Chapter 18-Collections of Sets

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12:58 AM

Collections of Sets:

It's like sets of sets.

Collections of sets are denoted using weird (script) letters like
$\mathscr{A}$

When a collection is the domain of a function, the function maps an entire subset to an output value.
For example, a collection of finite sets of integers can be the domain of a mean function, where the output of the function is the mean of whatever set inputted.
$\mathscr{D} = \{\{-12, 7, 9, 2\}, \{2, 3, 7\}, \{-10, -3, 10, 4\}, \{1, 2, 3, 6, 8\}\}$
$g(\{-12, 7, 9, 2\}) = 1.5 \text{ and } g(\{1, 2, 3, 6, 9\}) = 4.2$

Note that an empty set can also be put into another set. (and it also counts as an element in a collection)

A ***Powerset*** is a collection that contains all subsets of a specific set.
For example, suppose that A = {1,2,3}, then
$\mathbb{P}(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$
For a set with n elements, there $\mathbf{2^n}$ elements in its powerset.

Note that **all** powersets contain the empty set.
$\mathbb{P}(\emptyset) = \{\emptyset\}$

Powersets often appear as the *co-domain* (aka output) of functions which need to return a set of values rather than just a single value.
$\text{So } n(g) = \{h\}, \textbf{ not } n(g) = h$

The ***Partition*** of a set *A* are the non-overlapping subsets that include every original elements of *A.*

Recall an equivalence relation basically divides a set into different groups of numbers that are "equivalent."

This is related to partitioning!
For example, consider the below partition of integers:

$$\{\{0, 4, -4, 8, -8, \ldots\}, \{1, 5, -3, 9, -7, \ldots\},$$
$$\{2, 6, -2, 10, -6, \ldots\}, \{3, 7, -1, 11, -5, \ldots\}\}$$

(It is just the definition of congruence mod 4)

We could also write this partition as $\{[0], [1], [2], [3]\}$ since each equivalence class is a set of numbers.

Formally, a partition of a set A is a set of non-empty subsets of A which cover all the elements of A and which don't overlap. So, if the subsets in the partition are $A_1, A_2, \ldots A_n$, then they must satisfy three conditions:

1. covers all of A: $A_1 \cup A_2 \cup \ldots \cup A_n = A$
2. non-empty: $A_i \neq \emptyset$ for all i
3. no overlap: $A_i \cap A_j = \emptyset$ for all $i \neq j$.

Combinations:

The expression $\frac{n!}{k!(n-k)!}$ is often written $C(n, k)$ or $\binom{n}{k}$.

If we want to pick k objects from a list of n possible types, and we **allow duplicates** in the k objects, then the formula would be

$$\binom{k+n-1}{n-1}$$

Or the equivalent:

$$\binom{k+n-1}{k}$$

(source: trust me bro)

Some identities:

$$\binom{n}{k} = \binom{n}{n-k}$$

(look at the denominator of the definition of n choose k and you should know why)
(look harder if you still don't know)

(Pascal's identity) $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$

Binomial Theorem:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$