$A = k\langle x_i^g \rangle / (f_j) = \dfrac{k\langle x_1 \cdots x_g \rangle}{(f_1, \ldots, f_r)}$

HH1

$0 \to A \to A^{\# g} \to A^r \to \boxed{A^\vee}$

$a \mapsto \begin{pmatrix} [a, x_1] \\ \vdots \\ [a, x_g] \end{pmatrix}$

$\begin{pmatrix} a_1 \\ \vdots \\ a_g \end{pmatrix} \mapsto \sum a_i \frac{\partial}{\partial x_i} f_j \qquad \begin{pmatrix} \sum a_i \frac{\partial}{\partial x_i} f_1 \\ \\ \sum a_i \frac{\partial}{\partial x_i} f_r \end{pmatrix}$

$\boxed{a \dfrac{\partial}{\partial x_1}} (x_1 x_2 + x_2 x_1) = a x_2 + x_2 a$

$k\langle x_1 \cdots x_g \rangle / (f_1, \ldots, f_r)$ — groebner basis monomial order

$f_1 = m_1 - r_1 \qquad m_1$ is monomial

f_1 says $m_1 \mapsto$ replace with r_1

$m_1 > r_1$

$k \oplus k x \oplus k x^2 \simeq k[x] / (\underbrace{x^3}_{m_1} + \underbrace{x + 1}_{-r_1})$

$x^5 \mapsto x^2(-x - 1)$

~~$xy = yz$~~

~~4~~

$\boxed{\begin{aligned} yx &= xy + 3 \\ zy &= yz + 2 \end{aligned}}$

$zyx = z(yx) = \boxed{zxy + 3z}$

$= (zy)x = \boxed{yzx + 2x}$

$V = \#$ overlaps between m_i

$f_i = m_i - r_i$ $\qquad$ $m_{ij} = lm_j \neq m_i m_j$

$F_i = m_i - r_i + \varepsilon_i$ $\qquad$ $\varepsilon_i \in A$

$\begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_r \end{pmatrix} \in A^r \to A^V$

compute groebner basis

V many expressions in ε_i

in thesis.pdf $\qquad$ $\varepsilon_i \varepsilon_j = 0$

(S-polys) = (overlaps)

$$0 \to A \to A \otimes A_1^{!*} \to A \otimes A_2^{!*} \to A \otimes A_3^{!*}$$

$a \mapsto (a, x_1, \ldots)$ $\qquad$ ε_i's in A