Textbook Sections: 6.5, 8.1–8.5

Instructions: These problems are intended for **practice only.** They are not to be submitted for a grade. The purpose of this review sheet is to get you started in preparing for the exam and is not a comprehensive list of potential exam problems or topics . You should practice MANY MORE problems than appear on this review sheet (I recommend using the "Study Plan" feature in MyLabMath to generate more practice problems) as well as review lecture materials and the text for important formulas and definitions (a good place to start would be to review the lecture slides, quizzes, and worksheets).

Practice Problems:

I have included a few helpful identities for you below:

$$\sin^2\theta + \cos^2\theta = 1 \quad \tan^2\theta + 1 = \sec^2\theta \quad \cot^2\theta + 1 = \csc^2\theta \quad \sin^2\theta = \frac{1-\cos 2\theta}{2} \quad \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

1. A rectangular tank with base 5 ft by 8 ft and a height of 6 ft is filled halfway (to a height of 3 ft) with ethyl alcohol weighing 49.3 lb/ft^3. Set up BUT DO NOT EVALUATE an integral that would find how much **work** is required to pump all the liquid over the top of the tank. Provide correct units for final answer. (Note: you are given specific weight so you do not need to account for gravity.)
2. A bucket initially holding water weighing 45 lb is raised at a constant rate to a height of 20 ft. As the bucket is raised, water leaks out of the bottom at a constant rate. If the bucket has 15 lb of water remaining when it reaches the top, how much work did it take to raise the water? Set up BUT DO NOT EVALUATE an integral that would find how much **work** is required to pump all the liquid over the top of the tank. Provide correct units for final answer. (Neglect the weight of the bucket and rope.)
3. For each integral, *explain* your FIRST few steps in evaluating the integral and WHY (2 or 3 sentences is fine). **You do NOT need to evaluate the integrals** E.g., if you feel integration by parts is a good approach, state why and set up the IBP equation. Or if you feel partial fraction decomposition is needed, state why, and set up the PFD equation.
   (a) $\displaystyle\int \frac{3x^3-2}{x^2+4}\,dx$
   (b) $\displaystyle\int x^3\cos x\,dx$
   (c) $\displaystyle\int \cos^4 x\sin^7 x\,dx$
   (d) $\displaystyle\int \sec x(2-\sec x+3\tan x)\,dx$
   (e) $\displaystyle\int \frac{2\,dx}{x^3\sqrt{x^2-1}}$
   (f) $\displaystyle\int \frac{2x+1}{x^2-7x+12}\,dx$
   (g) $\displaystyle\int \frac{8x^2+8x+2}{(4x^2+1)^2}\,dx$
4. Now evaluate and simplify all integrals from Question 2.
5. Evaluate the following using **integration by parts.** Note the integrand is the *inverse sine* (i.e., arcsin) function, it is NOT $1/\sin x$. Show all work and use correct notation throughout! I am expecting clearly scaffolded, clean, and organized solutions for full credit.

$$\int \sin^{-1} x\,dx.$$

6. Evaluate the following using **partial fraction decomposition.** For full credit, all work must be shown and correct notation must be used throughout! I am expecting clearly scaffolded, clean, and organized solutions for full credit.

$$\int \frac{4x^2 + 4x + 2}{x + x^3}\, dx$$

7. Evaluate the following using **trigonometric substitution. Include a labeled triangle.** For full credit, all work must be shown and correct notation must be used throughout! I am expecting clearly scaffolded, clean, and organized solutions for full credit.

$$\int \frac{\sqrt{x^2 - 36}}{x}\, dx$$

8. Evaluate the following trigonometric integrals. Show your work carefully and use correct notation. Simplify your final answers as best as you can.
   (a) $\int \cos^3(6x)\, dx$
   (b) $\int \sin^2 x \cos^2 x\, dx$

9. The integral below can be evaluated using u-substitution, as I am sure you know.

$$\int \frac{x}{\sqrt{x^2 - 25}}\, dx$$

   (a) Let's pretend you did not notice that u-substitution would work. Instead evaluate this integral using using **trigonometric substitution.** Show all work, including a **labeled triangle** indicating how you selected your function for substitution.
   (b) Now let's pretend you finally noticed you could evaluate the integral directly using basic u-substitution. Do so, showing all steps and using correct notation throughout.
   (c) Compare your answers: Did you get the same answer both ways?