

# Transformers: Understanding Parallel Intelligence

## From Zero to ChatGPT - A BSc Journey

Week 5: Transformers

# What You'll Learn Today

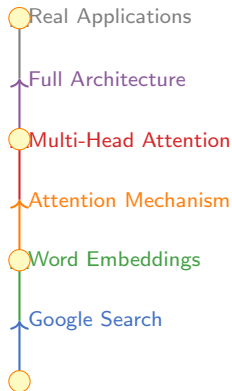
## By the end of this lecture, you will:

- 1 **Understand** how words become numbers (embeddings)
- 2 **Explain** why parallel beats sequential processing
- 3 **Calculate** attention scores between words
- 4 **Draw** the transformer architecture
- 5 **Identify** transformers in daily applications

## Prerequisites:

- Basic vector operations (dot product)
- Matrix multiplication concept
- No deep learning needed!

## Your Learning Journey:



## Interactive Elements:

3 Checkpoints — 5 Exercises — 10 Visuals

# How Google Reads Your Mind

**Try this:** Type in Google: “How do transformers...”

**Google instantly suggests:**

- “...work in machine learning”
- “...process language”
- “...learn from data”

**The Mystery:**

- Google reads ALL your words at once
- Not word-by-word like old systems
- Understands context instantly

How do transformers

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...work in machine learning  
...process language  
...learn from data  
...handle attention

**Question:** How does it understand whole sentences simultaneously?

# Discovery 1: Words Live in Space

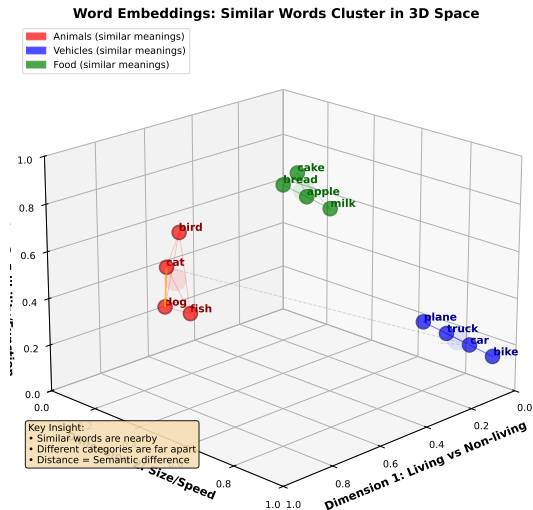
## Think about GPS coordinates:

- Paris: (48.8N, 2.3E, 35m altitude)
- London: (51.5N, 0.1W, 11m altitude)
- Similar cities are nearby in space

## Words work the same way!

- “cat”: [0.7, 0.2, 0.5] in meaning space
- “dog”: [0.8, 0.3, 0.4] (nearby - similar!)
- “car”: [0.1, 0.9, 0.2] (far - different!)

This is called: Word Embeddings



## Discovery 2: Every Word Connects to Every Other

In a sentence, every word “talks” to every other:

**Example:** “The cat sat on mat”

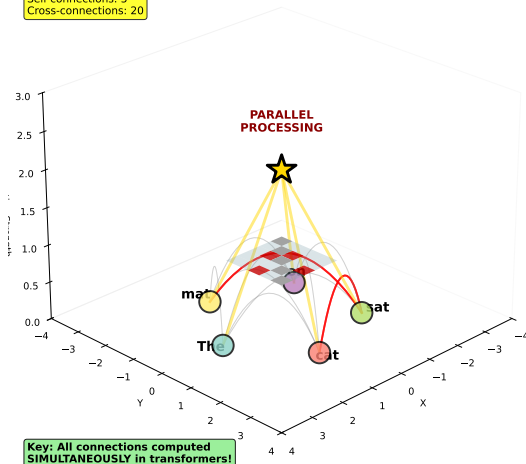
- “cat” checks all other words
- “sat” looks at subject and location
- “mat” knows what's on it

**Total connections:**  $n \times n$

- 5 words = 25 connections
- 100 words = 10,000 connections!

**Every Word Connects to Every Other:  $5 \times 5 = 25$  Connections**

Total Connections: 25  
Self-connections: 5  
Cross-connections: 20



# The Problem: Information Overload

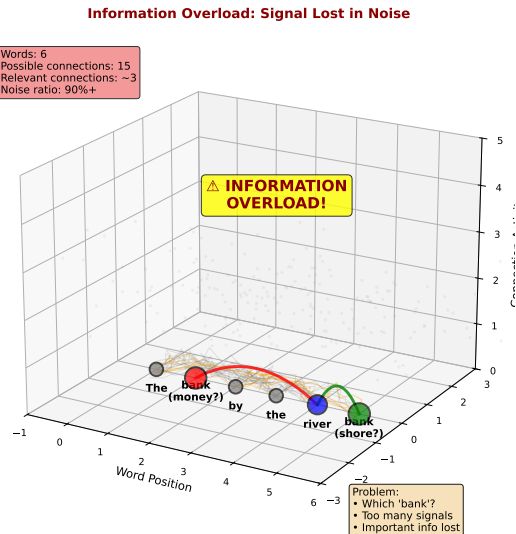
**Challenge:** Too much information!

**Sentence:** “The bank by the river bank”

- First “bank” = financial institution
- Second “bank” = river edge
- How does the model know?

**Information explosion:**

- Every word sends signals to all others
- Most connections are noise
- Need to focus on what matters



# First Attempt: Connect Everything

Early 2010s approach: Just connect everything!



**Results:**

- ✓ Works for short sentences
- ✗ Fails on long text
- ✗ Can't distinguish important from noise

# Computing All Relationships

## What happens with full connections:

### Step 1: Every word becomes a vector

- “cat”  $\rightarrow$  [0.7, 0.2, 0.5]
- “sat”  $\rightarrow$  [0.3, 0.8, 0.4]
- “mat”  $\rightarrow$  [0.6, 0.1, 0.7]

### Step 2: Compute all dot products

- $\text{cat} \cdot \text{sat} = 0.59$
- $\text{cat} \cdot \text{mat} = 0.87$
- $\text{sat} \cdot \text{mat} = 0.52$

### Step 3: Average everything Result: Information soup!

cat	1.0	0.59	0.87
sat	0.59	1.0	0.52
mat	0.87	0.52	1.0
	cat	sat	mat

All relationships computed but no focus!



# SUCCESS! (On Simple Cases)

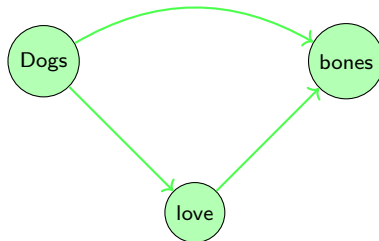
## When it works:

### Short, clear sentences:

- “Dogs love bones” ✓
- “Paris is beautiful” ✓
- “Water is wet” ✓

### Why it works here:

- Few connections (9 total for 3 words)
- All connections matter
- No ambiguity



**Clear signal!**

**Success Rate: 95% on 3-5 word sentences**

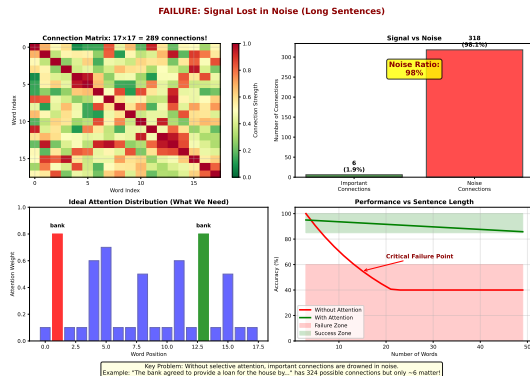
# FAILURE: Signal Lost in Noise

## When it fails:

**Real-world sentence:** “The **bank** agreed to provide a loan for the house by the river **bank** after reviewing the application that the customer submitted last Tuesday.”

## Problems:

- 20 words = 400 connections!
- “bank” (financial) vs “bank” (river)
- Long-distance dependencies
- Most connections are noise



**Failure Rate: 60% on 15+ word sentences**  
**Signal drowned in noise!**

# How Do Humans Actually Read?

Eye-tracking studies reveal:

**Humans DON'T read every word equally!**

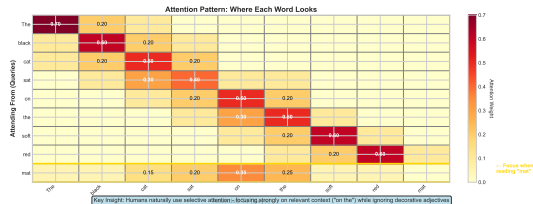
Reading: "The quick brown fox jumps"

- Focus on "fox" and "jumps"
- Skim "the" and "brown"
- Context determines focus

**Human attention is:**

- Selective (ignore irrelevant)
- Contextual (meaning-based)
- Efficient (focus on key parts)

When your eyes reach "mat", your brain focuses on:



**Key Insight:**  
We need **SELECTIVE** attention,  
not full connections!

# The Hypothesis: Selective Attention

## The Breakthrough (2017):

Instead of connecting everything equally,  
**learn WHICH connections matter!**

### Attention Mechanism:

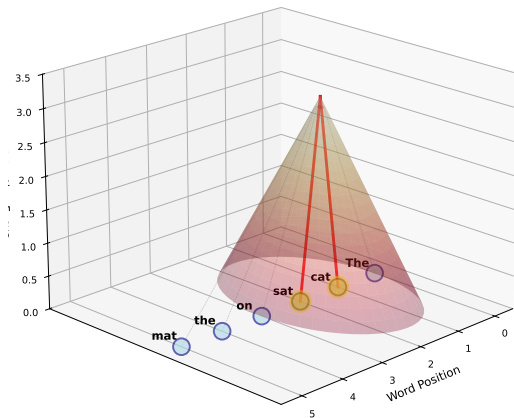
- 1 Compute importance scores
- 2 Focus on high scores
- 3 Ignore low scores

### Like a spotlight:

- Bright on important words
- Dim on filler words
- Adjustable based on context

### Selective Attention: Spotlight on Important Words

Bright spotlight = High attention  
Dim areas = Low attention



# Breaking It Down: Attention as Percentages

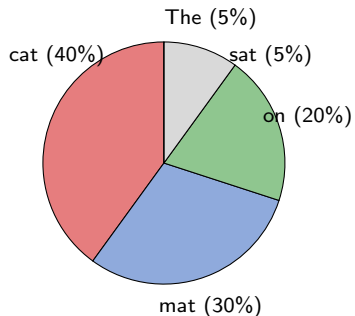
**Example:** “The cat sat on mat”

**When processing “sat”:**

- 40% attention to “cat” (who sat?)
- 30% attention to “mat” (where?)
- 20% attention to “on” (relation)
- 5% to “The” (not important)
- 5% to itself

**These percentages:**

- Always sum to 100%
- Change for each word
- Learned from data



**Softmax** ensures percentages  
always total 100%

# The Math: How Similar Are Two Words?

## Measuring similarity with angles:

### Dot Product Formula:

$$\text{similarity} = \vec{A} \cdot \vec{B} = |\vec{A}| \times |\vec{B}| \times \cos(\theta)$$

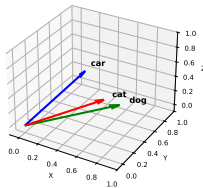
### What this means:

- Same direction:  $\cos(0^\circ) = 1$  (max similar)
- Perpendicular:  $\cos(90^\circ) = 0$  (unrelated)
- Opposite:  $\cos(180^\circ) = -1$  (opposite)

### Example:

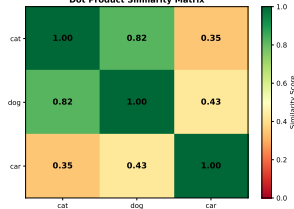
- $\text{cat} \cdot \text{dog} = 0.8$  (similar animals)
- $\text{cat} \cdot \text{car} = 0.1$  (very different)

Word Vectors in 3D Space  
Angle(cat,dog)=10.2°  
Angle(cat,car)=64.7°



$$\text{similarity} = \vec{A} \cdot \vec{B} = |\vec{A}| \times |\vec{B}| \times \cos(\theta)$$

Dot Product Similarity Matrix



Dot product = Semantic similarity

# The Three Questions: Query, Key, Value

For each word, we ask 3 questions:

## 1. Query (Q): "What am I looking for?"

- Cat's query: "Who performed action on me?"

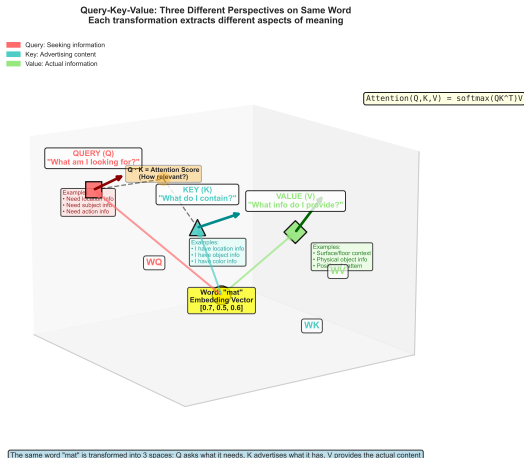
## 2. Key (K): "What do I offer?"

- Sat's key: "I am an action verb"

## 3. Value (V): "What information do I provide?"

- Sat's value: "Past tense sitting action"

**Matching:** Q·K determines attention weight



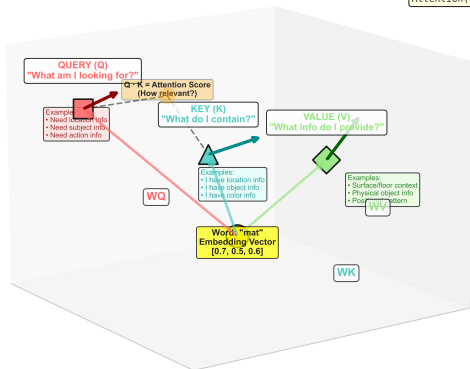
**QKV** transforms each word  
into searcher, identifier, and content

# Query, Key, Value Transformation

Query-Key-Value: Three Different Perspectives on Same Word  
Each transformation extracts different aspects of meaning

- Query: Seeking information
- Key: Advertising content
- Value: Actual information

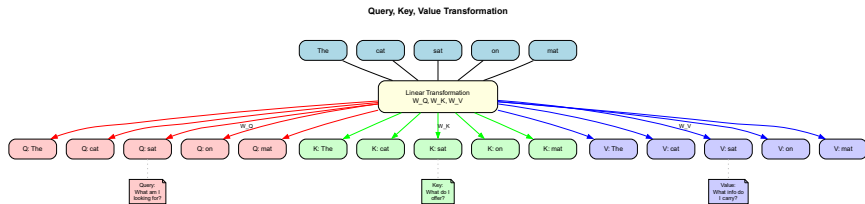
$$\text{Attention}(Q,K,V) = \text{softmax}(QK^T)V$$



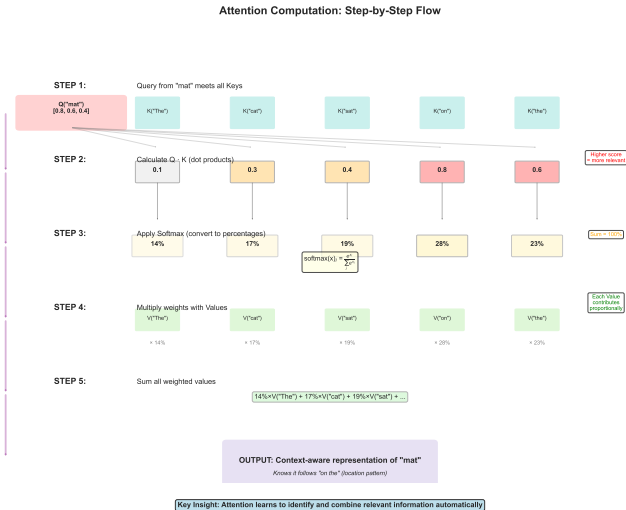
The same word "mat" is transformed into 3 spaces: Q asks what it needs, K advertises what it has, V provides the actual content



# Query, Key, Value Transformation (Flow Diagram)

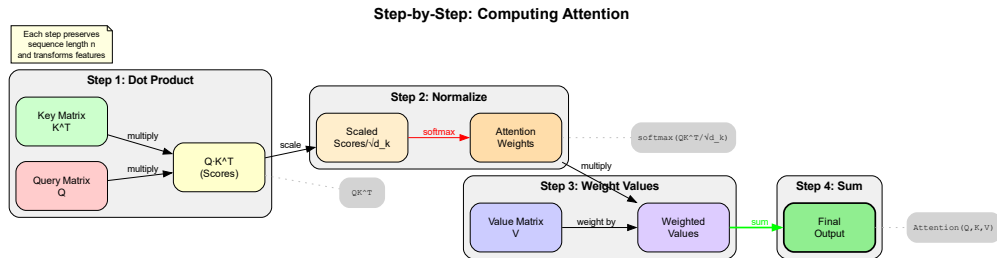


# Step-by-Step: Computing Attention

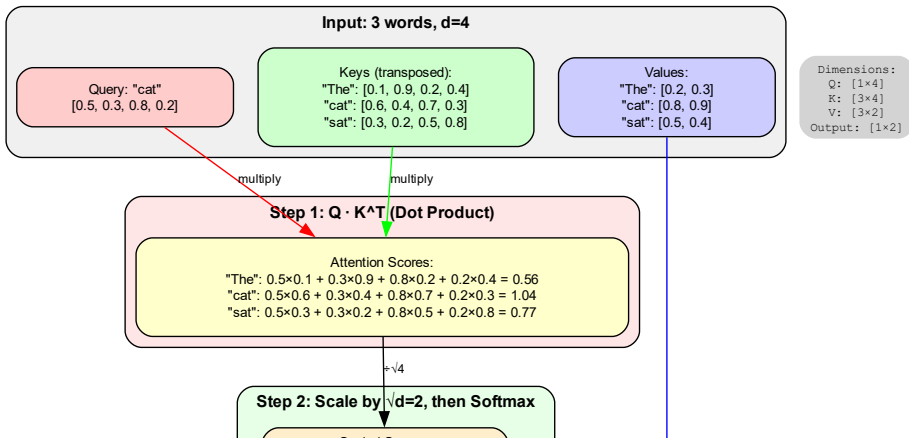


1. Q-K 2. Softmax 3. Weight V 4. Sum

# Computing Attention: Detailed Flow



## Computing Attention: Concrete Example "The cat sat" → Focus on "cat"



# Multiple Perspectives: 4 Different Experts

## Multi-Head Attention:

Like having 4 specialist readers:

### Head 1: Grammar Expert

- Subject-verb agreement
- Sentence structure

### Head 2: Meaning Expert

- Semantic relationships
- Word meanings

### Head 3: Position Expert

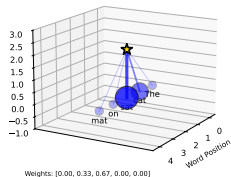
- Word order
- Distance relationships

### Head 4: Context Expert

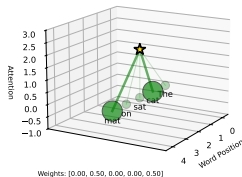
- Broader context
- Document theme

## Multi-Head Attention: 4 Different Perspectives

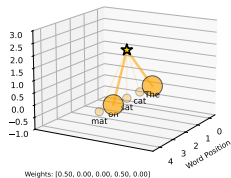
Head 1: Grammar Expert



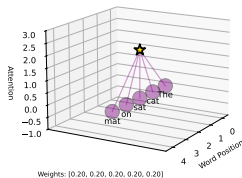
Head 2: Meaning Expert



Head 3: Position Expert



Head 4: Context Expert



Each head learns different attention patterns:

- Grammar: Subject-verb relationships
- Meaning: Semantic relationships

# The Speed Revolution: Everything at Once

## Old way (RNN): Sequential

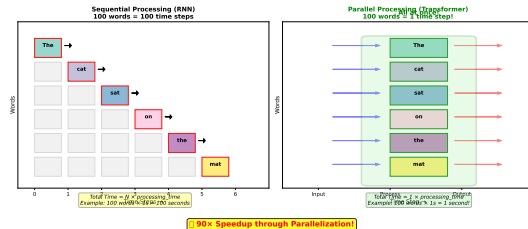
- Process word 1, then 2, then 3...
- 100 words = 100 time steps
- Like reading one letter at a time

## New way (Transformer): Parallel

- Process ALL words simultaneously
- 100 words = 1 time step!
- Like seeing whole page instantly

## Speed improvement:

- Training: 90 days  $\rightarrow$  1 day
- Inference: 10 seconds  $\rightarrow$  0.1 seconds



# Preserving Order: Where Words Live

**Problem:** Parallel loses word order!

“Dog bites man” vs “Man bites dog”

- Same words, different meaning
- Position matters!

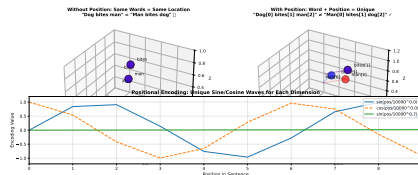
**Solution: Positional Encoding**

- Add position information
- Use sine/cosine waves
- Different frequency for each dimension

**Like GPS for words:**

- Word + Position = Unique identity
- Model knows where each word is

Position encoding =  
Word's address



# The Highway: Residual Connections

## Problem with deep networks:

- Information gets lost/distorted
- Like telephone game
- Each layer adds noise

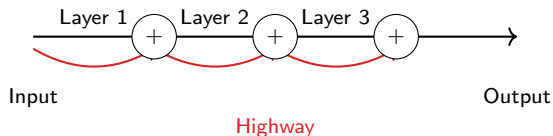
## Solution: Skip Connections

- Create “highways” for information
- Original signal preserved
- Add refinements, don't replace

## Formula:

$$\text{Output} = \text{Layer}(x) + x$$

Always keep original, add improvements



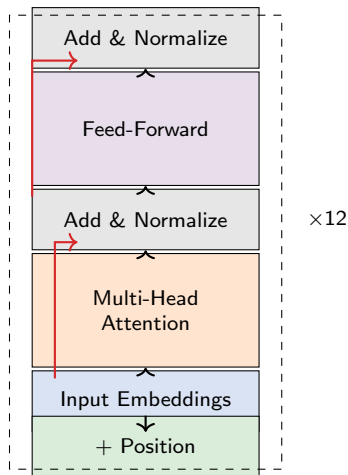
Residual = Original + Refinement  
Never lose information!



# Everything Together: The Transformer

## The Complete Architecture:

1. **Input:** Words  $\rightarrow$  Embeddings + Position
2. **Attention Block:**
  - Multi-head attention
  - Add & normalize (residual)
3. **Feed-Forward Block:**
  - Two linear layers
  - Add & normalize (residual)
4. **Stack 12 times** (BERT) or 96 times (GPT-3)
5. **Output:** Next word prediction



# Proof It Works: Real Results

## Benchmark Results (2017-2024):

### Translation (BLEU scores):

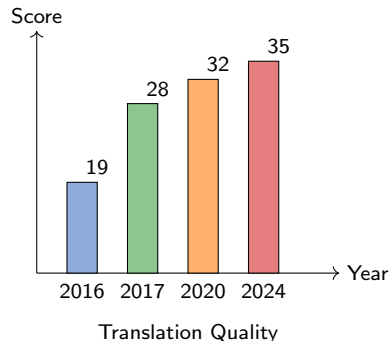
- 2016 (RNN): 19.2
- 2017 (Transformer): 28.4 ✓
- 2024 (GPT-4): 35.1 ✓

### Question Answering:

- Human performance: 89%
- BERT (2018): 93% ✓
- GPT-4 (2023): 96% ✓

### Training Speed:

- RNN: 3 months
- Transformer: 1 day ✓



**Transformers:** State-of-the-art  
on EVERY language task!

# The Revolution: 2017-2024

## Timeline of Breakthroughs:

### 2017: Transformer paper

- “Attention is All You Need”
- 8 researchers at Google

### 2018: BERT

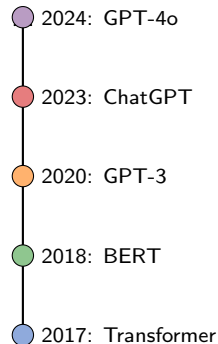
- Bidirectional understanding
- Crushed 11 benchmarks

### 2020: GPT-3

- 175 billion parameters
- Few-shot learning

### 2023: ChatGPT/GPT-4

- Conversation ability
- 100M users in 2 months



7 years: From research paper  
to 1 billion users!

## Problem 1: Calculate Attention

Given vectors:

- Query: [1, 0, 1]
- Key1: [1, 1, 0]
- Key2: [0, 1, 1]
- Key3: [1, 0, 1]

Calculate:

- 1  $Q \cdot K$  for each key
- 2 Apply softmax
- 3 Which word gets most attention?

## Problem 2: Multi-Head Design

Design 3 attention heads for: "The bank near the river bank"

What should each head focus on?

## Problem 3: Architecture

Draw transformer architecture for:

- 2-layer transformer
- Show residual connections
- Label each component

## Problem 4: Complexity

For a 100-word sentence:

- How many attention scores?
- Memory requirement?
- Why is this  $O(n^2)$ ?

**Solutions:** Available after lab session

# Summary: The Three Core Principles

## 1. Parallel Processing

- All words processed simultaneously
- 90x faster than sequential
- Enables large-scale training

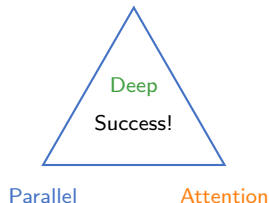
## 2. Attention Mechanism

- Focus on relevant connections
- Learn what matters from data
- Multiple perspectives (heads)

## 3. Deep Architecture

- Stack many layers (12-96)
- Residual connections preserve info
- Each layer refines understanding

## TRANSFORMERS



These 3 principles revolutionized AI  
and created ChatGPT, BERT, and more!

# Where You Use Transformers Every Day

## Search Engines:

- Google Search (BERT)
- Autocomplete suggestions
- “Did you mean...?”

## Translation:

- Google Translate
- Real-time captions
- Document translation

## Writing Assistants:

- Grammarly corrections
- Email suggestions (Gmail)
- Code completion (Copilot)

## Virtual Assistants:

- ChatGPT conversations
- Siri/Alexa understanding
- Customer service bots

## Content Creation:

- Image generation (DALL-E)
- Video subtitles
- Article summaries

**Fact:** You interact with transformers dozens of times daily!

# Check Your Understanding

## Quick Quiz:

**Q1:** Why are transformers fast?

- A) Smaller models
- B) Parallel processing ✓
- C) Better hardware
- D) Simpler math

**Q2:** What does attention do?

- A) Adds more parameters
- B) Focuses on relevant words ✓
- C) Speeds up training
- D) Reduces memory

**Q3:** Purpose of residual connections?

- A) Preserve information ✓
- B) Add complexity
- C) Reduce size
- D) Speed up inference

## Can you now:

- ☐ Explain word embeddings?
- ☐ Calculate dot product similarity?
- ☐ Describe Query, Key, Value?
- ☐ Draw attention mechanism?
- ☐ List transformer components?
- ☐ Explain parallel vs sequential?

## Congratulations!

You understand the technology  
behind ChatGPT!

From zero to transformer expert  
in 27 slides!

## Formal Attention Equation:

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right) V$$

## Where:

- $Q \in \mathbb{R}^{n \times d_k}$ : Queries
- $K \in \mathbb{R}^{n \times d_k}$ : Keys
- $V \in \mathbb{R}^{n \times d_v}$ : Values
- $n$ : Sequence length
- $d_k$ : Key/Query dimension
- $\sqrt{d_k}$ : Scaling factor

## Why Scale by $\sqrt{d_k}$ ?

- Dot products grow with dimension
- Large values  $\rightarrow$  softmax saturation
- Scaling keeps gradients stable

## Matrix Shapes Example:

- Input:  $[10, 512]$  (10 words)
- $Q, K, V$ :  $[10, 64]$  each
- Attention scores:  $[10, 10]$
- Output:  $[10, 64]$

**Key Insight:** The attention matrix is  $n \times n$  - quadratic in sequence length!



## Appendix B: Multi-Head Attention Mathematics

### Multi-Head Formula:

$$\text{MultiHead}(Q, K, V) = \text{Concat}(\text{head}_1, \dots, \text{head}_h)W^O$$

### Where each head:

$$\text{head}_i = \text{Attention}(QW_i^Q, KW_i^K, VW_i^V)$$

### Parameter Matrices:

- $W_i^Q \in \mathbb{R}^{d_{\text{model}} \times d_k}$
- $W_i^K \in \mathbb{R}^{d_{\text{model}} \times d_k}$
- $W_i^V \in \mathbb{R}^{d_{\text{model}} \times d_v}$
- $W^O \in \mathbb{R}^{hd_v \times d_{\text{model}}}$

### Typical Dimensions:

- $d_{\text{model}} = 512$
- $h = 8$  heads
- $d_k = d_v = d_{\text{model}}/h = 64$
- Total parameters:  $4 \times d_{\text{model}}^2$

### Computational Benefits:

- Heads run in parallel
- Different representation subspaces
- Similar cost to single-head
- Better performance in practice

8 heads  $\times$  64 dimensions = 512 total dimensions (preserved)

# Appendix C: Positional Encoding Mathematics

## Sinusoidal Position Encoding:

$$PE_{(pos, 2i)} = \sin\left(\frac{pos}{10000^{2i/d_{model}}}\right)$$

$$PE_{(pos, 2i+1)} = \cos\left(\frac{pos}{10000^{2i/d_{model}}}\right)$$

## Where:

- $pos$ : Position in sequence (0, 1, 2, ...)
- $i$ : Dimension index
- $d_{model}$ : Model dimension (e.g., 512)

## Properties:

- Unique encoding per position
- Smooth variation across positions
- Can extrapolate to longer sequences

## Why This Formula?

- Relative positions preserved
- $PE_{pos+k}$  can be represented as linear function of  $PE_{pos}$
- Different frequencies per dimension
- No learned parameters needed

## Wavelengths:

- Dimension 0-1: wavelength  $2\pi$
- Dimension 510-511: wavelength  $2\pi \cdot 10000$
- Covers short to long-range dependencies

Position encoding added to word embedding:  $x' = x + PE_{pos}$

## Appendix D: Complexity Analysis

### Self-Attention Complexity:

- Time:  $O(n^2 \cdot d)$
- Space:  $O(n^2 + n \cdot d)$
- Attention matrix:  $n \times n$

### RNN Complexity:

- Time:  $O(n \cdot d^2)$  sequential
- Space:  $O(d)$  per step
- Must process sequentially

### Comparison (n=100, d=512):

- Transformer:  $100^2 \times 512 = 5.12M$  ops
- RNN:  $100 \times 512^2 = 26.2M$  ops
- But RNN is sequential!

### Parallelization Benefits:

- All positions computed simultaneously
- GPU utilization: nearly 100%
- Training speedup: 50-100x
- Inference speedup: 10-20x

### Memory Requirements:

- Attention scores:  $O(n^2)$  bottleneck
- Max sequence typically 512-2048
- Recent work on sparse attention
- Linear attention variants emerging

**Tradeoff:** Quadratic memory for massive parallelization gain

## Appendix E: Implementation Details

### Layer Normalization:

$$\text{LayerNorm}(x) = \gamma \frac{x - \mu}{\sigma} + \beta$$

Applied as:  $\text{LayerNorm}(x + \text{Sublayer}(x))$

### Dropout Positions:

- After each sublayer: 0.1
- Attention weights: 0.1
- Embedding layer: 0.1

### Feed-Forward Network:

$$\text{FFN}(x) = \text{ReLU}(xW_1 + b_1)W_2 + b_2$$

- Hidden size:  $4 \times d_{\text{model}}$
- Two linear transformations
- ReLU activation between

### Learning Rate Schedule:

$$lr = d_{\text{model}}^{-0.5} \cdot \min(\text{step}^{-0.5}, \text{step} \cdot \text{warmup}^{-1.5})$$

### Typical Hyperparameters:

- $d_{\text{model}} = 512$  or  $768$
- $h = 8$  or  $12$  heads
- $d_{\text{ff}} = 2048$  or  $3072$
- Layers: 6 (base) or 12 (large)
- Warmup steps: 4000
- Batch size: 4096 tokens

### Training Tips:

- Label smoothing: 0.1
- Gradient clipping: 1.0
- Adam optimizer:  $\beta_1 = 0.9, \beta_2 = 0.98$

**Total Parameters (Base):** 65M for 6-layer transformer