Some Improvements in Quartz Crystal Circuit Elements

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The characteristics of the *Y*-cut quartz crystal plate are discussed. It is shown that by rotating a plate about the *X* axis special orientations are found for which the frequency spectrum is simplified, the temperature coefficient of frequency is reduced practically to zero and the amount of power that can be controlled without fracture of the crystal is increased. These improvements are obtained without sacrificing the advantages of the Y cut plate, i.e., activity and the possibility of rigid clamping in the holder.

THERE are at the present time two types of crystal quartz plates in general use as circuit elements for frequency stabilization at radio frequencies, namely, the *X*-cut and *Y*-cut.[1] This paper is concerned with the improved characteristics of plates having radically new orientations.

In its usual form the *Y*-cut plate is cut from the mother crystal, as shown in Fig. 1. The electric field is applied along the *Y* direction

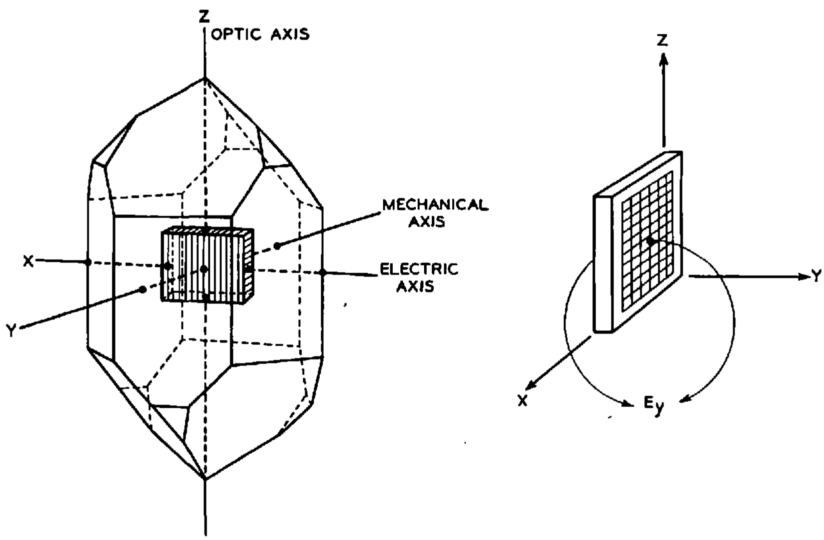

Fig. 1—Showing relation of Y-cut quartz crystal to the crystallographic axes.

and for high frequencies an x_y shear vibration is utilized.

The frequency of such a vibration is given by the expression:

$$f = \frac{1}{2l}\sqrt{\frac{c_{66}}{\rho}}, \tag{1}$$

[1] "Piezo-Electric Terminology," W. G. Cady, *Proc. I. R. E.*, 1930, p. 2136.

where c_{66} = the elastic constant for quartz connecting the X_y stress with an x_y strain $= 39.1 \times 10^{10}$ dynes per cm.[2]

ρ = the density of quartz = 2.65 gms. per cm.[3]

l = the thickness in cm.

On substituting the numerical values in equation (1), a frequency-thickness constant of 192 kc. per cm. is obtained which checks within 3 per cent the value of this constant found by experiment.

This x_y shear vibration is not appreciably affected when the plate is rigidly clamped, the clamping being applied either around the periphery if the plate is circular, or at the corners if square. Hence a mechanically rigid holder arrangement is possible which is particularly suitable for mobile radio applications.[2]

The temperature coefficient of frequency of this vibration is approximately + 85 parts/million/C.°, which means that for most applications it must be used in a thermostatically controlled oven. In operation, this comparatively large temperature coefficient is responsible for a major part of any frequency deviations from the assigned value.

Another important characteristic of the *Y*-cut crystal is the secondary frequency spectrum of the plate. This secondary spectrum consists of overtones of low frequency vibrations which are mechanically coupled to the desired vibration and cause discontinuities in the characteristic frequency-temperature and frequency-thickness curves of the crystal. In some instances these coupled secondary vibrations can be utilized to produce a low temperature coefficient over a limited temperature range.[3] But in general, at the higher frequencies (above one megacycle) this secondary spectrum is a source of considerable annoyance, not only in the initial preparation of a plate for a given frequency but in the added necessity for some form of temperature control. In practice, these plates are so adjusted that there are no discontinuities in the frequency-temperature characteristic in the region where they are expected to operate, but at high frequencies it is difficult to eliminate all of these discontinuities over a wide temperature range. If, then, for any reason the crystal must be operated without the temperature control, a frequency discontinuity with temperature may cause a large frequency shift greatly in excess of that to be expected from the normal temperature coefficient.

From the above considerations it may be concluded that the standard *Y*-cut plate has two distinct disadvantages: namely, a

[2] U. S. Patent No. 1883111, G. M. Thurston, Oct. 18, 1932. "Application of Quartz Plates to Radio Transmitters," O. M. Hovgaard, *Proc. I. R. E.*, 1932, p. 767.

[3] "Observations on Modes of Vibrations and Temperature Coefficients of Quartz Plates," F. R. Lack, *Proc. I. R. E.*, 1929, p. 1123; *Bell Sys. Tech. Jour.*, July, 1929.

temperature coefficient requiring close temperature regulation and a troublesome secondary frequency spectrum. Assuming that the temperature coefficient of the desired frequency could be materially reduced, the effect of any secondary spectrum must also be minimized before temperature regulation can be abandoned. In fact, from the standpoint of satisfactory production and operation of these crystal plates, it is perhaps more important that the secondary spectrum be eliminated than that the temperature coefficient be reduced.

The Secondary Spectrum

The secondary spectrum of these plates, as has been indicated above, is caused by vibrations of the same or of other types than the wanted vibration taking place in other directions of the plate and coupled to the wanted vibration mechanically. This condition of affairs exists in all mechanical vibrating systems but is complicated in the case of quartz by the complex nature of the elastic system involved.

Considering specifically the case of the Y-cut plate the desired vibration is set up through the medium of an x_y strain. Hence any coupled secondary vibrations must be set in motion through this x_y strain. Referring to the following elastic equations for quartz (in these equations X, Y and Z are directions coincident with the crystallographic axes; see Fig. 1 and Appendix),

$$\left.\begin{aligned}
-X_x &= c_{11}x_x + c_{12}y_y + c_{13}z_z + c_{14}y_z \\
-Y_y &= c_{12}x_x + c_{22}y_y + c_{23}z_z + c_{24}y_z \\
-Z_z &= c_{13}x_x + c_{23}y_y + c_{33}z_z \\
-Y_z &= c_{14}x_x + c_{24}y_y \qquad\quad + c_{44}y_z \\
-Z_x &= \qquad\qquad\qquad\qquad\qquad + c_{55}z_x + c_{56}x_y \\
-X_y &= \qquad\qquad\qquad\qquad\qquad + c_{56}z_x + c_{66}x_y
\end{aligned}\right\} \quad (2)$$

it will be seen that by reason of the constant c_{56} an x_y strain will set up a stress in the Z_x plane which in turn will produce a z_x strain. Hence the x_y and z_x strains are coupled together mechanically, the value of the constant c_{56} being a measure of that coupling.

High order overtones of vibrations resulting from this z_x strain constitute the major part of the secondary frequency spectrum of these plates.

The technique for dealing with this secondary spectrum in the past has been the proper choice of dimensions. At high frequencies these overtones occur very close together and when one set is moved out of the range by grinding a given dimension another set will appear. Some benefit is obtained with the clamped holder, which tends to inhibit certain types of transverse vibrations; but as indicated above,

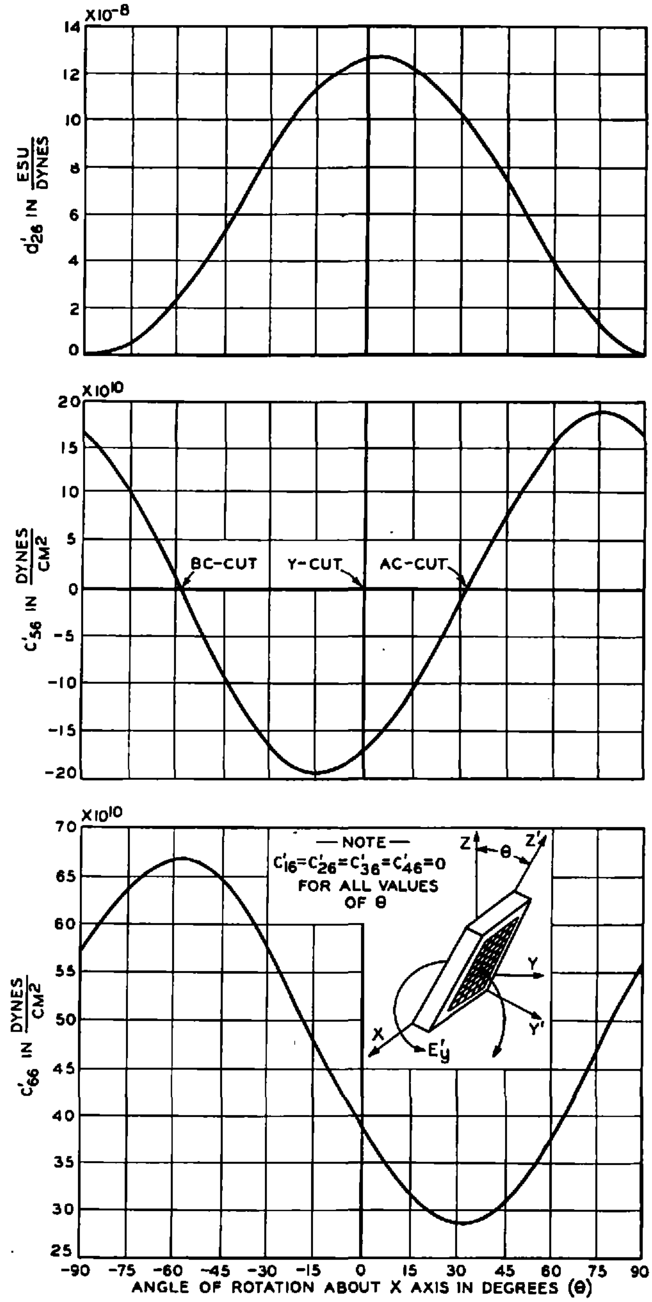

Fig. 2—c_{66}', c_{56}', and d_{26}' as a function of rotation about the X axis.

the elimination of the effects of the coupled secondary frequency spectrum over a wide temperature range is a difficult matter.

Another method has been developed recently for dealing with these coupled vibrations.[4] This consists of reducing, by a change in orientation, the magnitude of the elastic constant responsible for the coupling. If the orientation of the crystal plate be shifted with respect to the crystallographic axes then, in general, the elastic constants with reference to the axes of the plate will vary. The direct constants (c_{11}', c_{22}', $\cdots$) which represent the longitudinal and shear moduli will of course vary in magnitude only, while the cross constants (c_{ij}', $\cdots$) will vary both as to magnitude and sign. There is a possibility therefore that the proper choice of orientation of the plate will reduce c_{56}' to zero without at the same time introducing other couplings.

Figure 2 shows graphically the variation of c_{66}' and c_{56}' as a function of rotation about the X axis. These have been calculated by means of the equations given in the appendix. It will be seen that at approximately $+ 31°$ and $- 60°$ c_{56}' becomes zero. Here then are two orientations for which the coupling between the x_y and z_x strains should be zero.

In shifting the orientation of the plate the necessity for exciting the wanted vibration piezo-electrically must not be lost sight of. Hence in addition to computing the values of the elastic constants the variation of the piezo-electric moduli as a function of orientation must also be examined. Figure 2 also shows the effect of rotation about the X axis on d_{26}' (the constant connecting the E_y' electric field with the x_y' strain). It will be seen that at both $+ 31°$ and $- 60°$ the x_y' vibration can be excited piezo-electrically but it is to be expected that a plate cut at $- 60°$ will be relatively inactive,[5] for d_{26}' at this point is only 20 per cent of its value for the Y-cut plate. On the other hand at $+ 31°$ a plate would be practically equivalent to the Y-cut as far as activity is concerned. The frequency of the x_y' vibration for these special orientations can be calculated by means of equation (1) substituting for c_{66} the value of c_{66}' for the given angle as read from the curve of Fig. 2.

[4] The expression of the coupling between two modes of vibration in quartz in terms of the elastic constants was first suggested in 1930 by Mr. W. P. Mason of the Bell Telephone Laboratories.

[5] The word "activity" is a rather loose term used by experimenters in this field to describe the ease with which a given vibration can be excited in a particular circuit. It is often spoken of in terms of the grid current that is obtained in that circuit or the amount of feedback necessary to produce oscillation. It can better be expressed quantitatively as the coupling between the electrical and mechanical systems (not to be confused with the mechanical coupling between different vibrations described above) which is a simple function of the piezo-electric and elastic moduli of the vibration involved and the dielectric constant of the crystal plate.

For the purpose of identification the plate cut at + 31° has been designated as the *AC*-cut and the plate cut at − 60° the *BC*-cut. Crystal plates having these orientations have been made up and tested. It is evident from the frequency-temperature and frequency-thickness characteristics of both cuts that a simplification of the frequency spectrum results from the reduction in coupling to secondary

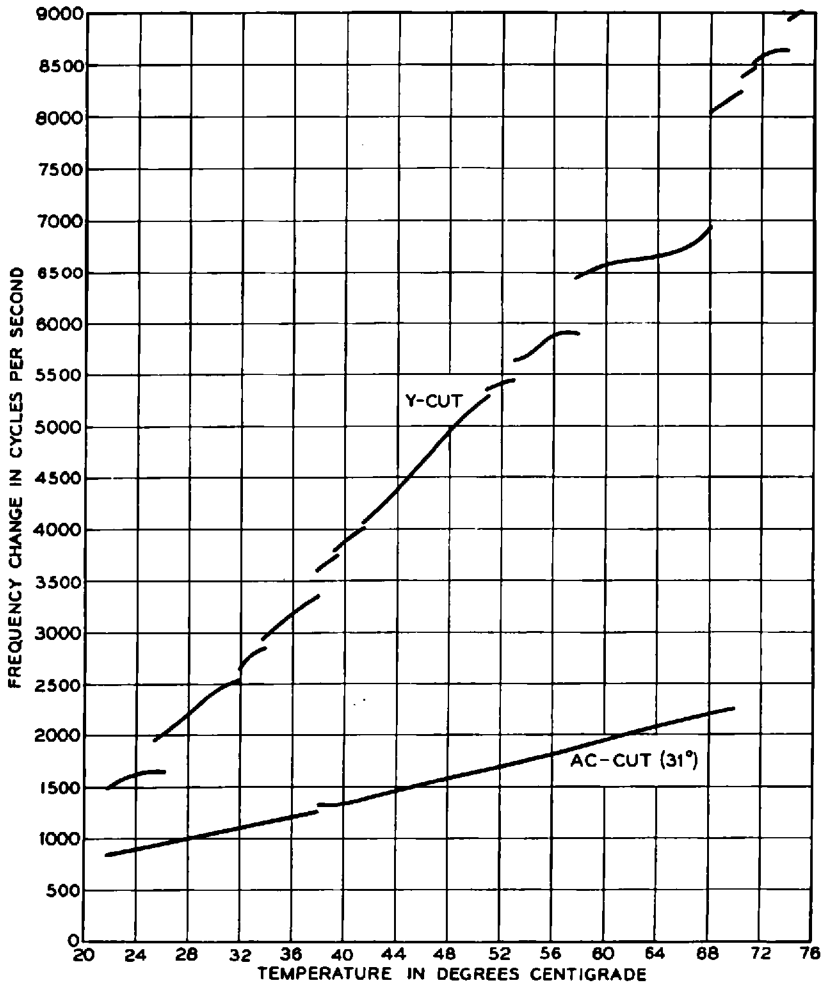

Fig. 3—Frequency-temperature characteristics of AC-cut and Y-cut plates of same frequency and area.

Frequency 1600 KC.
Dimensions:

Y-cut	y = 1.22 mm.	x = 38 mm.	z = 38 mm.
AC-cut (31°)	y' = 1.00 mm.	x = 38 mm.	z' = 38 mm.

vibrations. Frequency discontinuities of the order of a kilocycle or more which are a common occurrence with the *Y*-cut plate have disappeared and frequency-temperature curves that are linear over a considerable temperature range can be obtained without much difficulty. This is illustrated by Fig. 3 which shows frequency-temperature characteristics for both *AC*-cut and *Y*-cut plates of the same frequency and area. The *AC*-cut plate can be clamped to the same extent as the *Y*-cut plate.

There is still some coupling remaining to certain secondary frequencies. These frequencies are difficult to identify but are thought to be caused by overtones of flexural vibrations set up by the x_y' shear itself and hence would be unaffected by the reduction of c_{56}'. These remaining frequencies do not cause much difficulty above 500 kc. For the *AC*-cut (+ 31°) plate the temperature coefficient of frequency is + 20 cycles/million/C.°, while for the *BC*-cut (− 60°) plate it is − 20 cycles/million/C.°.

In addition to these calculations for the x_y' vibration in plates rotated about the *X* axis, a detailed study has been made of other types of vibration and rotation about the other axes. For high frequencies nothing has been found to compare with the reduction in complexity of frequency spectrum obtainable with these two orientations.

Temperature Coefficients

This study has produced in the *AC*-cut a new type of plate which has superior characteristics to the standard *Y*-cut: i.e., a simplified frequency spectrum and a lower temperature coefficient. The values of the temperature coefficients obtained for these new orientations are significant and suggest that perhaps other orientations can be chosen for which the temperature coefficient will be zero. With the measured values of the temperature coefficients for the different orientations and the c_{66}' equation (Appendix) it is possible to compute the temperature coefficient for any angle. Figure 4 shows graphically the results of such a computation for an x_y' vibration as a function of rotation about the *X* axis. It will be seen that at approximately + 35° and − 49° the x_y' vibration will have a zero temperature coefficient of frequency.

This curve has been checked experimentally, the check points being indicated on the curve. Concentrating on a plate cut at + 35°, which has been designated the *AT*-cut, it will be seen that this type of plate offers considerable possibilities. Figure 5 shows the frequency-temperature curves of a 2-megacycle *AT*-cut plate and a standard Y-cut of the same frequency and area. These curves not only illustrate the reduction in temperature coefficient but also show that in the AT-cut plate the secondary frequency spectrum has been eliminated over the temperature range of the test. This is to be expected, for 35° is close to the 31° zero coupling point; hence such coupling as does exist is small in magnitude.

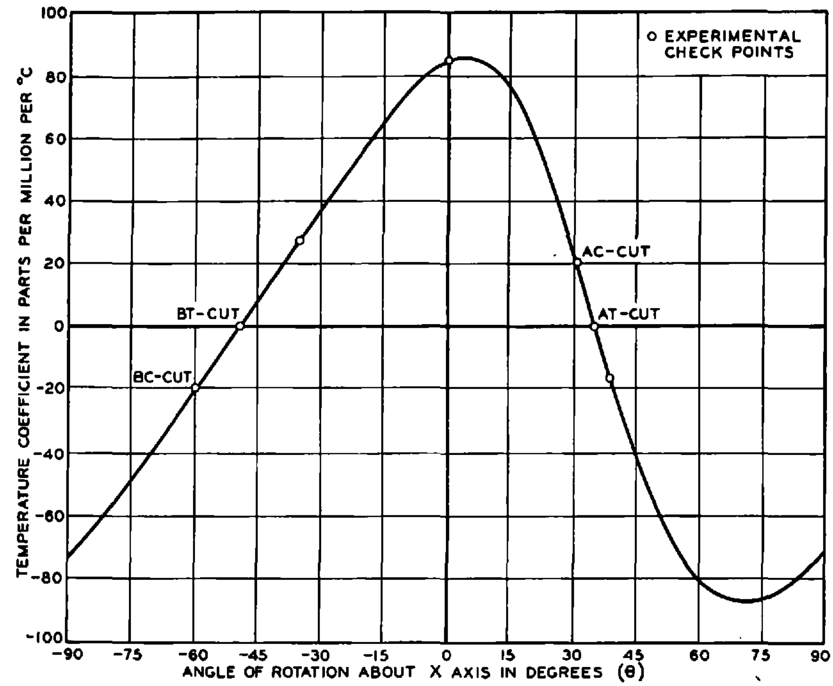

Fig. 4—Temperature coefficient of frequency of the vibration depending upon c_{66}' as a function of rotation about the X axis.

These AT-cut plates can be produced with a sufficiently low temperature coefficient so that for most applications the temperature regulating system can be discarded, and in addition a simplification of the secondary frequency spectrum is obtained. Furthermore, the advantages of the Y-cut plate, i.e., clamping and activity, have not been sacrificed.

Additional tests on AT-cut plates indicate that it will be possible to use them to control reasonable amounts of power without danger of fracture. At 2 megacycles, 50-watt crystal oscillators would appear to be practical and in some experimental circuits the power output has been run up to 200 watts without fracturing the crystal. The explanation for this lies in the fact that the reduction in magnitude of the coupling to transverse vibrations has reduced the transverse stresses which in the *Y*-cut plate are responsible for the fractures.

Experimental crystals of this type have been produced in the frequency range from 500 kc. to 20 megacycles. The possibility of high frequencies, together with the elimination of the temperature

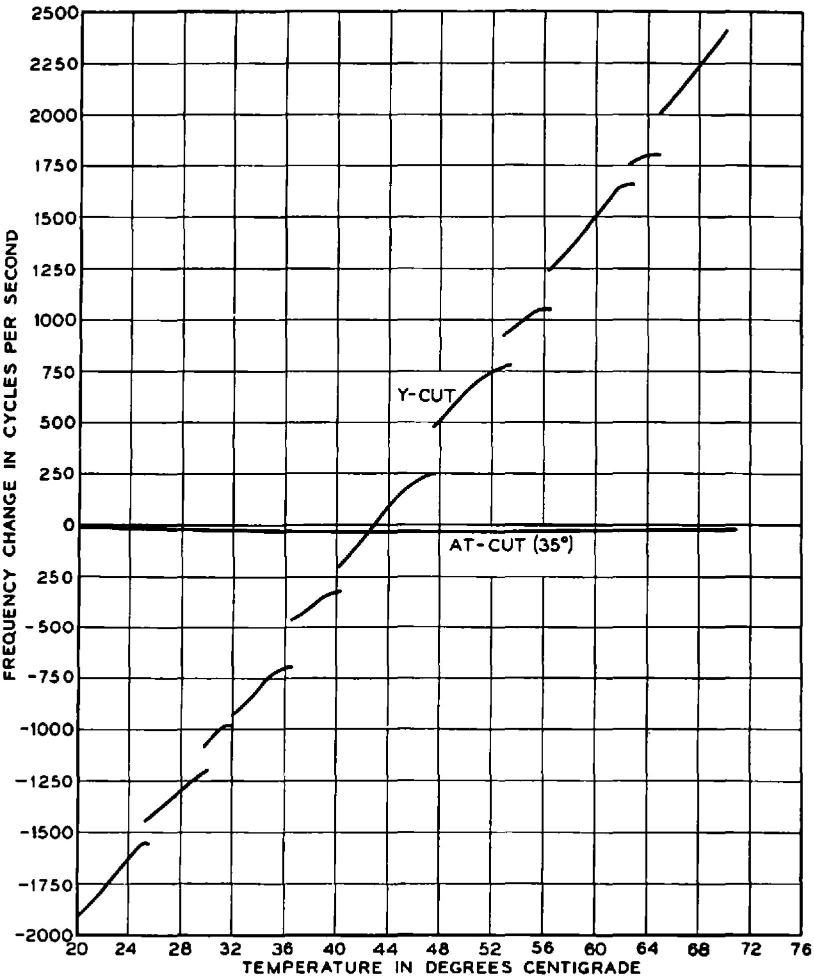

Fig. 5—Frequency-temperature characteristics of AT-cut and Y-cut plates of same frequency and area.

Frequency 1000 KC.
Dimensions:

Y-cut	y = 1.970 mm.	x = 38 mm.	z = 38 mm.
AT-cut (35°)	y' = 1.675 mm.	x = 38 mm.	z' = 38 mm.

control and the increase in the amount of power that may be controlled, should result in a considerable simplification of future short wave radio equipment.

APPENDIX

Elastic Equations

The general elastic equations for any crystal are given below, X', Y' and Z' representing any orthogonal set of axes.

$$\left.\begin{aligned}
-X_x' &= c_{11}'x_x' + c_{12}'y_y' + c_{13}'z_z' + c_{14}'y_z' + c_{15}'z_x' + c_{16}'x_y' \\
-Y_y' &= c_{12}'x_x' + c_{22}'y_y' + c_{23}'z_z' + c_{24}'y_z' + c_{25}'z_x' + c_{26}'x_y' \\
-Z_z' &= c_{13}'x_x' + c_{23}'y_y' + c_{33}'z_z' + c_{34}'y_z' + c_{35}'z_x' + c_{36}'x_y' \\
-Y_z' &= c_{14}'x_x' + c_{24}'y_y' + c_{34}'z_z' + c_{44}'y_z' + c_{45}'z_x' + c_{46}'x_y' \\
-Z_x' &= c_{15}'x_x' + c_{25}'y_y' + c_{35}'z_z' + c_{45}'y_z' + c_{55}'z_x' + c_{56}'x_y' \\
-X_y' &= c_{16}'x_x' + c_{26}'y_y' + c_{36}'z_z' + c_{46}'y_z' + c_{56}'z_x' + c_{66}'x_y'
\end{aligned}\right\} \quad (3)$$

When in quartz X', Y' and Z' coincide with the crystallographic axes of the material (X the electric axis, Y the mechanical axis, and Z the optic axis), equation (3) reduces to equation (2) of the text. In addition the following relations exist between the constants of equation (2) because of conditions of symmetry

$$c_{11} = c_{22}, \qquad c_{44} = c_{55}, \qquad c_{66} = (c_{11} - c_{12})/2, \qquad c_{13} = c_{23}$$
$$c_{14} = -c_{23} = c_{56}.$$

The numerical values of these constants have been determined experimentally by Voigt[6] and others.

$$\begin{aligned}
c_{11} &= 85.1 \times 10^{10} \frac{\text{dy.}}{\text{cm.}^2} & c_{12} &= 6.95 \times 10^{10} \frac{\text{dy.}}{\text{cm.}^2}, \\
c_{33} &= 105.3 \times 10^{10} \frac{\text{dy.}}{\text{cm.}^2} & c_{13} &= 14.1 \times 10^{10} \frac{\text{dy.}}{\text{cm.}^2}, \\
c_{44} &= 57.1 \times 10^{10} \frac{\text{dy.}}{\text{cm.}^2} & c_{14} &= 16.8 \times 10^{10} \frac{\text{dy.}}{\text{cm.}^2}, \\
c_{66} &= 39.1.
\end{aligned}$$

Using these constants it is possible to calculate the c_{ij}' for any orientation by means of transformation equations.[7] The expressions giving c_{16}', c_{26}', $\cdots$ c_{66}' (the constants relating to the x_y' strain) in terms of the c_{ij} for rotation about the X axis, are given below, θ being the

[6] W. Voigt, "Lehrbuch der Kristallphysik," 1928, p. 754.
[7] A. E. H. Love, "Mathematical Theory of Elasticity," 4th ed., p. 43.

angle between the Z' and Z axis (Fig. 2).

$$\begin{aligned} c_{16}' &= c_{26}' = c_{36}' = c_{46}' = 0, \\ c_{56}' &= c_{14}(\cos^2\theta - \sin^2\theta) + (c_{66} - c_{44})\sin\theta\cos\theta, \\ c_{66}' &= c_{44}\sin^2\theta + c_{66}\cos^2\theta - 2c_{14}\sin\theta\cos\theta. \end{aligned} \tag{4}$$

Piezo-Electric Equations

The inverse piezo-electric relations for the X', Y', Z' system of axes can be expressed by the following equations:

$$\left.\begin{aligned} x_x' &= d_{11}'E_x' + d_{21}'E_y' + d_{31}'E_z' \\ y_y' &= d_{12}'E_x' + d_{22}'E_y' + d_{32}'E_z' \\ z_z' &= d_{13}'E_x' + d_{23}'E_y' + d_{33}'E_z' \\ y_z' &= d_{14}'E_x' + d_{24}'E_y' + d_{34}'E_z' \\ z_x' &= d_{15}'E_x' + d_{25}'E_y' + d_{35}'E_z' \\ x_y' &= d_{16}'E_x' + d_{26}'E_y' + d_{36}'E_z' \end{aligned}\right\} \tag{5}$$

When in quartz X', Y', Z' coincide with the crystallographic axes, eq. 5 reduces to the following:

$$\left.\begin{aligned} x_x &= d_{11}E_x \\ y_y &= -\,d_{11}E_x \\ z_z &= 0 \\ y_z &= d_{14}E_x \\ z_x &= \qquad -\,d_{14}E_y \\ x_y &= \qquad -\,2d_{11}E_y \end{aligned}\right\} \tag{6}$$

where

$$d_{11} = -\,6.36 \times 10^{-8}\frac{\text{esu}}{\text{dyne}},$$

$$d_{14} = 1.69 \times 10^{-8}\frac{\text{esu}}{\text{dyne}}.$$

For rotation about the X axis,

$$d_{26}' = (d_{14}\sin\theta - 2d_{11}\cos\theta)\cos\theta. \tag{7}$$