MME 9621: Computational Methods in Mechanical Engineering

Assignment 3

(**Due date: 15 March 2024**, **Friday**. *Submit through owl*)

Note:

- *(i) For all of the problems, comment on accuracy and cost of computing.*
- *(ii) Plot the structure of the coefficient Matrix for Problem 1.*
- *(iii) Please submit your computer programs with necessary outputs.*
- *(iv) Mark distribution: Problem 1 (40), Problem 2 (60).*

1. Consider a steady incompressible laminar flow through the parallel plate channel (plane Poiseuille flow) as shown in Figure 1. The fluid has properties with density ρ and dynamic viscosity μ. Assuming the y-velocity component v = 0, and the flow is fully developed, the governing equation reads as,

$$\mu \frac{d^2u}{dy^2} + \frac{12\mu\, u_{av}}{H^2} = 0 \qquad (1)$$

where u is the x-velocity component, u_{av} is the average inlet fluid velocity.

Considering a constant pressure gradient ($\frac{\partial p}{\partial x} = -K$) is applied in flow direction x, u_{av} is defined as

$$u_{av} = \frac{H^2}{12\mu}\left(-\frac{\partial p}{\partial x}\right) = \frac{K\,H^2}{12\mu}$$

(a) Descritize equation (1) using the Finite Difference method, and obtain a recurrence formula for the uniform grid shown in Figure 2. Use *second-order central difference* formula.

(b) Now, apply the boundary conditions, u =0 at y =H/2, and y = -H/2

(c) Write all the resulting algebraic equations for n number of nodes into the following matrix form

$$\mathrm{A}u = \mathrm{b}$$

where $u = [u_1\ u_2\ ...\ u_n]^T$, A is a (n×n) matrix, b is (n×1) matrix. **Use number of nodes *n* = 5.**

(d) Considering H = 0.1 m, ρ = 1.2 kg/m^3, μ = 4×10^{-5} $kg/m/s$, u_{av} = 0.1 m/s,
write a computer program (in MATLAB) to solve the algebraic equations obtained in (c).
Use number of nodes *n* = 5.

(e) Plot the obtained numerical solution (velocity) and the analytic solution as a function of y. Analytical solution reads,

$$u = \frac{3}{2}u_{av}\left(1 - \frac{y^2}{(H/2)^2}\right)$$

(f) Check whether the maximum velocity (i.e., velocity at y =0) that you obtained from the numerical solution has the magnitude close to $u_{max} = \frac{3}{2}u_{av}$.

(g) Obtain the grid independent solution by changing number of nodes n.

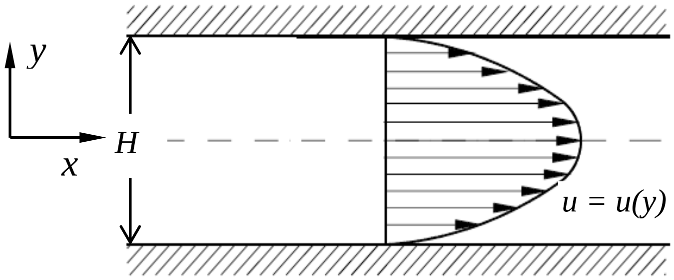

Figure 1. Plane Poiseuille flow.

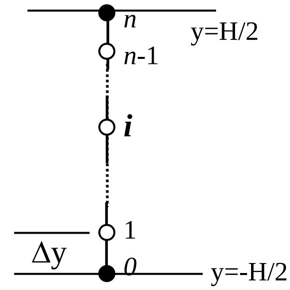

Figure 2. Uniform grid for full-channel height (*H*).

(h) Solve the problem using MATLAB 'bvp4c' function.

2. Consider the Couette flow characterized by the parabolic equation,

$$\frac{\partial u}{\partial t} - \nu \frac{\partial^2 u}{\partial y^2} = 0$$

with the geometry given in Figure 3. Assume the kinematic viscosity ν is constant and u is $u(y,t)$.

Initial conditions at $t = 0$ $\begin{cases} u = u_0, & at\, y = 0 \\ u = 0, & 0 < y \le h \end{cases}$

Boundary conditions at $t > 0$ $\begin{cases} u = u_0, & at\, y = 0 \\ u = 0, & at\, y = h \end{cases}$

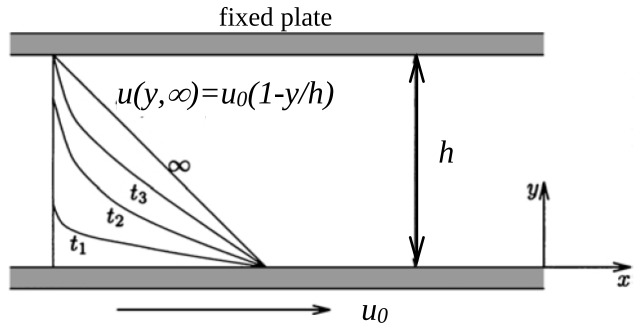

Figure 3. Transient Couette flow.

(a) Discretize the above governing equation using the finite difference method, and obtain a recurrence formula for a uniform space-grid.
Use *second-order central difference* formula for space discretization.
For time discretization use the following methods,
(i) Explicit,
(ii) Implicit,
(iii) Crank-Nicolson.

Write the required matrices for number of space nodes $\boldsymbol{M} = \mathbf{5}$ (Note: for space grid i =0,1,2,...M).

(b) Considering, the plate spacing h =40 mm, ν = 0.000217 m^2/s, and u_0 = 40 m/s, write required computer programs (in MATLAB) to solve the algebraic equations obtained in (a). **Obtain a grid independent solution by changing *M*.**

(c) Solve the problem using MATLAB pdepe function.

(d) Plot the velocity distribution $u(y,t)$ obtained from (b) and (c) at time t = 0.1, 0.3, 0.6, 2 sec in a *single* graph.
