Linear Control Systems

Lab Experiment 5: Performance of First order and second order systems

Objective: The objective of this exercise will be to study the performance characteristics of first and second order systems using MATLAB.

List of Equipment/Software

Following equipment/software is required:

- MATLAB

Category Soft-Experiment

Deliverables

A complete lab report including the following:

- Summarized learning outcomes.
- MATLAB scripts and their results for Exercise 1 & 2 should be reported properly.

Overview First Order Systems:
An electrical RC-circuit is the simplest example of a first order system. It comprises of a resistor and capacitor connected in series to a voltage supply as shown below on Figure 1.

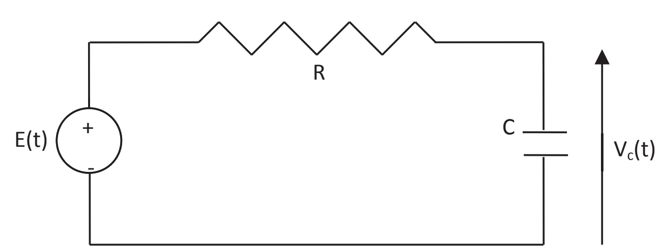


Figure 1: RC Circuit

If the capacitor is initially uncharged at zero voltage when the circuit is switched on, it starts to charge due to the current 'i' through the resistor until the voltage across it reaches the supply voltage. As soon as this happens, the current stops flowing or decays to zero, and the circuit becomes like an open circuit. However, if the supply voltage is removed, and the circuit is closed, the capacitor will discharge the energy it stored again through the resistor. The time it takes the capacitor to charge depends on the time constant of the system, which is defined as the time taken by the voltage across the capacitor to rise to approximately 63% of the supply voltage. For a given RC-circuit, this time constant is $\tau = RC$. Hence its magnitude depends on the values of the circuit components.

The RC circuit will always behave in this way, no matter what the values of the components. That is, the voltage across the capacitor will never increase indefinitely. In this respect we will say that the system is passive and because of this property it is stable.

For the RC-circuit as shown in Fig. 1, the equation governing its behavior is given by

$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} E \quad \text{where } v_c(0) = v_0 \tag{1}$$

where $v_c(t)$ is the voltage across the capacitor, R is the resistance and C is the capacitance. The constant $\tau = RC$ is the time constant of the system and is defined as the time required by the system output i.e. $v_c(t)$ to rise to 63% of its final value (which is E). Hence the above equation (1) can be expressed in terms of the time constant as:

$$\tau \frac{dv_c(t)}{dt} + v_c(t) = E \quad \text{where } v_c(0) = v_0 \tag{1}$$

Obtaining the transfer function of the above differential equation, we get

$$\frac{V_c(s)}{E(s)} = \frac{1}{\tau s + 1} \tag{2}$$

where τ is time constant of the system and the system is known as the first order system. The performance measures of a first order system are its time constant and its steady state.

Exercise 1:

a) Given the values of R and C, obtain the unit step response of the first order system.
   a. R=2KΩ and C=0.01F
   b. R=2.5KΩ and C=0.003F
b) Verify in each case that the calculated time constant ($\tau = RC$) and the one measured from the figure as 63% of the final value are same.
c) Obtain the steady state value of the system.

Overview Second Order Systems:

Consider the following Mass-Spring system shown in the Figure 2. Where K is the spring constant, B is the friction coefficient, x(t) is the displacement and F(t) is the applied force:

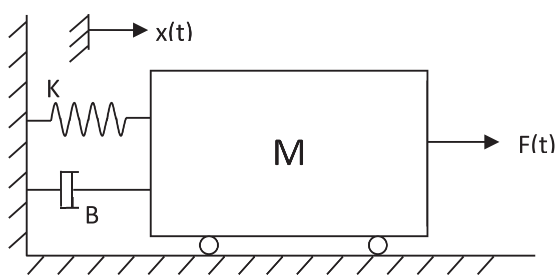


Figure 2. Mass-Spring system

The differential equation for the above Mass-Spring system can be derived as follows

$$M \frac{d^2x(t)}{dt^2} + B \frac{\partial x(t)}{\partial t} + Kx(t) = F(t)$$

Applying the Laplace transformation we get

$$(Ms^2 + Bs + K) * X(s) = F(s)$$

provided that, all the initial conditions are zeros. Then the transfer function representation of the system is given by

$$TF = \frac{Output}{Input} = \frac{F(s)}{X(s)} = \frac{1}{(Ms^2 + Bs + K)}$$

The above system is known as a second order system.

The generalized notation for a second order system described above can be written as

$$Y(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} R(s)$$

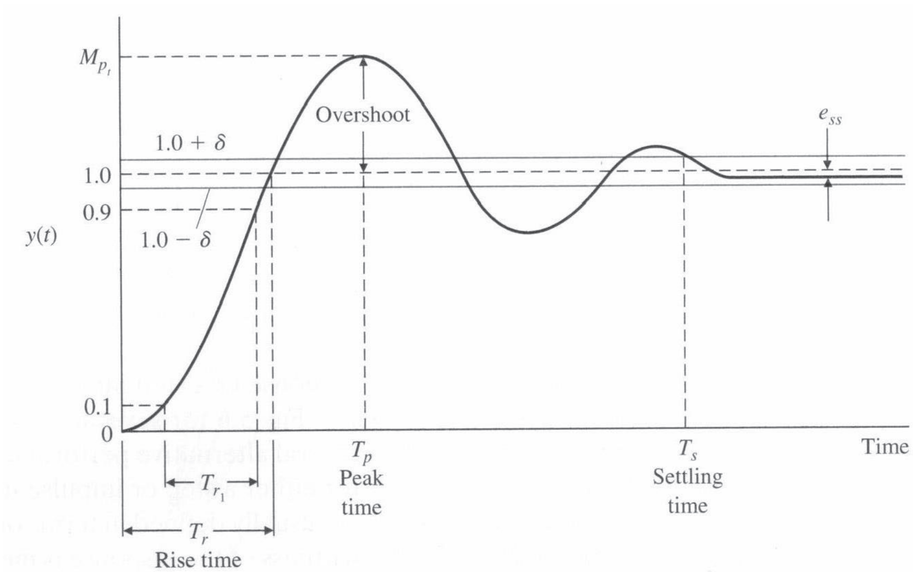
With the step input applied to the system, we obtain

$$Y(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

for which the transient output, as obtained from the Laplace transform table (Table 2.3, Textbook), is

$$y(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2}\, t + \cos^{-1}(\zeta))$$

where $0 < \zeta < 1$. The transient response of the system changes for different values of damping ratio, ζ. Standard performance measures for a second order feedback system are defined in terms of step response of a system. Where, the response of the second order system is shown below.



The performance measures could be described as follows:

Rise Time: The time for a system to respond to a step input and attains a response equal to a percentage of the magnitude of the input. The 0-100% rise time, T_r, measures the time to 100% of the magnitude of the input. Alternatively, T_{r1}, measures the time from 10% to 90% of the response to the step input.

Peak Time: The time for a system to respond to a step input and rise to peak response.

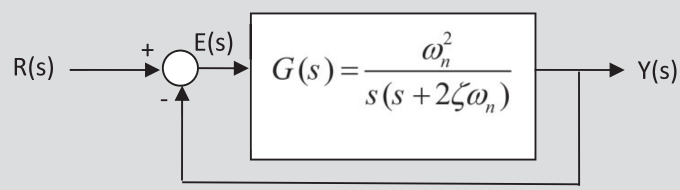
Overshoot: The amount by which the system output response proceeds beyond the desired response. It is calculated as

$$\text{P.O.}=\frac{M_{p_t}-f\upsilon}{f\upsilon}\times 100\%$$

where M_{Pt} is the peak value of the time response, and fv is the final value of the response.
Settling Time: The time required for the system's output to settle within a certain percentage of the input amplitude (which is usually taken as 2%). Then, settling time, T_s, is calculated as

$$T_s = \frac{4}{\zeta\omega_n}$$

Exercise 2: Effect of damping ratio ζ on performance measures. For a single-loop second order feedback system given below



Find the step response of the system for values of ωn = 1 and ζ = 0.1, 0.4, 0.7, 1.0 and 2.0. Plot all the results in the same figure window and fill the following table.

ζ	Rise time	Peak Time	% Overshoot	Settling time	Steady state value
0.1					
0.4					
0.7					
1.0					
2.0					