

LSTM Primer: Next Word Prediction

A Comprehensive Introduction to Long Short-Term Memory Networks

BSc Level - No Prerequisites Required

September 27, 2025

From Autocomplete to Modern Language Models

Part 1: The Problem (9 slides)

- 1. Introduction: The Autocomplete Challenge
- 2. The LSTM Revolution (2015-2018)
- 3. Scientific Breakthroughs & Innovations
- 4. Course Roadmap & Learning Path
- 5. N-gram Baseline Models
- 6. Why N-grams Fail
- 7. The Memory Problem

Part 2: First Attempts (5 slides)

- 8. Recurrent Neural Networks (RNN)
- 9. RNN Concrete Example
- 10. The Vanishing Gradient Problem
- 11. Why RNNs Forget (Paris Example)

Part 3: The LSTM Solution (14 slides)

Part 4: Training (8 slides)

- 20. Training LSTMs (BPTT)
- 21. Training Progression Visualization
- 22. Why LSTMs Work (Gradient Highway)
- 23. Checkpoint: Gradient Highway
- 24. Hyperparameter Selection Guide
- 25. Regularization & Stability Techniques
- 26. Common Pitfalls & Debugging

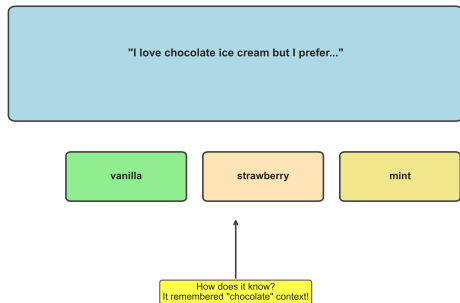
Part 5: Variants & Applications (12 slides)

- 27. GRU: Simplified LSTM Alternative
- 28. Bidirectional LSTM
- 29. Architectural Extensions (Stacked, Attention, Peephole)
- 30. Modern Context: Where LSTMs Fit (2020-2025)
- 31. Model Comparison Table

The Autocomplete Challenge

The Problem: Predicting What Comes Next

Your Phone Predicts the Next Word



The Challenge:

- Context can be very long
- Meaning changes with history

What You See:

The LSTM Revolution (2015-2018)

LSTMs Changed How We Interact with Technology

Google Translate (2016):

- Switched from phrase-based to LSTM
- Translation errors dropped 60%
- First human-quality translations
- 100+ languages supported
- Newspaper headline: “AI achieved breakthrough”

Speech Recognition:

- Siri, Alexa, Google Assistant
- Word error rate halved
- Real-time transcription enabled
- Accent-independent understanding
- Made voice interfaces practical

Text Prediction:

- Gmail Smart Compose

Why It Was Revolutionary:

Memory Breakthrough:

- **Before LSTMs:** 5-10 word memory
- **After LSTMs:** 50-100+ word memory
- **10x improvement** in context length
- First practical long-range modeling

Key Capabilities Unlocked:

- Understand full sentences
- Connect distant information
- Capture linguistic structure
- Handle complex grammar

Industry Transformation:

- Translation industry disrupted

Why LSTMs Matter for Research

Scientific Impact:

- **50,000+ papers:** Citing LSTM architecture
- **Hochreiter & Schmidhuber (1997):** Original breakthrough
- **Breakthrough:** Solved vanishing gradient problem
- **Foundation:** Enabled Transformers, BERT, GPT
- **Legacy:** Changed how we think about sequences
- **Recognition:** Nobel/Turing Award worthy

Conceptual Innovations:

- **Gating mechanisms**
 - Learned information control
 - Forget, input, output gates
 - Neural decision-making
- **Memory highway**
 - Direct gradient path via addition

Why Study LSTMs in 2025:

Educational Value:

- Perfect introduction to sequence models
- Intuitive gate mechanisms
- Clear mathematical structure
- Understand attention/Transformers better
- Foundation for all modern architectures

Practical Relevance:

- Still state-of-the-art for time series
- Mobile/edge deployment (efficient)
- Streaming/real-time processing
- Understanding gradient flow
- Debugging modern models

What We'll Cover in This Course

Part 1: The Problem (3 slides)

- **Why autocomplete is hard**
 - Need long context
 - Example: "I grew up in Paris ... speak fluent ___"
- **N-gram baseline**
 - Count-based approach
 - Fixed 1-2 word window
- **Why N-grams fail**
 - Can't see beyond 2 words
 - Combinatorial explosion
 - No generalization

Part 2: First Attempts (3 slides)

- **Recurrent Neural Networks**
 - Hidden state as memory
 - Can theoretically remember everything
- **Vanishing gradient problem**

Part 3: The LSTM Solution (7 slides)

- **Architecture overview**
 - Three gates + cell state
 - Memory highway concept
- **Gate mechanisms**
 - Forget gate: What to remove
 - Input gate: What to add
 - Output gate: What to reveal
- **Mathematics**
 - All six equations
 - Concrete numerical examples
 - Gradient flow analysis
- **Why it works**
 - Addition preserves gradients
 - 50-100+ step memory

Part 4: Training & Applications (7 slides)

Simple Idea: Count What Usually Comes Next

How It Works:

Step 1: Look at training data

- Count every word pair (bigram)
- Count every word triple (trigram)
- Store frequency tables

Step 2: Make predictions

- Look at last 1-2 words
- Find in frequency table
- Pick most common next word

Example Training Data:

"I love chocolate. I love pizza. I love ice cream."

Bigram Counts:

- "I love" → 3 times
- "love chocolate" → 1 time

Prediction Process:

Input: "I"

- Check bigram table
- "I love" appears 3 times
- Predict: "love"

Input: "I love"

- Check trigram table
- Three options (1 count each)
- Pick randomly or use context

The Math:

$$P(\text{word}_t \mid \text{word}_{t-1}, \text{word}_{t-2}) = \frac{\text{count}(w_{t-2}, w_{t-1}, w_t)}{\text{count}(w_{t-2}, w_{t-1})}$$

Why It's Popular:

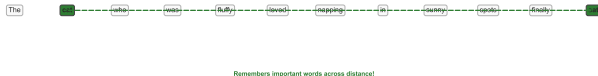
Why N-grams Fail: The Context Window Problem

Fatal Limitation: Can Only See 1-2 Words Back

N-Gram: Fixed 2-Word Window (Forgets "cat")



LSTM: Selective Memory (Remembers "cat")



The Window Problem:

"I grew up in Paris. I went to school there for 12 years. I learned to speak fluent ___"

- **Trigram sees:** "speak fluent ___"

Three Fatal Flaws:

1. Limited Context:

- Only 1-2 words visible
- Long-range dependencies impossible

The Memory Problem: What Should We Remember?

Insight from Human Reading

Imagine Reading a Novel:

Chapter 1: "Alice was born in London in 1985. She had a happy childhood."

Chapter 3: "After graduating from university, Alice moved to New York."

Chapter 7: "Now 38 years old, Alice reflected on her life in ---"

What You Remember:

- Alice (main character)
- Born in London
- Moved to New York
- Currently 38 years old

What You Forgot:

- "had a happy childhood"
- "graduating from university"

Key Insight: Memory is selective

Human Memory Strategy:

- 1 **Decide** what's important
- 2 **Keep** relevant information
- 3 **Forget** irrelevant details
- 4 **Update** as story progresses

What We Need in AI:

Forget Mechanism:

- Remove outdated information
- Clear memory when topic changes
- Example: Forget "chocolate" after period

Input Mechanism:

- Decide what to store

Idea: Maintain a Hidden State as Memory

The RNN Concept:

- **Hidden state** h_t = memory
- Update memory at each word
- Use memory to make predictions
- Memory flows through time

The Math:

$$h_t = \tanh(W_h h_{t-1} + W_x x_t + b)$$

$$y_t = \text{softmax}(W_y h_t + b_y)$$

Where:

- h_t = hidden state (memory) at time t
- h_{t-1} = previous memory
- x_t = current word embedding
- y_t = prediction probabilities

How It Processes Sequences:

Input: "I love chocolate"

Step 1: Process "I"

- $h_0 = [0, 0, 0, \dots]$ (initial state)
- $h_1 = \tanh(W_h h_0 + W_x [\text{embed}(\text{"I"})] + b)$
- Predict next word from h_1

Step 2: Process "love"

- Use h_1 from previous step
- $h_2 = \tanh(W_h h_1 + W_x [\text{embed}(\text{"love"})] + b)$
- Now h_2 contains info about "I love"

Step 3: Process "chocolate"

- $h_3 = \tanh(W_h h_2 + W_x [\text{embed}(\text{"chocolate"})] + b)$
- h_3 should remember full sequence

Walking Through an Actual RNN Computation

Setup:

Input sequence: "I love"

Dimensions:

- Hidden size: 3 (toy example)
- Word embeddings: 2-dimensional

Weights (simplified):

$$W_h = \begin{bmatrix} 0.5 & -0.2 & 0.3 \\ 0.1 & 0.4 & -0.1 \\ -0.3 & 0.2 & 0.6 \end{bmatrix}$$

$$W_x = \begin{bmatrix} 0.8 & 0.3 \\ -0.2 & 0.5 \\ 0.4 & -0.1 \end{bmatrix}$$

Word embeddings:

- "I" = [1.0, 0.5]

Step 1: Process "I"

$$h_0 = [0, 0, 0]$$

$$W_x x_1 = \begin{bmatrix} 0.8 & 0.3 \\ -0.2 & 0.5 \\ 0.4 & -0.1 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 0.95 \\ 0.05 \\ 0.35 \end{bmatrix}$$

$$h_1 = \tanh([0, 0, 0] + [0.95, 0.05, 0.35])$$

$$h_1 = \tanh([0.95, 0.05, 0.35])$$

$$h_1 = [0.74, 0.05, 0.34]$$

Step 2: Process "love"

$$W_h h_1 = \begin{bmatrix} 0.5 & -0.2 & 0.3 \\ 0.1 & 0.4 & -0.1 \\ -0.3 & 0.2 & 0.6 \end{bmatrix} \begin{bmatrix} 0.74 \\ 0.05 \\ 0.34 \end{bmatrix} = \begin{bmatrix} 0.46 \\ 0.04 \\ -0.01 \end{bmatrix}$$

The Vanishing Gradient Problem

Why RNNs Can't Learn Long-Term Dependencies

Training Neural Networks:

Forward Pass:

- Input \rightarrow Hidden layers \rightarrow Output
- Compute prediction
- Calculate loss (error)

Backward Pass (Backpropagation):

- Compute gradient of loss w.r.t. weights
- Flow gradient backward through network
- Update weights to reduce error

The Problem in RNNs:

Gradient at step t depends on all previous steps:

$$\frac{\partial L}{\partial W_h} = \sum_{t=1}^T \frac{\partial L_t}{\partial W_h}$$

Why It Vanishes:

Typical values:

- $\tanh'(x) \leq 1$ (often ≈ 0.5)
- If $\|W_h\| < 1$, products shrink
- After n steps: $\approx 0.5^n \cdot \|W_h\|^n$

The Numbers:

Steps	Gradient	% Remaining
1	0.90	90%
5	0.59	59%
10	0.35	35%
20	0.12	12%
50	0.005	0.5%
100	0.000027	0.0027%

The Impact:

- Gradient becomes infinitesimally small

Why RNNs Forget: The Paris Example

Concrete Example of Memory Decay

The Sentence:

"I grew up in Paris. I went to school there. I learned to speak fluent ___"

What Happens in RNN:

Word 1-5: "I grew up in Paris"

- h_5 encodes this information
- "Paris" stored in hidden state
- Looks promising!

Word 6-15: "I went to school there"

- h_{15} updates with new words
- Previous h_5 gets multiplied by W_h ten times
- Information about "Paris" weakens
- $h_{15} \approx$ mostly recent words

Word 16-21: "I learned to speak fluent"

The Math Behind Forgetting:

Hidden state update:

$$h_t = \tanh(W_h h_{t-1} + W_x x_t)$$

After n steps, contribution from h_0 :

$$h_n \approx (W_h)^n h_0 + \text{recent terms}$$

If largest eigenvalue of W_h is λ :

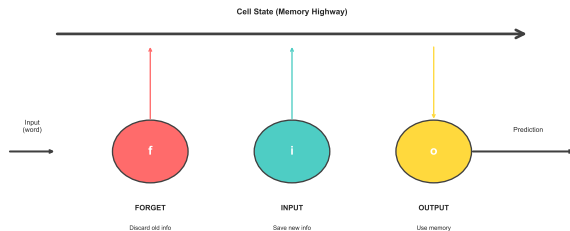
- $\lambda < 1$: Exponential decay
- $\lambda = 0.9$ typical
- After 20 steps: $0.9^{20} = 0.12$
- Information multiplied by 0.12

During Training:

Gradient from step 21 to step 5:

Long Short-Term Memory: Gated Memory Cells

LSTM Cell: Three Gates Control Memory



Like Traffic Lights: Red (forget) • Green (input) • Yellow (output)

The Three Gates:

Checkpoint: Understanding LSTM Architecture

Test Your Understanding

Quick Quiz:

Question 1: What's the key difference between RNN and LSTM?

- A) LSTM has more parameters
- B) LSTM uses addition instead of multiplication
- C) LSTM is faster to train
- D) LSTM uses different activation functions

Question 2: Which gate controls what enters memory?

- A) Forget gate
- B) Input gate
- C) Output gate
- D) Cell gate

Question 3: When does the forget gate activate strongly (high value)?

Answers:

Answer 1: B - LSTM uses addition for cell state

- Cell state: $C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$
- Addition creates direct gradient path
- No repeated matrix multiplication
- This is the key innovation!

Answer 2: B - Input gate

- Controls how much new information to store
- High value = store strongly
- Low value = ignore
- Works with candidate cell state \tilde{C}_t

Answer 3: C - During normal continuation

- High forget gate = keep memory

Forget Gate: What Should We Remove?

The Equation:

$$f_t = \sigma(W_f[h_{t-1}, x_t] + b_f)$$

Where:

- f_t = forget gate activations (0 to 1)
- h_{t-1} = previous hidden state
- x_t = current input word
- σ = sigmoid function
- $[h_{t-1}, x_t]$ = concatenation

What Sigmoid Does:

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

- Maps any number to (0,1)
- $z \rightarrow -\infty$: $\sigma(z) \rightarrow 0$ (forget)

Concrete Example:

Input sequence: "I love chocolate. But I prefer"

At word "chocolate":

- $f_t = [0.95, 0.92, 0.88, \dots]$
- Keep most information
- Normal sentence continuation

At word "." (period):

- $f_t = [0.1, 0.2, 0.15, \dots]$
- Forget most of previous sentence!
- Topic might change
- New sentence starting

At word "But":

- $f_t = [0.3, 0.4, 0.25, \dots]$
- Contrast coming

The Input Gate: Adding New Information

Input Gate: What Should We Store?

Two Equations:

Input Gate:

$$i_t = \sigma(W_i[h_{t-1}, x_t] + b_i)$$

Controls *how much* to add (0 to 1)

Candidate Memory:

$$\tilde{C}_t = \tanh(W_C[h_{t-1}, x_t] + b_C)$$

What content to potentially add (-1 to 1)

Combined Update:

$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$$

- First term: What to keep from past
- Second term: What to add from present
- Element-wise operations

Concrete Example:

Input: “I grew up in Paris”

At word “I”:

- $i_t = [0.3, 0.2, 0.1, \dots]$ (low)
- Common word, not much info
- $\tilde{C}_t = [0.5, -0.2, 0.1, \dots]$
- Minimal storage

At word “Paris”:

- $i_t = [0.95, 0.92, 0.88, \dots]$ (high!)
- Important location word
- $\tilde{C}_t = [0.8, 0.7, -0.3, \dots]$
- Strong encoding of “Paris”
- Will be useful later

At word “grew”:

The Output Gate: Revealing Memory

Output Gate: What Should We Use Now?

The Equations:

Output Gate:

$$o_t = \sigma(W_o[h_{t-1}, x_t] + b_o)$$

Controls how much memory to expose (0 to 1)

Hidden State (Output):

$$h_t = o_t \odot \tanh(C_t)$$

- $\tanh(C_t)$: Squash cell state to $(-1, 1)$
- $o_t \odot$: Filter what's revealed
- h_t : Working memory for prediction

Final Prediction:

$$y_t = \text{softmax}(W_y h_t + b_y)$$

Concrete Example:

Sequence: "I grew up in Paris. I learned to speak fluent
---"

At word "learned":

- $o_t = [0.3, 0.4, 0.2, \dots]$ (low)
- Not predicting yet
- Don't need full memory
- Just process the word

At word "fluent":

- $o_t = [0.9, 0.85, 0.92, \dots]$ (high!)
- About to predict language
- Need location information
- Recall "Paris" from C_t
- h_t contains relevant context

Cell State C_t : The Key Innovation

The Complete Update:

$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$$

What Makes This Special:

RNN Update:

$$h_t = \tanh(W_h h_{t-1} + W_x x_t)$$

- Matrix multiplication by W_h
- Nonlinear tanh
- Information transformed
- Gradient must flow through both

LSTM Cell State:

$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$$

The Highway Analogy:

RNN (Local Roads):

- Information stops at every step
- Gets transformed each time
- Slow, lossy transmission
- Limited range

LSTM (Highway):

- Direct express lane (C_t)
- Minimal transformation
- Fast, lossless transmission
- Long-range connectivity

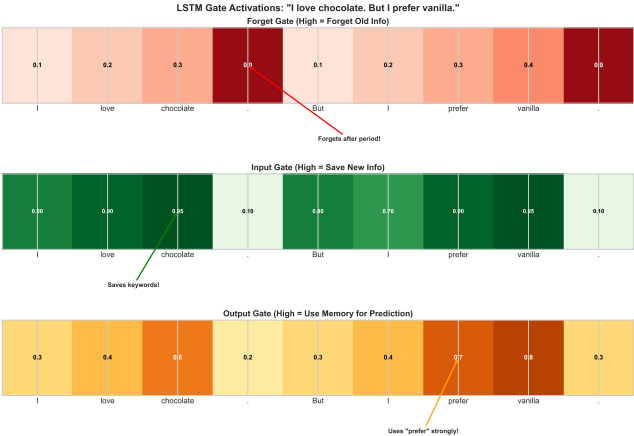
Numerical Comparison:

Remembering information from 50 steps back:

RNN:

Gate Activation Patterns: Real LSTM Behavior

Visualizing How Gates Actually Work



Key Observations:

Forget Gate (Top):

What We're Looking At:

Full LSTM Computation with Numbers

All Equations:

$$f_t = \sigma(W_f[h_{t-1}, x_t] + b_f)$$

$$i_t = \sigma(W_i[h_{t-1}, x_t] + b_i)$$

$$\tilde{C}_t = \tanh(W_C[h_{t-1}, x_t] + b_C)$$

$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$$

$$o_t = \sigma(W_o[h_{t-1}, x_t] + b_o)$$

$$h_t = o_t \odot \tanh(C_t)$$

Dimensions (example):

- Vocabulary: 10,000 words
- Embedding: 100 dims
- Hidden/Cell: 256 dims
- W_f, W_i, W_C, W_o : 256×356
- $[h_{t-1}, x_t]$: 356 dims (256+100)

Concrete Numbers:

Input: "love" (word embedding)

Step 1: Forget gate

```
f_t = sigmoid([0.5, -0.2, 0.8, ...])  
      = [0.62, 0.45, 0.69, ...]
```

Step 2: Input gate & candidate

```
i_t = sigmoid([1.2, 0.8, -0.5, ...])  
      = [0.77, 0.69, 0.38, ...]  
C_tilde = tanh([0.6, -0.3, 0.9, ...])  
          = [0.54, -0.29, 0.72, ...]
```

Step 3: Update cell state

```
C_t = [0.62*0.5, 0.45*0.3, ...]  
      + [0.77*0.54, 0.69*(-0.29), ...]  
      = [0.73, 0.14, ...]
```

Step 4: Output & hidden

```
o_t = sigmoid([0.9, 1.1, -0.2, ...])  
      = [0.71, 0.75, 0.45, ...]  
h_t = [0.71*tanh(0.73), ...]  
      = [0.44, ...]
```

Training LSTMs: Backpropagation Through Time

How LSTMs Learn from Data

Training Process:

1. Forward Pass:

- Process entire sequence
- Compute predictions at each step
- Calculate loss (cross-entropy)

$$L = - \sum_{t=1}^T \log P(w_t \mid w_1, \dots, w_{t-1})$$

2. Backward Pass:

- Compute gradients of loss
- Flow backward through time
- Update all weight matrices

3. Weight Update:

Why LSTM Gradient Flow Works:

Key Gradient:

$$\frac{\partial C_t}{\partial C_{t-1}} = f_t$$

- Simple element-wise multiplication
- No matrix multiplication
- If $f_t \approx 1$: Perfect transmission
- Gradient preserved across time

Training Challenges:

- **Sequence length:** Longer = more memory
- **Batch size:** Typically 32-128 sequences
- **Learning rate:** Must be carefully tuned
- **Gradient clipping:** Prevent explosions

Hyperparameters:

Training Progression: Watching the LSTM Learn

How Performance Improves Over Time

LSTM Training: Watching It Learn

Epoch 1: Random Initialization

Input: "I love chocolate"

Prediction: "xjwkq"

Loss: 8.5 (Gibberish)

Epoch 10: Learning Letters

Input: "I love chocolate"

Prediction: "cream"

Loss: 2.1 (Better!)

Epoch 50: Understanding Context

Input: "I love chocolate"

Prediction: "ice cream"

Loss: 0.4 (Good!)

Epoch 200: Fluent Generation

Input: "I love chocolate"

Prediction: "Ice cream
and strawberry cake"

Loss: 0.08 (Excellent!)

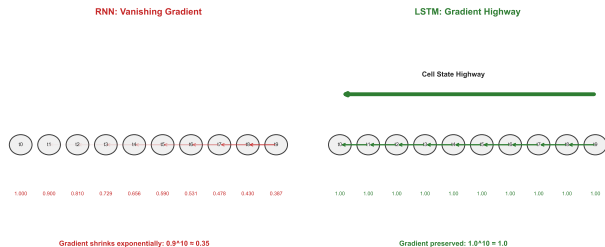
Epoch 1 (Random):

- Loss: 8.5 (very high)

What This Shows:

Why LSTMs Work: The Gradient Highway

Direct Comparison: RNN vs LSTM



Numerical Evidence:

Gradient after n steps:

Steps	RNN	LSTM
10	0.35	0.90
20	0.12	0.82
50	0.005	0.61
100	0.000027	0.37

Why It Works:

Additive Updates:

$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$$

- Addition creates direct path
- No repeated transformations
- Information preserved

Checkpoint: Why the Gradient Highway Works

Test Your Understanding of LSTM Training

Quick Quiz:

Question 1: What causes vanishing gradients in RNNs?

- A) Too many parameters
- B) Repeated matrix multiplication
- C) Wrong learning rate
- D) Long sequences

Question 2: How does LSTM solve this?

- A) Larger matrices
- B) Addition instead of multiplication
- C) Better initialization
- D) More layers

Question 3: If forget gate $f_t = 1.0$ always, what happens?

- A) Network can't learn

Answers:

Answer 1: B - Repeated matrix multiplication

- Each step: $\text{gradient} \times W_h$
- After n steps: $(W_h)^n$
- If $\|W_h\| < 1$: Exponential decay
- This is the core problem

Answer 2: B - Addition for cell state

- $C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$
- Gradient: $\frac{\partial C_t}{\partial C_{t-1}} = f_t$
- Element-wise, no matrix
- Direct path preserved

Answer 3: B - Perfect gradient flow

- $f_t = 1 \Rightarrow \text{gradient multiplied by 1}$

Choosing the Right LSTM Configuration

Hidden Size (Memory Capacity):

General Guidelines:

- **Start with 256** (good default)
- **Small data (< 10MB):** 128
- **Medium data (10-100MB):** 256-512
- **Large data (> 100MB):** 512-1024

Trade-offs:

- Larger = better memory, more capacity
- Larger = slower training, more overfitting risk
- Rule of thumb: Match to sequence complexity

Number of Layers (Depth):

Recommendations:

- **Start with 2 layers** (sweet spot)

Learning Rate:

Safe Defaults:

- **Adam optimizer:** 0.001 (standard)
- **SGD with momentum:** 0.01-0.1
- **RMSprop:** 0.001

Troubleshooting:

- **Loss oscillates:** Reduce by 10x
- **Slow convergence:** Increase by 3x
- **NaN loss:** Too high, reduce significantly

Learning Rate Schedules:

- Reduce by 0.5x every 10 epochs
- Or use ReduceLROnPlateau
- Cosine annealing for fine-tuning

Preventing Overfitting and Training Instability

Dropout (Essential):

How to Apply:

- Between LSTM layers: 0.2-0.5
- NOT within LSTM cell (breaks gradient flow)
- After each LSTM layer (except last)
- Higher for small datasets (0.4-0.5)

Effects:

- Prevents co-adaptation
- Forces redundant representations
- Typical improvement: 2-5% accuracy
- Critical for $\leq 100K$ examples

Gradient Clipping (Critical):

Why Needed:

Sequence Length Management:

Recommendations:

- Start with 50-100 tokens
- Truncate longer sequences (BPTT)
- Pad shorter sequences to batch
- Sort by length, batch similar lengths

Trade-offs:

- Longer = better context, more memory/time
- Shorter = faster, worse long-term dependencies
- Use Truncated BPTT for very long sequences

Training Time and Monitoring:

Expect:

- 10-100 epochs typical

What Can Go Wrong and How to Fix It

Problem 1: Loss Not Decreasing

Symptoms:

- Loss stays constant
- Or decreases very slowly
- Predictions remain random

Possible Causes & Fixes:

- **Learning rate too low**
 - Try: 10x higher (0.01 instead of 0.001)
- **Vanishing gradients** (still possible!)
 - Check gradient norms
 - Reduce sequence length
 - Initialize forget gate bias to 1.0
- **Bug in data pipeline**
 - Verify labels match inputs
 - Check for shuffling errors

Problem 2: Exploding Gradients

Problem 3: Overfitting

Symptoms:

- Train loss low, val loss high
- Perfect on training data
- Poor on validation/test

Fixes:

- Increase dropout (0.3 \rightarrow 0.5)
- Reduce model size
- Get more training data
- Early stopping
- L2 regularization

Problem 4: Predictions Uniform

Symptoms:

Gated Recurrent Unit - Fewer Gates, Similar Performance

Key Idea: Simplify LSTM architecture

Main Differences from LSTM:

- Only 2 gates (vs 3 in LSTM)
- No separate cell state
- Fewer parameters (25% reduction)
- Faster training
- Often comparable performance
- Popular for quick experiments

GRU Equations:

$$z_t = \sigma(W_z[h_{t-1}, x_t]) \quad (\text{update gate})$$

$$r_t = \sigma(W_r[h_{t-1}, x_t]) \quad (\text{reset gate})$$

$$\tilde{h}_t = \tanh(W[r_t \odot h_{t-1}, x_t])$$

$$h_t = (1 - z_t) \odot h_{t-1} + z_t \odot \tilde{h}_t$$

When to Use GRU:

Advantages:

- Faster to train (fewer parameters)
- Less memory usage
- Good starting point
- Often matches LSTM performance
- Easier to tune

Choose GRU if:

- Limited compute budget
- Quick experimentation needed
- Dataset not huge
- Speed more important than last 1% accuracy

Choose LSTM if:

- Need maximum performance

Processing Sequences in Both Directions

Key Idea: See future context too

Architecture:

- Two LSTMs running simultaneously
- Forward LSTM: \vec{h}_t (left-to-right)
- Backward LSTM: \overleftarrow{h}_t (right-to-left)
- Concatenate outputs at each step

$$h_t = [\vec{h}_t; \overleftarrow{h}_t]$$

Example: “The cat sat on the ___”

- **Forward:** Sees “The cat sat on the”
- **Backward:** Sees full sentence from end
- **Combined:** Full context for each word

Advantages:

Limitations:

- **Cannot generate:** Needs full input first
- **2x slower:** Two LSTMs to run
- **2x memory:** Double hidden states
- **No real-time:** Must wait for complete sequence

Use Cases:

Perfect for:

- Sentence classification (sentiment, intent)
- Named entity recognition (NER)
- Part-of-speech (POS) tagging
- Question answering (full question available)
- Any classification with full sequence

NOT for:

Advanced LSTM Configurations

Stacked/Deep LSTMs:

- Multiple LSTM layers stacked vertically
- Layer 1 output becomes Layer 2 input
- Hierarchical feature learning
- 2-4 layers typical (diminishing returns after 4)

Why Stacking Works:

- **Layer 1:** Low-level patterns
- **Layer 2:** Mid-level features
- **Layer 3:** High-level abstractions
- Similar to deep CNNs

Attention Mechanism:

Problem: Encoder-decoder bottleneck

- Single vector must encode entire source

Attention Equations:

$$\alpha_{t,i} = \frac{\exp(\text{score}(h_t, h_i))}{\sum_j \exp(\text{score}(h_t, h_j))}$$

$$c_t = \sum_i \alpha_{t,i} h_i$$

- $\alpha_{t,i}$ = attention weights
- c_t = context vector
- Led directly to Transformers

Peephole Connections:

- Gates see cell state directly
- $f_t = \sigma(W_f[h_{t-1}, x_t, C_{t-1}])$
- Better timing information
- Marginal improvements (1-2%)
- Rarely used in practice

LSTMs in the Transformer Era

Transformers Dominate NLP:

- **BERT** (2018): Bidirectional pretraining
- **GPT** (2018+): Autoregressive generation
- **T5, BART** (2019): Unified text-to-text
- **ChatGPT** (2022): Massive scale

Why Transformers Won NLP:

- **Parallel training:** All positions at once
- **Self-attention:** Direct connections
- **Scalability:** 100B+ parameters
- **Pre-training:** Transfer learning
- **Long context:** 128K+ tokens

Where LSTMs Still Excel:

- **Time series forecasting**

Hybrid Architectures:

- **CNN + LSTM:** Video/audio analysis
 - CNN extracts spatial features
 - LSTM models temporal dynamics
- **LSTM + Attention:** Improved seq2seq
 - Encoder: Bidirectional LSTM
 - Decoder: LSTM + attention
 - Pre-Transformer standard
- **Transformer + LSTM:** Hybrid models
 - Transformer encoder
 - LSTM decoder (faster inference)

Research Directions:

- **Linear RNNs:** $O(1)$ memory complexity
- **State Space Models:** S4, Mamba
- **Efficient Transformers:** Linformer, Performer
- **Hardware optimization:** Custom LSTM chips
- **Continual learning:** Streaming updates

Decision Framework for Sequence Modeling

Comparison: N-gram vs RNN vs LSTM

Feature	N-gram	RNN	LSTM
Memory Type	Fixed window	Fading	Selective
Long Context	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Parameters	Few	Moderate	Many
Training Speed	Fast	Medium	Slow
Vanishing Gradient	N/A	Yes <input type="checkbox"/>	Solved <input type="checkbox"/>
Best For	Short (2-3 words)	Medium (10 words)	Long (50+ words)
Example	"I love..."	"The cat sat..."	"The cat, who was...finally..."

Practical Recommendations:

Use LSTM when:

- Time series prediction

Decision Tree:

Question 1: Need bidirectional context?

The Idea That Changed Everything

The Problem with LSTMs:

In sequence-to-sequence tasks:

- Encoder compresses entire source into one vector
- Decoder must use this single vector
- Bottleneck for long sequences
- Information loss

The Attention Solution:

Instead of single vector:

- Keep all encoder hidden states
- Decoder “attends” to relevant parts
- Weighted combination at each step
- Dynamic focus

Attention Score:

Example: Translation

English: “The cat sat on the mat”

French: “Le chat ___”

When generating “assis” (sat):

- High attention on “sat” (0.7)
- Medium attention on “cat” (0.2)
- Low attention on “the”, “on” (0.05 each)
- Almost zero on “mat” (0.01)

Why It Matters:

- Solves long sequence problem
- Interpretable (visualize attention)
- State-of-the-art 2015-2017
- Direct path to Transformers

Transformers (2017)

How LSTMs Transformed Natural Language Processing

Machine Translation:

The Breakthrough (2016):

- Google Translate switches to LSTM
- Translation errors dropped 60% overnight
- First human-quality translations
- 100+ languages supported
- Newspaper: "AI achieved breakthrough"

How It Works:

- Encoder LSTM: Read source sentence
- Decoder LSTM: Generate translation
- Attention mechanism: Focus on relevant parts
- Beam search: Find best translation

Impact:

- Billions of translations daily

Text Understanding:

- **Sentiment analysis:** Product reviews, social media
- **Named entity recognition:** Extract names, locations
- **Question answering:** Early chatbot systems
- **Document classification:** Topic categorization
- **Intent detection:** Virtual assistants
- **Relation extraction:** Knowledge graphs

Text Generation:

- **Story writing:** Character-level models
- **Code completion:** Early IDE assistants
- **Email auto-complete:** Gmail Smart Compose
- **Dialogue systems:** Conversational AI
- **Summarization:** Abstractive summaries

Beyond Text: Multimodal Sequence Modeling

Speech Recognition:

The Revolution:

- Siri, Alexa, Google Assistant
- Word error rate halved (2015-2017)
- Real-time transcription enabled
- Accent-independent understanding
- Background noise robustness

Architecture:

- Bidirectional LSTM (see full context)
- CTC loss (alignment-free training)
- Multi-layer stacking (hierarchical features)
- Combined with CNN for spectrograms

Audio Applications:

Computer Vision:

Video Understanding:

- **Video captioning:** Describe video content
- **Action recognition:** Identify activities
- **Video prediction:** Forecast future frames
- **Temporal modeling:** Frame relationships
- **Event detection:** Anomaly in surveillance

Architecture: CNN+LSTM

- CNN: Extract spatial features per frame
- LSTM: Model temporal dynamics
- Attention: Focus on important frames
- End-to-end training possible

Image-Text Tasks:

Where LSTMs Excel at Temporal Prediction

Financial Forecasting:

- **Stock prediction:** Price movements, volatility
- **Trading algorithms:** Automated strategies
- **Risk assessment:** Portfolio optimization
- **Market trends:** Pattern recognition
- **Fraud detection:** Transaction anomalies

Why LSTMs Work Here:

- Capture long-term trends
- Handle non-stationary data
- Model complex dependencies
- Real-time prediction capability

Infrastructure Monitoring:

- **Weather forecasting:** Temperature, precipitation

Industrial Applications:

- **Anomaly detection:** Equipment monitoring
- **Predictive maintenance:** Failure prediction
- **Quality control:** Defect detection
- **Sensor data analysis:** IoT systems
- **Process optimization:** Manufacturing

Key Advantages:

- Sequential data naturally handled
- Variable-length sequences
- Missing data tolerance
- Multi-variate time series
- Uncertainty quantification

Comparison with Alternatives:

Medical AI and Historical Impact

Healthcare Applications:

Patient Monitoring:

- ICU vital signs time series
- Early warning systems
- Sepsis prediction
- Mortality risk assessment

Clinical Analysis:

- **ECG analysis:** Arrhythmia detection
- **Disease progression:** Trajectory modeling
- **Medical imaging:** Temporal scan sequences
- **EEG analysis:** Seizure prediction

Research Applications:

- **Drug discovery:** Molecular sequences
- **Protein folding:** Structure prediction

Impact Statistics:

Research Impact:

- **50,000+ papers:** Citing LSTM architecture
- **Hochreiter & Schmidhuber (1997):** Original paper
- **Foundation:** Enabled modern sequence models
- **Turing Award** consideration-worthy

Industry Impact:

- **Google Translate:** 60% error reduction (2016)
- **Speech recognition:** Word error rate halved
- **Billions of users:** Daily interactions
- **Economic value:** Tens of billions USD

Current Status (2025):

- **NLP:** Transformers dominate
- **Speech:** Hybrid LSTM+Transformer

Building LSTM Models in Practice

Model Definition:

```
import torch
import torch.nn as nn

class LSTMModel(nn.Module):
    def __init__(self, vocab_size,
                  embed_dim, hidden_dim):
        super().__init__()
        self.embedding = nn.Embedding(
            vocab_size, embed_dim)
        self.lstm = nn.LSTM(
            embed_dim, hidden_dim,
            num_layers=2, dropout=0.3,
            batch_first=True)
        self.fc = nn.Linear(
            hidden_dim, vocab_size)

    def forward(self, x):
        embed = self.embedding(x)
        lstm_out, (h_n, c_n) = self.lstm(embed)
        output = self.fc(lstm_out)
        return output

model = LSTMModel(10000, 100, 256)
```

Training Loop:

```
optimizer = torch.optim.Adam(
    model.parameters(), lr=0.001)
criterion = nn.CrossEntropyLoss()

for epoch in range(num_epochs):
    for batch in dataloader:
        input_seq, target_seq = batch

        # Forward
        output = model(input_seq)
        loss = criterion(
            output.view(-1, vocab_size),
            target_seq.view(-1))

        # Backward
        optimizer.zero_grad()
        loss.backward()

        # Gradient clipping
        torch.nn.utils.clip_grad_norm_(
            model.parameters(), 1.0)

        optimizer.step()
```

Key Parameters:

- num_layers=2: Stack 2 LSTMs

Summary: The Problem and LSTM Solution

From Memory Problems to the LSTM Breakthrough

The Problem We Solved:

Challenge: Next word prediction needs long context

- Example: “I grew up in Paris . . . speak fluent ___”
- Need to remember “Paris” 18 words back
- Connect location to language

Previous Attempts Failed:

- **N-grams:** Fixed 1-2 word window
 - Can't see beyond immediate context
 - Combinatorial explosion
- **RNNs:** Vanishing gradients
 - Memory limited to 5-10 words
 - Gradient: $0.5^{50} \approx 10^{-15}$

What We Needed:

- Selective long-term memory (50-100+ words)
- Learned control (what to remember/forget)

The LSTM Solution:

Three Gates Control Flow:

- **Forget gate** f_t : Remove old information
- **Input gate** i_t : Add new information
- **Output gate** o_t : Reveal information
- **Cell state** C_t : Long-term storage

The Mathematical Key:

$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$$

Addition (not multiplication) = gradient highway

Why It Works:

- Cell state uses direct addition
- No repeated matrix multiplication
- Gradient: $\frac{\partial C_t}{\partial C_{t-1}} = f_t \approx 1$

Why LSTMs Matter and How to Use Them

Historical Impact:

- **Breakthrough era:** 2015-2018
- **Google Translate:** 60% error reduction
- **Speech recognition:** Word error rate halved
- **Foundation:** Led to Transformers
- **Citations:** 50,000+ papers

Key Innovations:

- **Cell state:** Separate memory path
- **Gating:** Learned information control
- **Gradient highway:** No vanishing
- **Modular:** Stackable architecture

When to Use LSTMs (2025):

- **Time series:** Still state-of-the-art

Practical Implementation Tips:

Architecture:

- **Start:** 2 layers, 256 hidden units
- **Small data:** 1 layer, 128 units
- **Large data:** 3-4 layers, 512-1024 units

Training:

- **Optimizer:** Adam ($\text{lr}=0.001$)
- **Gradient clipping:** Essential (1.0-5.0)
- **Dropout:** Between layers (0.3-0.5)
- **Batch size:** 32-128
- **Sequence length:** 50-100 tokens

Debugging:

- **Monitor gate activations**
- **Check gradient norms**

Foundational Resources for LSTM Understanding

Must-Read Blog Posts:

1. Colah's Blog: "Understanding LSTM Networks"

- Best visual explanation available
- Step-by-step diagrams
- Intuitive gate explanations
- URL: colah.github.io
- **Start here!**

2. Karpathy's Blog: "Unreasonable Effectiveness of RNNs"

- Character-level text generation
- Real code examples
- Intuitive explanations
- URL: karpathy.github.io
- Shows what's possible

Key Research Papers:

Original Paper:

- Hochreiter & Schmidhuber (1997)
- "Long Short-Term Memory"
- Neural Computation
- Introduced LSTM architecture

Important Extensions:

- Graves (2013): Generating sequences with RNNs
- Cho et al. (2014): GRU architecture
- Bahdanau et al. (2014): Attention mechanism
- Graves et al. (2013): Speech recognition with LSTMs

Online Courses:

- **Stanford CS224n**: NLP with Deep Learning

Hands-On Learning Resources

Code Tutorials:

- **PyTorch LSTM Tutorial**
 - Official documentation
 - Complete examples
 - Best starting point
- **TensorFlow RNN Guide**
 - Keras API examples
 - Production-ready code
- **Keras LSTM Examples**
 - High-level API
 - Quick prototyping

Practice Datasets:

- **Penn Treebank:** Language modeling benchmark
- **IMDB Reviews:** Sentiment analysis (50K reviews)
- **Time Series:** Stock prices, weather data
- **Text8:** Character-level modeling (100MB text)

Your Learning Path:

- 1 **Implement from scratch** (1-2 days)
 - Understand every equation
 - Debug by hand
- 2 **Train on Penn Treebank** (1 day)
 - Language modeling task
 - Monitor perplexity
- 3 **Visualize gate activations** (half day)
 - See what gates learn
 - Build intuition
- 4 **Compare with GRU** (1 day)
 - Same task, different architecture
 - Understand trade-offs
- 5 **Study Transformers** (1 week)
 - Next evolution
 - Self-attention mechanism
- 6 **Explore modern architectures** (ongoing)
 - BERT, GPT, T5
 - Apply to real problems