Homework 4

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Problem 1

$$P(open|u) = P(open|u, open)P(open) + P(open|u, closed)P(closed) = 1 \times 0.5 + 0.8 \times 0.5 = 0.9$$

Problem 2

at start

$Bel(x_0 = open) = 0.5 \quad Bel(x_0 = closed) = 0.5$

at time $t = 1$

sensor data $z_1 = open$, action $u_1 = do\ nothing$.

$$\begin{aligned} Bel(X_1 = open) &= \eta P(Z_1 = open|X_1 = open) \int Bel(x_0)P(X_1 = open|U_1 = do\ nothing, x_0)dx_0 \\ &= \eta P(Z_1 = open|X_1 = open)(Bel(X_0 = open)P(X_1 = open|U_1 = do\ nothing, X_0 = open) \\ &\quad + Bel(X_0 = closed)P(X_1 = open|U_1 = do\ nothing, X_0 = closed)) \\ &= \eta \cdot 0.8 \cdot (0.5 \cdot 1 + 0.5 \cdot 0) = 0.4\eta \end{aligned}$$

Similarly,

$$\begin{aligned} Bel(X_1 = closed) = \eta P(Z_1 = open|X_1 = closed)(Bel(X_0 = open)P(X_1 = closed|U_1 = do\ nothing, X_0 = open) \\ + Bel(X_0 = closed)P(X_1 = closed|U_1 = do\ nothing, X_0 = closed)) \\ = \eta \cdot 0.3 \cdot (0.5 \cdot 0 + 0.5 \cdot 1) = 0.15\eta \end{aligned}$$

$\eta = (0.4 + 0.15)^{-1} = \frac{1}{0.55}$, $Bel(X_1 = open) = 0.4/0.55 = 0.73, \quad Bel(X_1 = closed) = 0.15/0.55 = 0.27$

at time t = 2

$Bel(X_2 = open) = \eta \cdot 0.8(0.73 \cdot 1 + 0.27 \cdot 0.9) = 0.778\eta$

$Bel(X_2 = open) = \eta \cdot 0.3(0.73 \cdot 0 + 0.27 \cdot 0.1) = 0.778\eta = 0.008\eta$

$\eta = \frac{1}{0.786}$

Final distribution:

$Bel(X_2 = open) = 0.99 \quad Bel(X_2 = closed) = 0.01$

Problem 3

(1) Markov model

$$\begin{bmatrix} r1 \\ r2 \\ r3 \\ r4 \end{bmatrix} = \begin{bmatrix} 0.9 & 0 & 0 & 0.1 \\ 0 & 0.8 & 0.1 & 0.1 \\ 0 & 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.7 \end{bmatrix} \cdot \begin{bmatrix} r1' \\ r2' \\ r3' \\ r4' \end{bmatrix}$$

(2)

All rooms have probability of **0.25** at final stable state

(3)

let ***P(R$_i$)*** denote the probability of the robot being in room ***i***.

D: robot is going through a door

D$_{i,j}$: robot is going from room i to room j

$$P(D_{i,j}) = P(R_i) \cdot 0.1$$

$$P(D) = \sum_{(i,j)\in S} (P(D_{i,j}) + P(D_{j,i})) = 0.1 \cdot P(R_1) + 0.2 \cdot P(R_2) + 0.2 \cdot P(R_3) + 0.3 \cdot P(R_4) = \frac{1}{5}$$
$$where \quad S = \{(1,4),(2,4),(2,3),(3,4)\}$$

$$\text{Because } D_{1,4} \in D, \quad P(D_{1,4}, D) = P(D_{1,4}) = P(R_1) \cdot 0.1 = \frac{1}{40}$$

$$P(D_{1,4}|D) = \frac{P(D, D_{1,4})}{P(D)} = \frac{\frac{1}{40}}{\frac{1}{5}} = \frac{1}{8}$$

$$\text{Similarly, } P(D_{4,1}|D) = \frac{1}{8}$$

$$P(D_{1,4}, D_{4,1}|D) = P(D_{1,4}|D) + P(D_{4,1}|D) = \frac{1}{4}$$

The answer is $\frac{1}{4}$