MME 9621: Computational Methods in Mechanical Engineering

Assignment 2

(**Due date: 01 March 2024**, **Friday**. *Submit through owl*)

Note:

(i) For all of the problems, comment on accuracy and cost of computing.

(ii) Plot the structure of the coefficient Matrix for Problem 1.

(iii) Please submit your computer programs with necessary outputs. Also attach the Excel file if you use any.

(iv) Mark distribution: Problem 1 (40), Problem 2(30), Problem 3 (30)

1. The Figure 1 shows the axial force F_i in each of the 21 member pin-connected truss. The system of equations can be obtained by using the mechanical equilibrium principle of the system with 21 equations as follows. Solve the system of equation to find the unknown axial forces.

Use (i) Jacobi method, (ii) Gauss-Seidel method, (iii) SOR with ω<1,
(iv) SOR with ω>1, and (v) MATLAB '\'.

Comment on the suitability of methods used in (i) to (iv) for solving this problem.

$$-F_1 - aF_3 = 0$$
$$-F_2 + F_6 - aF_3 = 0$$
$$F_5 + aF_3 = 0$$
$$-F_5 - aF_7 = 0$$
$$-F_4 + F_8 + aF_7 = 0$$
$$-F_6 + F_{10} - aF_7 = 0$$
$$F_9 + aF_7 = 0$$
$$-F_{10} - aF_{11} = 0$$
$$F_{12} + aF_{11} = 0$$
$$-F_{12} + F_{16} = 0$$
$$F_{13} = 0$$
$$bF_{14} + aF_{15} - aF_{11} - F_9 = 0$$
$$cF_{14} + aF_{15} + aF_{11} + F_{13} - F_8 = 0$$
$$bF_{18} - bF_{14} + 0.7433F_{19} = 0$$
$$cF_{18} - cF_{14} + F_{17} + dF_{19} = 8000$$
$$F_{20} - F_{16} - aF_{15} = 0$$
$$-F_{17} - aF_{15} = 0$$
$$-F_{20} - 0.7433F_{19} = -5000$$
$$-F_{21} - dF_{19} = 0$$
$$\boxed{bF_{18} = 10000\cos 60^0}$$
$$F_{21} - cF_{18} = 10000\sin 60^0$$

Consider, a=0.7071, b=0.9806,
c =0.1961, d=0.669.

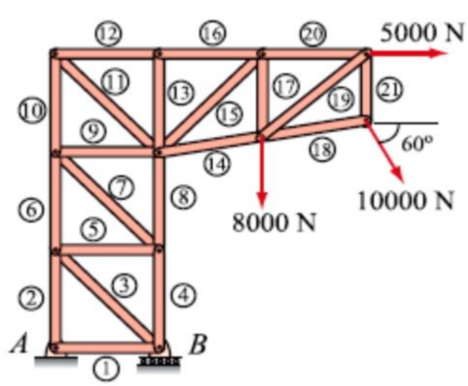

Figure 1.

2. Consider the conduction heat transfer of a rod as shown in Figure 2a. There is heat transfer by convection along the surface of the rod. Considering the radiation heat loss the governing equation describing the system takes the following form,

$$\frac{d^2T}{dx^2} + h_p\left(T_\infty - T\right) + \sigma\left(T_\infty^4 - T^4\right) = 0$$

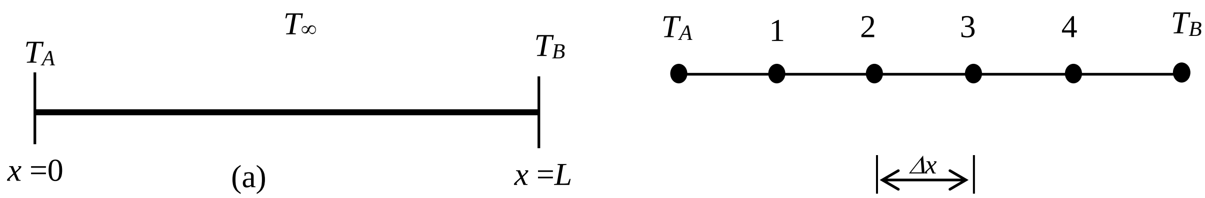

Figure 2. Heat conduction in a rod.

To solve the above governing equation, a discretization approach with four nodes is used (see Figure 2b), and the following system of four non-linear equations is obtained,

$$\begin{aligned}
-T_A + \left(2 + h_p\Delta x^2\right)T_1 + \sigma\Delta x^2\, T_1^4 - T_2 &= h_p\Delta x^2\, T_\infty + \sigma\Delta x^2\, T_\infty^4 \\
-T_1 + \left(2 + h_p\Delta x^2\right)T_2 + \sigma\Delta x^2\, T_2^4 - T_3 &= h_p\Delta x^2\, T_\infty + \sigma\Delta x^2\, T_\infty^4 \\
-T_2 + \left(2 + h_p\Delta x^2\right)T_3 + \sigma\Delta x^2\, T_3^4 - T_4 &= h_p\Delta x^2\, T_\infty + \sigma\Delta x^2\, T_\infty^4 \\
-T_3 + \left(2 + h_p\Delta x^2\right)T_4 + \sigma\Delta x^2\, T_4^4 - T_B &= h_p\Delta x^2\, T_\infty + \sigma\Delta x^2\, T_\infty^4
\end{aligned}$$

Consider, length of the rod L =10 m, h_p = 0.05 m^{-2}, σ = 2.7×10^{-9} K^{-3}m^{-2} , T_∞= 200 K, T_A= 300 K, T_B= 400 K and Δx =2m

(a) Solve the above system using (i) multivariate Newton's method, (ii) Broyden's method, (iii) MATLAB '*fsolve*' function.

(b) Plot the temperature distribution of the rod as a function of x.

3. Engineers and scientists use mass-spring models to gain insight into the dynamics of structures under the influence of disturbances such as earthquakes. Figure 3 shows such a representation for a three-story building. For this case, the analysis is limited to horizontal motion of the structure. Using Newton's second law, force balance can be developed for this system as

$$\begin{aligned}
\frac{d^2x_1}{dt^2} &= -\frac{k_1}{m_1}x_1 + \frac{k_2}{m_1}\left(x_2 - x_1\right) \\
\frac{d^2x_2}{dt^2} &= \frac{k_2}{m_2}\left(x_1 - x_2\right) + \frac{k_3}{m_2}\left(x_3 - x_2\right) \\
\frac{d^2x_3}{dt^2} &= \frac{k_3}{m_3}\left(x_2 - x_3\right)
\end{aligned}$$

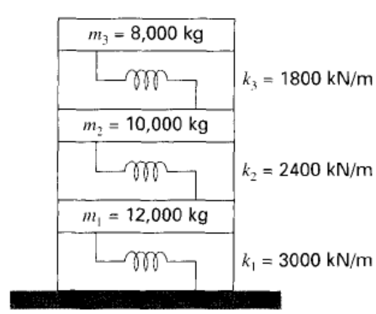

Figure 3

where, x_i = displacement of the mass m_i (i = 1,2,3).

Simulate the dynamics of this structure from $t = 0$ to 20 s, given the initial condition that the velocity of the ground floor is $dx_1/dt = 1$ m/s, and all other initial values of displacements and velocities are zero. Note: use k values as kN/m.

Present your results as two time-series plots of (a) displacements and (b) velocities. In addition, develop a three-dimensional phase-plane plot of the displacements.

Use (i) RK4 method (vectorized form) and (ii) MATLAB ‘ode45’ function.

Note: *compare the accuracy, step size, and computational time for this problem.*
