree", group))
# group does not have cycle
# look at edges in edges land.
                   558
559 ^
                              potential_root_nodes[[group]] <- unique(c(potential_root_nodes[[group]], node))</pre>
                       }
}
# look at edges in edges entering groups
for (edge in out_edge_groups_s[[group]]){
    # extract number from edge
    number <- gregexpr("[0-9]+", edge)
    result <- regmatches(edge, number)
    #get first number (the from node)
    node <- as.numeric((result[[1]][2]))
    # add node to potential root nodes list
    potential_root_nodes[[group]] <- unique(c(potential_root_nodes[[group]], node))
}</pre>
 561 +
562
 563
564
 565
 566
567
 568
 569 -
570
571
572 +
                        # if there are no nodes for the group in potential_root_nodes list
if (length(potential_root_nodes[[group]]) == 0){
    # add every node in the group to potential_root_nodes list
    for (node in tree_cycle_graph_list[[group]]){
     # nolly keep using noder.
                       # only keep unique nodes
potential_root_nodes[[group]] <- unique(c(potential_root_nodes[[group]], node))
}
</pre>
575
576
577 -
```

```
### create best root node analysis list (keeps track of best root node, best tree value, and best tree statistics)
### create best root node analysis list (keeps track of best root node, best tree value, and best tree statistics)
### group index
### grou
```

Next, the program determines which communities are line branch graphs by comparing every community against the properties of an ideal line branch graph.

Like the analyzation of the cloistered graph and tree graph, I created several helper functions to assist with the analysis. The first helper function finds the longest line that ends with a given node in a group. The second function finds every reversed node connection within the same group. The third helper function examines the group in its line branch structure and calculates specific statistics, which will be discussed in Section 3.1. The fourth helper function evaluates these statistics to determine how similar the line branch structure is to an ideal line branch graph.

```
642 - #LINE BRANCH GRAPH
643
644 → # function to find longest line in group
645 + ```{r}
646 - longest_line <- function(node, group_node_connections, visited = c()) {
      # if the initial node has been visited
648 -
      if (node %in% visited){
     # return length of 0 and an empty path
649
       return(list(length = 0, path = c()))
650
651 -
652
      # initial node has not been visited so add node to visited
653
654
      visited <- c(visited, node)
655
65.6
      # if node does not have connections to other nodes
      if (is.null(group_node_connections[[as.character(node)]]))
657
        # return length of 1 (just initial node) and the node as the path
658
        return(list(length = 1, path = c(node)))
659
660
      # longest line length
661
      max_length <- 0
662
      # path of longest line
663
664
      max_path <- c()
665
      # explore all nodes connected to node
666
     for (near_node in group_node_connections[[as.character(node)]]) {
667 +
668
        # recursive call
669
        line <- longest_line(near_node, group_node_connections, visited)
670
        # if the length of the line is greater than max_length
671 +
       if (line$length > max_length) {
672
       # set max_length to the line's length
         max_length <- line$length
673
674
         # set max_path to the line's path
675
          max_path <- line$path
       }
676 -
677 - }
678
679
       # return max_length + 1 to include current node and max_path
680
       return(list(length = max_length + 1, path = c(node, max_path)))
681 - }
682 - ` `
```

```
684 → # create reverse group node connections list
685 + ```{r}
686 # function to create reverse group node connections
687 reverse_connections <- function(connections) {
    # initialize reverse group node connections list
      reversed <- list()
       # iterate through all from nodes in connections list (from nodes: nodes that point to other nodes)
691 - for (from in names(connections)) {
       # iterate through all to nodes (to nodes: nodes that are pointed to)
693 +
        for (to in connections[[from]]) {
         # if to node is not in the names of reversed list
694
          if (!as.character(to) %in% names(reversed)) {
695 +
696
            # initialize nodes pointed at to node in reversed list
             reversed[[as.character(to)]] <- c()
697
698 -
699
          # add from node to reversed list at index to node
700
          reversed[[as.character(to)]] <- c(reversed[[as.character(to)]], as.numeric(from))
701 -
702 -
703
      # return reversed list
704
      return(reversed)
705 ^ }
706
707 # create reversed connections
708 reversed_gn_connections <- reverse_connections(group_node_connections)
```

```
711 - # function to find statistics
712 - ```{r}
713 - lb_statistics <- function(line, group_nodes, group_node_connections){
714
       # set branch_nodes to number of nodes connected to last node in line
715
       branch_nodes <- group_node_connections[[as.character((tail(line, n=1)))]]</pre>
716
717
       # set line_error to (number of nodes in group) - ((number of nodes in line) + (branching nodes))
718
      line_error <- length(group_nodes) - (length(line) + length(branch_nodes))</pre>
719
       # set llrill to (number of nodes in line) / (2*log(length(branch_nodes) + 1) + 5)
       llrill <- sprintf("%i/%i", length(line), as.integer((2*log(length(branch_nodes) + 1) + 5)))</pre>
720
721
722
       # initialize branch_continuation to 0
723
       branch_continuation <- 0
724
       # iterate through all branch_nodes
725 -
      for (node in branch_nodes){
726
        # add number of nodes connected to node to branch_continuation
727
        branch_continuation <- branch_continuation + length(group_node_connections[[as.character(node)]])
728 4 }
729
      # initialize branch_positions
730
731
     branch_positions <- c()
       # set line_index to 0
732
733
       line_index <- 0
734
       # iterate through nodes in line
735
736 - for (node in line[-length(line)]){
737
        # increase line_index by 1
738
         line_index <- line_index + 1
         # if the node has more than 1 connection in group
739
740 -
         if (length(group_node_connections[[as.character(node)]]) > 1){
741
          # add position relative to line to branch_positions
742
           branch_positions <- c(branch_positions, sprintf("%i/%i", line_index, length(line)))
743
744 -
745 -
      if (length(branch_positions) == 0){
746 -
747
         branch_positions = 0
748 -
749
750
      # return statistics
       return (c(line_error, llrill, branch_continuation, branch_positions))
751
752 4 }
753 🔺
```

```
755 \star # function to find value of line branch based on analysis 756 \star ```{r}
757 - lb_analysis <- function(lb_statistic){
        # split llrill into numerator and denominator by /
758
        fraction <- strsplit(lb_statistic[2], "/")[[1]]</pre>
75.9
        # get numerator of fraction
760
        line_length <- as.numeric(fraction[1])</pre>
761
762
        # get denominator of fraction
763
        i_line_length <- as.numeric(fraction[2])
764
        # calculate <u>llrill</u>
       llrill <- line_length / i_line_length
765
766
       # calculate how many branch nodes there are in group (isolate branch_nodes in branch_nodes \leftarrow exp((i_line_length - 5)/2)-1
767
768
        # calculate line_error such that line_error is relative to number of nodes in ideal line length + branching nodes line_error <- as.integer(lb_statistic[1]) / (i_line_length + as.integer(branch_nodes))
769
770
771
772
773
        # calculate branch_continuation relative to number of nodes in line
        branch_continuation <- as.integer(lb_statistic[3]) / line_length</pre>
774
775
        # initialize penalty_sum
776
        penalty_sum <- 0
        # iterate through positions of branching nodes in line relative to line
777
778 +
        for (position_r in lb_statistic[4]){
          # split position relative into numerator and denominator by /
fraction <- strsplit(position_r, "/")[[1]]</pre>
779
780
781
          # get position (numerator)
782
          position <- as.numeric(fraction[1])</pre>
          # add ((number of nodes in line) - (position of branching node in line)) / (number of nodes in line) to penalty_sum
783
          penalty_sum <- penalty_sum + ((line_length - position) / line_length)</pre>
784
786
787
        # calculate branch_position (1 - penalty_sum / (number of branching nodes in line))
788
        branch_positions <- 1 - penalty_sum/length(lb_statistic[4])</pre>
789
790
        # line error has weight of 0.4
791
        line_error_weight <- (1 - line_error) * 0.4
792
          11rill has weight of 0.3
793
        llrill_weight <- llrill * 0.3
794
        # branch_continuation has weight of 0.2
795
       branch_continuation_weight <- (1 - branch_continuation) * 0.2 # branch_positions has weight of 0.1
796
797
       branch_positions_weight <- branch_positions * 0.1
798
799
       # sum up weights to calculate lb_statistic and return max(0, weights)
        return (max(0, (line_error_weight + llrill_weight + branch_continuation_weight + branch_positions_weight)))
801 - }
802 🛦
```

Using these four helper functions, the program then performs the line branch graph identification analysis on every group within the network.

```
804 - # LINE BRANCH GRAPH ANALYSIS

    '```{r}
    print("------ Line Branch Graph Analysis ------")

                      # create new from_to list for finding cycle
lb_cycle_graph_list <- list()</pre>
  810 # iterate through nodes in same group
811 - for (group in unique(new_g_nodes$group)){
  812
                          # add nodes in same group to lb_cycle_graph_list
lb_cycle_graph_list[[as.character(group)]] <- new_g[[as.character(group)]]</pre>
813 | b_cycle_graph_list[[as.charact 814 - } 815 | 816 # create potential end nodes list 817 | potential_end_nodes <- list() 818 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 819 | 8
  319 # get each group in lb_cycle_graph_list
820 • for (group in names(lb_cycle_graph_list)){
821 # group must have more than 1 node
  822 - if (length(lb_cycle_graph_list[[group]]) > 1) {
                          # print statement print(sprintf(" Group %s has a cycle so not a line branch", group))
# group does not have cycle
} else {
                        if (has_cycle(lb_cycle_graph_list[[group]], group_node_connections)) {
  828 +
                                         potential_end_nodes[[group]] <- lb_cycle_graph_list[[group]]</pre>
  829
  830 -
832 - }
                # create best start node analysis list (keeps track of best start node, best line branch value, and best line branch statistics) bstart_path <- list()
836

837 # iterate through groups in potential_end_nodes list
838 * for (group in names(potential_end_nodes)){
839 # path of longest line
840 temp_longest_path <- c()
841 # most branching nodes
842 largest_branch <- 0
843 # longest_path with most branching
844 longest_path <- list()
845 # iterate through nodes in groups in potential end
 844 | longest_path <- Inst()

845  # iterate through nodes in groups in potential_end_nodes

846  * for (node in potential_end_nodes[[group]]){

847  # if the node has more branches than largest_branch
                                if (length(group_node_connections[[as.character(node)]]) > largest_branch){
    # set largest_branch to the number of branches of the node
    largest_branch - length(group_node_connections[[as.character(node)]])
    # set the longest_path to the longest line of nodes in the group that ends with node
    longest_path <- longest_line(node, reversed_gn_connections)Spath
    # if the node has equal number of branches to largest_branch
    # lelse if (length(group_node_connections)[as.character(node)]]) == largest_branch){
    # find the longest line of nodes in the group that ends with node
    temp_longest_path <- longest_line(node, reversed_gn_connections)Spath
    # if the length of this line is greater than the length of longest_path
    if (length(temp_longest_path) > length(longest_path)){
        # set longest_path to the line
        longest_path <- temp_longest_path
    }</pre>
  848 -
  856
 862 ^
863 ^
                          # save path using best start node to bstart_path bstart_path[[group]] <- c(rev(longest_path))
  864
  868 bstart node analysis <- list()
                   # iterate through groups in potential_end_nodes inst

for (group in names(potential_end_nodes)){

# get line branch statistic using path in bstart_path at the group as the start node

| b_statistic <- lb_statistic(bstart_path[[group]], lb_cycle_graph_list[[group]], group_node_connections)

# get line branch value from lb_analysis

# get line branch value from lb_analysis
                          # get line branch value from lb_analysis
lb_value <- lb_analysis(lb_statistic)
# add line branch value and lb_statistic to bstart_node_analysis
bstart_node_analysis[[group]] <- c(lb_value, lb_statistic, bstart_path[[group]])</pre>
  876
                  bstart_node_analysis[[group]] <- c(lb_value, lb_statistic, bstart_path[[group]])
# Group is analysis
print(sprintf("Group %s:", group))
# if the line branch analysis for the group is greater than or equal to 0.8 then the group is a line branch
if (lb_value >= 0.7){
# print statements
print(sprintf(" Group %s is a line branch with node %i being the start node", group, as.numeric(bstart_node_analysis[[group]][6])))
last_i <- length(bstart_node_analysis[[group]])
print(sprintf(" The line is from node %s", paste(as.character(as.integer(bstart_node_analysis[[group]][6:last_i])), collapse=" to ")))
print(sprintf(" Node %s then branches into nodes %s", as.character(as.integer(bstart_node_analysis[[group]][last_i])),
psste(as.character(group_node_connections[[as.character(bstart_node_analysis[[group]][last_i]))]),
collapse=", ")))
  877
  878
  885
                  collapse=",")))
print(sprintf(" %f%% of the group shares the same properties as those of an ideal line branch", 100°as.numeric(bstart_node_analysis[[group]][1])))
print(sprintf(" %i Line Errors, %s (%f%%) LLRBN, %i Branch Continuation, %s Branch Positions", as.numeric(bstart_node_analysis[[group]][2]), bstar
100°as.numeric(eval(parse(text = bstart_node_analysis[[group]][3]))), as.numeric(bstart_node_analysis[[group]][4]), bstart_node_analysis[[group]][5]))
}else(
# group to the transfer of the group is less than 0.8 then the group is not a line branch
# group to the transfer of the group is less than 0.8 then the group is not a line branch
                                 # print statements
print(sprintf(" | last_i <- length(b)</pre>
   890
                    # print statements
print(sprintf(" Group %s is not a line branch and has a structure closest to a line branch when node %i is the start node", group, as.numeric(bstart_node_analysis[[group]][6])))
last_i <- length(bstart_node_analysis[[group]])
print(sprintf(" The line is from node %s", paste(as.character(as.integer(bstart_node_analysis[[group]][6:last_i])),
print(sprintf(" Node %s then branches into nodes %s", as.character(as.integer(bstart_node_analysis[[group]][last_i])))
paste(as.character(group_node_connections[[as.character(bstart_node_analysis[[group]][last_i]))]),
paste(as.character(group_node_connections[[as.character(bstart_node_analysis[[group]][last_i]))])),
  891
```

2.4 What does it do

The program allows for both visualization and analysis of networks by graphing the entire network and identifying subgraphs among communities within the network. The program also incorporates a community detection algorithm to split the network into distinct communities. The network is then visually represented as a directed graph, highlighting the connections between nodes (users) and the direction of the connection (flow of the information).

Once the network is split into communities, the program studies the communities by analyzing properties to identify subgraphs. By analyzing structural properties and the flow of information, the program calculates if a community is a cloistered graph and assesses the similarities of communities to the ideal form of tree and line branch graphs. The program is a dynamic tool designed to visualize the network, analyze its structure, and evaluate the flow of information.

2.5 How does it work

1 load packages
2 read csv file

To provide a clear understanding of how the program operates and identifies subgraphs within the network, the following lines of pseudocode break down and explain each segment of the code and its underlying ideas

```
3 split data in csv file into nodes that point to other nodes (from id list) and nodes that
are pointed to (to id list)
4 iterate through every row in csv file to create directed list (from to list) using from
and to id list
### GRAPHING
5 create a graph object (from igraph package)
6 apply Louvain algorithm on graph object to create new graph object
7 iterate through nodes in graph object to find unique nodes id's and number of nodes in
each group
8 iterate through unique node id's to find node labels (node id but as a char)
9 iterate through number of nodes in each group to find node groups (group that node
belongs to)
10 iterate through unique node id's to create node titles (title of the node)
11 create node data frame that holds the following info
     unique node id's
     node labels
     node groups
     node titles
```

12 iterate through from id list and add 1 to every node id to create from id plus one list (nodes/indexing in R must start at 1)

- 13 iterate through to id list and add 1 to every node id to create to id plus one list
- 14 iterate through from id plus one list to create edge labels (every edge between two nodes)
- 15 iterate through from id plus one list to create edge titles (title of every edge)
- 16 create edge data frame that holds the following info

from id plus one list to id plus one list edge labels edge titles

17 using node data frame and edge data frame, graph network using visNetwork 18 using graph object, graph network using igraph

ANALYSIS

CLOISTERED GRAPH

- 19 iterate through from to list and find all node connections for every node
- 20 iterate through every edge
- 21 if nodes in edges belong to the same group
- 22 add the edge to same group edge list
- 23 add nodes to group node connections list
- 24 else
- 25 add edge to different group edge list
- 26 iterate through graph object to find maximum possible number of edges (MPNE) for every group
- 27 iterate through every group
- 28 find number of edges leaving/entering group
- find number of edges in group
- calculate how close group is to a complete graph using number of edges in group and MPNE (percentage)
- 31 find how many edges are missing for group to be a complete graph
- 32 if number of edges leaving/entering a group is less than 2% of MPNE
- then group is a cloistered graph
- 34 print statistics and analysis
- 35 else
- 37 print statistics and analysis
- if percentage is 100% and group is a cloistered graph and group is disconnected
- then group is an ideal cloistered graph

```
Progress Report 3
                                          McGill
                                                                               COMP 400
101
            set nol to number of nodes on level
102
            if nol is less than or equal to lnol
103
                 increase height balance by 1
104
            else
105
                 set lnol to nol
       set left nodes list to left half of children of root node
106
       set total left nodes to number of left nodes
107
108
       set right nodes list to right half of children of root node
       set total right nodes to number of right nodes
109
110
      while left nodes list is not empty
           if first node in left nodes list has children
111
                 append children to left nodes list
112
113
                 add number of children to total left nodes
114
            pop off first node in left nodes list
115
      while right nodes list is not empty
116
            if first node in right nodes list has children
                 append children to right nodes list
117
118
                 add number of children to total right nodes
119
            pop off first node in right nodes list
       set nrin = (total number of nodes in group) / (2 ^ (number of levels in tree) - 1)
120
       return (extra nodes, missing nodes, level error, total left nodes, total right
121
nodes, nrin, height balance)
122 function tree_analysis(tree statistics)
      tree_analysis analyzes statistics of a tree and gives a rating of how well the tree
123
is
124
       :tree statistics: statistics of a tree as a vector
       :return: float
125
     .....
126
       get left nodes and right nodes from tree statistics (total left nodes and total
right nodes)
127
       if left nodes is less than right nodes
128
            side balance = left nodes / right nodes
129
       else
            side balance = right nodes / left nodes
130
131
       calculate nrin as a float
132
       get node error = extra nodes + missing nodes
```

```
135
      nrin has weight of 0.15
136
       side balance has weight of 0.15
137
      height balance has weight of 0.1
138
      find value of node error given its weight and its relative value
139
      find value of level error given its weight and its relative value
      find value of nrin given its weight
140
      find value of side balance given its weight
141
      find value of height balance given its weight and its relative value
142
142
       sum up values
      return max(0, sum of values)
144
145 create potential root nodes list
146 iterate through every group in network
       if group has more than one node
147
148
            check if group has a cycle by calling has cycle(group, group node connections)
149
                 if group has a cycle then group is not a tree
150
            else
151
                 iterate through edges leaving/entering group
152
                      add nodes in group connected to other groups to potential root nodes
list
153
                 if there are no nodes in potential root nodes list
154
                      add every node in group to potential root nodes list
155 create best root node analysis list
156 iterate through every group
157
        set best tree statistics, best tree analysis, and best node to 0
        iterate through nodes in potential root nodes list that belong to same group
158
159
             create tree by calling create_tree(list(), 1, node)
             get statistics of tree by calling tree statistics(tree,
160
group node connections)
161
             get analysis of tree by calling tree_analysis(tree statistics)
             if tree analysis > best tree analysis
162
                  set best tree statistics to tree statistics
163
                  set best tree analysis to tree analysis
164
165
                  set best node to node
166
       save best node, best tree analysis, and best tree statistics to best root node
analysis list at index group
```

133

134

node error has weight of 0.35

level error has weight of 0.25

```
if best tree analysis is greater than or equal to 0.7
167
168
            then group is a tree with best node being the root node
169
            print statistics and analysis
170
       else
171
            group is not a tree but best node being the root node leads to a tree structure
closest to an ideal tree compared to other nodes in group as the root node
            print statistics and analysis
172
# LINE BRANCH GRAPH
173 function longest_line(node, group_node_connections, visited = c())
       longest line finds the longest line of nodes in a group that ends with node
174
       :node: node id as an int of last node in longest line of nodes for a group
175
       :group node connections: group node connections list
176
       :visited: visited nodes in group as a list
177
       :return: list with length of longest line and nodes in longest line
178
     .....
       if node is in visited list
179
180
            return (length = 0, path = c())
181
       add node to visited list
182
       if node does not have connections in group
            return (length = 1, path = c(node))
183
       set max length and max path to 0
184
185
       iterate through nodes that node is connected to in group
            recursively call longest_line(near_node, group_node_connections, visited)
186
            if the length of line is greater than max length
187
                 set max length to length of line
188
189
                 set max path to path of line
190
       return (length = max length, path = c(node, max path))
191 function reverse_connections(connections)
192
       reverse connections reverses a connections list (reverses 'from nodes' -> 'to nodes'
to 'to nodes' -> 'from nodes')
       :connections: connections list
193
194
       :return: connections list but reversed
     .. .. ..
195
       initialize reversed list
196
       iterate through all from nodes in connections list
```

iterate through all to nodes that are connected to from node add to node and its from node to reversed connections list

199 return reversed connections list

200 create reversed group node connections by calling reverse_connections(group node connections)

```
201 function lb_statistics(line, group_nodes, group_node_connections)
202
      lb statistics finds the statistics of a line branch
       :line: line of nodes as a vector of ints
203
204
       :group nodes: all nodes in the group as a vector of ints
       :group node connections: group node connections list
205
206
       :return: statistics of a line branch as a vector
     .....
       set branching nodes to the connections of last node in line
207
208
       set line error = group_nodes - ((number of nodes in line) + (number of branching
nodes))
       set llrill = (number of nodes in line) / (2*log(number of branching nodes + 1) + 5)
209
210
       initialize branch continuation to 0
            iterate through all nodes in branching nodes
211
            add number of connections each node has to branch continuation
212
213
       initialize branch positions to 0
            iterate through all nodes in line
214
                 if node has more than one connection in group
215
                      add the position of node to branch positions
216
217
       return (line error, llrill, branch continuation, branch positions)
218 function lb analysis(lb statistic)
219
       lb_analysis analyzes statistics of a line branch and gives a rating of well the line
branch is
220
       :lb statistic: statistics of a line branch as a vector
       :return: float
221
222
       set line length to numerator of llrill
```

add line branch analysis and statistic and path to best start node analysis list at

if line branch analysis is greater than or equal to 0.7
then group is a line branch
print statistics and analysis
else
group is not a line branch

group is not a line branch print statistics and analysis

267

index group

III. Detailed Project Description

3.1 Domain Model

1. Cloistered Graph

Definitions:

Cloistered Graph: Group that has few to none edges leaving or entering the group

Ideal Cloistered Graph: Complete graph that is disconnected

MPNE: Maximum possible number of edges in a group. Calculated by taking every node in group and finding all possible connections between every node in same group.

Method to identify cloistered graphs: Compare Number of Edges to Max Possible Number of Edges (MPNE)

- 1. Find how many edges are in the group
- 2. Find the number of edges that leave/enter the group
- 3. Calculate MPNE
- 4. If number of edges leaving/entering the group is less than 2% of MPNE of that group, then group is a cloistered group. If greater than 2% of MPNE, then group is not a cloistered group
- 5. Divide number of edges by MPNE. This percentage represents how close the group is to that of a complete graph.

Example:

Group has 12 nodes with 11 inner group edges (1 node connects to the other 11 nodes in the group). The group also has 2 outer group edges.

11 edges inside group

2 edges leaving/entering the group

MPNE: 12 nodes * (12 nodes - 1 node) / 2 = 66 edges

Because 2 / 66 * 100% = 3.03% (> 2%) of nodes leave/enter the group, the group is not a cloistered group

Accuracy Percentage: 11 edges / MPNE * 100% = 16.67% Group is 16.67% of a complete graph

2. Tree

Definitions:

Ideal Tree: A noncyclic graph where the graph develops from a single node (root node) and each node splits into 2 child nodes

Tree Finder Algorithm (TFA): Properties to check/find to identify whether a group is a tree What the TFA checks for and analyzes:

- Node Error
- Level Error
- Side Balance
- Height Balance
- Nodes Relative to Number of Ideal Nodes

Node Error: Sum of missing nodes and extra nodes that should not exist in a tree (if there is an extra node that should not exist, then assume for the remaining levels that the deeper nodes should have two additional child nodes to ensure that the error is not counted more than once). This statistic helps identify deviations from an ideal tree by analyzing node counts at ever level.

Level Error: Number of levels with an incorrect number of nodes. For each level, calculate the expected number of nodes based on the previous level (e.g. if level 2 has 3 nodes, level 3 should have 6 nodes). This statistic is assessed on a per-level basis and evaluates how closely the tree follows the property of each parent node having two children nodes.

Side Balance: Determines how evenly distributed the nodes are between the left and right sides of the tree. Split the root node into two sides: left and right. Then count the number of nodes on each side. This statistic finds how balanced the tree is in terms of side distribution.

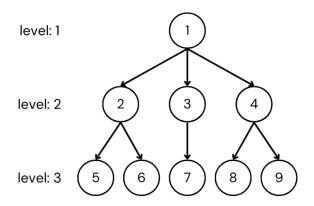
Height Balance: Evaluates the distribution of nodes across the levels of the tree. Iterate through all levels and find how many levels have fewer nodes than the largest number of nodes observed so far on a level (Inol). This statistic determines whether the tree continues to grow in terms of number of nodes per level.

Nodes Relative to Number of Ideal Nodes (NRIN): Total number of nodes in tree relative to number of nodes in an ideal tree with the same depth. This statistic determines how close the tree is to an ideal tree by comparing the actual number of nodes to the theoretical ideal number of nodes given the depth level.

Method to identify trees:

- a. Check for cycles: if the group contains a cycle, it cannot be a tree
- b. Identify potential root nodes: create a list of potential root nodes for each group if no cycle exists in that group
- c. Evaluate each root node: for each node in the group's potential root nodes list
 - i. Consider current node to be root node
 - ii. Create tree structure using root node
 - iii. Apply TFA on tree structure to get statistics
 - iv. Compare statistics to best statistics observed so far. If statistics are better, then save statistics as best statistics and save root node as best root node
- d. Repeat until group potential root nodes exhausted: repeat step c) until list of nodes from same group in potential root nodes list is exhausted
- e. Output results: from the list of best statistics, identify the group and output the best node and statistics that form a tree closest to an ideal tree when the best node is used as the root node

Example:



Node Error: 2

Node 1 should only have 2 children (has 1 extra child). Node 3 should have 2 children (has 1 missing child) 1 extra child + 1 missing child = 2 node errors

Level Error: 2

Level 2 should have 2 nodes (has 3 nodes) Level 3 should have 6 nodes (has 5 nodes) Level 2 and Level 3 have errors, so Level Error = 2 Side Balance = 1

Both Left Side and Right Side have same number of nodes

Height Balance: 3

Number of nodes increases every level, because there are 3 levels then Height Balance = 3

NRIN: 9/7

9 total nodes and ideal tree of height 3 should have 7 nodes

Value of Tree: (1 - (2/7)) * (0.35) + (1 - (2/3)) * 0.25 + 9/7 * 0.15 + 1 * 0.15 + (1 - (3/3)) * 0.1 = 0.67619047619

3. Line Branch

Definitions:

Ideal Line Branch: a sequence of nodes connected in a path sequence where the final node in the path branches into multiple children nodes

Line Branch Finder Algorithm (LBFA): Properties to check/find to identify whether a group is a line branch

What the LBFA checks for and the analyzes:

- Line Error
- Line Length Relative to Ideal Line Length
- Branch Continuation
- Branch Positions

Line Error: Finds how many errors there are in the line (nodes in the line should not branch before the final node in line). Calculate the number of nodes that branch prematurely, as well as the total number of nodes in branches that should not exist. This statistic determines how much the line branch deviates from an ideal line branch.

Line Length Relative to Ideal Line Length (LLRILL): Compares actual number of nodes in a line to the ideal line length. The ideal line length L is calculated by

$$L = a \cdot log(B + 1) + c$$

where a is the scaling factor, B is the number of branching nodes after the final node in the line, and c is the baseline line length. This statistic evaluates how accurately the group aligns with the ideal line length.

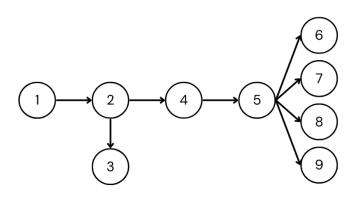
Branch Continuation: Finds whether additional branching occurs after the initial branching from the final node in the line. This statistic determines how closely the group follows the structure of an ideal line branch.

Branch Positions: Finds the positions of nodes in the line that branch early. This statistic will tell the user whether branching occurs, and if so, whether it happens earlier or later in the line branch.

Method to identify line branch:

- a. Check for cycles: if a cycle exists in the group, it cannot be a line branch
- b. Identify potential ending nodes: create a list of potential ending nodes for each group if no cycle exists for that group. Then initialize most branching nodes and best path (longest path in the group that ends the line with the node that led to most branching nodes) and iterate over each node in the group's potential ending nodes list
 - i. Consider current node as the ending node of the line
 - ii. Find number of branching nodes ending node has
 - iii. If number of branching nodes is greater than most branching nodes, get longest path in the group that ends with the ending node and set best path to the current path and set most branching nodes to number of branching nodes from ending node. If number of branching nodes is same as most branching nodes, get longest path in the group that ends with the ending node. Compare the length of longest path to length of best path. If longest path is longer than best path, then set best path to longest path and most branching nodes to number of branching nodes from ending node
- c. Evaluate each best path for each group: apply LBFA on each best path for each group to get statistics and analysis.
- d. Output results: identify each group and output the path and statistics that form a line branch closest to an ideal line branch

Example:



Line Error: 1 Node 2 branches to Node 3

LLRILL: 4/(2*log(4+1) + 5) = 4/(2*log(5)+5)4 Nodes in line Ideal length of line = a*log(B+1)+cwhere a = 2, B = 4, c = 5 Branch Continuation = 0

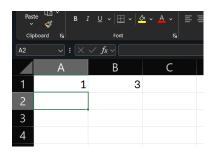
Branch Positions: 2/4 Node 2 branches and is second node in line

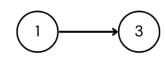
Value of Line Branch: (1 - (1/(2*log(5)+5+4)))*0.4 + (4/(2*log(5)+5))*0.3 + (1 - (0/4))*0.2 + (1 - ((4-2)/4)/4)*0.1 = 0.83659121245

3.2 Deployment Description

This section outlines how to run the program, including organizing the CSV file, understanding the libraries used, and executing the program.

The program requires a CSV file as input to represent the network. The file must follow a specific structure for the data to be properly processed and analyzed. The first column should contain the source nodes, representing the nodes that initiate connections (nodes that point towards other nodes). The second column should contain the target nodes, representing the nodes that receive connections (nodes that are pointed towards). Therefore, each row in the CSV file corresponds to a directed edge in the network, where connections point from the source nodes to target nodes. For example, a row with 1 in the first column and 3 in the second column represents node 1 pointing to node 3. This format for the CSV file must be maintained to ensure no errors occur during the program's execution.





The program utilizes several R libraries to carry out data manipulation, graph analysis, and visualization

Tidyverse: R packages that assist with data wrangling, parsing, reading, and writing.

igraph: Allows for graph visualisation and graph analysis

visNetwork: Allows for interactive graph visualization as well as dynamic network diagrams

MASS: R packages that include statistical tools and functions used to analyze data

Steps to run the program (R Markdown File)

- 1. Prepare the CSV file and specify file path in the program
- 2. Install required libraries
- 3. Run each code chunk to view analysis and results

3.3 Information for Next Development Team

This project was done individually and, while it explores a similar topic to that of another team (the project done on Information Spread Models on X), it branches away significantly in its approach and focus. The emphasis here was on visually representing a network and identifying subgraphs within a network.

For more information about this project or any inquiries, contact Colin Song Email: colin.song@mail.mcgill.ca

IV. Experiment or Test Cases

Describe Experiment and Results

The primary focus of this project was to visually represent the network and identify subgraphs within a network. As a result, extensive testing was not a central component of this project. However, deeper analysis was conducted on cloistered graphs, tree graphs, and line branch graphs. After defining these subgraphs, I evaluated their characteristics to understand the differences in their structures and how these subgraphs influence information flow.

This analysis involved studying the structural properties of each subgraph, using metrics such as connectivity and node distribution. For cloistered graphs, I analyzed the density of internal connections and the role of gatekeeper nodes. Analysis revealed that cloistered graphs often create self-contained cycles of information with limited spread beyond the group. For tree graphs, I examined properties such as height and branching patterns. The analysis showed that the branching factor and depth of the tree significantly influences the spread of information, with bottlenecking at critical nodes slowing the spread of further information. For line branch graphs, I focused on the length of the line and the branching behavior. I found that the linear structure of a line branch allowed for predictable spread of information, however, the line often ended shortly and soon branched into many nodes. The line branch is a structure where the source of the information is less influential than the final user, who interacts with and impacts many of their followers. Although these analyses provide valuable information into the nature of the subgraphs, more testing must be done to further understand the impact of these subgraphs and improve the program's capabilities.

VI. Reflection

6.1 What would you do differently (what have you learned negative)

My primary goals for this project were to develop a program to visually represent a network and implement three algorithms to identify three subgraphs: cloistered graphs, tree graphs, and line branch graphs. While I was successful in achieving these goals, I ended up coding a program of 900 lines. I often found myself getting lost in

the complexity of the code as well as the sheer number of variables used in the program. This made the code harder to follow and required me to rewrite certain sections of the program multiple times to fix issues or improve the clarity.

Looking back, I would focus on better organization for not only the coding process but also for the planning process of implementing the three algorithms. Although I had clear definitions and steps for each algorithm, I did not effectively plan out how to translate those steps into code. This created numerous challenges during the development of the program, as I often found myself unsure of how to proceed with the next step. I would have also dedicated more time to designing the structure of the code and planning how to implement the algorithms programmatically. This approach would have relieved much of the stress I experienced from being overwhelmed by the complexity of the code and ensured a smoother implementation process.

6.2 What would you do again (what have you learned positive)

Looking back, one decision I would make again is choosing to code this project in R. R proved to be an excellent choice due to many tools for statistical analysis, data visualization, and graph modeling the software offers. Libraries such as *igraph* and *tidyverse* allowed for constructing and analyzing the network, while *visNetwork* allowed for effective and interactive visualizations. These tools streamlined many aspects of the project and let me focus more on my primary goals.

Another decision I would repeat again is the decision to create my own definitions for the subgraphs—cloistered graphs, tree graphs, and line branch graphs—and to develop properties for analyzing these structures. The definitions and properties I came up with proved to be the foundation of this project and provided a unique perspective for evaluating groups of nodes within a network. By breaking down the steps to analyze each subgraph, I was able to have a better understanding of not only the influences subgraphs have on the network but how information flows. The process of defining these subgraphs provided clarity and a consistent method of analyzing communities.

Finally, the algorithms I designed to identify and classify subgraphs within a network is a decision I would make again. By breaking down the steps systematically, I developed a procedure to identify any community as a specific subgraph, given it matches the properties of the ideal subgraph it is being identified as. Although the algorithms can be improved, they currently succeed in achieving the goals I set out and demonstrated to be very effective when analyzing large complex network structures. These decisions not only contributed to the success of the project but also provides a strong foundation for future development.

Future Work

Things to Fix

One thing to fix involves tuning the parameters used in the program. One example of this is when calculating the ideal line length for a line branch graph. The values of the scaling factor a and the baseline length c in the formula $L = a \cdot log(B+1) + c$ require further tuning to align with real-world data. Testing numerous datasets would help determine optimal values for these parameters, ensuring the formula better represents realistic line structures. This adjustment would improve the accuracy of the line branch graph analysis.

One major challenge I faced during my research was graphing an entire network. Early on, I attempted to graph a dataset with one million connections, which took the program an hour to complete. However, the resulting graph was unreadable as such a dense network appeared as a blob of colors. Networks with many nodes quickly became visually overwhelming, making it hard to distinguish meaningful patterns or relationships. The only fix I had around this issue was to focus on smaller areas of the network to make graphing and analyzing more manageable.

Lastly, the need for new data remains an unresolved issue. The current project heavily relies on existing datasets, which may limit the scope of analysis and the ability to simulate diverse scenarios. Developing a method to generate or acquire new data—such as creating datasets, simulating user interactions, or collaborating with data providers—would expand the project's capabilities and allow for more robust testing of the algorithms. Access to new and varied data would also enable the program to adapt to evolving network dynamics and remain relevant for future applications.

Features to Add/Future Work

1. Testing and Tuning Parameters

Future development should involve more extensive testing to fine-tune the formula for the ideal line length. The current formula, $L = a \cdot log(B+1) + c$, uses parameters a (scaling factor) and c (baseline length). I currently have a set to be 2 and c set to be 5, however these values should be adjusted to better reflect real-world network properties.

Weights used when analyzing tree and line branch graphs should also be adjusted. These weights determine whether a group is fit to be a tree or a line branch graph. Due to the significant role these weights play in identifying subgraphs within a network, they must be tested rigorously against large datasets to ensure accurate identification

Experimenting with these parameters can help refine the identification process to ensure it aligns more closely with observed patterns in social media networks. Testing various datasets and edge cases will provide valuable insights into how well the program adapts to different types of networks.

2. Using the Program to Evaluate Information Spread

The program has the potential to simulate and evaluate how information flows in networks. Future teams can build on my program to analyze the flow of information through specific subgraphs, focusing on factors such as:

- Speed of Propagation: How quickly information reaches distant nodes.
- Influence of Key Nodes: Identifying nodes that play pivotal roles in spreading information.
- Impact of Communities: How tightly-knit groups influence or impede the flow of information.

These factors can be extended to simulate real-world scenarios, such as tracking viral tweets or examining how misinformation spreads. By using the program to analyze real data, the algorithms could be tested in more practical scenarios and reveal how information flows when subgraphs are identified. This could include studying how tightly connected cloistered graphs gatekeep information, how hierarchical tree structures spread information, and how line branch graphs influence the reach and longevity of information. Analyzing real data would uncover patterns in information spread that are unique to real-world networks.

3. Adding More Properties to Subgraphs

To further solidify the definitions of each subgraph, additional properties and metrics can be incorporated into the analysis. Examples include:

- Clustering Coefficient: Measure how tightly nodes are connected within a subgraph.
- Edge Weight Analysis: Include weights on edges to represent the strength of relationships or interaction frequencies.
- Centrality Metrics: Analyze measures such as betweenness or eigenvector centrality to better understand the roles of nodes within subgraphs.

Improving the subgraph properties will provide a more comprehensive understanding of their structure and behavior, enabling more accurate identification and deeper analysis. While the current metrics provide a strong foundation, adding more properties would help solidify the definitions of cloistered graphs, tree graphs, and line branch graphs. Incorporating metrics could provide deeper insights into the behavior of each subgraph. This would not only improve the identification process but also offer a better understanding of how these subgraphs function within larger networks.

Suggestions

- 1. Plan ahead: Before starting, organize the steps required to complete the project and outline each task
- 2. Research: Take the time to read articles to become familiar with the topics involved in the project
- 3. Focus on your goals: Always relate your work back to the project's main objective

VII. Supplemental Materials

During the course of this project, several materials and resources were studied that were ultimately not used for the final implementation of my research. However, this information could be valuable for future work. These include unused statistical metrics and hydrating tweets using Twarc.

Unused Statistical Metrics

In the process of analyzing subgraphs, I gathered information about several additional statistical metrics that were not incorporated into the project. These metrics could provide deeper insights into network behavior and help refine subgraph analysis.

1. Modularity

 Compares edges inside communities to edges between communities. Quantifies how well a graph can be divided into separate communities

2. Number of Communities

 Total number of distinct communities in network. Large number of communities may indicate highly fragmented network. Small may indicate more centralized structure

3. Distribution of Community Size

Distribution of sizes of detected communities. Shows presence of large communities vs. small

4. Average # of Edges in Community

Average number of edges in communities. Shows how well-connected per node in community

5. Number of In-Degrees and Out-Degrees

 Number of edges coming to a vertex and number of edges coming out of vertex. Can find users who are at central areas within community

6. Conductance

 Ratio of in-degrees to total number of edges. Lower conductance indicates more tightly-knit, welldefined community, higher indicates community has more connections outside

7. Conductance of Whole Network

- Sum of conductance values of all communities. Measures how well network is divided into communities. Lower value suggests well-defined community structures

8. Edge Density

 Ratio of number of edges to max number of possible edges. An idea of how dense a graph is in terms of edge connectivity

9. Clustering Coefficient

 The degree to which nodes in a community tend to cluster together. High coefficient suggests that nodes form tight-knit groups

10. Eigenvector Centrality

- Finds centrality for a node, proportional to other nodes. Nodes that are pointed to by "important" nodes will also be considered "important"

11. Betweenness Centrality

 Measure of centrality based on shortest path. Node with value will have more information passed through

12. Intra-Community/Inter-Community Interactions

 Ratio of edges within communities to edges between communities. High ratios suggest strong community boundaries, low ratios suggest overlap/interaction between communities

13. Overlapping Communities

Identifies nodes that belong to multiple communities. Reveals users that participate in various groups

Hydrate Tweets

Another supplemental material involves the use of *Twarc*, a Python library for working with Twitter data. *Twarc* allows for hydrating tweet IDs, which retrieves the full metadata of tweets from their IDs (this requires an X Developer Account and access to the X API). While this approach was not utilized in this project due to limited API call rates, I invested considerable time exploring it and believed it had the potential to provide valuable datasets.

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