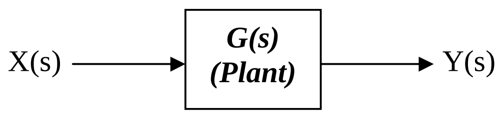
Lab # 6: Design via Root Locus

Open loop transfer function:

The open loop transfer function is a transfer function that represents a system and relates the output Y(s) to the input X(s) as a ratio.

$$H(s) = \frac{Y(s)}{X(s)}$$

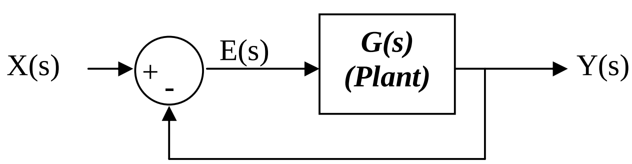
In Control Theory, the transfer function of the system that will be controlled is called a *plant* and sometimes denoted as *G(s).*



Example of an open loop system: Consider a motor system and we want to control its speed, so if you input the system with a suitable voltage to rotate 1000 rpm, the motor should rotate with speed 1000 rpm … etc. But if the motor at any moment face some disturbance or noise or a huge load such that its speed become lower than 1000 rpm, in this case the motor will not correct this error and will rotate with speed lower than 1000 rpm. So the main disadvantage of an open loop system is the absence of sensitivity to disturbance and inability to correct its behavior for this disturbance.

Closed loop transfer function:

The closed loop system overcomes the disadvantage that existed in the open loop systems.



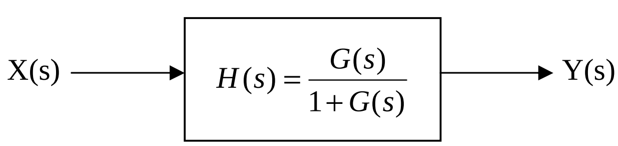
Where ***E(s) = L {e(t)}*** : the error signal between the input and output.

$$G(s) = \frac{Y(s)}{E(s)}$$

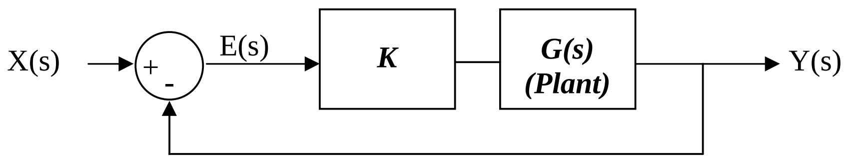
$$E(s) = X(s) - Y(s)$$

$$H(s)_{Forallsystem} = \frac{Y(s)}{X(s)} = \frac{G(s)}{1+G(s)}$$

Then the transfer function *H(s)* represent the closed loop system and relate *Y(s)* and *X(s)* by another way.



In the previous work we considered the gain of the forward and feedback paths is 1 (unity), but here we will remain the feedback gain unity and change the forward path gain with K gain as follows:



The new transfer function of the closed loop system is :

$$KG(s) = \frac{Y(s)}{E(s)}$$

$$E(s) = X(s) - Y(s)$$

$$H(s)_{Forallsystem} = \frac{Y(s)}{X(s)} = \frac{KG(s)}{1 + KG(s)}$$

$$H(s) = \frac{KG(s)}{1 + KG(s)}$$

In this case K is called the ***Proportional Gain or Proportional Controller.***

The effect of proportional gain on the system specifications (overshooting, settling time, rise time, peak time, steady state error).

A speed DC Motor transfer function will be our case study to study the effect of K gain on the behavior of the system.

$$G(s) = \frac{W(s)}{V(s)} = \frac{K_m}{(LS + R)(JS + K_f) + K_m K_b}$$

Let: R= 2.0 % Ohms
L= 0.5 % Henrys
K_m = 0.015 % torque constant
K_b = 0.015 % emf constant
K_f = 0.2 % Nms
J= 0.02 % kg.m^2/s^2

$$G(s) = \frac{1.5}{S^2 + 14S + 40.02}$$

Then the Closed loop transfer function is :

$$H(s) = \frac{KG(s)}{1 + KG(s)} = \frac{1.5K}{S^2 + 14S + (40.02 + 1.5K)}$$

Now let us assume some values of K and plot the step response and analyze the changes in the response specifications.

K= 0.1 , 1 , 10 ,100 , 1000

```
K=0.1;
step(feedback(K*tf([1.5],[1 14 40.02]),1))
```

Try to fill this table ….

	K= 0.1	K= 1	K= 10	K= 100	K= 1000	comment
Ts						
Tr						
OS%						
Yss						
S.S.E						

Conclusion:

The previous results say that the proportional controller has an effect on overshooting and settling time such that increasing the K gain will increase the overshooting and decrease the settling time and rise time. In the other hand decreasing the K gain will decrease the overshooting and increase the settling time and rise time.

How can we choose the suitable value of K Controller … ?

There are many methods for designing the K controller as :

1. Root locus technique
2. Frequency Response technique
3. State Space technique

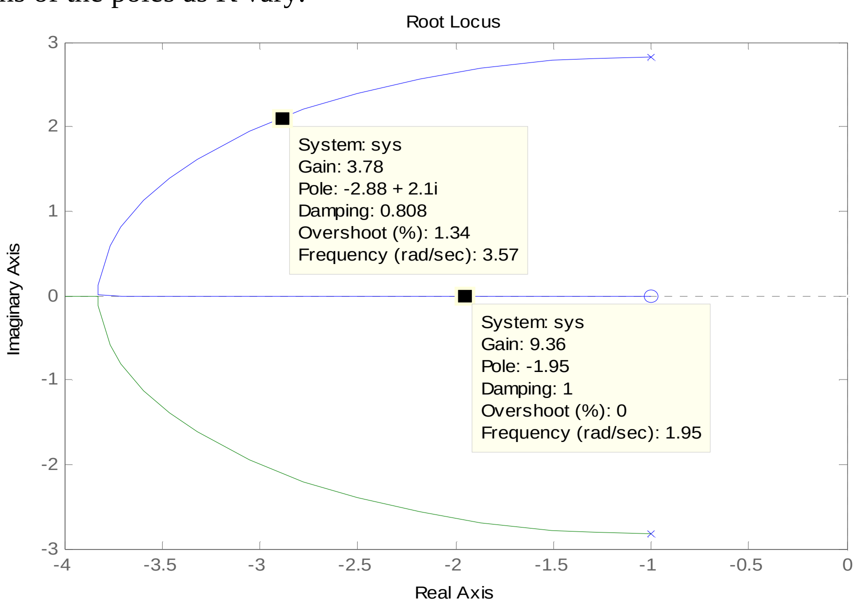
And here we will cover the first technique only (*Root Locus*).

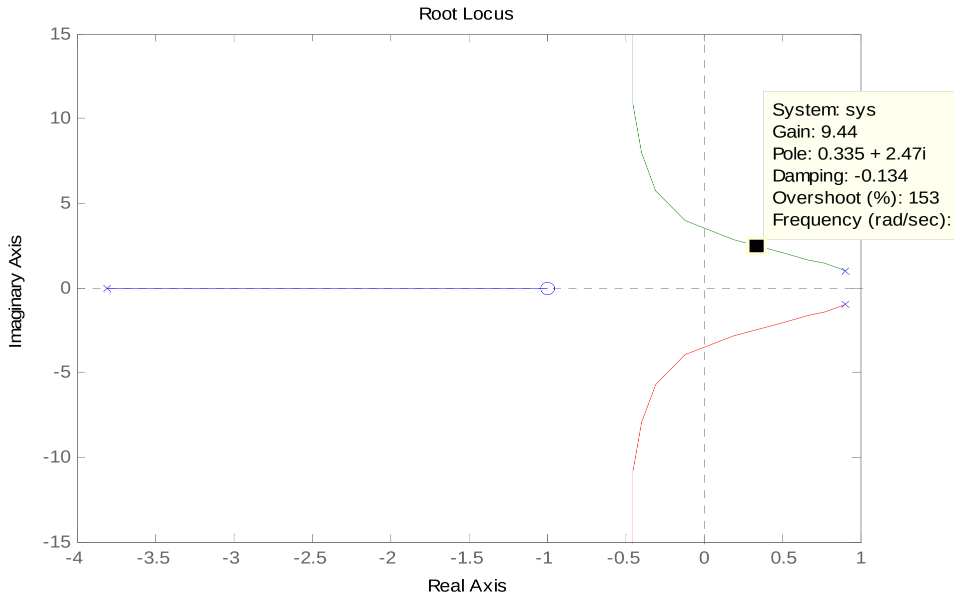
What is Root Locus … ?

As system parameter *k* varies over a continuous range of values, the root locus diagram shows the trajectories of the closed-loop poles of the feedback system. Typically, the root locus method is used to tune the loop gain of a SISO control system by specifying a designed set of closed-loop pole locations.

Then, Root Locus technique produces a plot that shows the locations of poles of a closed loop system on the S-Plane as K varies and from this plot we can choose the suitable K that meet our specification conditions.

For example below we see two plots of root locus. The lines are represent the locations of the poles as K vary.





As we note in the first locus, implemented by Matlab , the plot tell us that when the gain K = 3.78 the response will have overshooting 1.34% and damping ratio 0.8 and when K = 9.36 the response will have no overshooting. Also the locus say that all values of K will not make the system unstable and will remain in the stable region.

The second locus tell us when the gain K = 9.44 the system will be unstable because the poles become in RHP , so we can conclude that the proportional controller may drive the system from the unstable mode to a stable one and vise versa.

Example 1:

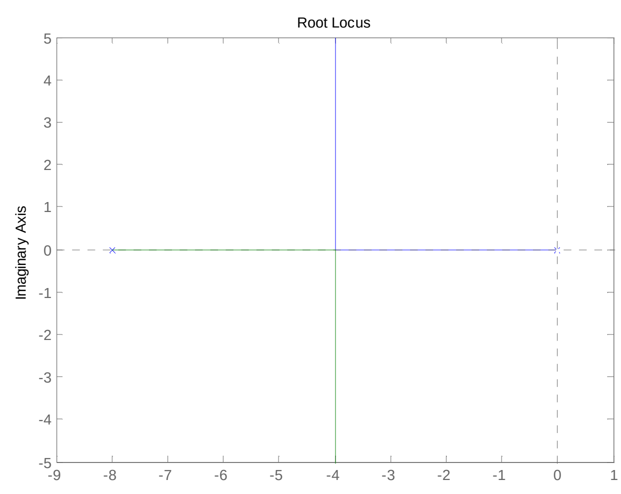
Using Matlab plot the root locus for the T.F and design K controller to get 5% O.S

$$G(s) = \frac{3}{s(s+8)}$$

- At 5 % O.S we find that $\zeta = 0.689$

Matlab Code:

```
num=[3]
den=[1 8 0]
sys=tf(num,den)
rlocus(sys)
```

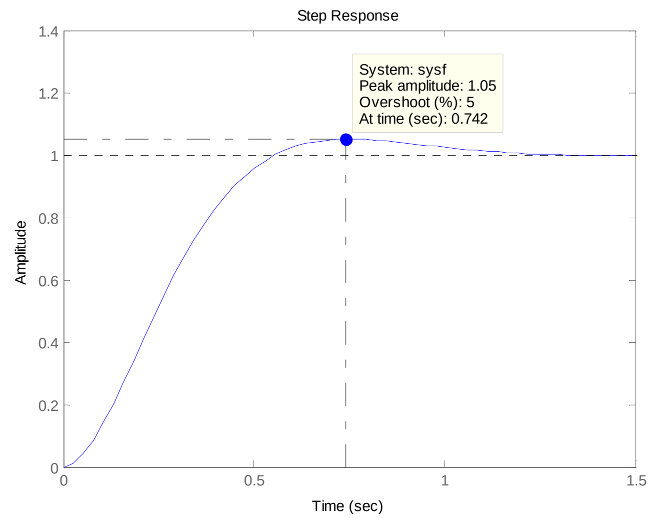


From the plot of root locus we find that if K = 11.2 then the O.S will be 5 %

Obtain the step response and check your result

Matlab Code:

```
num=[3]
den=[1 8 0]
sys=tf(num,den)
k=11.2
sysf=feedback(k*sys,1)
step (sysf)
```

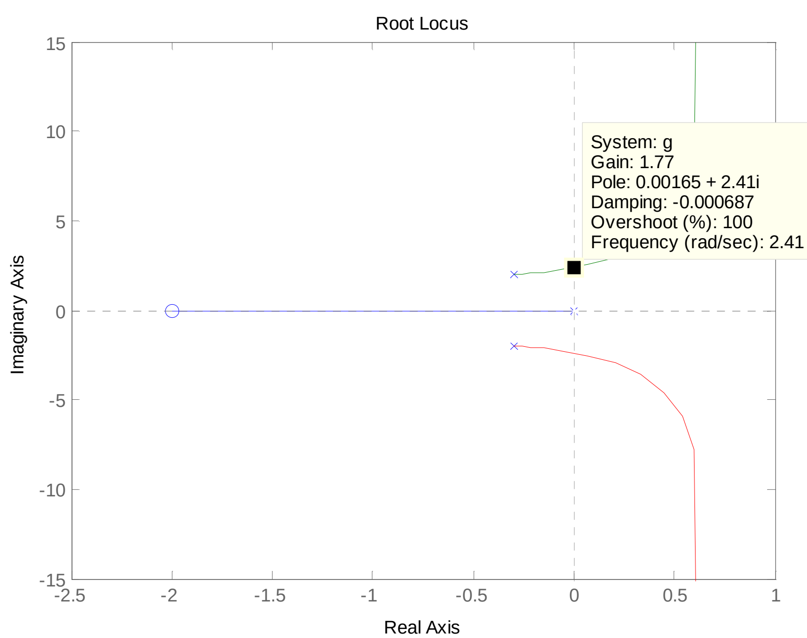


Example 2:

For the following T.F Plot the root locus of this system and obtain the value range of the proportional gain to make the system:

1. Stable
2. Unstable
3. Margin

$$G(s) = \frac{s+2}{s(s^2+0.6s+4)}$$

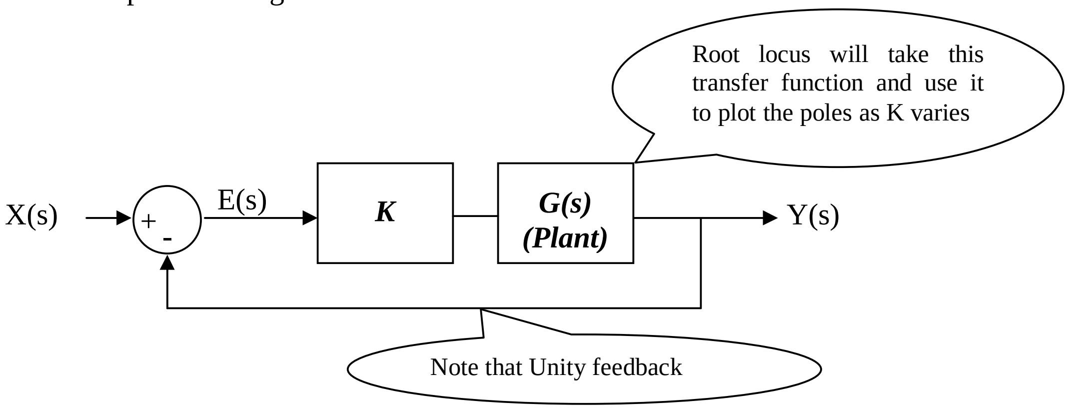


From the Root Locus we find that:

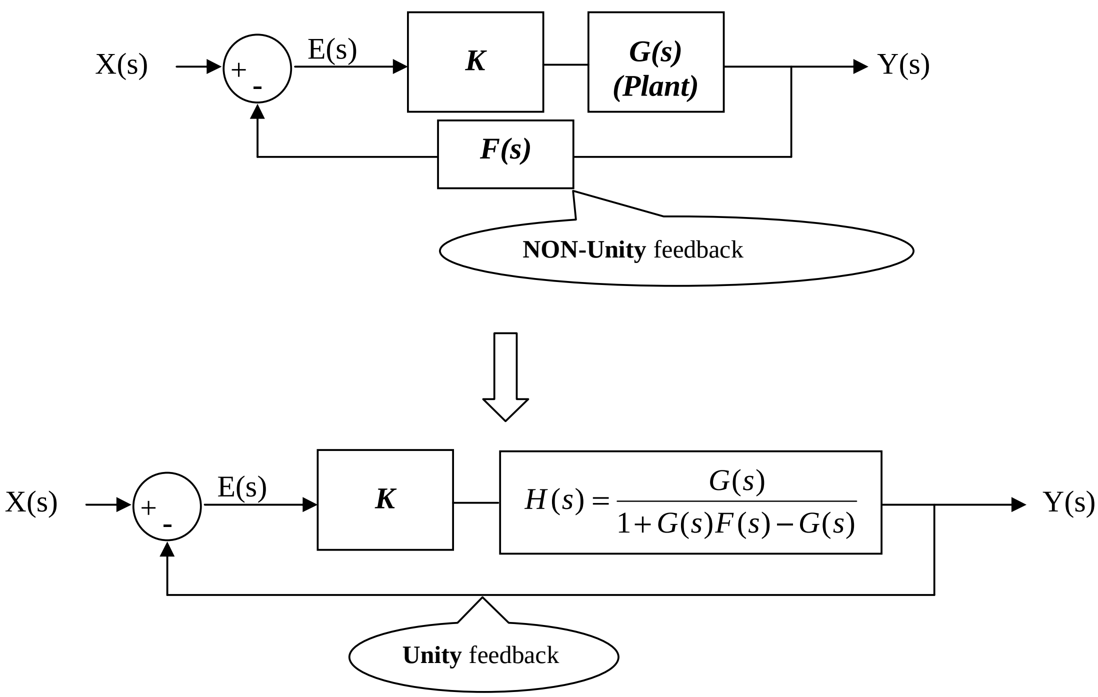
1- The system is stable for K<1.77
2- The system is Margin stable for K = 1.77
3- The system is unstable for K >1.77

❖ Extra About root locus :

We knew that root locus plot the poles for an *closed loop transfer function* that will be controlled by Proportional controller and the system must have unity feedback path as in figure below.



But if the system does not has unity feedback as in figure below, so we should convert it to another has unity feedback by the solution below.



❖ Matlab Command :

- ❖ **rlocus (sys) :** This command used to plot the root locus of the open loop transfer function which will be controlled by K controller.
- ❖ **[Wn,Z] = damp(sys)**: Compute damping factors and natural frequencies.
- ❖ **sgrid** : Generate an s-plane grid.
- ❖ **sgrid (z,wn)** : Generate an s-plane grid of constant damping factors and natural frequencies.

Exercise:

1. (**Feedback Transfer function**)

By Matlab, for the open loop transfer function G(s),

$$G(s) = \frac{1}{s^2 + s + 1}$$

a. Plot the step response of the open loop system.
b. Plot the closed loop transfer function with unity feedback.
c. Record your notes about the open and closed loop system.
d. Obtain the closed loop transfer function with feedback $F(s) = \frac{1}{s+1}$

2. (**Stability Proportional Controller**)

By Matlab, for the open loop transfer function G(s)

$$G(s) = \frac{s+1}{s(s^2 + 0.5s + 1)}$$

a. Plot the open loop step response and comment on it.
b. Plot the closed loop transfer function and comment on it.
c. Plot the root locus of this system and obtain the value range of the proportional gain to make the system:
   1. Stable
   2. Unstable
   3. Margin