

Inverse Finite Element Method for Beam Analysis

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Abstract—This report gives an overview of the Inverse Finite Element Method (iFEM) for beam analysis, including its theoretical foundations, implementation details, and potential applications. The iFEM approach allows for the reconstruction of structural displacements and strains from measured data, providing a powerful tool for structural health monitoring and damage detection in beam-like structures.

Index Terms—iFEM, Inverse Finite Element Method, Beam Analysis

I. INTRODUCTION

The main objective of this report is to give a general overview of the possibilities given by the Inverse Finite Element Method (iFEM) for beam structures. The iFEM is a computational technique that allows for the reconstruction of structural displacements and strains from measured data, providing a powerful tool for structural health monitoring, damage detection and model testing in experimental environment. The aim is also to identify the limits of such a tool in regards of mesh, noise and model error sensitivities.

II. METHODOLOGY

A. Geometry of the problem

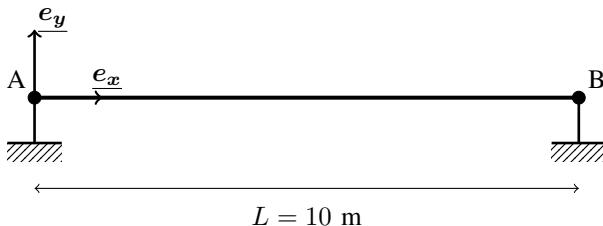


Fig. 1. Simply supported beam configuration

The work carried out on all of this report was done on a simply supported beam clamped at both ends as shown in figure 1. The length of the beam is 10 m. Different load scenarios were tested such as point load and distributed load. This means that the following boundary conditions were applied:

- Point A: $u_x = u_y = 0, \theta_z = 0$
- Point B: $u_x = u_y = 0, \theta_z = 0$

Where u_x and u_y are the displacements in the x and y directions respectively.

B. Element type

Euler Beam elements were used for the discretization of the beam structure. These elements are suitable for slender beams where shear deformation is negligible. Using 4 Gauss Point for the shape functions integration. The residual is following the principle of virtual work.

$$R = \int_L \delta \epsilon^T \sigma dL - \int_L \delta u^T b dL - \sum_{i=1}^{nLoad} \delta u_i^T Q_i \quad (1)$$

More info on the specificity of the beam

C. Material properties

Unit mass, unit local inertia, Young's modulus of $210e9$ Pa and a cross section of 0.01 m^2 were used for the beam. Make a table of the properties used.

TABLE I
MATERIAL PROPERTIES

Property	Value	Units
Axial Stiffness, EA	10^6	m/N
Bending Stiffness along e_y , EI_1	10^5	N.m^2
Bending Stiffness along e_z , EI_2	10^5	N.m^2
Torsional Stiffness, GJ	10^6	N.m^2
Linear mass along main axis, μ_1	1.0	kg/m
Mass moment of inertia around main axis, ι_1	1.0	kg.m^3

D. Load Scenarios

To investigate the limits of the iFEM approach for beam analysis, several load scenarios were considered. These scenarios include both static and dynamic loading conditions to assess the method's performance under different circumstances.

1) *Nodal load (Static/Dynamic/Quasi-static)*: A local load is applied at the mid span to the i-th node of the beam as shown in figure 2. Depending on the studied case \underline{F} can take different forms such as a step load or a sinusoidal load.

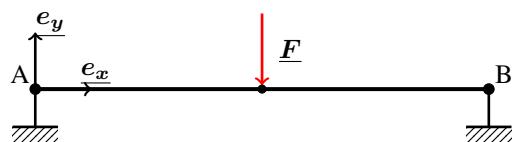


Fig. 2. Local nodal load configuration

2) *Distributed load: Uniform (Dynamic)*: Distributed loads are quite common in structural applications. A uniform distributed load is applied along the entire length of the beam as shown in figure 3. The load intensity can vary with time, allowing for dynamic analysis of the beam under such loading conditions.

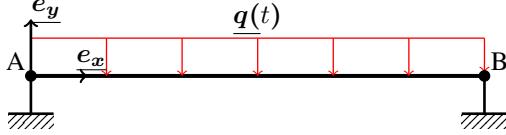


Fig. 3. Uniform distributed load configuration

3) *Distributed load: 2nd mode shape (Dynamic)*: To get near free vibration condition, a sinusoidally distributed load is applied along the entire length of the beam as shown in figure 4. The load intensity varies with both time and position along the beam, following the second mode shape of vibration.

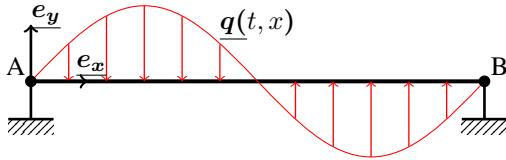


Fig. 4. Sinusoidally distributed load configuration

E. Inverse simulation

The typical costs used were... We pretend that all nodes have a known cost and all.

III. RESULTS

A. Inverse crime

Everything is perfect, no noise, same mesh, same model.

B. Mesh sensibility

Local node disappears, uniform mesh coarsening, knowledge of the displacement of only some nodes

C. Noise sensibility

D. Model knowledge

Inexact knowledge of the material properties (E , I , ρ , A), from XU to XUA.

E. Time discretization sensibility

Shannon and potential difference in discretization between interpolated measurements and inverse method

IV. CONCLUSION