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MAT300 HW#2

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1. Derive $\frac{d}{dt}B_i^d(t) = d\left(B_{i-1}^{d-1}(t) - B_i^{d-1}(t)\right)$

$$\frac{d}{dt}\left[\binom{d}{i}(1-t)^{d-i}t^i\right] = d\left(B_{i-1}^{d-1}(t) - B_i^{d-1}(t)\right)$$

$$i*\binom{d}{i}(1-t)^{d-i}t^{i-1} + (-1)(d-i)*\binom{d}{i}(1-t)^{d-i-1}t^i = d\left(B_{i-1}^{d-1}(t) - B_i^{d-1}(t)\right)$$

$$d\left(\frac{(d-1)!}{(i-1)!d-i!}(1-t)^{d-i}t^{i-1} - \frac{(d-1)!}{i!(d-i-1)!}(1-t)^{d-i-1}t^i\right) = d\left(\binom{d-1}{i-1}(1-t)^{d-i}t^{i-1} - \binom{d-1}{i}(1-t)^{d-1-i}t^i\right)$$

$$d\left(\frac{(d-1)!}{(i-1)!d-i!}(1-t)^{d-i}t^{i-1} - \frac{(d-1)!}{i!(d-i-1)!}(1-t)^{d-i-1}t^i\right) = d\left(\frac{(d-1)!}{(i-1)!d-i!}(1-t)^{d-i}t^{i-1} - \frac{(d-1)!}{i!(d-i-1)!}(1-t)^{d-i-1}t^i\right) \checkmark$$

2. Derive $\frac{d}{dt}C_i^d(t) = d * B_{i-1}^{d-1}(t)$

$$\frac{d}{dt}\left(\sum_{j=i}^{d} B_j^d(t)\right) = d * B_{i-1}^{d-1}(t)$$

$$\sum_{j=i}^{d}\frac{d}{dt}B_j^d(t) = d * B_{i-1}^{d-1}(t)$$

$$d\left(B_{i-1}^{d-1}(t) - B_i^{d-1}(t) + B_i^{d-1}(t) - B_{i+1}^{d-1}(t) + \cdots - B_d^{d-1}(t)\right) = d * B_{i-1}^{d-1}(t)$$

A lot of stuff cancels out in the sum above leaving us…

$d * B_{i-1}^{d-1}(t) - B_d^{d-1}(t) = d * B_{i-1}^{d-1}(t)$

Out of bounds gives us a zero if $I < 0$ or $I > d$

$d * B_{i-1}^{d-1}(t) - 0 = d * B_{i-1}^{d-1}(t) \checkmark$

3.

a)	$a = \begin{bmatrix} 1 & 2 & 2^2 \\ 1 & 3 & 3^2 \\ 1 & 5 & 5^2 \end{bmatrix}$ Det(a)=(5-3)*(5-2)*(3-2) = 2 * 3 * 1 = 6
b)	$a = \begin{bmatrix} 1 & 4 & 4^2 & 4^3 & 4^4 \\ 1 & 5 & 5^2 & 5^3 & 5^4 \\ 1 & 6 & 6^2 & 6^3 & 6^4 \\ 1 & 7 & 7^2 & 7^3 & 7^4 \\ 1 & 9 & 9^2 & 9^3 & 9^4 \end{bmatrix}$ Det(a) = (9-7)*(9-6)*(9-5)*(9-4)*(7-6)*(7-5)*(7-4)*(6-5)*(6-4)*(5-4) = 2 * 3 * 4 * 5 * 2 * 3*2=1440
c)	$a = \begin{bmatrix} 1 & 4 & 4^2 & 4^3 & 4^4 \\ 0 & 1 & 2*4 & 3*4^2 & 4*4^3 \\ 0 & 0 & 2 & 6*4 & 12*4^2 \\ 0 & 0 & 0 & 6 & 24*4 \\ 1 & 3 & 3^2 & 3^3 & 3^4 \end{bmatrix}$ $\text{Det}(a) = 3!! * (b-a)^{4*1} = 12(4-3) = 12$

4. Find the interpolating polynomial for the data below in three different forms: i) standard basis, ii) Lagrange basis, and iii) Newton Form. (You may a use a symbolic algebra tool.)
$t0 = -1, t1 = 2, t2 = 3.\ g(-1) = 1, g(2) = -1, g(3) = 2.$

i. $\begin{bmatrix} 1 & -1 & 1 & 1 \\ 1 & 2 & 4 & -1 \\ 1 & 3 & 9 & 2 \end{bmatrix} \Rightarrow (RREF) \Rightarrow \begin{bmatrix} 1 & 0 & 0 & \frac{-3}{2} \\ 0 & 1 & 0 & \frac{-19}{12} \\ 1 & O & 1 & \frac{11}{12} \end{bmatrix}$

ii. L(t) = 1*(t-2/-1-2)*(t-3) + -1*(t+1/2+1)*(t-3/2-3) + 2*(t+1/3+1)*(t-2/3-2)

$= 1\cdot\left(\frac{t-2}{-1-2}\right)\cdot\left(\frac{t-3}{-1-3}\right) + (-1)\cdot\left(\frac{t+1}{2+1}\right)\cdot\left(\frac{t-3}{2-3}\right) + 2\cdot\left(\frac{t+1}{3+1}\right)\cdot\left(\frac{t-2}{3-2}\right)$

$= \left(\frac{4t-8*3t-9}{-12}\right) + (-1)\cdot\left(\frac{t+1\cdot 9-3t}{3}\right) + 2\cdot\left(\frac{t+1\cdot 4t-8}{4}\right)$

$= -t^2 + 5t - 6 + t^2 - 2t - 3 + 2t^2 - 2t - 4$

$= 2t^2 + t - 13$

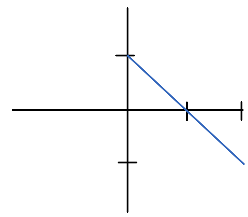
iii. Newton Form

$t_0=-1$	$a_0=g(t_0)=1$	$a_1=\frac{g(t_1)-g(t_0)}{t_1-t_0}=\frac{-1-1}{2+1}=-\frac{2}{3}$	$a_2=\frac{\frac{g(t_2)-g(t_1)}{t_2-t_1}-\frac{g(t_1)-g(t_0)}{t_1-t_0}}{t_2-t_0}=\frac{3+\frac{2}{3}}{3+1}=\frac{11}{12}$
$t_1=2$	$g(t_1)=-1$	$\frac{g(t_2)-g(t_1)}{t_2-t_1}=\frac{2+1}{3-2}=3$	
$t_2=3$	$g(t_2)=2$		$p(t)=a_2(t-t_0)(t-t_1)+a_1(t-t_0)+a_0$ $=\frac{11}{12}(t+1)(t-2)-\frac{2}{3}(t+1)+1=\frac{11}{12}(t^2-t-2)-\frac{8}{12}(t+1)+1$ $=\frac{11}{12}t^2-\frac{19}{12}t-\frac{3}{2}$

5. Let $g(t)=\cos(\frac{\pi}{2}t)$. In each part use the Newton form to find the quadratic interpolating polynomial p(t) which agrees with the function g(t) for each of the sequences [t0, t1, t2]: Sketch three graphs, one for each polynomial together with the function g(t).

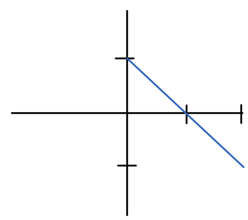
i. [0,1,2]

$t_0=0$	$a_0=g(t_0)=1$	$a_1=\frac{g(t_1)-g(t_0)}{t_1-t_0}=\frac{-1}{1}=-1$	$a_2=\frac{\frac{g(t_2)-g(t_1)}{t_2-t_1}-\frac{g(t_1)-g(t_0)}{t_1-t_0}}{t_2-t_0}=\frac{-1+1}{2}=0$
$t_1=1$	$g(t_1)=0$	$\frac{g(t_2)-g(t_1)}{t_2-t_1}=\frac{-1}{1}=-1$	
$t_2=2$	$g(t_2)=-1$		$p(t)=a_2(t-t_0)(t-t_1)+a_1(t-t_0)+a_0$ $=0*(t-t_0)(t-t_1)+(-1)*(t-0)+1$ $=1-t$



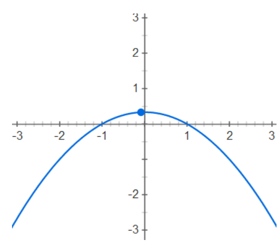
ii. $[-1,0,2]$

$t_0=-1$	$a_0=g(t_0)=0$	$a_1=\frac{g(t_1)-g(t_0)}{t_1-t_0}=\frac{1}{-1}=-1$	$a_2=\frac{\frac{g(t_2)-g(t_1)}{t_2-t_1}-\frac{g(t_1)-g(t_0)}{t_1-t_0}}{t_2-t_0}=\frac{-1+1}{2+1}=0$
$t_1=0$	$g(t_1)=1$	$\frac{g(t_2)-g(t_1)}{t_2-t_1}=\frac{-2}{2}=-1$	
$t_2=2$	$g(t_2)=-1$		$p(t)=a_2(t-t_0)(t-t_1)+a_1(t-t_0)+a_0$ $=0*(t-t_0)(t-t_1)+(-1)*(t+1)+0$ $=1-t$



iii. $[-1,1,2]$

$t_0=-1$	$a_0=g(t_0)=0$	$a_1=\frac{g(t_1)-g(t_0)}{t_1-t_0}=0$	$a_2=\frac{\frac{g(t_2)-g(t_1)}{t_2-t_1}-\frac{g(t_1)-g(t_0)}{t_1-t_0}}{t_2-t_0}=\frac{-1}{2+1}=-\frac{1}{3}$
$t_1=1$	$g(t_1)=0$	$\frac{g(t_2)-g(t_1)}{t_2-t_1}=\frac{-1}{1}=-1$	
$t_2=2$	$g(t_2)=-1$		$p(t)=a_2(t-t_0)(t-t_1)+a_1(t-t_0)+a_0$ $=-\frac{1}{3}*(t+1)(t-1)+0*(t+1)+0$ $=\frac{1}{3}-\frac{1}{3}t^2$



6. Let $g(t)=\frac{1}{t}$ and $h(t)=\frac{1}{t^2}$ and $f(t)=g(t)h(t)=\frac{1}{t^3}$. Find the divided differences $[-1,1,2]f$, $[-1,1,2]g$ and $[-1,1,2]h$ by computing the divided difference table in each case. Then verify Leibniz's Formula for computing $[-1,1,2]f$ with the summation of products from g and h.

i. $[-1,1,2]f$, $f(t)=g(t)h(t)=\frac{1}{t^3}$

$t_0=-1$	$a_0=f(t_0)=\frac{1}{(-1)^3}=-1$	$a_1=\frac{f(t_1)-f(t_0)}{t_1-t_0}=\frac{1+1}{1+1}=1$	$a_2=\frac{\frac{f(t_2)-f(t_1)}{t_2-t_1}-\frac{f(t_1)-f(t_0)}{t_1-t_0}}{t_2-t_0}=\frac{-\frac{7}{8}-1}{2+1}=-\frac{5}{8}$
$t_1=1$	$f(t_1)=\frac{1}{(1)^3}=1$	$\frac{f(t_2)-f(t_1)}{t_2-t_1}=\frac{\frac{1}{8}-1}{2-1}=-\frac{7}{8}$	
$t_2=2$	$f(t_2)=\frac{1}{(2)^3}==\frac{1}{8}$		$p(t)=a_2(t-t_0)(t-t_1)+a_1(t-t_0)+a_0$ $=-\frac{5}{8}*(t+1)(t-1)+1*(t+1)-1=-\frac{15}{24}*(t^2-1)+t$ $=\frac{5}{8}+t-\frac{5}{8}t^2$

ii. $[-1,1,2]g$, $g(t)=\frac{1}{t}$

$t_0=-1$	$a_0=g(t_0)=-1$	$a_1=\frac{g(t_1)-g(t_0)}{t_1-t_0}=\frac{1+1}{1+1}=1$	$a_2=\frac{\frac{g(t_2)-g(t_1)}{t_2-t_1}-\frac{g(t_1)-g(t_0)}{t_1-t_0}}{t_2-t_0}=\frac{-\frac{1}{2}-1}{2+1}=-\frac{1}{2}$

t_1 =1	$g(t_1) = 1$	$\frac{g(t_2)-g(t_1)}{t_2-t_1} = \frac{\frac{1}{2}-1}{2-1} = -\frac{1}{2}$	
t_2 =2	$g(t_2) = \frac{1}{2}$		$p(t) = a_2(t-t_0)(t-t_1) + a_1(t-t_0) + a_0$ $= -\frac{1}{2} * (t+1)(t-1) + 1 * (t+1) - 1 = (\frac{1}{2} - \frac{1}{2}t^2) + t$ $= 1 + t - \frac{1}{2}t^2$

iii. $[-1,1,2]h$, $h(t) = \frac{1}{t^2}$

t_0 =-1	$a_0 = h(t_0) = \frac{1}{(-1)^2} = 1$	$a_1 = \frac{h(t_1)-h(t_0)}{t_1-t_0} = \frac{0}{1+1} = 0$	$a_2 = \frac{\frac{h(t_2)-h(t_1)}{t_2-t_1} - \frac{h(t_1)-h(t_0)}{t_1-t_0}}{t_2-t_0} = \frac{-\frac{3}{4}-0}{2+1} = -\frac{1}{4}$
t_1 =1	$h(t_1) = \frac{1}{(1)^2} = 1$	$\frac{h(t_2)-h(t_1)}{t_2-t_1} = \frac{\frac{1}{4}-1}{2-1} = -\frac{3}{4}$	
t_2 =2	$h(t_2) = \frac{1}{(2)^2} = \frac{1}{4}$		$p(t) = a_2(t-t_0)(t-t_1) + a_1(t-t_0) + a_0$ $= -\frac{1}{4} * (t+1)(t-1) + 0 * (t+1) + 1$ $= \frac{5}{4} - \frac{1}{4}t^2$

iv. $[t_i, t_{i+1}, \dots, t_{i+k}]f = \sum_{r=i}^{i+k} ([t_i, \dots, t_r]g)([t_r, \dots, t_{i+k}]h)$

$[t_0, t_1, t_2]f = [t_0]g[t_0, t_1, t_2]h + [t_0, t_1]g[t_1, t_2]h + [t_0, t_1, t_2]g[t_2]h$

$[-1,1,2]f = (-1)(-\frac{1}{4}) + (1)(-\frac{3}{4}) + (-\frac{1}{2})(\frac{1}{4}) = \frac{1}{4} - \frac{3}{4} - \frac{1}{8} = \frac{2-6-1}{8} = -\frac{5}{8}$ ✓

7. Let $g(t) = t-2$ and $h(t) = (t-2)_+^3$ and $f(t) = g(t)h(t) = (t-2)_+^4$. Find the divided differences [1,3,4]f, [1,3,4]g and [1,3,4]h by computing the divided difference table in each case. Then verify Leibniz's Formula for computing [1,3,4]f with the summation of products from g and h.

i. $[1,3,4]f$, $f(t) = g(t)h(t) = (t-2)_+^4$

t_0 =1	$a_0 = f(t_0) = 0$	$a_1 = \frac{f(t_1)-f(t_0)}{t_1-t_0} = \frac{1-0}{3-1} = \frac{1}{2}$	$a_2 = \frac{\frac{f(t_2)-f(t_1)}{t_2-t_1} - \frac{f(t_1)-f(t_0)}{t_1-t_0}}{t_2-t_0} = \frac{15-\frac{1}{2}}{4-1} = \frac{29}{6}$
t_1 =3	$f(t_1) = (3-1)^4 = 1$	$\frac{f(t_2)-f(t_1)}{t_2-t_1} = \frac{16-1}{4-3} = 15$	
t_2 =4	$f(t_2) = (4-2)^4 = 16$		$p(t) = a_2(t-t_0)(t-t_1) + a_1(t-t_0) + a_0$ $= \frac{29}{6} * (t+1)(t-1) + \frac{1}{2} * (t+1) + 0$ $= \frac{29}{6}t^2 + \frac{1}{2}t - \frac{13}{3}$

ii. $[1,3,4]g$, $g(t) = t-2$

t_0 =1	$a_0 = g(t_0) = -1$	$a_1 = \frac{g(t_1)-g(t_0)}{t_1-t_0} = \frac{1+1}{3-1} = 1$	$a_2 = \frac{\frac{g(t_2)-g(t_1)}{t_2-t_1} - \frac{g(t_1)-g(t_0)}{t_1-t_0}}{t_2-t_0} = \frac{1-1}{4-1} = 0$
t_1 =3	$g(t_1) = 1$	$\frac{g(t_2)-g(t_1)}{t_2-t_1} = \frac{2-1}{4-3} = 1$	
t_2 =4	$g(t_2) = 2$		$p(t) = a_2(t-t_0)(t-t_1) + a_1(t-t_0) + a_0$ $= 0 * (t+1)(t-1) + 1 * (t+1) - 1$ $= t$

iii. $[1,3,4]h$, $h(t) = (t-2)_+^3$

t_0 =1	$a_0 = h(t_0) = 0$	$a_1 = \frac{h(t_1)-h(t_0)}{t_1-t_0} = \frac{1-0}{3-1} = \frac{1}{2}$	$a_2 = \frac{\frac{h(t_2)-h(t_1)}{t_2-t_1} - \frac{h(t_1)-h(t_0)}{t_1-t_0}}{t_2-t_0} = \frac{7-\frac{1}{2}}{4-1} = \frac{13}{6}$
t_1 = 3	$h(t_1) = (3-2)^3 = 1$	$\frac{h(t_2)-h(t_1)}{t_2-t_1} = \frac{8-1}{4-3} = 7$	
t_2 =4	$h(t_2) = (4-2)^3 = 8$		$p(t) = a_2(t-t_0)(t-t_1) + a_1(t-t_0) + a_0$ $= \frac{13}{6} * (t+1)(t-1) + \frac{1}{2} * (t+1) + 0$ $= \frac{13}{6}t^2 + \frac{1}{2}t - \frac{5}{3}$

iv. $[t_i, t_{i+1}, \dots, t_{i+k}]f = \sum_{r=i}^{i+k} ([t_i, \dots, t_r]g)([t_r, \dots, t_{i+k}]h)$

$[t_0, t_1, t_2]f = [t_0]g[t_0, t_1, t_2]h + [t_0, t_1]g[t_1, t_2]h + [t_0, t_1, t_2]g[t_2]h$

$[1,3,4]f = (-1)(\frac{13}{6}) + (1)(7) + (0)(8) = 7 - \frac{13}{6} = \frac{42-13}{6} = \frac{29}{6}$ ✓