Unsupervised Learning
& naïve bayes

Vladislav Goncharenko
MIPT, 2019

Outline

1. Naïve Bayes
2. Unsupervised learning
    a. Maniflod learning

Naïve Bayes

Naive assumption
of features independence
leads to simple
and easy to calculate result

$$P(y|x_1,\dots,x_n) = P(y)\cdot\frac{P(x_1,\dots,x_n|y)}{P(x_1,\dots,x_n)}$$

$$\boxed{P(x_i|y,x_1,\dots,x_{i-1},x_{i+1},\dots,x_n) = P(x_i|y)}$$

$$P(y|x_1,\dots,x_n) = P(y)\cdot\frac{\prod_i P(x_i|y)}{P(x_1,\dots,x_n)}$$

$$\boxed{P(x_1,\dots,x_n) \equiv const}$$

$$\hat{y} = \arg\max_y P(y)\cdot\prod_i P(x_i|y)$$

What $P(x_i|y)$ really is?

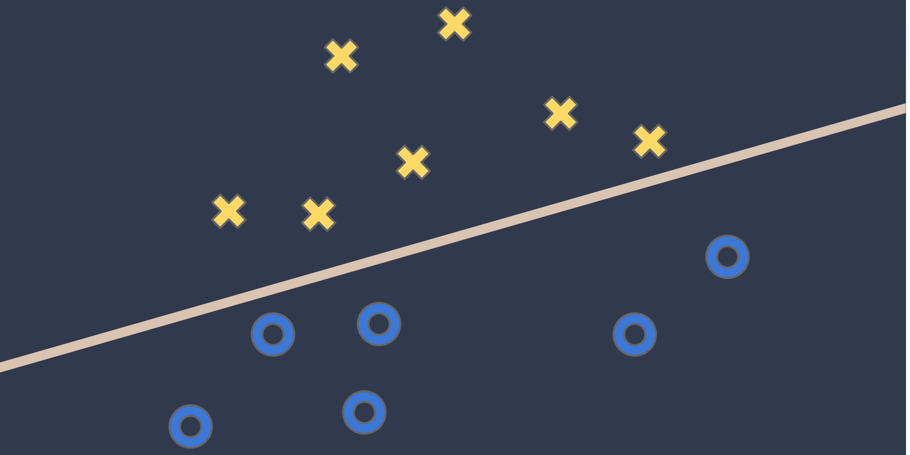
Typical likelihood of the features

1. Gaussian
2. Multinomial
3. Bernoulli

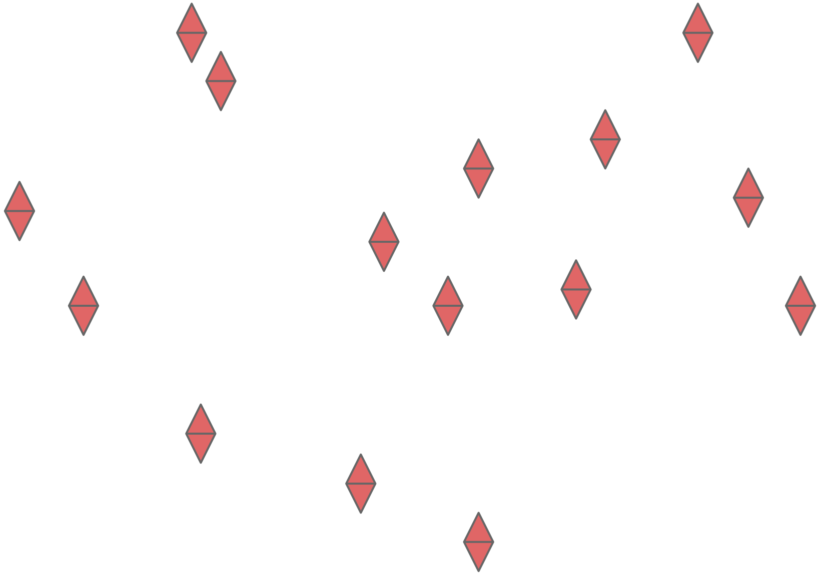
$$P(x_i \mid y) = \frac{1}{\sqrt{2\pi\sigma_y^2}} \exp\left(-\frac{(x_i - \mu_y)^2}{2\sigma_y^2}\right)$$

$$P(x_i \mid y) = P(i \mid y)x_i + (1 - P(i \mid y))(1 - x_i)$$

Supervised learning



Unsupervised learning



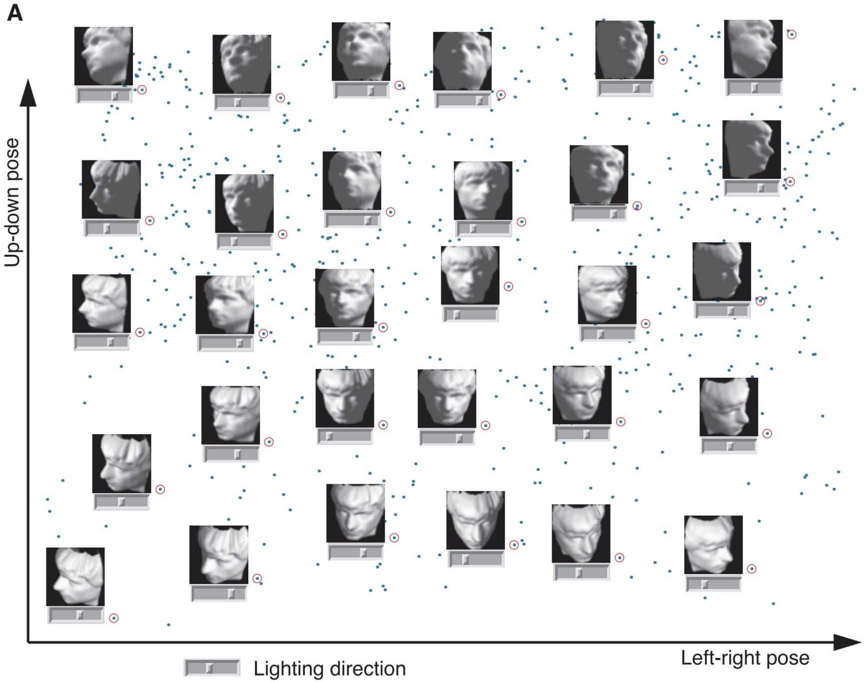
Manifold learning

Manifold assumption

The data lie approximately
on a manifold of much lower dimension
than the input space

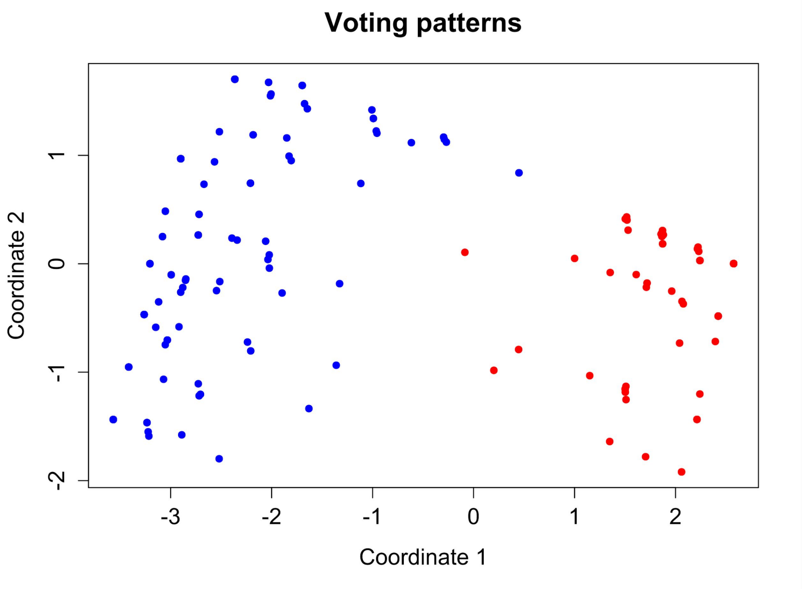
So problem dimensionality could be
(non-)linearly reduced

Approach doesn't require any labels



[Tenenbaum, de Silva, Langford
A Global Geometric Framework for Nonlinear Dimensionality Reduction]

Multi-dimensional Scaling (MDS)



[Voting patterns in the United States House of Representatives]

Goal:

Linearly embed to given lower space

Solution:

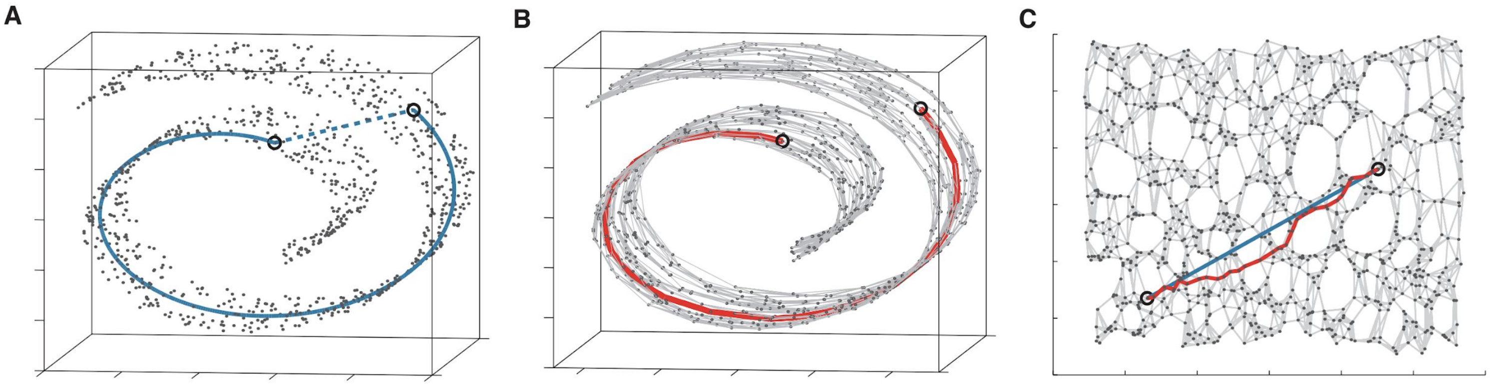
PCA

$$L = ||D_x - D_y||_2 \to \min_{y=Ax}$$

$$y = \Lambda^{1/2} V^T$$

Params: p - target dimensionality

Isomap



Now make distancies geodesic!
And measure distances on the produced graph

Params:

n - number of neighbours to connect
p - dimensionality of manifold

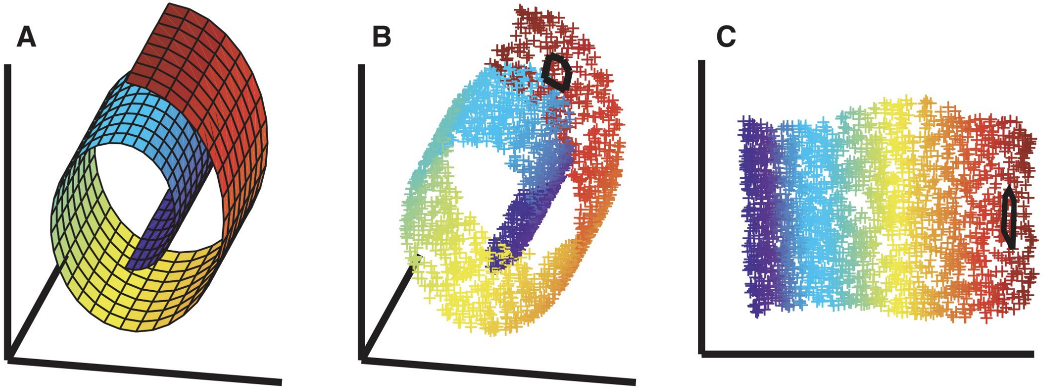
[Original article]

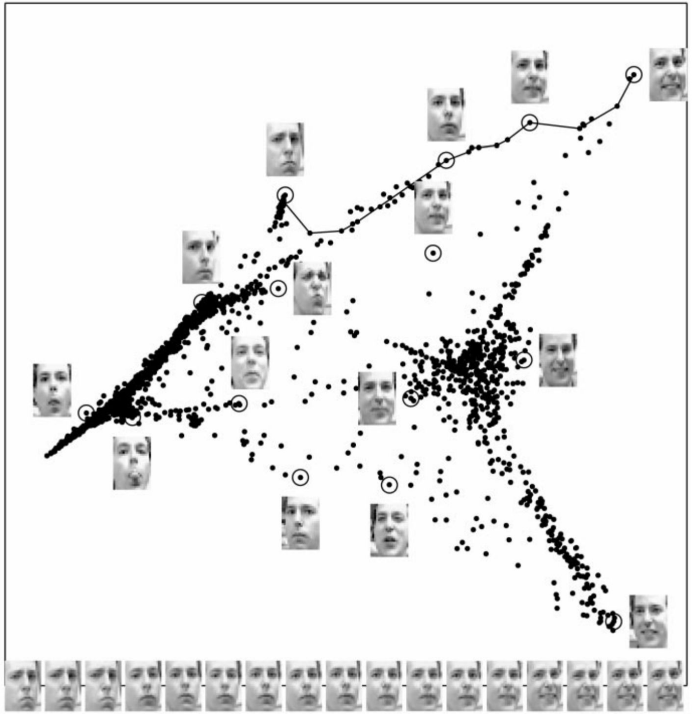
Locally linear embedding (LLE)

Idea:

Smooth manifold can be locally approximated linearly.
Linear peases can be flattened

[Original article]





Locally linear embedding (LLE)

Two steps of embedding and two objective functions:

1. estimate point by its K neighbours

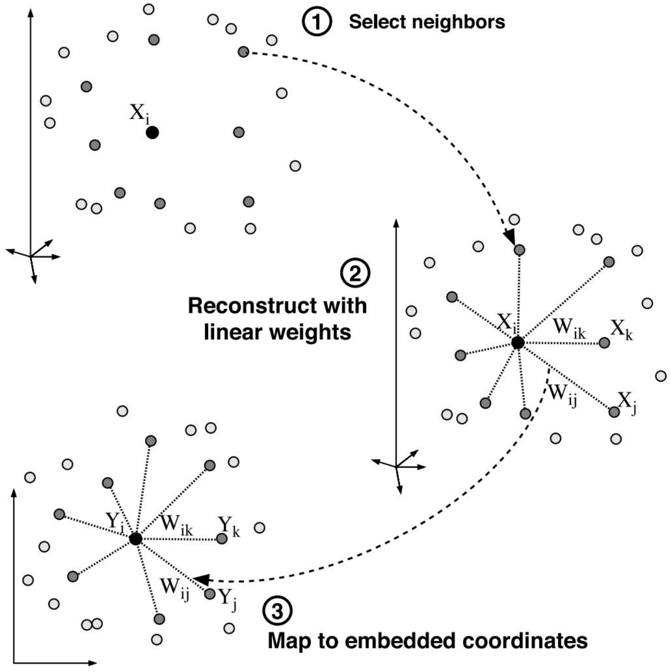
$$\varepsilon(W) = \sum_{i=1}^{n} ||x_i - \sum_{j=1}^{K} W_{ij} x_j||^2$$

2. Estimate new points based on known relations

$$\Phi(Y) = \sum_{i=1}^{n} ||y_i - \sum_{j=1}^{n} W_{ij} y_j||^2$$

Params:

n - number of neighbours to connect
p - dimensionality of manifold



Many more...

- Hessian Eigenmapping
- Spectral Embedding
- Local Tangent Space Alignment
- Riemannian Geometry
- ……

t-SNE

t-distributed $\underbrace{\text{Stochastic Neighbor Embedding}}_{\text{SNE}}$

original article

Stochastic Neighbor Embedding

Idea:

Convert pairwise distances to probabilities

$$p_{j|i} = \frac{\exp(-\frac{||x_i - x_j||^2}{2\sigma_i^2})}{\sum_{k \neq i} \exp(-\frac{||x_i - x_k||^2}{2\sigma_i^2})}$$

asymmetric probability
of object i chooses j as its neighbour

$$q_{j|i} = \frac{\exp(-||y_i - y_j||^2)}{\sum_{k \neq i} \exp(-||y_i - y_k||^2)}$$

the same in target space

Let’s construct embedding s.t. this distributions are close.
What are close distributions?

Kullback–Leibler divergence

$$D_{KL}(P \parallel Q) = \sum_{i,j} p_{j|i} \log \frac{p_{j|i}}{q_{j|i}}$$

Suspiciously similar to Shannon entropy

[Learn more]

Stochastic Neighbor Embedding

$$p_{j|i} = \frac{\exp(-\frac{||x_i - x_j||^2}{2\sigma_i^2})}{\sum_{k \neq i} \exp(-\frac{||x_i - x_k||^2}{2\sigma_i^2})}$$

$$q_{j|i} = \frac{\exp(-||y_i - y_j||^2)}{\sum_{k \neq i} \exp(-||y_i - y_k||^2)}$$

$$D_{KL}(P \parallel Q) \to \min_{Y}$$

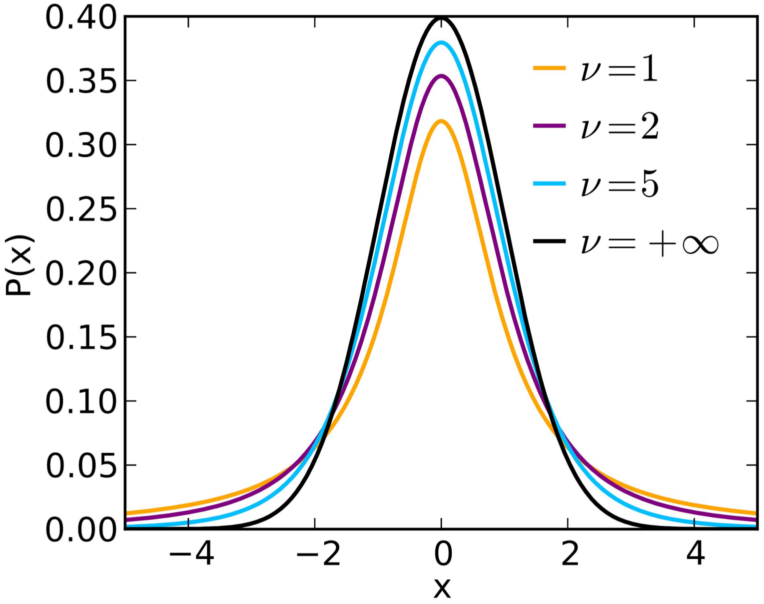
t-distributed Stochastic Neighbor Embedding

Patches over SNE:

1. make distribution symmetric
2. make it decrease faster than Gaussian (use Student's t-distribution)

$$q_{ij} = \frac{\frac{1}{1+||y_i - y_j||^2}}{\sum_{k \neq i} \frac{1}{1+||y_i - y_k||^2}}$$

Original article



Links

1. Good lecture on MDS, Isomap, LLE
2. Lecture on t-SNE (this one is good too)
3. Slides about clusterization
4. Metrics in clusterization
5. Slides about ICA