# Chapter 10-2-way Bounding

Monday, January 2, 2023
12:23 AM

***2-Way Bounding:***

Create bounds from both directions to ensure the accuracy of a proof. (Using 2 sub proofs)
For example: to prove $F(x) = k$, we can prove $F(x) \geq k$ **and** $F(x) \leq k$.
*(kind of like sandwich/squeeze theorem, Calc 2 moment)*

***Markers*** are used to mark out outlines of things to cut out (cookie dough, cloth for clothing, etc.)
*Need to be as efficient as possible.*

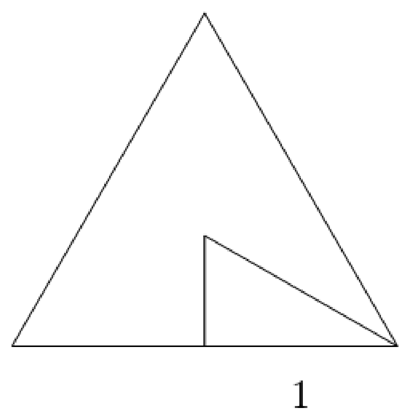
Things like markers cannot be calculated directly, but through 2-way bounding, sometimes we can make a ***Tight*** bound and make the upper and lower bounds meet.

*2-Way Bounding is helpful when trying to prove that there is a maximum or minimum of something.*

**Claim 36** *Suppose that $T$ is an equilateral triangle with sides of length 2 units We can place a maximum of four points in the triangle such that every pair of points are more than 1 unit apart.*

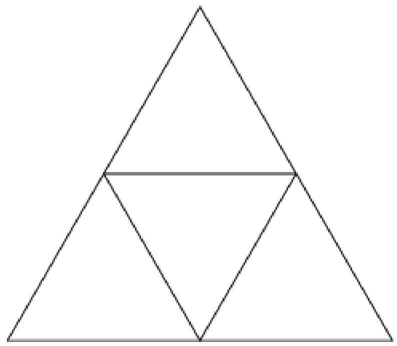
To show that 4 is the maximum number of points we can place with the required separations, we need to show that it's possible to place 4 points, but it is not possible to place 5 points. It's easiest to tackle these two subproblems separately, using different techniques.

> Proof: To show that the maximum is at least four, notice that we can place three points at the corners of the triangle and one point in the center. The points at the corners are two units apart. To see that the point in the center is more than one unit from any corner, notice that the center, the corner, and the midpoint of the side form a right triangle.




The hypotenuse of this triangle connects the center point to the corner point. Since one leg of the triangle has length 1, the hypotenuse must have length greater than 1.

To show that the maximum number of points cannot be greater than four, divide up the triangle into four smaller equilateral triangles as follows:



Suppose we tried to place five or more points into the big triangle. Since there are only four small triangles, by the pigeonhole principle, some small triangle would have to contain at least two points. But since the small triangle has side length only 1, these points can't be separated by more than one unit.

The *Coloring* of a graph assigns a color to each node of G but two adjacent nodes cannot be the same color.
If G can be colored with k colors, we say G is *k-colorable.*

$\chi(G)$

is the smallest number of colors needed to color G.
*A complete graph of Kn requires n colors, since all nodes are adjacent to each other.*

Coloring is often used to optimize algorithms. For example, in code compiling, each variable is considered a node at compile time, and the colors available are the register available. When 2 variables(nodes) need to be used at the same time, they cannot be assigned the same color (register), so we connect them with an edge. As we assign colors (registers), we try to use as little different colors (registers) as possible, and take out nodes(variables) if necessary to optimize the compiling process.

2-way bounding proofs are also used to prove that two sets are equal.
show that $A \subseteq B$ and $B \subseteq A$

**Claim 37** *Let* $A = \{15p + 9q \mid p, q \in \mathbb{Z}\}$ *Then* $A = \{$*multiples of 3*$\}$.

Proof:

(1) Show that $A \subseteq \{\text{multiples of } 3\}$.

Let $x$ be an element of $A$. By the definition of $A$, $x = 15s + 9t$, for some integers $s$ and $t$. But then $x = 3(5s + 3t)$. $4s + 3t$ is an integer, since $s$ and $t$ are integers. So $x$ is a multiple of 3.

(2) Show that $\{\text{multiples of } 3\} \subseteq A$.

Notice that (*) $15 \cdot (-1) + 9 \cdot 2 = 3$.

Let $x$ be a multiple of 3. Then $x = 3n$ for some integer $n$. Substituting (*) into this equation, we get $x = (15 \cdot (-1) + 9 \cdot 2)n$. So $x = 15 \cdot (-n) + 9 \cdot (2n)$. So $x$ is an element of $A$.

Since we've shown that $A \subseteq \{\text{multiples of } 3\}$ and $\{\text{multiples of } 3\} \subseteq A$, we can conclude that $A = \{\text{multiples of } 3\}$.