

Problem formation

The process to get the optimal J as a function of T , is shown as follow:
Remark: the final speed is assumed to be zero.

Let $\Delta x = P_f - P_0$
 $\Delta y = P_f - P_0$
 $\Delta z = P_f - P_0$

$\alpha_1 = -\frac{12}{T^3} \Delta P_x + \frac{6}{T^2} \Delta V_x$
 $\beta_1 = \frac{6}{T^2} \Delta P_x - \frac{2}{T} \Delta V_x$

$\alpha_1 = -\frac{12}{T^3} (P_f - V_0 T - P_0) + \frac{6}{T^2} (-V_0)$
 $= -\frac{12}{T^3} (-P_f + V_0 T + P_0) - \frac{6}{T^2} V_0$
 $\beta_1 = \frac{6}{T^2} (P_f - V_0 T - P_0) - \frac{2}{T} (-V_0)$
 $= \frac{6}{T^2} (P_f - V_0 T - P_0) + \frac{2}{T} V_0$

$\alpha_1^2 = \frac{36}{T^6} (4\Delta x^2 - 4\Delta x V_0 T + V_0^2 T^2)$
 $\beta_1^2 = \frac{4}{T^4} (9\Delta x^2 - 12\Delta x V_0 T + 4V_0^2 T^2)$
 $\beta_1^2 = g(T) = \frac{1}{3} \alpha_1^2 T^2 + \beta_1^2 T$
 $= \frac{12}{T^3} (4\Delta x^2 - 4\Delta x V_0 T + V_0^2 T^2)$
 $+ \frac{12}{T^3} (-6\Delta x^2 + 7\Delta x V_0 T - 2V_0^2 T^2)$
 $+ \frac{4}{T^3} (9\Delta x^2 - 12\Delta x V_0 T + 4V_0^2 T^2)$
 $= \frac{12}{T^3} (4\Delta x^2 - 4\Delta x V_0 T + V_0^2 T^2 - 6\Delta x^2 + 7\Delta x V_0 T - 2V_0^2 T^2 + 3\Delta x^2 - 4\Delta x V_0 T + \frac{4}{3} V_0^2 T^2)$
 $= \frac{12}{T^3} (\Delta x^2 - \Delta x V_0 T + \frac{1}{3} V_0^2 T^2)$
 $= \frac{12}{T^3} \Delta x^2 - \frac{12\Delta x V_0}{T^2} + \frac{4V_0^2}{T}$

Final equation (Note that T is bigger than zero):

$g(T) = -3 \frac{12}{T^4} \Delta x^2 - (-2) \frac{12\Delta x V_0}{T^3} - \frac{4V_0^2}{T^2}$
 $= -\frac{36}{T^4} \Delta x^2 + \frac{24\Delta x V_0}{T^3} - \frac{4V_0^2}{T^2}$

$\dot{J} = 1 - \frac{36}{T^4} (\Delta x^2 + \Delta y^2 + \Delta z^2)$
 $+ \frac{24}{T^3} (\Delta x V_{x0} + \Delta y V_{y0} + \Delta z V_{z0})$
 $- \frac{4}{T^2} (V_{x0}^2 + V_{y0}^2 + V_{z0}^2) = 0$

$\Leftrightarrow T^4 - 4(V_{x0}^2 + V_{y0}^2 + V_{z0}^2) T^2$
 $+ 24(\Delta x V_{x0} + \Delta y V_{y0} + \Delta z V_{z0}) T$
 $- 36(\Delta x^2 + \Delta y^2 + \Delta z^2) = 0$

Validation: Ref: https://blog.csdn.net/fb_941219/article/details/102984587

It proofs that the equation is correct.

$\Delta X = P_{xj}$
 $V_{x0} = \frac{1}{5}$

$$\frac{12}{T^3} P_{xj}^2 - \frac{12}{T^2} P_{xj} \cdot \frac{1}{5} + \frac{4}{T} \cdot \frac{1}{25}$$

$$= \frac{12}{T^3} P_{xj}^2 - \frac{12}{5T^2} P_{xj} + \frac{4}{25T}$$

$$J = T + \frac{1}{5} \left(\frac{12}{T^3} P_{xj} + \frac{12}{5T^2} \right) + \frac{1}{5} \left(\frac{12}{T^3} P_{xj} + \frac{12}{5T^2} \right) + \frac{1}{5} \left(\frac{12}{T^3} P_{xj} + \frac{12}{5T^2} \right)$$

$$A = \frac{1}{5} \left(\frac{12}{T^3} P_{xj} + \frac{12}{5T^2} \right) + \frac{1}{5} \left(\frac{12}{T^3} P_{xj} + \frac{12}{5T^2} \right) + \frac{1}{5} \left(\frac{12}{T^3} P_{xj} + \frac{12}{5T^2} \right)$$

$$B = \frac{1}{5} \left(\frac{12}{T^3} P_{xj} + \frac{12}{5T^2} \right) + \frac{1}{5} \left(\frac{12}{T^3} P_{xj} + \frac{12}{5T^2} \right) + \frac{1}{5} \left(\frac{12}{T^3} P_{xj} + \frac{12}{5T^2} \right)$$

$$C = \frac{1}{5} \left(\frac{12}{T^3} P_{xj} + \frac{12}{5T^2} \right) + \frac{1}{5} \left(\frac{12}{T^3} P_{xj} + \frac{12}{5T^2} \right) + \frac{1}{5} \left(\frac{12}{T^3} P_{xj} + \frac{12}{5T^2} \right)$$

$$J = T + \frac{1}{5} \left(\frac{12}{T^3} P_{xj} + \frac{12}{5T^2} \right) + \frac{1}{5} \left(\frac{12}{T^3} P_{xj} + \frac{12}{5T^2} \right) + \frac{1}{5} \left(\frac{12}{T^3} P_{xj} + \frac{12}{5T^2} \right)$$

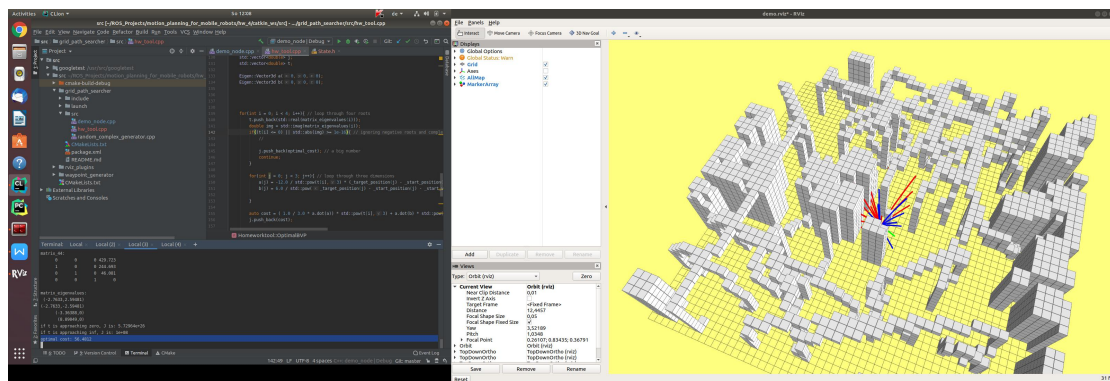
$$J' = \frac{1}{5} \left(\frac{12}{T^3} P_{xj} + \frac{12}{5T^2} \right) + \frac{1}{5} \left(\frac{12}{T^3} P_{xj} + \frac{12}{5T^2} \right) + \frac{1}{5} \left(\frac{12}{T^3} P_{xj} + \frac{12}{5T^2} \right)$$

Analytic Solution to the quadratic equation

A comparison matrix is used, based on:

https://blog.csdn.net/fb_941219/article/details/102984587

Programming and Simulation



Suggestions

It would be better to have more descriptions for the assignment. The instruction is rather unclear. It took me a lot of time to understand why the code combines forward integration with BVP.

Future work

It will be interesting to get the optima of J using convex optimization.