Answers to questions in Lab 1: Filtering operations

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Instructions: Complete the lab according to the instructions in the notes and respond to the questions stated below. Keep the answers short and focus on what is essential. Illustrate with figures only when explicitly requested.
Good luck!

Question 1: Repeat this exercise with the coordinates p and q set to (5, 9), (9, 5), (17, 9), (17, 121), (5, 1) and (125, 1) respectively. What do you observe?
Answers:
All the plots are shown in Figure 1-1 to Figure 1-6.

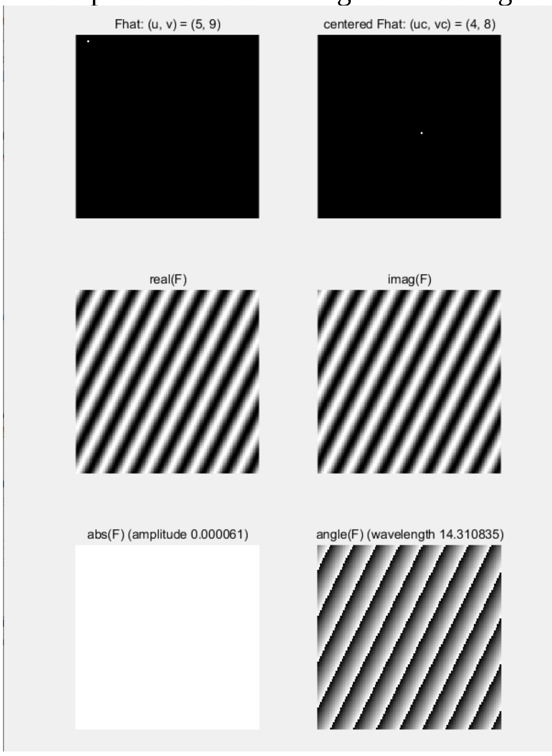

Figure 1-1. Coordinates (5, 9)

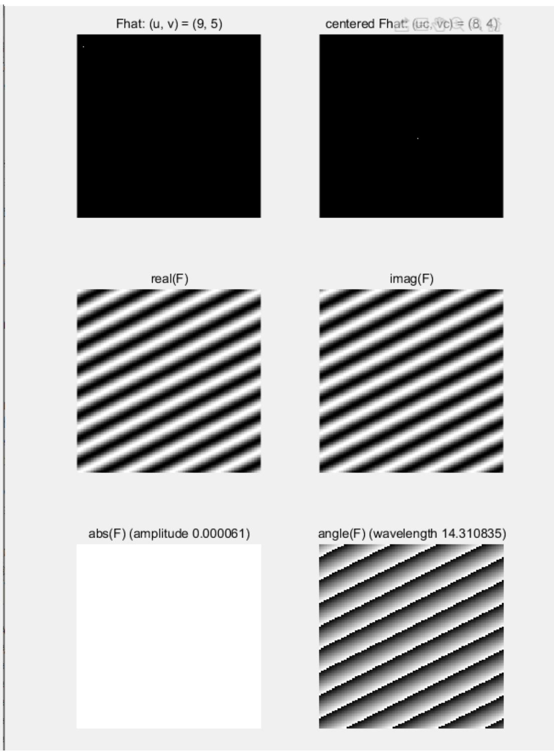

Figure 1-2. Coordinates (9, 5)

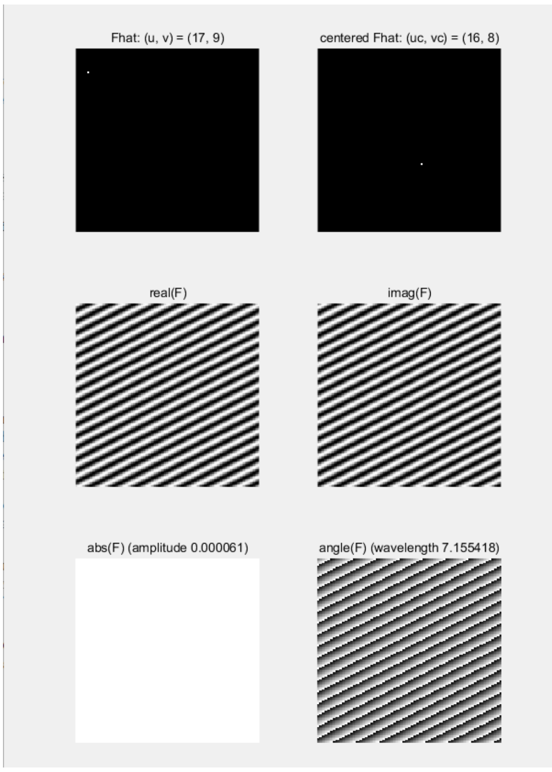

Figure 1-3. Coordinates (17, 9)

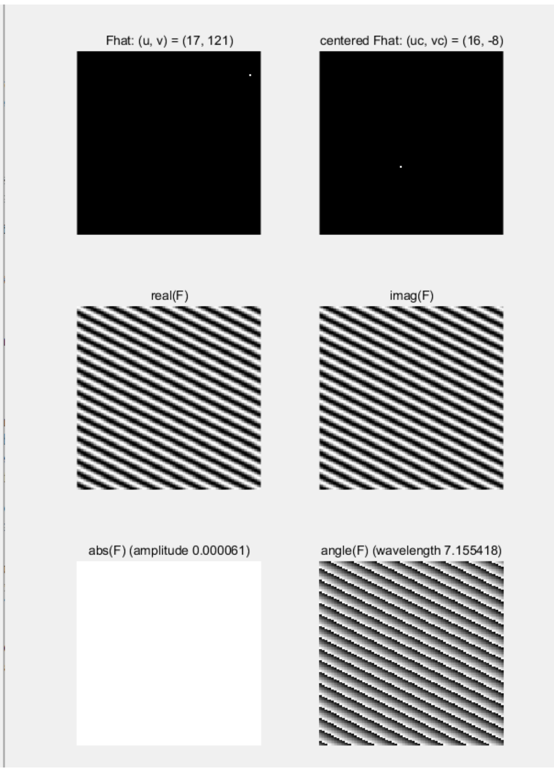

Figure 1-4. Coordinates (17, 121)

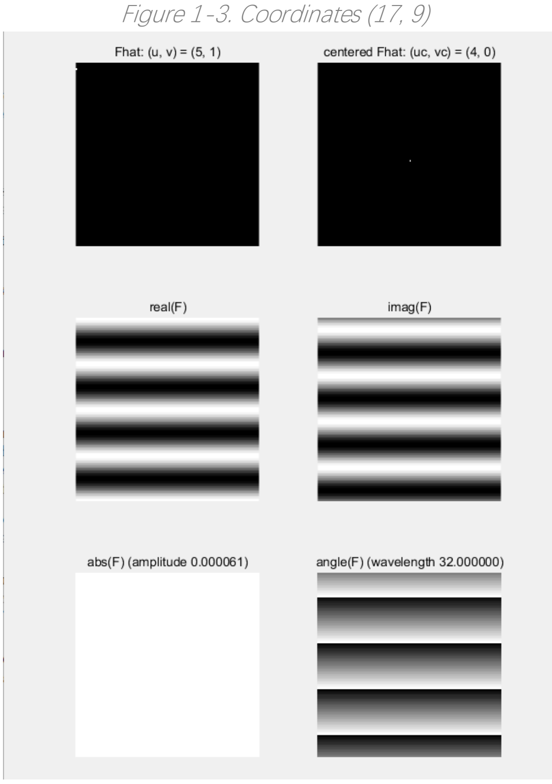

Figure 1-5. Coordinates (5, 1)

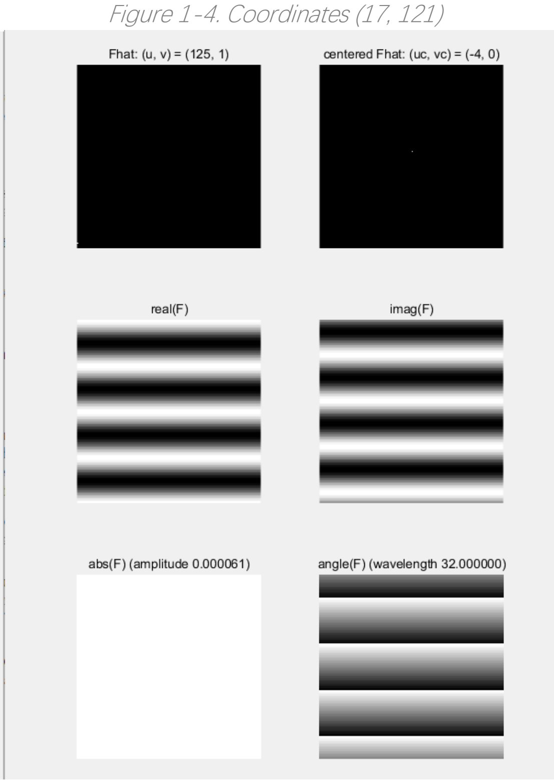

Figure 1-6. Coordinates (125, 1)

From these plots, I observed that:

- For coordinates (5, 9) and (9, 5), the centered coordinates are (4, 8) and (8, 4), which are symmetrical with respect of line u=v. The plots of real part of F, the imaginary part of F and the angles are symmetrical with the respect to line u=v.
- For coordinates (17, 9) and (17, 121), the centered coordinates are (16, 8) and (16, -8), which are symmetrical with respect to the horizontal axis. The plots of them are also symmetrical with respect to the horizontal axis.

- For coordinates (5, 1) and (125, 1), the centered coordinates are (4, 0) and (-4, 0), which have a phase π. The plots of them are the same.
- The frequency will be higher, and the wave length will be shorter, when the distance between centered coordinates and origin are longer.
- No matter where is coordinate is, magnitude plot will always be the same.

Question 2: Explain how a position (p, q) in the Fourier domain will be projected as a sine wave in the spatial domain. Illustrate with a MATLAB figure.

Answers:

Intuitively, in the plot of $\hat{F}$, coordinate (u, v) in Fourier domain means the frequency of the origin figure in horizontal direction is u and the frequency in vertical direction is v. The figure in spatial domain should be a sine wave in the direction of (u, v).

The mathematical proof is shown below.

According to the inversion theorem,

$$\mathrm{F(x)} = F_D^{-1}(\hat{F})(x) = \frac{1}{N} \sum_{u \in [0 \ldots N-1]^2} \hat{F}(u) e^{\frac{2\pi i u^T x}{N}}$$

According to Euler's law, new equation can be derived from it,

$$\mathrm{F(x)} = \frac{1}{N} \sum_{u \in [0 \ldots N-1]^2} \hat{F}(u) \left(\cos\left(\frac{2\pi u^T x}{N}\right) + \mathrm{i} \cdot \sin\left(\frac{2\pi u^T x}{N}\right) \right)$$

Note: In the equations above, u represents $\begin{bmatrix} u \\ v \end{bmatrix}$ and x represents $\begin{bmatrix} x \\ y \end{bmatrix}$.

From the equation above, we can see that the real and the imaginary part in spatial domain are sine wave.

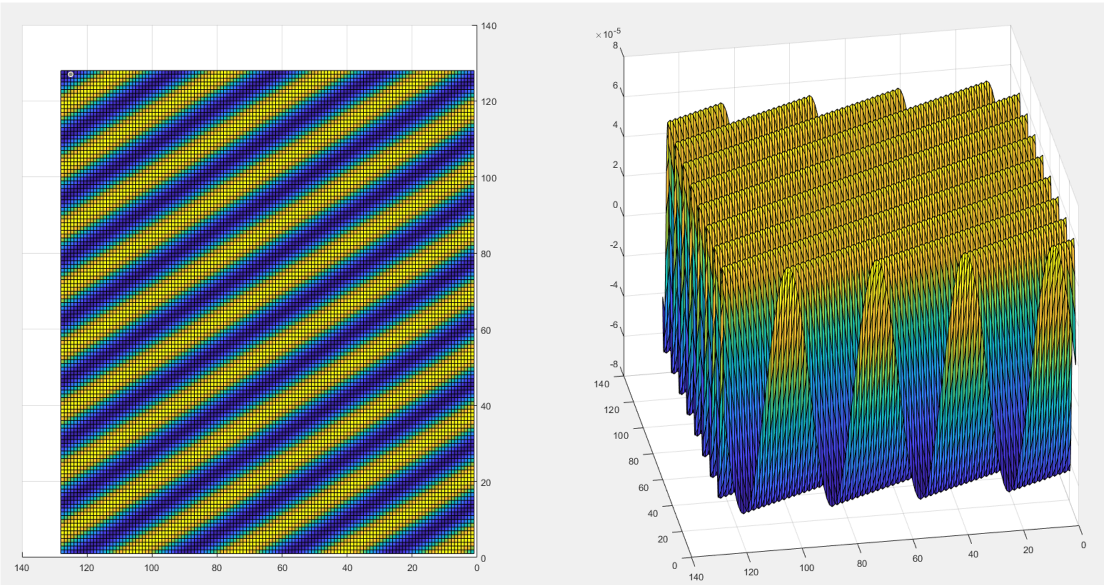

Figure 2. The surface of real(F) with coordinate (5, 9)

Question 3: How large is the amplitude? Write down the expression derived from Equation (4) in the notes. Complement the code (variable amplitude) accordingly.

Answers:

According to the equation in Question 2,

$$\mathrm{F(x)} = \frac{1}{N} \sum_{u \in [0 \ldots N-1]^2} \hat{F}(u) \left(\cos\left(\frac{2\pi u^T x}{N}\right) + \mathrm{i} \cdot \sin\left(\frac{2\pi u^T x}{N}\right) \right)$$

The amplitude is $\frac{1}{N} \max\left(\hat{F}(u)\right)$ according to the theorem.

However, in this case, the amplitude should be $\frac{1}{N^2} \max\left(\hat{F}(u)\right)$ due to MATLAB calculate inverse Fourier transform with a factor $\frac{1}{N^2}$.

Because $\max\left(\hat{F}(u)\right) = 1$, the amplitude is $\frac{1}{N^2}$.

Question 4: How does the direction and length of the sine wave depend on p and q? Write down the explicit expression that can be found in the lecture notes. Complement the code (variable wavelength) accordingly.

Answers:

It said in the lecture notes that the wavelength of the sinusoid is:

$$\lambda = \frac{1}{\sqrt{u^2 + v^2}} = \frac{2\pi}{\sqrt{{\omega_1}^2 + {\omega_2}^2}}$$

where (u, v) are the frequencies along (r, c) and the periods are $\frac{1}{u}$ and $\frac{1}{v}$.

In this case,

$$\omega_1 = 2\pi\frac{u}{N}, \omega_2 = 2\pi\frac{v}{N}$$

The wavelength will be calculated as:

$$\lambda = \frac{2\pi}{\sqrt{{\omega_1}^2 + {\omega_2}^2}}$$

It is illustrated that the wavelength will be shorter when the distance between (u, v) and (0, 0) gets longer.

The line consisted by points which can make $u^T x = 0$ is perpendicular with the direction of sine waves, which means $ux + vy = 0$ or $y = -\frac{u}{v}x$.

Take the perpendicular line, the direction is $y = \frac{v}{u}x$.

Question 5: What happens when we pass the point in the center and either p or q exceeds half the image size? Explain and illustrate graphically with MATLAB!

Answers:

Some tests are shown as below.

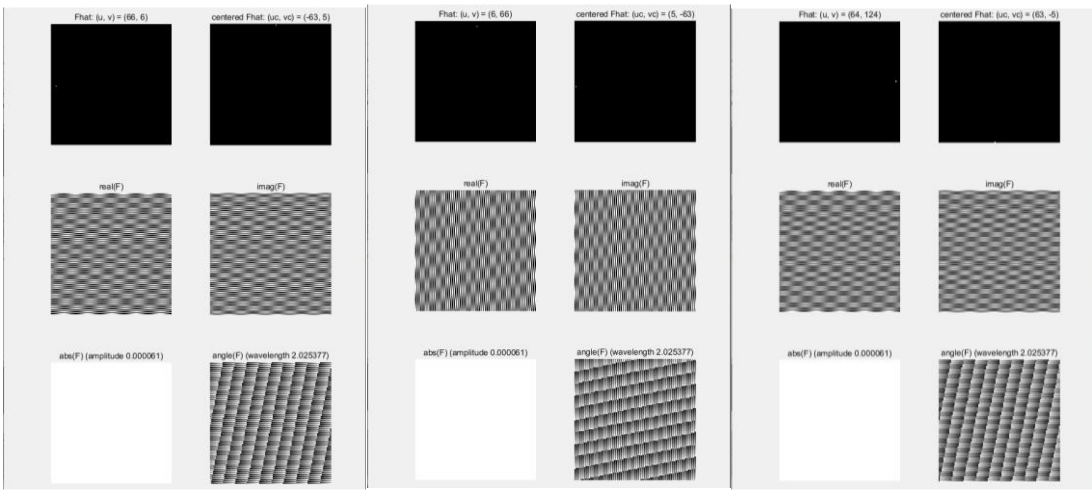

Figure 3. Some tests for specific coordinates

The shift procedure turns the range of coordinate $[0, size - 1]$ into $\left[-\frac{size}{2}, \frac{size}{2} - 1\right]$.

Due to the first index of list is 1 in MATLAB, if p or q exceeds half the image size, the coordinate will be calculated as:

$$x = x - 1 - size$$

Question 6: What is the purpose of the instructions following the question *What is done by these instructions?* in the code?

Answers:

The purpose of the codes is to calculate the centered coordinate (uc, vc), which is centered by shifting 1st and the 3rd quadrant, 2nd and the 4th quadrant.

It turns the range of coordinate $[0, size-1]$ into $\left[-\frac{size}{2}, \frac{size}{2}-1\right]$.

1.4 Linearity

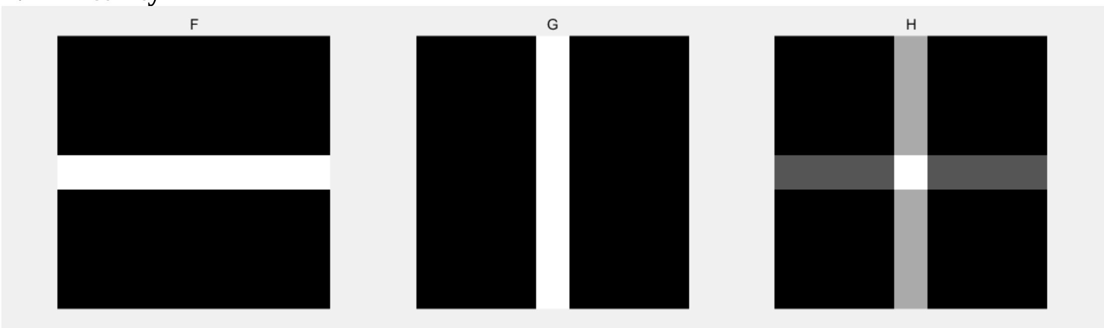

Figure 4-1. The Origin Figures of F, G and H

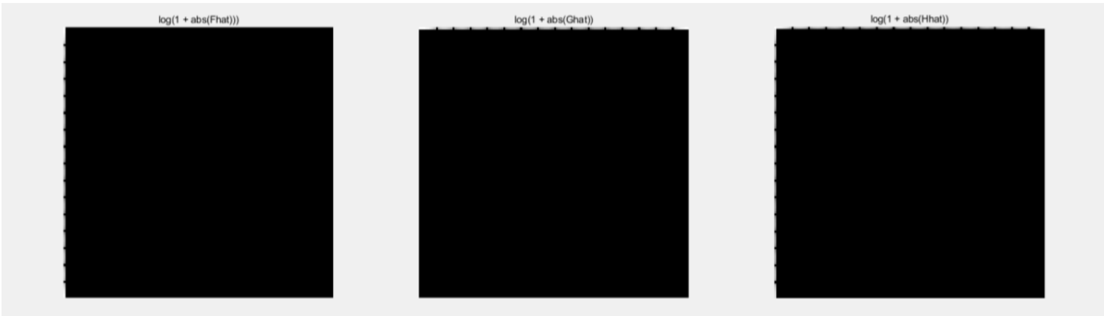

Figure 4-2 The Fourier Spectrums of F, G and H

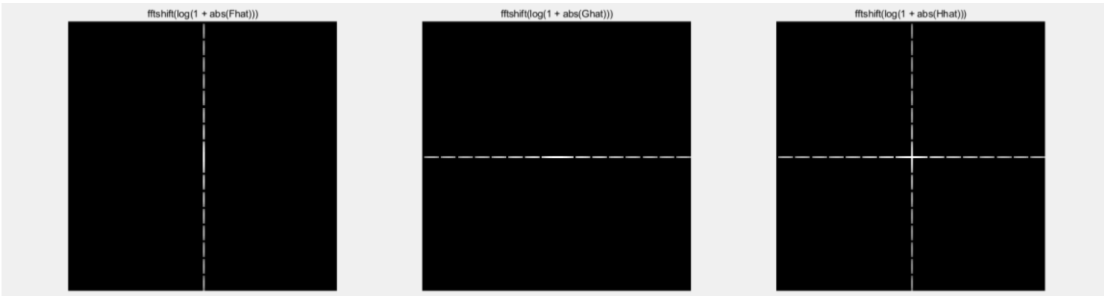

Figure 4-3 Centered Fourier Spectrums of F, G and H

Question 7: Why are these Fourier spectra concentrated to the borders of the images? Can you give a mathematical interpretation? Hint: think of the frequencies in the source image and consider the resulting image as a Fourier transform applied to a 2D function. It might be easier to analyze each dimension separately!

Answers:

The uncentered spectrums' origin is at the top-left corner. F has no frequency along horizontal direction, therefore x in the spectrum is always 0. That is the reason why the spectrum of F is the line on the left.

The mathematical explanation is shown below:

The Fourier transform of a quadratic image is

$$\hat{F}(\mathrm{u}) = F_D(F)(u) = \frac{1}{N}\sum_{u\in[0\ldots N-1]^2} F(x)e^{\frac{-2\pi i u^T x}{N}}$$

Note: In the equation above, u represents $\begin{bmatrix} u \\ v \end{bmatrix}$ and x represents $\begin{bmatrix} x \\ y \end{bmatrix}$.

Consider F as an example,

$$\hat{F}(\mathrm{u}) = \frac{1}{128}\left(\sum_{x=56}^{71} e^{\frac{-2\pi i ux}{128}}\right)\left(\sum_{y=0}^{127} e^{\frac{-2\pi i ux}{128}}\right)$$

According to Dirac delta function,

$$\delta(\mathrm{u}) = \sum_{x=0}^{N-1} e^{\frac{-2\pi iux}{N}} = \begin{cases} 1, u = 0 \\ 0, u \neq 0 \end{cases}$$

That is the reason why Fourier spectra concentrated to the borders.

Question 8: Why is the logarithm function applied?

Answers:

The absolute values of Fourier spectra are quite small and near to each other. Using logarithm functions can make the spectra more distinguishable.

	1	2	3
1	2.0480e+03 + 0.0000e+00i	0.0000 + 0.0000i	0.0000 + 0.0000i
2	-1.9954e+03 - 4.8983e+01i	0.0000 + 0.0000i	0.0000 + 0.0000i
3	1.8424e+03 + 9.0510e+01i	0.0000 + 0.0000i	0.0000 + 0.0000i
4	-1.6032e+03 - 1.1826e+02i	0.0000 + 0.0000i	0.0000 + 0.0000i
5	1.2996e+03 + 1.2800e+02i	0.0000 + 0.0000i	0.0000 + 0.0000i
6	-9.5880e+02 - 1.1826e+02i	0.0000 + 0.0000i	0.0000 + 0.0000i
7	6.1017e+02 + 9.0510e+01i	0.0000 + 0.0000i	0.0000 + 0.0000i
8	-2.8230e+02 - 4.8983e+01i	0.0000 + 0.0000i	0.0000 + 0.0000i
9	0.0000 + 0.0000i	0.0000 + 0.0000i	0.0000 + 0.0000i
10	2.1813e+02 + 4.8983e+01i	0.0000 + 0.0000i	0.0000 + 0.0000i
11	-3.6133e+02 - 9.0510e+01i	0.0000 + 0.0000i	0.0000 + 0.0000i
12	4.2732e+02 + 1.1826e+02i	0.0000 + 0.0000i	0.0000 + 0.0000i
13	-4.2196e+02 - 1.2800e+02i	0.0000 + 0.0000i	0.0000 + 0.0000i
14	3.5797e+02 + 1.1826e+02i	0.0000 + 0.0000i	0.0000 + 0.0000i
15	-2.5296e+02 - 9.0510e+01i	0.0000 + 0.0000i	0.0000 + 0.0000i
16	1.2698e+02 + 4.8983e+01i	0.0000 + 0.0000i	0.0000 + 0.0000i
17	0.0000 + 0.0000i	0.0000 + 0.0000i	0.0000 + 0.0000i
18	-1.1050e+02 - 4.8983e+01i	0.0000 + 0.0000i	0.0000 + 0.0000i
19	1.9137e+02 + 9.0510e+01i	0.0000 + 0.0000i	0.0000 + 0.0000i
20	-2.3494e+02 - 1.1826e+02i	0.0000 + 0.0000i	0.0000 + 0.0000i
21	2.3947e+02 + 1.2800e+02i	0.0000 + 0.0000i	0.0000 + 0.0000i

Figure 5-1. Fhat Values

	1	2	3	4	5
1	7.6251	0	0	0	0
2	7.5994	0	0	0	0
3	7.5206	0	0	0	0
4	7.3831	0	0	0	0
5	7.1754	0	0	0	0
6	6.8743	0	0	0	0
7	6.4262	0	0	0	0
8	5.6613	0	0	0	0
9	0	0	0	0	0
10	5.4142	0	0	0	0
11	5.9229	0	0	0	0
12	6.0967	0	0	0	0
13	6.0912	0	0	0	0
14	5.9349	0	0	0	0
15	5.5972	0	0	0	0
16	4.9207	0	0	0	0
17	0	0	0	0	0
18	4.8030	0	0	0	0
19	5.3598	0	0	0	0
20	5.5760	0	0	0	0
21	5.6078	0	0	0	0

Figure 5-2. log(1 + abs(Fhat)) Values

Question 9: What conclusions can be drawn regarding linearity? From your observations can you derive a mathematical expression in the general case?

Answers:

From the matrixes, we can know that:

$$\hat{H}(u,v) = \mathcal{F}\big(\mathrm{F}(x,y) + 2G(x,y)\big) = \hat{F}(x,y) + 2\hat{G}(x,y)$$

More generally, both Fourier transform and inverse Fourier transform have homogeneity and additivity:

$$\mathcal{F}(\mathrm{a}F_1 + bF_2) = \mathrm{a}\mathcal{F}(F_1) + \mathrm{b}\mathcal{F}(F_2)$$
$$\mathcal{F}^{-1}\big(\mathrm{a}\mathcal{F}(F_1) + \mathrm{b}\mathcal{F}(F_2)\big) = \mathrm{a}F_1 + bF_2$$

1.5 Multiplication

Given the frequency spectrum FREQSPEC showfs(FREQSPEC, RES, FOURSPECMAX) displays a compressed version of the corresponding Fourier spectrum (magnitude) as a gray-level image.

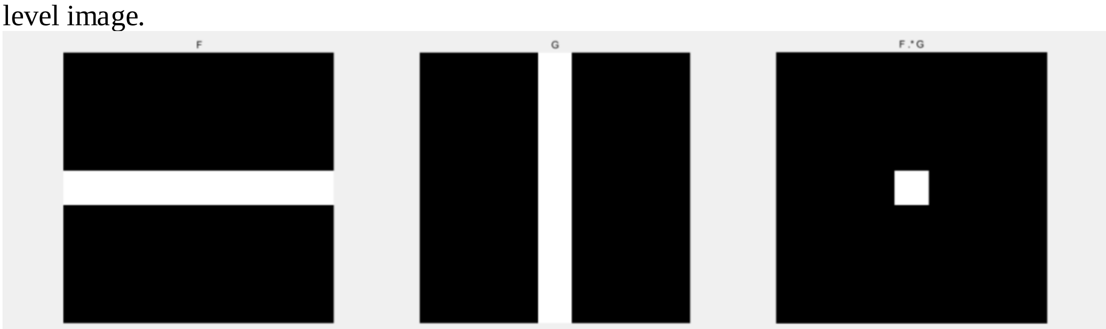

Figure 6-1. The Origin Figures of F, G and F . G*

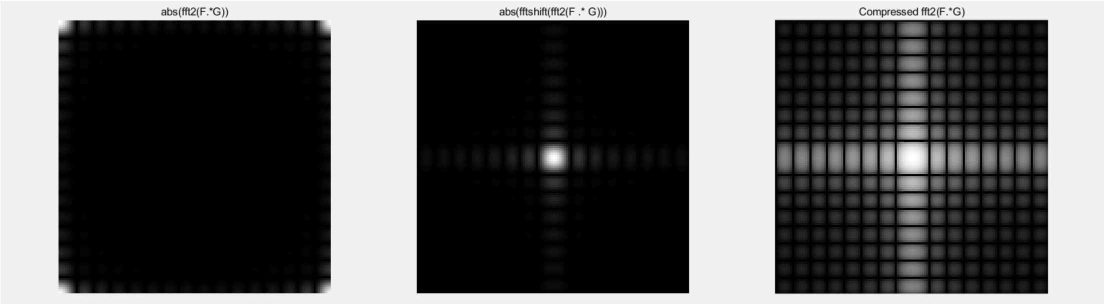

Figure 6-2. Fourier Spectrum, Centered Spectrum and Compressed Spectrum

Question 10: Are there any other ways to compute the last image? Remember what multiplication in Fourier domain equals to in the spatial domain! Perform these alternative computations in practice.

Answers:

We can use convolution to calculate the last image.

Convolution in Fourier domain equals to multiplication in spatial domain.

The results are shown below.

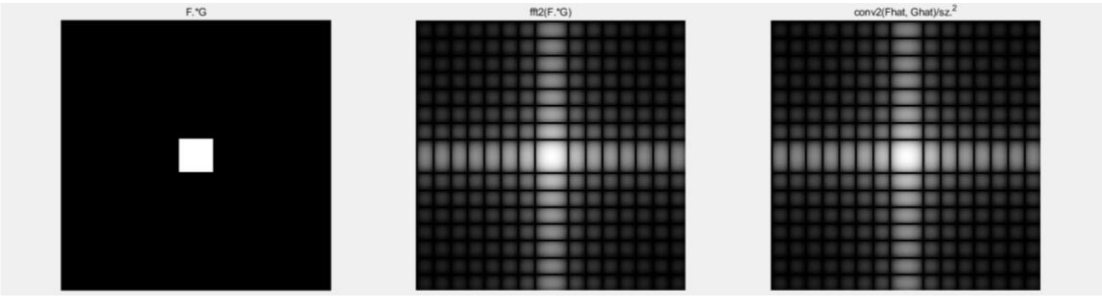

Figure 7. The Spectrum of Multiplication and the Convolution of Spectrum

Reference:

https://ww2.mathworks.cn/matlabcentral/answers/87413-how-do-i-have-to-normalize-fft-and-ifft-when-i-am-using-the-convolution-theorem

1.6 Scaling

The results are shown as below (Figure 8).

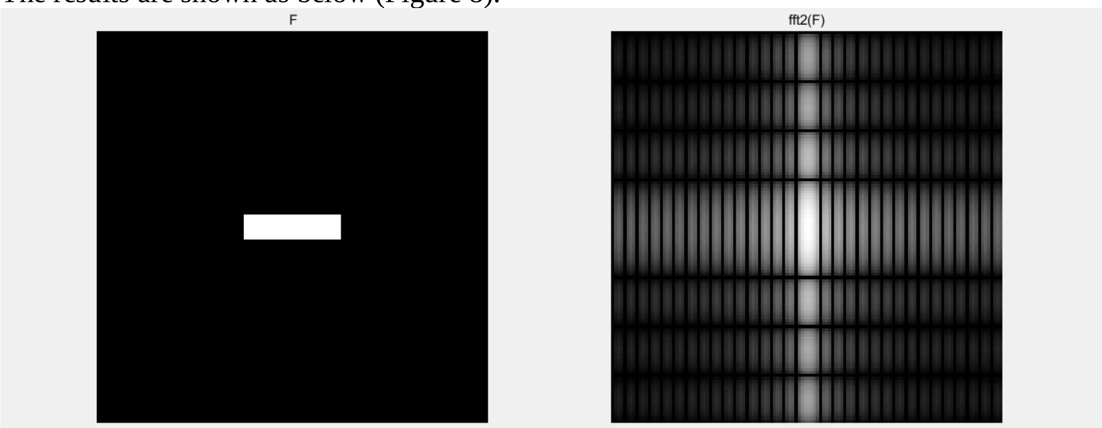

Figure 8. The Spectrum after Scaling

Question 11: What conclusions can be drawn from comparing the results with those in the previous exercise? See how the source images have changed and analyze the effects of scaling.

Answers:

The former one has a 16×16 square in the middle, and this one has a 8×32 rectangle in the middle, which means

$$x_{new} = 2x_{old}, y_{new} = \frac{1}{2} y_{old}$$

According to

$$\hat{F}(\mathrm{u}) = F_D(F)(u) = \frac{1}{N} \sum_{u \in [0 \ldots N-1]^2} F(x) e^{\frac{-2\pi i u^T x}{N}}$$

Note: In the equation above, u represents $\begin{bmatrix} u \\ v \end{bmatrix}$ and x represents $\begin{bmatrix} x \\ y \end{bmatrix}$.

The spectra points which has the same value of $u^T x$ will have the same value of $\hat{F}$. Therefore,

$$u_{new} = \frac{1}{2} u_{old}, v_{new} = 2v_{old}$$

1.7 Rotation

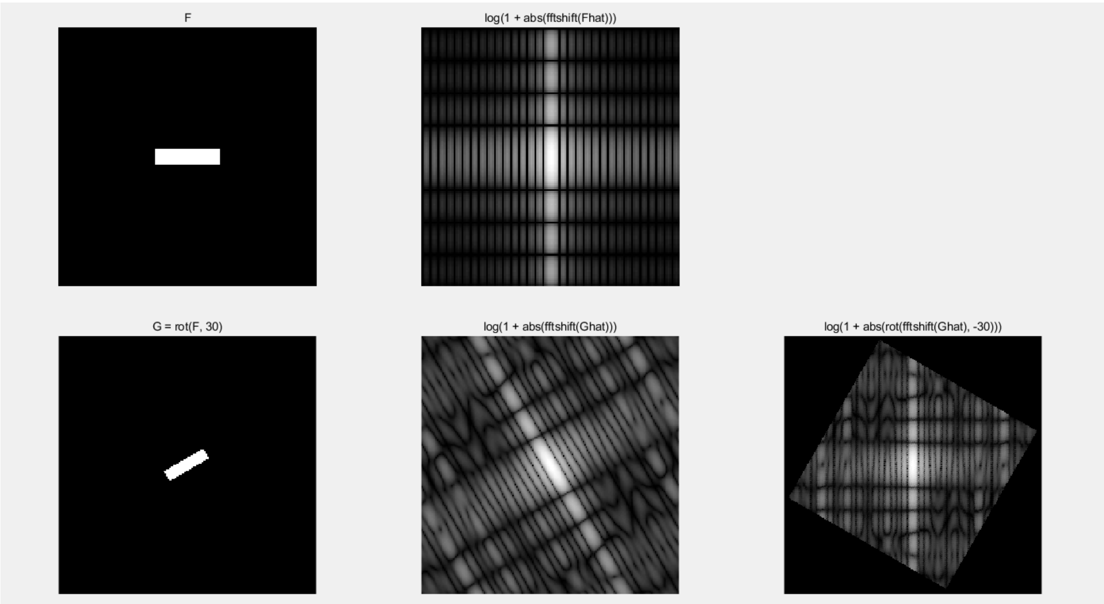

Figure 9-1. The Spectrum after Rotated 30°

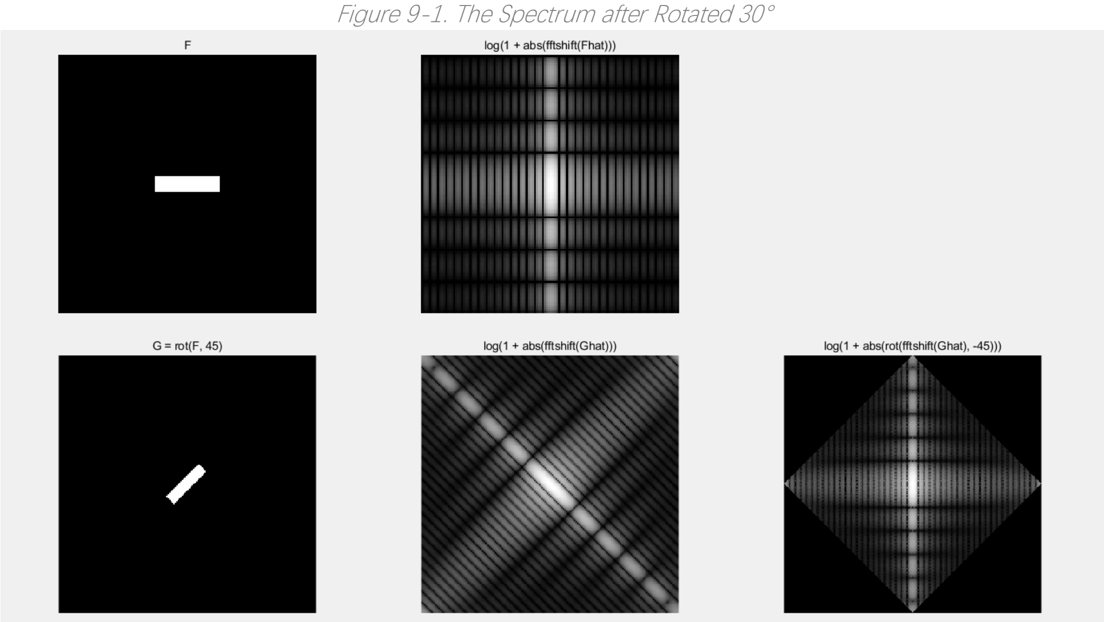

Figure 9-2. The Spectrum after Rotated 45°

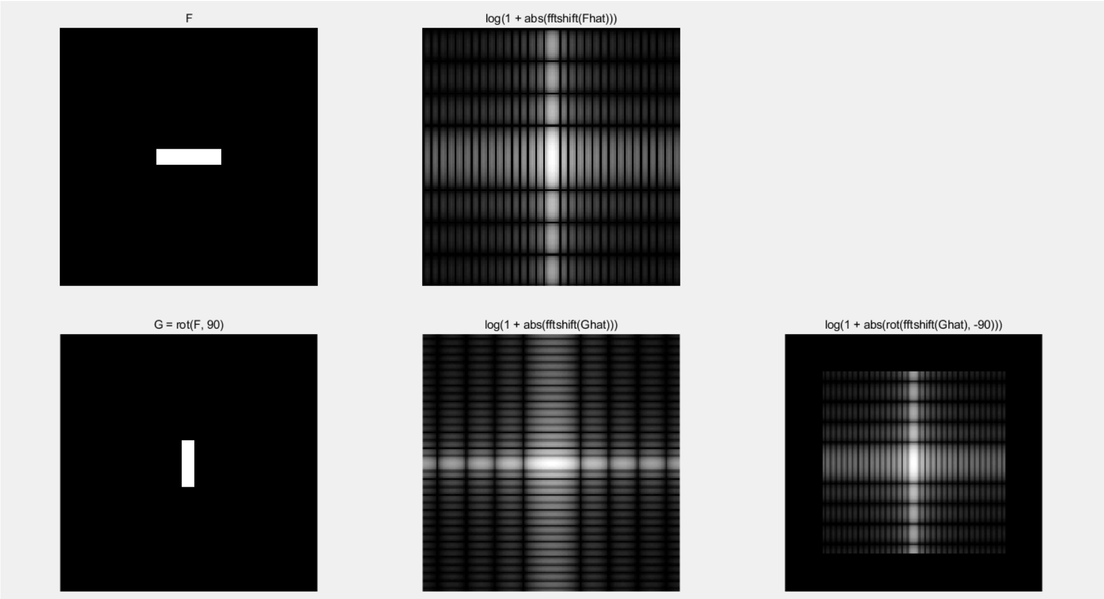

Figure 9-3. The Spectrum after Rotated 90°

Question 12: What can be said about possible similarities and differences? Hint: think of the frequencies and how they are affected by the rotation.

Answers:

The Fourier spectrum of F and the spectrum after rotated, Fourier transform and rotated back has the same structure.

We know that

$$\begin{bmatrix} x_{rotated} \\ y_{rotated} \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

According to

$$\hat{F}(\mathrm{u,v}) = \frac{1}{N} \sum_{u\in[0\ldots N-1]^2} F(x,y) e^{\frac{-2\pi i(ux+vy)}{N}}$$

We can get

$$\hat{F}(\mathrm{u}_r, v_r)$$

$$= \frac{1}{N} \sum_{u\in[0\ldots N-1]^2} F(x\cos\theta - y\sin\theta\,, x\sin\theta + y\cos\theta) e^{\frac{-2\pi i(ux\cos\theta - uy\sin\theta + vx\sin\theta + vy\cos\theta)}{N}}$$

$$= \frac{1}{N} \sum_{u\in[0\ldots N-1]^2} F(\mathrm{x}_r, y_r) e^{\frac{-2\pi i(x(u\cos\theta + v\sin\theta) + y(v\cos\theta - u\sin\theta))}{N}}$$

Set

$$\begin{bmatrix} u_{rotated} \\ v_{rotated} \end{bmatrix} = \begin{bmatrix} u\cos\theta + v\sin\theta \\ -u\sin\theta + v\cos\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

Therefore, the rotation angle of the spectrum is the same as the rotation angle of the origin image + (π – θ).

1.8 Information in Fourier Phase and Magnitude

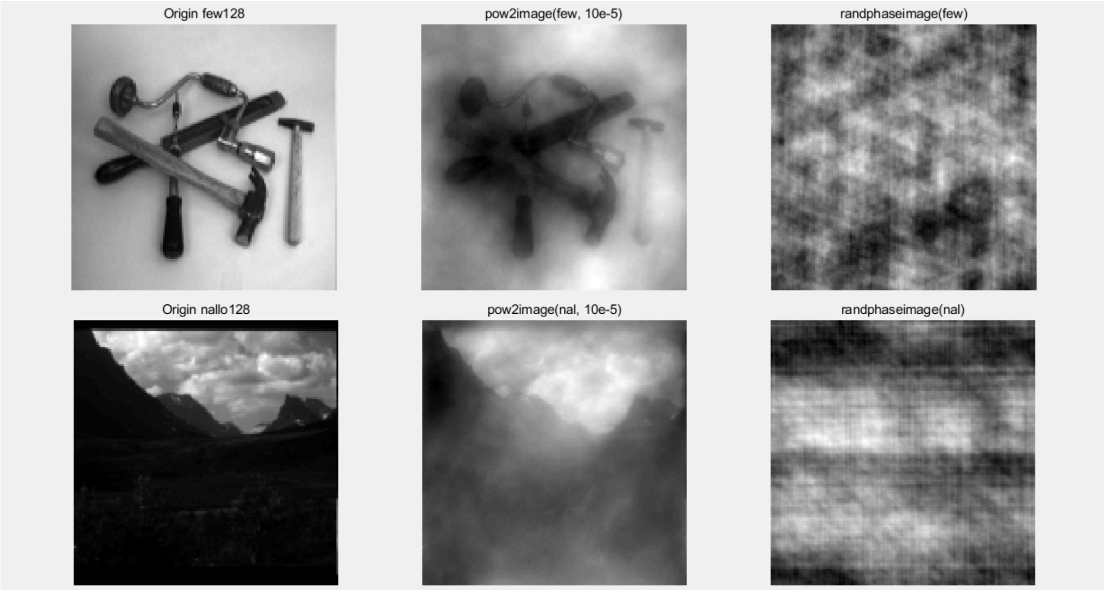

Figure 10. Original Image, Fourier Magnitude and Another with Random Phase

Question 13: What information is contained in the phase and in the magnitude of the Fourier transform?

Answers:

The magnitude contains information of pixel value or brightness.

The phase in Fourier domain contains the information of pixels' locations.

As shown in Figure 10, when the phase is randomly picked, the pixels' location will be shifted.

Question 14: Show the impulse response and variance for the above-mentioned t-values. What are the variances of your discretized Gaussian kernel for t = 0.1, 0.3, 1.0, 10.0 and 100.0?

Answers:

The impulse responses are shown as below.

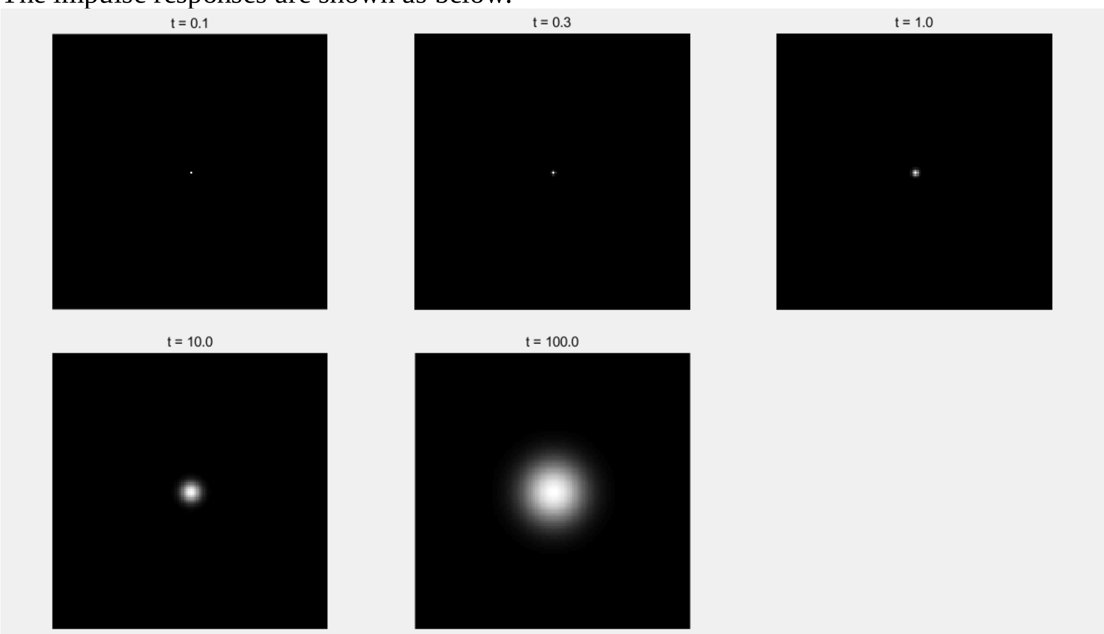

Figure 11-1. Impulse Response for Different t-values

The variances are shown as below

	1
1	[0.0133,1.9559e-14;1.9559e-14,0.0133]
2	[0.2811,1.7596e-15;1.7596e-15,0.2811]
3	[1.0000,2.1444e-15;2.1444e-15,1.0000]
4	[10.0000,1.0679e-15;1.0679e-15,10.0000]
5	[100.0000,8.6991e-17;8.6991e-17,100.0000]

Figure 11-2. Variance for Different t-values

According to the result,

$$\mathrm{T} \approx \mathrm{covarience}$$

Question 15: Are the results different from or similar to the estimated variance? How does the result correspond to the ideal continuous case? Lead: think of the relation between spatial and Fourier domains for different values of t.

Answers:

t	Estimated variances	Results
0.1	$\begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$	$\begin{bmatrix} 0.0133 & 1.9559 \times 10^{-14} \\ 1.9559 \times 10^{-14} & 0.1 \end{bmatrix}$
0.3	$\begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}$	$\begin{bmatrix} 0.2811 & 1.7596 \times 10^{-15} \\ 1.7596 \times 10^{-15} & 0.2811 \end{bmatrix}$
1.0	$\begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \end{bmatrix}$	$\begin{bmatrix} 1.0000 & 2.1444 \times 10^{-15} \\ 2.1444 \times 10^{-15} & 1.0000 \end{bmatrix}$
10.0	$\begin{bmatrix} 10.0 & 0 \\ 0 & 10.0 \end{bmatrix}$	$\begin{bmatrix} 10.0000 & 1.0679 \times 10^{-15} \\ 1.0679 \times 10^{-15} & 10.0000 \end{bmatrix}$
100.0	$\begin{bmatrix} 100.0 & 0 \\ 0 & 100.0 \end{bmatrix}$	$\begin{bmatrix} 100.0000 & 8.6991 \times 10^{-17} \\ 8.6991 \times 10^{-17} & 100.0000 \end{bmatrix}$

Chart1. The Estimated Variance and Results

Ideal variances in continuous case and the results are shown above.

After comparing the contents above, we found that there are differences when $\mathrm{t} < 1$. The differences get larger when t get nearer to 0.

When $\mathrm{t} \geq 1$, the result are basically the same with the estimated variance. (Some numbers are small enough to be ignored).

Question 16: Convolve a couple of images with Gaussian functions of different variances (like t = 1.0, 4.0, 16.0, 64.0 and 256.0) and present your results. What effects can you observe?

Answers:

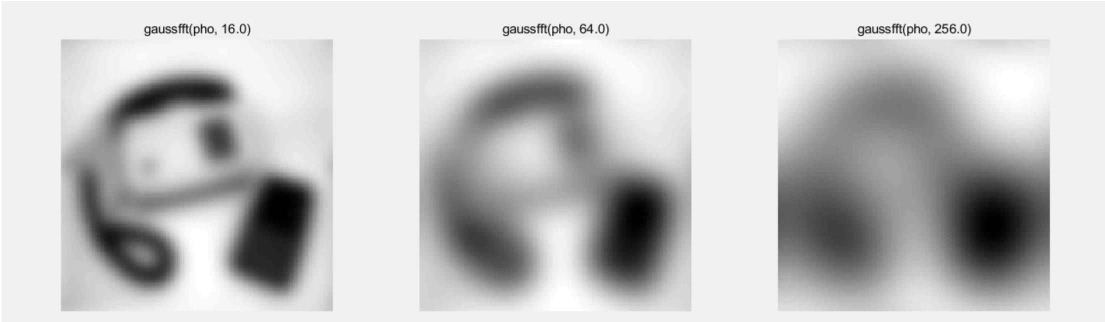

Figure 12-1. phonecalc128 and Its Convolved Image

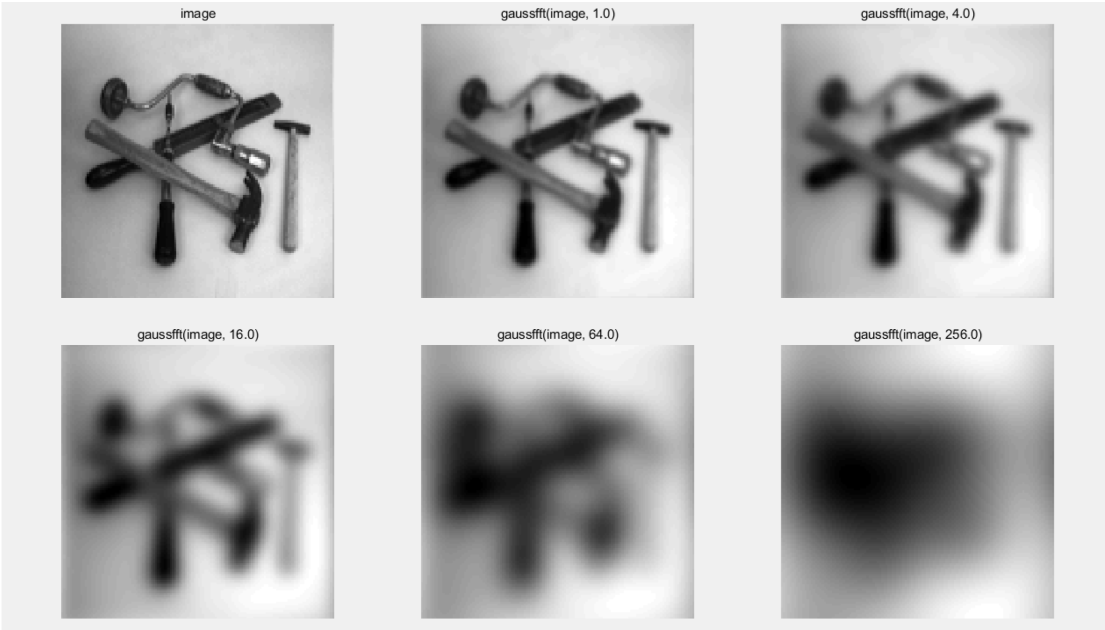

Figure 12-2. vew128 and Its Convolved Image

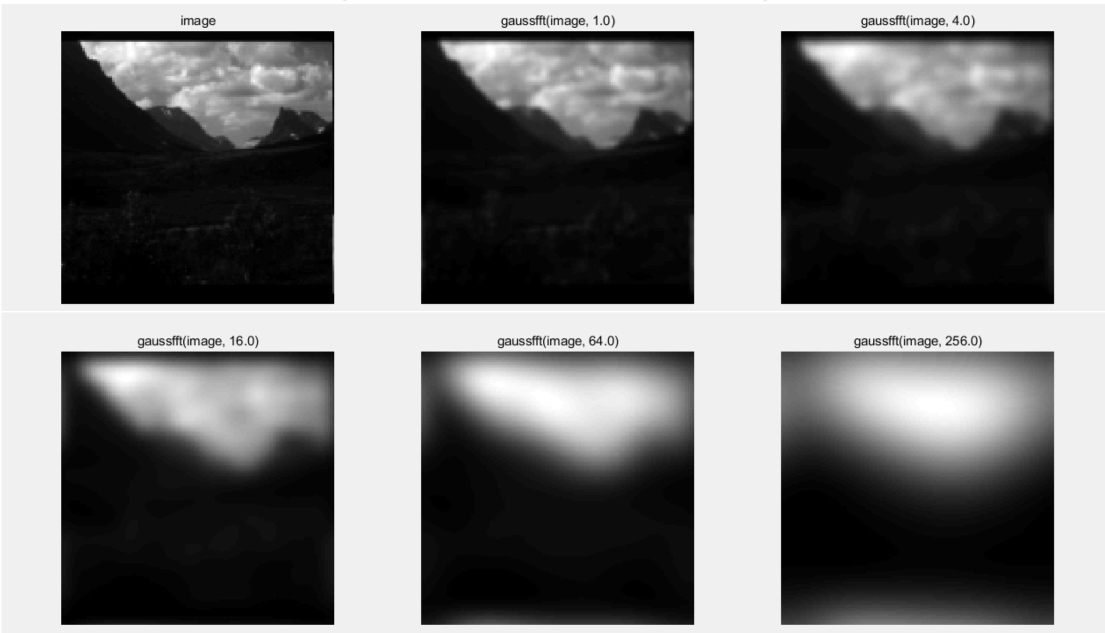

Figure 12-3. nallo128 and Its Convolved Image

As shown in the instances above, the images become blurrier when t increases.
A small t value only removes the highest frequency and the image remains sharp.
A large t value has a smaller cut-off frequency, more information in high frequency which contains detail is lost. Only information with low frequency which is the outline remains.

3.1 Smoothing of noisy data

Figure 13-1. The Original and Noised Figures

Function gaussnoise (INPIC, SDEV, ZMIN, ZMAX) adds white (uncorrelated) Gaussian noise with standard deviation SDEV to INPIC. If the arguments ZMIN and ZMAX are specified, the output values are truncated to the range [ZMIN, ZMAX].

Function sapnoise (INPIC, FRAC, ZMIN, ZMAX) adds salt and pepper noise to an image by resetting a fraction FRAC/2 of the pixels to ZMIN and a similar fraction to ZMAX in a pixel-to-pixel independent manner.

Figure 14-1. Reduce Gaussian Noise with Gaussian Smoothing

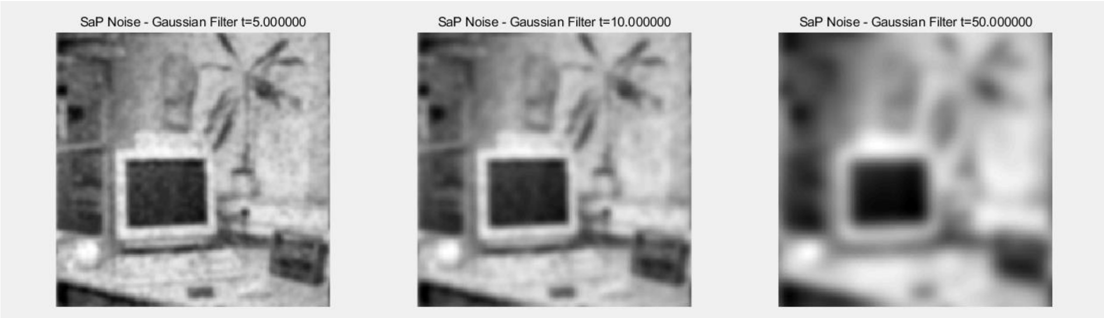

Figure 14-2. Reduce Salt and Pepper Noise with Gaussian Smoothing

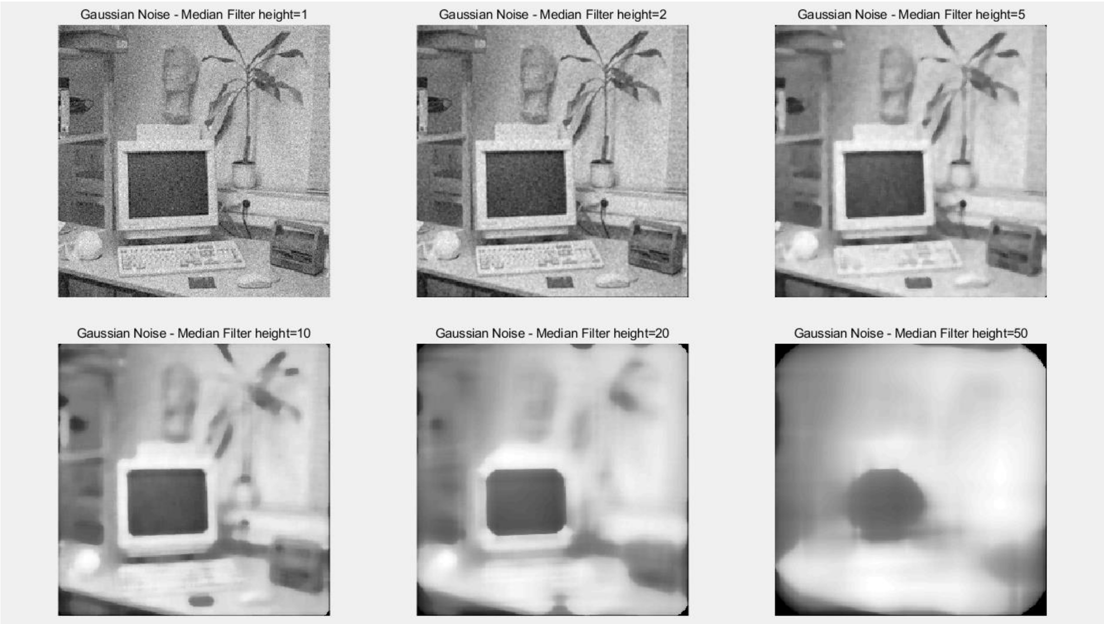

Figure 15-1. Reduce Gaussian Noise with Median Filtering

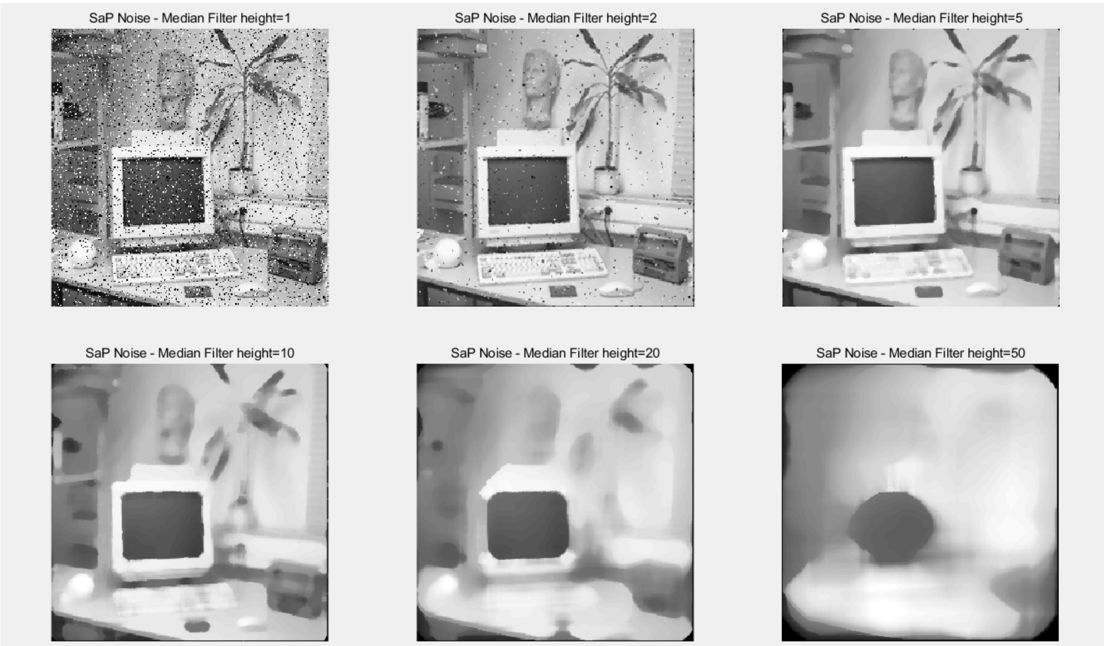

Figure 15-2. Reduce Salt and Pepper Noise with Median Filtering

Function ideal(IMAGE, CUTOFF, TYPE) filters the image IMAGE with an ideal high-pass or low-pass filter with cut-off frequency CUTOFF cycles per pixel and returns the resulting image FILTIM along with the modulation transfer function MTF.

The filter is high-pass or low-pass depending on whether TYPE = 'h' or TYPE = 'l' respectively. If TYPE is omitted, it is set to 'l'.

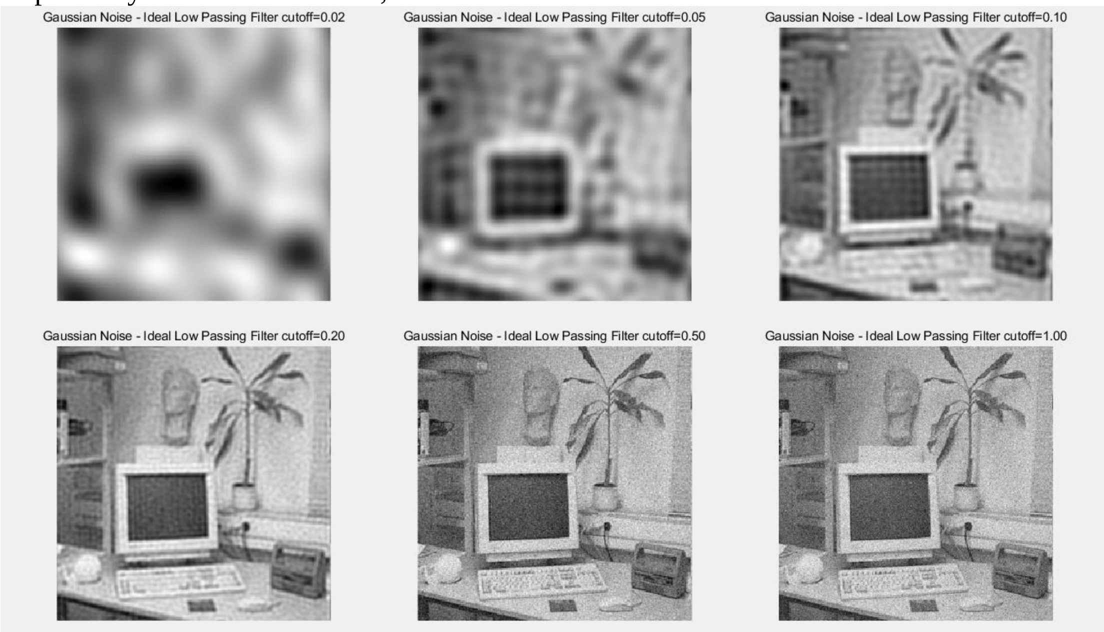

Figure 16-1. Reduce Gaussian Noise with Low Passing Filter

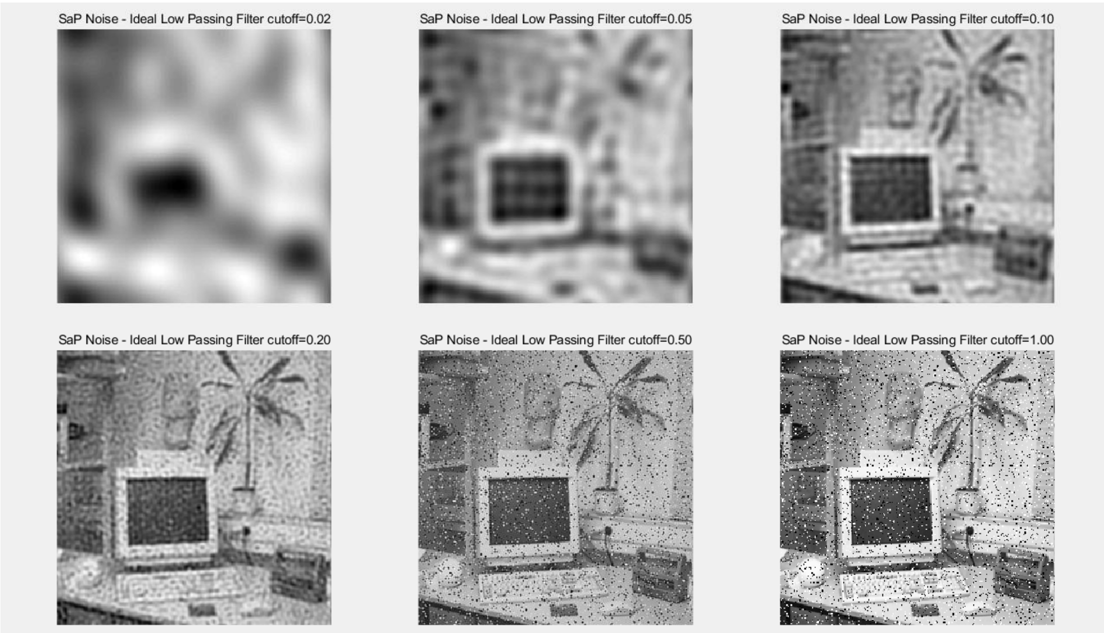

Figure 16-2. Reduce Salt and Pepper Noise with Low Passing Filter

Question 17: What are the positive and negative effects for each type of filter? Describe what you observe and name the effects that you recognize. How do the results depend on the filter parameters? Illustrate with MATLAB figure(s).

Answers:

The positive and negative effects for each type of filter:

Smoothing Method	Advantages	Disadvantages
Gaussian Smoothing	1. Very effective to remove Gaussian noise 2. Computationally efficient 3. Rotationally symmetric	1. Takes much time 2. Reduce details
Median Filter	1. No need to generate new pixel	1. Good at removing salt and

	value 2. Easy to implement 3. Can remove extreme values more effectively due to less sensitive to extreme values	pepper noise, but not all types of noise
Ideal Low Passing Filter	1. Easy to implement and understand	1. Image distortion and frequency leakage 2. Poor performance due to less smooth

Chart 2-1. Comparison of Different Smoothing Method

Smoothing Method	Effects of Parameters	Similarities
Gaussian Smoothing	The larger the variance is, the smoother and blurrier the image is.	All the smoothing methods need a compromised parameter to get the best result.
Median Filter	With a larger height, the image gets smoother and blurrier and the feeling of daub gets stronger.	
Ideal Low Passing Filter	The images get smoother and blurrier with a smaller cut-off frequency.	

Chart 2-2. Effects of Parameters of Different Smoothing Method

Reference:

[A COMPARATIVE STUDY OF DIFFERENT NOISE FILTERING TECHNIQUES IN DIGITAL IMAGES. Sanjib Das, Jonti Saikia, Soumita Das and Nural Goni. International Journal of Engineering Research and General Science Volume, Issue, September - October 2015 ISSN 2091-2730]

Question 18: What conclusions can you draw from comparing the results of the respective methods?

Answers:

Gaussian convolution is the most effective method to remove Gaussian noise.

The performance of median filter to reduce different kinds of noise are almost the same. When the height is too large, there will be a strong feeling of daub.

There will be noticeable grids on images proceeded by ideal low pass filter. The reason is when applying the inversed Fourier transform to ideal low pass filter, there will be some waves on the spatial domain. (The 1D case is shown in Figure 17-1 and 17-2)

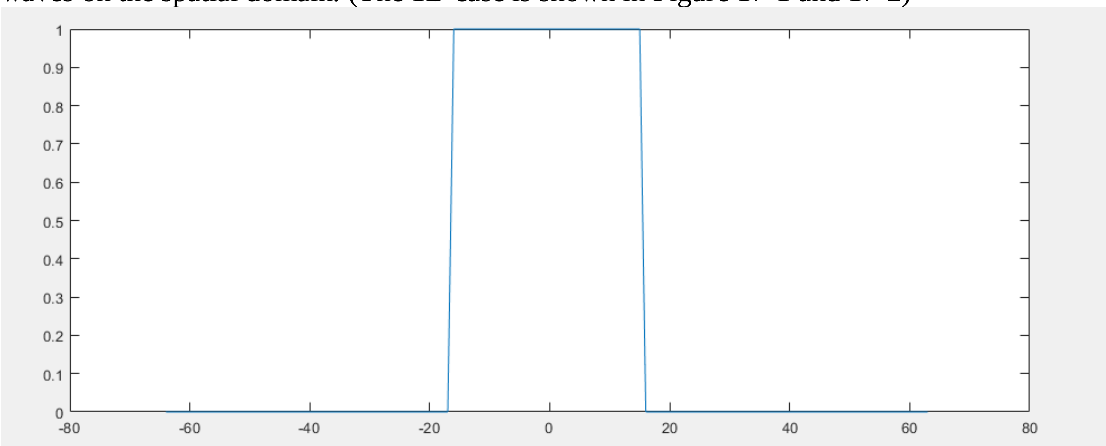

Figure 17-1. The Ideal Low Pass Filter on Fourier Domain

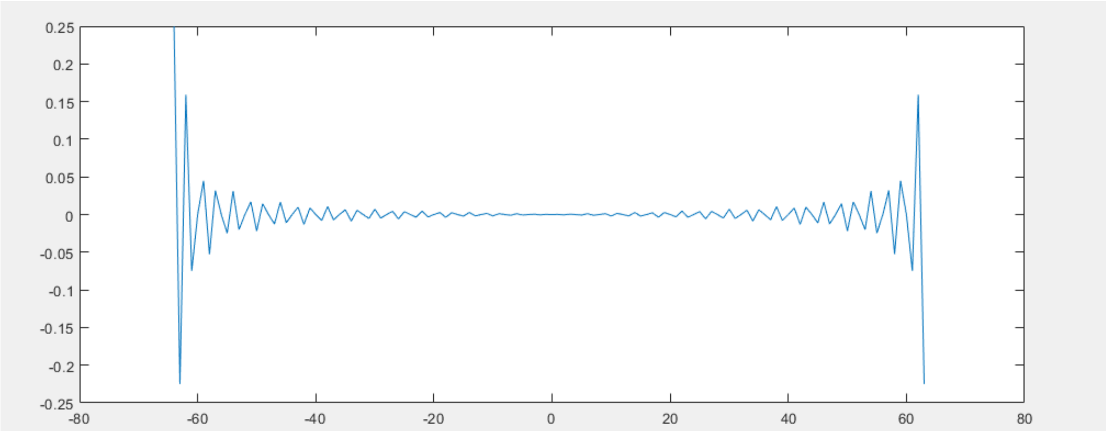

Figure 17-2. The Inversed Fourier Transform of Ideal Low Pass Filter

3.2 Smoothing and Subsampling

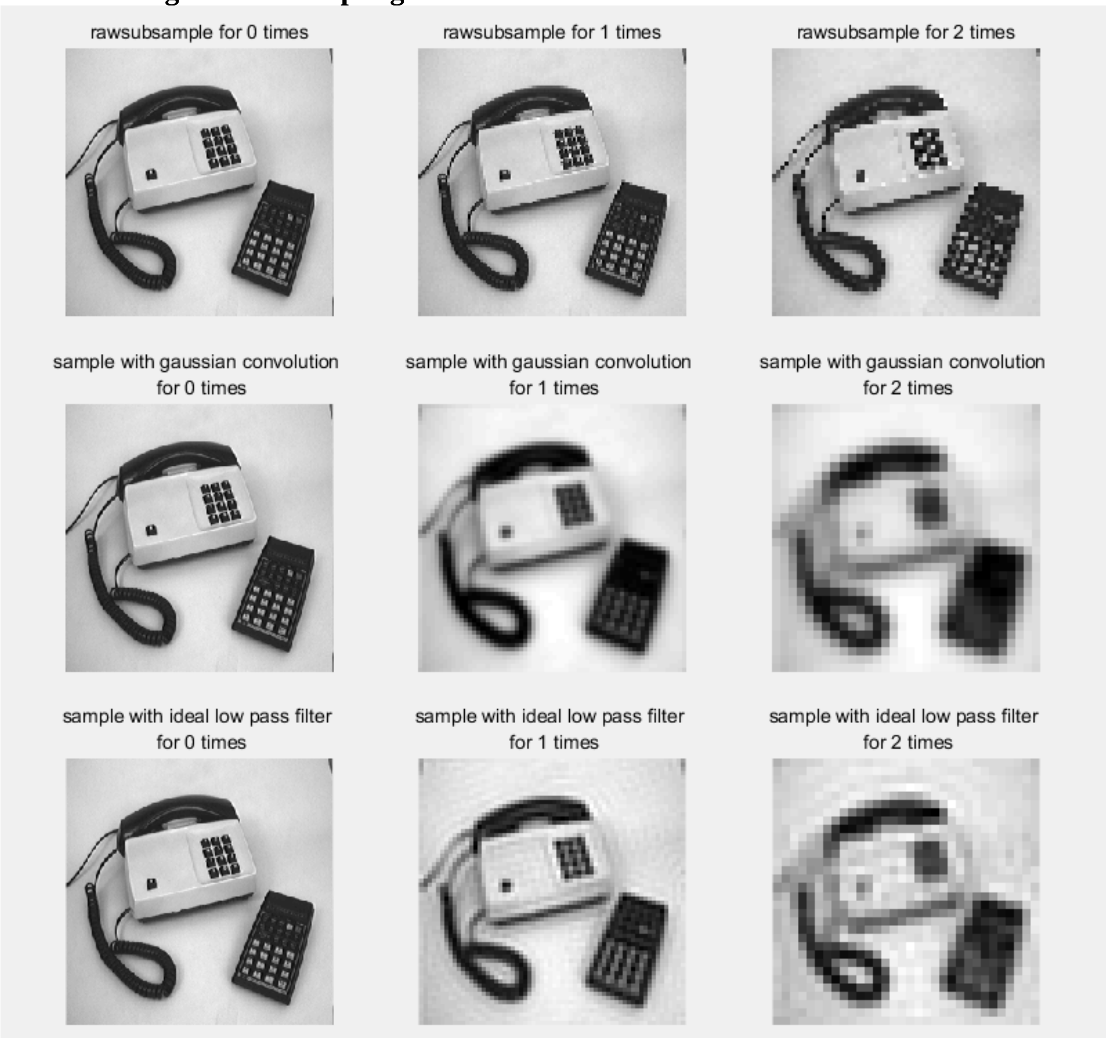

Figure 18-1. Subsampling with Different Methods

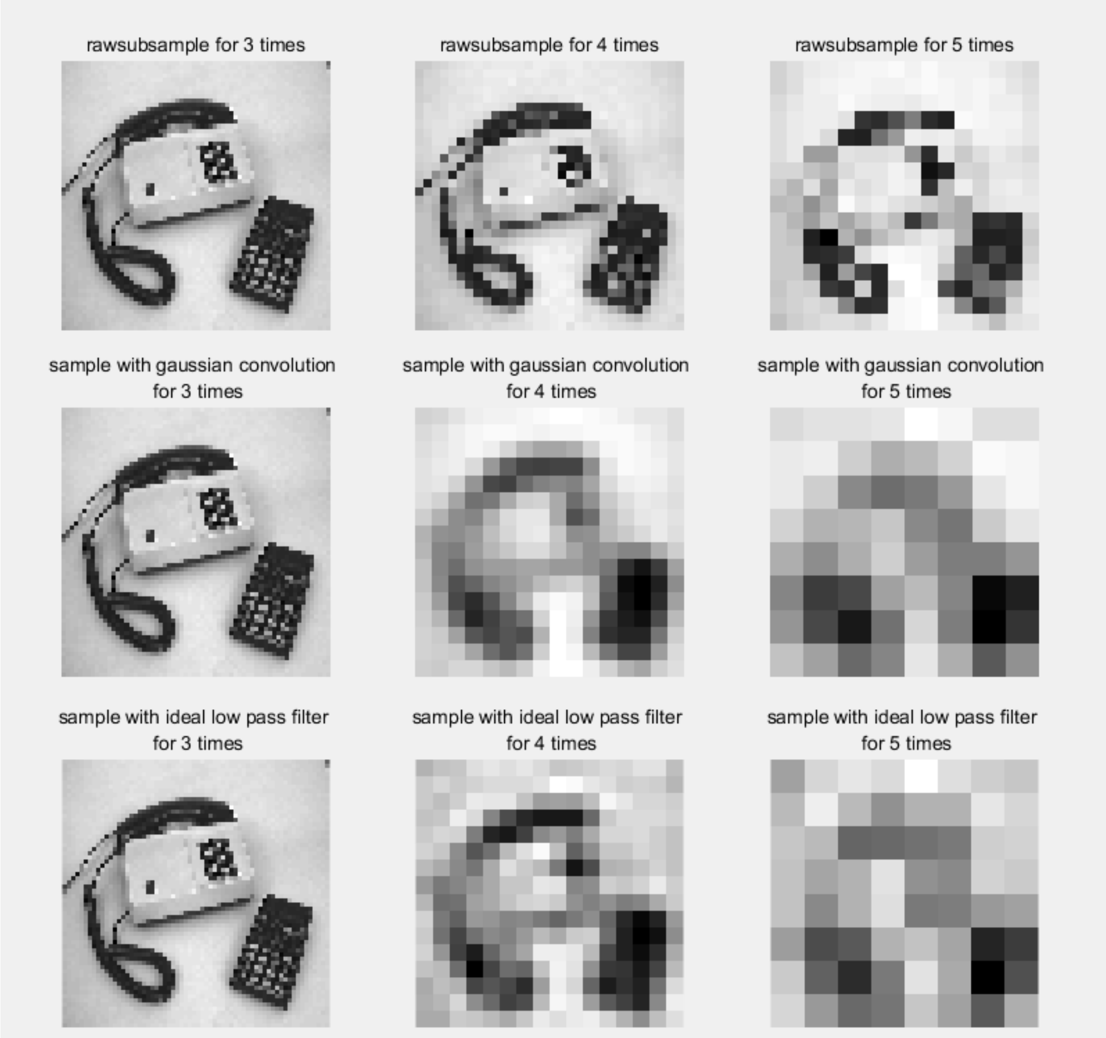

Figure 18-2. Subsampling with Different Methods

Question 19: What effects do you observe when subsampling the original image and the smoothed variants? Illustrate both filters with the best results found for iteration i = 4.

Answers:

The image subsampled from the origin image lost much information, but it is still sharp. Besides, there are distortions by the procedure.

Subsampling after Gaussian convolution makes it smoother.

Subsampling after ideal low pass filtering falls in between.

Question 20: What conclusions can you draw regarding the effects of smoothing when combined with subsampling? Hint: think in terms of frequencies and side effects.

Answers:

Subsampling is a procedure to depress the frequency of high frequency components, so it brings distortion or noise to the image.

Subsampling after smoothing can reduce the loss of frequency of high frequency sections, which means more information can be kept, especially the components with high frequency.