Consider $\Lambda = (\lambda_0, \ldots, \lambda_{r-1})$ where $\lambda_i \vdash n_i$ and $\sum_{i=0}^{r-1} n_i = n$. Recall the definition of the higher Specht polynomials for $G_{r,n}$, consider $T, S \in \mathrm{ST}(\Lambda)$ where $T = (T_0, \ldots, T_{r-1})$, then $\varepsilon_T = \varepsilon_{T_0} \cdots \varepsilon_{T_{r-1}}$. Recall that

$$\hat{F}_T^S = (\varepsilon_T . x_T^S)(\prod_{j=0}^{r-1}(\prod_{m \in T_j} x_m)^j))$$

Firstly note that

$$(\prod_{j=0}^{r-1}(\prod_{m \in T_j} x_m)^j) = (\prod_{j=0}^{r-1}(\prod_{m \in T_j} x_m^j))$$

Consider $0 \leqslant i \leqslant r-1$ and $c \in C_{T_i}$ and $r \in R_{T_i}$, then c and r are only permuting the entries in T_i. Therefore when applied to the product $x_{m_0}^i \cdots x_{m_k}^i$ where $\{m_0, \ldots, m_k\}$ are all the entries of T_i, it has to be the same

$$cr(((\prod_{m \in T_i} x_m^i)) = ((\prod_{m \in T_i} x_{cr(m)}^i) = ((\prod_{m \in T_i} x_m^i)$$

This means that since cr is only permuting the entries of this tableaux when applied to the product we get that

$$cr(\prod_{j=0}^{r-1}(\prod_{m \in T_j} x_m^j)) = \prod_{j=0}^{r-1} cr(\prod_{m \in T_j} x_m^j) = \prod_{j=0}^{r-1}(\prod_{m \in T_j} x_m^j)$$

Therefore we can apply ε_{T_i} to $\hat{F_T^S}$ and we will get the following

$$\begin{aligned}
\varepsilon_{T_i}(\hat{F}_T^S(\prod_{j=0}^{r-1}(\prod_{m \in T_j} x_m)^j)) &= \sum_{c \in C_{T_i}} \sum_{r \in T_{T_i}} \mathrm{sgn}(c) cr(\hat{F}_T^S(\prod_{j=0}^{r-1}(\prod_{m \in T_j} x_m)^j)) \\
&= \sum_{c \in C_{T_i}} \sum_{r \in T_{T_i}} (\mathrm{sgn}(c) cr(\hat{F}_T^S))(cr(\prod_{j=0}^{r-1}(\prod_{m \in T_j} x_m)^j)) && \text{since } cr(fg) = c(r(f)r(g)) = cr(f)cr(g) \\
&= \sum_{c \in C_{T_i}} \sum_{r \in T_{T_i}} (\mathrm{sgn}(c) cr(\hat{F}_T^S))(\prod_{j=0}^{r-1}(\prod_{m \in T_j} x_m)^j) && \text{by the argument above} \\
&= (\sum_{c \in C_{T_i}} \sum_{r \in T_{T_i}} (\mathrm{sgn}(c))(\hat{F}_T^S))(\prod_{j=0}^{r-1}(\prod_{m \in T_j} x_m)^j) \\
&= (\varepsilon_{T_i} \varepsilon_{T_0} \cdots \varepsilon_{T_{r-1}} . x_T^S)(\prod_{j=0}^{r-1}(\prod_{m \in T_j} x_m)^j) \\
&= (\varepsilon_{T_0} \cdots \varepsilon_{T_i} \varepsilon_{T_i} \cdots \varepsilon_{T_{r-1}} . x_T^S)(\prod_{j=0}^{r-1}(\prod_{m \in T_j} x_m)^j) && \text{since } \varepsilon_{T_i}\varepsilon_{T_j} \text{ commute if } i \neq j \\
&= (\varepsilon_{T_0} \cdots \varepsilon_{T_{r-1}} . x_T^S)(\prod_{j=0}^{r-1}(\prod_{m \in T_j} x_m)^j) && \text{since } \varepsilon_{T_i}\varepsilon_{T_i} = \varepsilon_{T_i} \\
&= \hat{F}_T^S
\end{aligned}$$

Therefore the following holds

$$\varepsilon_T \hat{F_T^S} = \hat{F_T^S}$$