

MME 9621: Computational Methods in Mechanical Engineering
Assignment 4

(Due date: 29 March 2024, Friday. Submit through owl)

Note:

- (i) For all of the problems, comment on accuracy and cost of computing.
- (ii) Please submit your computer programs with necessary outputs.
- (iii) Mark distribution: Problem 1 (60), Problem 2 (40).

1. A large flat plate at temperature $T = 925\text{K}$ is placed in front of a second large flat plate. The heated plate emits radiation to the second plate and the second plate reflects some radiation back to the heated plate. The emissivity as a function of wavelength of the heated plate is given by equation (1) where the wavelength λ is measured in micrometers.

$$\epsilon(\lambda) = \begin{cases} 0.850 \left(1 - \frac{\lambda}{10.5}\right) & 0 \leq \lambda \leq 10.5 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

The reflectivity of the second plate as a function of wavelength λ is given by equation (2),

$$\rho(\lambda) = \begin{cases} 0.35 & \text{for } 0 \leq \lambda \leq 4.5 \\ 0.82 & \text{for } \lambda > 4.5 \end{cases} \quad (2)$$

The energy flux of radiation being absorbed by the first plate is given by equation (3),

$$Q_{\text{abs}} = \rho(\lambda) \int_0^{\infty} \epsilon^2(\lambda) E(\lambda, T) d\lambda \quad (3)$$

$$\text{where,} \quad E(\lambda, T) = \frac{2\pi C_1}{\lambda^5 \left(e^{\frac{C_2}{\lambda T}} - 1 \right)}$$

$$\text{with} \quad C_1 = hc_0^2, \quad C_2 = \frac{hc_0}{k_B} \quad (4)$$

Considering $c_0 = 2.9979 * 10^8 \text{ m/s}$, $h = 6.626 * 10^{-34} \text{ Js}$ and $k_B = 1.3806 * 10^{-23} \text{ J/K}$,

calculate the energy flux absorbed by the first plate.

Use (a) MATLAB function and (b) Simpson's 1/3 rule.

2. An automobile is modeled as shown in Figure 1 with mass M , mass moment of inertia J_G , suspension system springs of k_1 , k_2 and k_3 with weights m_1 and m_2 . Four degrees of freedom are defined for the system as x_1 , x_2 , x_3 and θ . The governing equations for the free harmonic oscillations with variables $x_r = X_r e^{i\omega t}$ ($r=1,2,3$) and $\theta = \Theta e^{i\omega t}$ are as follows:

$$\begin{aligned} -m_1 \omega^2 X_1 + (k_1 + k_2) X_1 - k_2 X_3 + k_2 l_1 \Theta &= 0 \\ -m_2 \omega^2 X_2 + (k_1 + k_3) X_2 - k_3 X_3 - k_3 l_2 \Theta &= 0 \\ -M \omega^2 X_3 - k_2 X_1 - k_3 X_2 + (k_2 + k_3) X_3 + (k_3 l_2 - k_2 l_1) \Theta &= 0 \\ -J_G \omega^2 \Theta + k_2 l_1 X_1 - k_3 l_2 X_2 + (k_3 l_2 - k_2 l_1) X_3 + (k_2 l_1^2 + k_3 l_2^2) \Theta &= 0 \end{aligned}$$

If $k_1 = 18,000$ N/m, $k_2 = 20,000$ N/m, $k_3 = 20,000$ N/m, $l_1 = 1.0$ m, $l_2 = 1.5$ m, $M = 1000$ kg, $m_1 = 100$ kg, $m_2 = 200$ kg, radius of gyration $r = 0.9$ m and $J_G = Mr^2$,

find the natural frequencies of the system.

Use (a) Q-R factorization, (b) MATLAB 'eig' function.

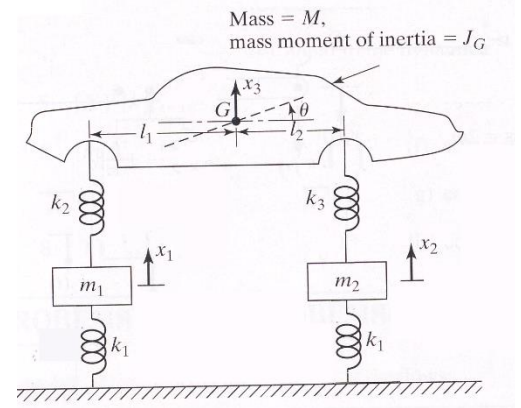


Figure 1.
