# Natural Language Processing

## Week 4: Sequence-to-Sequence Models

Breaking the Fixed-Length Barrier

# Learning Objectives

**By the end of this lecture, you will:**

1. Understand why translation is hard for neural networks
2. Design encoder-decoder architectures
3. Identify the information bottleneck problem
4. Master the attention mechanism
5. Implement your own seq2seq model

**Prerequisite**

**Required Knowledge:**

- RNNs and LSTMs (Week 3)
- Backpropagation basics
- Softmax function
- Python/NumPy

**Time Allocation:**

- Part 1: 15 min
- Part 2: 20 min
- Part 3: 15 min
- Part 4: 20 min
- Exercises: 20 min

# Week 4 Overview

# Build Your Intuition: The Translation Problem

**Imagine you're translating a book from English to French. Would you:**

- A) Translate word-by-word in order?
- B) Read the whole sentence, understand it, then write in French?
- C) Look at chunks of 5 words at a time?

**Think:** Why doesn't option A work?

**Example - Word-by-word translation fails:**

- English: "I gave her the book yesterday"
- French: "Je lui ai donné le livre hier"
- Word-by-word back: "I her have given the book yesterday"

**The word order completely changes between languages!**

# Why Can't We Just Use RNNs?

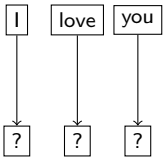
**Key Question:** *You learned RNNs last week. Why can't we use them for translation?*

**The Fundamental Problem:**

- RNNs expect: Input length = Output length
- Translation needs: Input length $\neq$ Output length

**Concrete Example:**

- EN: "I love you" (3 words)
- FR: "Je t'aime" (2 words)
- JP: "Aishiteru" (1 word)
- Which output position gets which input?




Fixed mapping!

# The Length Mismatch: Real Data

**Let's look at actual translation pairs:**

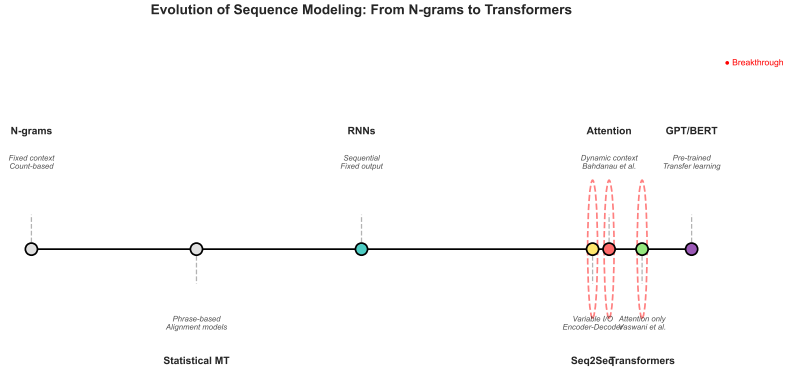
| English | Target Language | EN Words | Target Words |
|---|---|---|---|
| I love you | Je t'aime (French) | 3 | 2 |
| I love you | Ich liebe dich (German) | 3 | 3 |
| I love you | Aishiteru (Japanese) | 3 | 1 |
| I love you | Wo ai ni (Chinese) | 3 | 3 |
| I love you | Te amo (Spanish) | 3 | 2 |
| *Average length ratio: 3:2.2 (varies by 40%!)* | | | |

**Common Mistake: "Just pad shorter sequences"**

- Where to pad? Beginning? End? Middle?
- Model doesn't know target length beforehand
- "Je [PAD] t'aime" $\neq$ "Je t'aime [PAD]"

**Question:** If padding doesn't work, what's the solution? *Hint: How do human translators handle this?*

# Evolution of Translation Approaches




## Key Insights

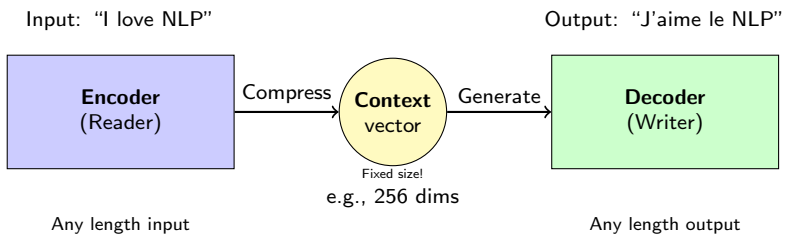
- **1950s-1990s**: Rule-based (dictionaries + grammar)

# The Brilliant Insight: Two-Stage Process

**Think about how YOU translate:**

1. **Read** and **understand** the entire sentence
2. Form a mental **representation** of the meaning
3. **Generate** the translation from that understanding

**The Seq2Seq Solution (mimics human process):**



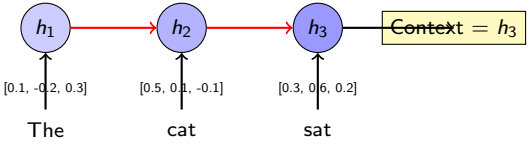

**Key Question:** Why do we need TWO networks instead of modifying one RNN?

# Building Intuition: The Encoder

**The encoder is like a reader that builds understanding:**

- Reads words one by one (like you reading this)
- Updates its understanding with each word
- Final understanding = complete meaning

**Step-by-step encoding of "The cat sat":**




**Example - Track how the hidden state changes:**

- "The" → General/article context
- "The cat" → Animal/subject identified
- "The cat sat" → Complete action understood

# Encoder Mathematics (With Intuition)

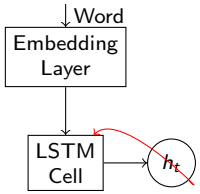
**What happens at each step:**

**For each input word $x_t$ at time $t$:**

$$h_t^{enc} = \text{LSTM}(x_t, h_{t-1}^{enc})$$

Breaking this down:

- $x_t$ = current word (embedded as vector)
- $h_{t-1}^{enc}$ = what we understood so far
- $h_t^{enc}$ = updated understanding




**Concrete dimensions:**

- Word embedding: $x_t \in \mathbb{R}^{100}$
- Hidden state: $h_t \in \mathbb{R}^{256}$
- Context: $c = h_T^{enc} \in \mathbb{R}^{256}$

**Quick note:** Processing 10 words $\rightarrow$ 10 hidden states (one per word)

# Encoder Implementation (Simplified)

**Let's implement what we just learned** - it's simpler than you think!

```
1  class Encoder:
2      def __init__(self, vocab_size, hidden_dim):
3          # Two components only!
4          self.embedding = Embedding(vocab_size, 100)
5          self.lstm = LSTM(100, hidden_dim)
6
7      def forward(self, sentence):
8          # sentence = ["I", "love", "NLP"]
9
10         # Start with zero understanding
11         hidden = zeros(hidden_dim)  # [0,0,...,0]
12
13         # Process each word
14         for word in sentence:
15             # Convert word to vector
16             embed = self.embedding[word]  # 100d
17
18             # Update our understanding
19             hidden = self.lstm(embed, hidden)  # 256d
20
21         # Final understanding
22         context = hidden
23         return context  # This is all decoder gets!
```

**Line-by-line walkthrough:**

- **Lines 3-4**: Just 2 components!
- **Line 10**: Start knowing nothing
- **Lines 13-17**: Core loop
  - Get word vector
  - Update understanding
  - Keep only latest
- **Line 20**: Final state = context

**Key Insight**

Context size is **always the same**:

- 3 words → 256 dims
- 100 words → Still 256 dims!

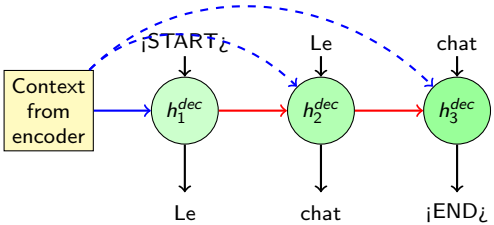
*This fixed size is both a strength and weakness...*

# Building Intuition: The Decoder

**The decoder is like a writer that generates from understanding:**

- Starts with the context (understanding)
- Generates one word at a time
- Each word depends on context + previous words

**Generation process for "Le chat":**



# Decoder Mathematics (With Intuition)

**Generation at each step:**

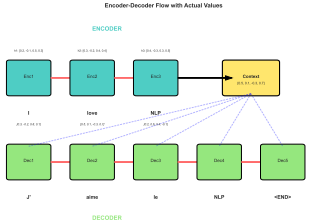
**For each output position $t$:**

$$h_t^{dec} = \text{LSTM}(y_{t-1}, h_{t-1}^{dec}, c)$$

$$P(y_t|y_{<t}, c) = \text{softmax}(W \cdot h_t^{dec} + b)$$

Breaking this down:

- $y_{t-1}$ = previous word we generated
- $c$ = context from encoder (always same!)
- $h_t^{dec}$ = decoder's current state
- $P(y_t|...)$ = probability of each word




**Concrete example:**

- Generating "chat" after "Le"
- Previous: $y_{t-1}$ = "Le" $\rightarrow$ [0.2, 0.1, ...]
- Context: $c$ = [0.3, 0.6, 0.2, ...] (256d)
- Output: P("chat") = 0.7, P("chien") = 0.2, ...

# Training Trick: Teacher Forcing

**Problem:** How do we train when the model makes mistakes early on?

**During Training (Teacher Forcing):**

- Feed the TRUE previous word
- Not the model's prediction
- Speeds up training dramatically

Example: Teaching "Le chat noir"

1. Input: ¡START¿ → Predict: "Le"
2. Input: "Le" (true) → Predict: "chat"
3. Input: "chat" (true) → Predict: "noir"

**During Testing (No Teacher):**

- Feed MODEL's previous prediction
- No true words available!
- Errors can accumulate

Example: Generating translation

1. Input: ¡START¿ → Generates: "Le"
2. Input: "Le" (generated) → Generates: "chat"
3. Input: "chat" (generated) → Generates: "noir"

**Important Distinction**

**Common mistake:** "Teacher forcing at test time"
Remember: During testing, you don't have the correct translation to feed back!

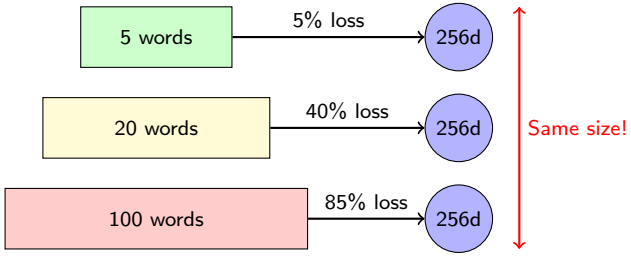
# The Compression Problem

**Imagine compressing a book into a single paragraph:**

- Short story (5 pages) → Paragraph: Works well!
- Novel (300 pages) → Paragraph: Loses details
- Encyclopedia → Paragraph: Impossible!

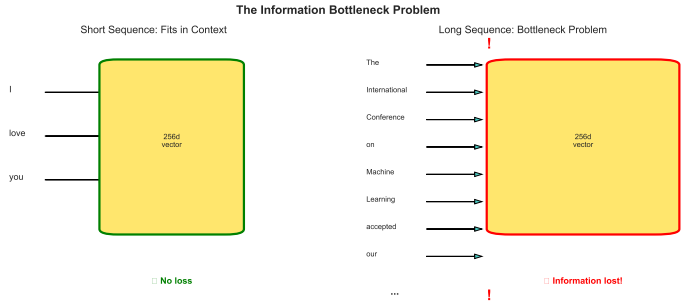
*Same problem with seq2seq: Longer input → More information loss*

**The Bottleneck Visualization:**




**Question:** Why can't we just use a bigger context vector?

# Information Theory Analysis




## Key Insights

**The bottleneck effect:**

- Short sentences (5 words) → Easy to compress
- Medium sentences (20 words) → Some loss but manageable
- Long sentences (100 words) → Major information loss
- **Rule of thumb:** Performance drops significantly after 30 words

# Where Information Gets Lost

**Let's trace what happens to a long sentence:**

*"The International Conference on Machine Learning, which is one of the premier venues for presenting research in machine learning and attracts submissions from researchers around the world, accepted our paper."*

**What the context vector captures:**

**Y Preserved:**

- General topic (ML conference)
- Sentiment (positive - accepted)
- Basic structure (statement)

**X Lost:**

- "International" detail
- "premier venues" specificity
- "researchers around the world"
- Exact conference name

**Experimental Results (Bahdanau et al., 2015):**

| Sentence Length | BLEU Score | Quality |
|---|---|---|
| ¡ 10 words | 35.2 | Excellent |
| 10-20 words | 28.5 | Good |
| 20-30 words | 19.3 | Mediocre |
| ¿ 30 words | 9.7 | Poor |

**Performance drops 72% for long sentences!**

# How Humans Translate (The Key Insight)

**When you translate "The black cat sat on the mat" to French:**

- For "Le" → You look at "The"
- For "chat" → You look at "cat"
- For "noir" → You look at "black"
- You DON'T look at all words equally!

**Let's track what we look at:**

| Generating | Looking at | Attention Weight |
|---|---|---|
| "Le" | Mainly "The" | 0.8 on "The", 0.2 others |
| "chat" | Mainly "cat" | 0.7 on "cat", 0.3 others |
| "noir" | Mainly "black" | 0.6 on "black", 0.4 others |
| "s'est assis" | Mainly "sat" | 0.9 on "sat", 0.1 others |
| "sur" | Mainly "on" | 0.8 on "on", 0.2 others |
| "le" | Mainly "the" | 0.7 on "the", 0.3 others |
| "tapis" | Mainly "mat" | 0.85 on "mat", 0.15 others |

**Key Insight:** What if the decoder could do this too - look back at specific encoder states?

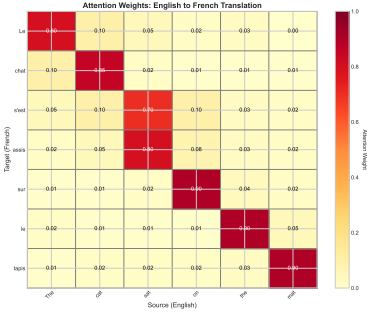
# The Attention Solution

**Instead of one context vector:**

- Keep ALL encoder hidden states
- Let decoder choose what to look at
- Different focus for each output word

**The 3-step attention process:**

1. **Score:** How relevant is each encoder state?
$$e_{ti} = \text{score}(h_t^{dec}, h_i^{enc})$$
2. **Normalize:** Convert to probabilities
$$\alpha_{ti} = \frac{\exp(e_{ti})}{\sum_j \exp(e_{tj})}$$
3. **Combine:** Weighted sum
$$c_t = \sum_i \alpha_{ti} \cdot h_i^{enc}$$




**Example:** For "chat", attention weights = [0.1, 0.7, 0.2]

- 10% focus on "The"
- **70% focus on "cat"** ← Makes sense!
- 20% focus on "sat"

# Attention Calculation: Step by Step

**Let's calculate attention for generating "chat":**

**Step 1: Score each source word**

Current decoder state: $h_2^{dec} = [0.5, -0.2, 0.8]$

| Word | Hidden State | Score |
|---|---|---|
| "The" | [0.1, 0.2, 0.1] | 0.09 |
| "cat" | [0.8, 0.1, 0.7] | 0.94 |
| "sat" | [0.2, 0.3, 0.2] | 0.20 |

**Step 2: Apply softmax**

$$\alpha_1 = \frac{e^{0.09}}{e^{0.09} + e^{0.94} + e^{0.20}} = 0.27$$

$$\alpha_2 = \frac{e^{0.94}}{\ldots} = 0.63$$

$$\alpha_3 = \frac{e^{0.20}}{\ldots} = 0.10$$

**Step 3: Weighted combination**

$c_2 = 0.27 \cdot h_1^{enc} +$
$0.63 \cdot h_2^{enc} +$
$0.10 \cdot h_3^{enc}$

**Result:**

- 63% attention on "cat"
- Correct word alignment!
- Context is mostly "cat"

# Attention Implementation

```
1  def attention(decoder_hidden, encoder_outputs):
2      """
3      decoder_hidden: current state [256]
4      encoder_outputs: all states [seq_len, 256]
5      """
6      scores = []
7
8      # Step 1: Score each encoder output
9      for enc_out in encoder_outputs:
10         # Dot product similarity
11         score = dot(decoder_hidden, enc_out)
12         scores.append(score)
13
14     # Step 2: Normalize with softmax
15     scores = array(scores)
16     exp_scores = exp(scores - max(scores))
17     weights = exp_scores / sum(exp_scores)
18
19     # Step 3: Weighted combination
20     context = zeros_like(decoder_hidden)
21     for i, enc_out in enumerate(encoder_outputs):
22         context += weights[i] * enc_out
23
24     return context, weights
25
26 # Usage in decoder:
27 for t in range(max_length):
28     context, attn = attention(hidden, all_enc)
29     # Use context instead of fixed vector!
```

**Key improvements:**

- **Line 9-11**: Score relevance
- **Line 15-16**: Softmax for probabilities
- **Line 19-21**: Custom context

**Example with 10 source words:**

- 10 attention weights
- Sum to 1.0
- Different for each output!

**Important Note:**

- **Q:** "Does attention look at future words?"
- **A:** No! Only at encoder (source) states, never future target words.

# Types of Attention Mechanisms

**Three ways to compute attention scores:**

**1. Dot Product (Luong)**

$$e_{ti} = h_t^{dec} \cdot h_i^{enc}$$

- Simplest and fastest
- No parameters to learn
- Works well in practice

**2. Scaled Dot Product**

$$e_{ti} = \frac{h_t^{dec} \cdot h_i^{enc}}{\sqrt{d}}$$

- Used in Transformers
- Prevents large values
- More stable gradients

**3. Additive (Bahdanau)**

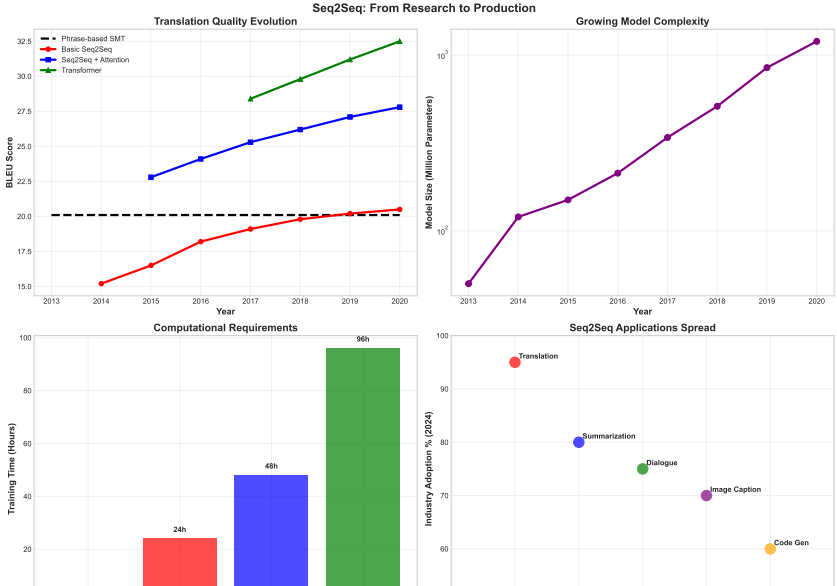
$$e_{ti} = v^T \tanh(W_1 h_t^{dec} + W_2 h_i^{enc})$$

- Original attention paper
- More parameters
- More flexible

**Performance comparison:**

| Type | BLEU | Speed |
|---|---|---|
| Dot Product | 31.2 | Fast |
| Scaled | 31.5 | Fast |
| Additive | 31.7 | Slower |

# The Impact of Attention



# Complete Mathematical Formulation

**Encoder Equations:**

$$h_t^{enc} = \text{LSTM}^{enc}(E^{enc}(x_t), h_{t-1}^{enc}) \quad \text{(Process each word)} \quad (1)$$

$$H^{enc} = [h_1^{enc}, h_2^{enc}, \ldots, h_T^{enc}] \quad \text{(Keep all states)} \quad (2)$$

**Decoder with Attention:**

$$e_{ti} = \text{score}(h_{t-1}^{dec}, h_i^{enc}) \quad \text{(Relevance scores)} \quad (3)$$

$$\alpha_{ti} = \frac{\exp(e_{ti})}{\sum_{j=1}^{T} \exp(e_{tj})} \quad \text{(Attention weights)} \quad (4)$$

$$c_t = \sum_{i=1}^{T} \alpha_{ti} h_i^{enc} \quad \text{(Context vector)} \quad (5)$$

$$h_t^{dec} = \text{LSTM}^{dec}([E^{dec}(y_{t-1}); c_t], h_{t-1}^{dec}) \quad \text{(Update decoder)} \quad (6)$$

$$P(y_t | y_{<t}, X) = \text{softmax}(W_o h_t^{dec} + b_o) \quad \text{(Output probabilities)} \quad (7)$$

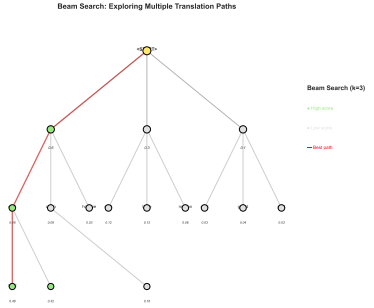
**Training Objective:**

$$\mathcal{L} = -\sum_{t=1}^{T'} \log P(y_t^* | y_{<t}^*, X) \text{(Cross-entropy loss)} \quad (8)$$

# Beam Search Algorithm

**Problem:** Greedy decoding (always pick highest probability) is suboptimal
**Solution:** Keep top-k hypotheses at each step

```python
def beam_search(encoder_outputs, beam_size=3):
    # Start with single hypothesis
    beams = [([<START>], 0.0)]

    for t in range(max_length):
        new_beams = []

        for sequence, score in beams:
            if sequence[-1] == <END>:
                completed.add((sequence, score))
                continue

            # Get probabilities for next word
            probs = decode_step(sequence, encoder_outputs)

            # Keep top k words
            top_words = top_k(probs, beam_size)

            for word, prob in top_words:
                new_seq = sequence + [word]
                new_score = score + log(prob)
                new_beams.append((new_seq, new_score))

        # Keep top k beams overall
        beams = sorted(new_beams, key=score)[:beam_size]

    return best_completed()
```




**Example with `beam_size=2`:**

- Start: “Le” (0.7), “Un” (0.3)
- After “Le”: “chat” (0.6), “chien” (0.1)
- After “Un”: “chat” (0.2), “animal” (0.1)
- Keep: “Le chat” (0.42), “Un chat” (0.06)

# BLEU Score: Evaluating Translation Quality

**BLEU = Bilingual Evaluation Understudy**

$$\text{BLEU} = \text{BP} \cdot \exp\left(\sum_{n=1}^{4} w_n \log p_n\right)$$

Where:

- $p_n$ = precision of n-grams
- $w_n$ = weights (usually 0.25 each)
- BP = brevity penalty (penalizes short translations)

**Concrete Example:**

- Reference: "The cat sat on the mat"
- Hypothesis: "The cat is on the mat"

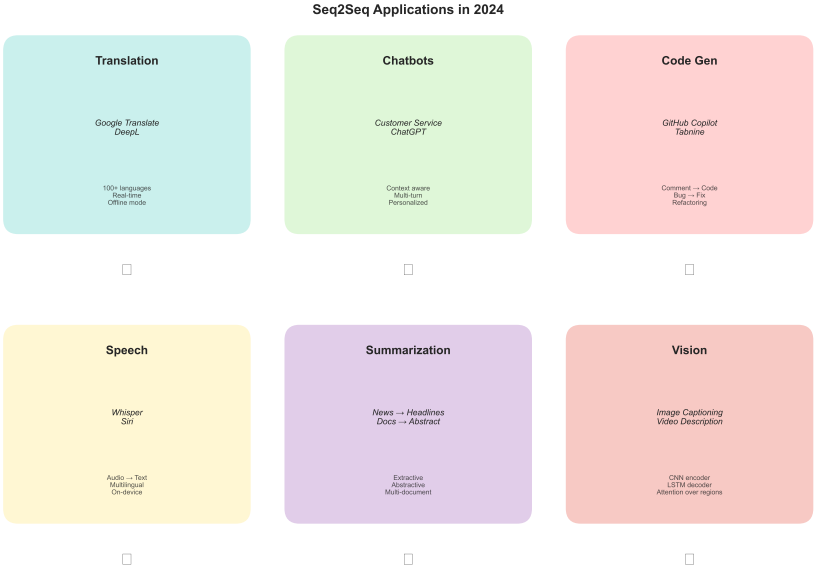
| N-gram | Matches | Total | Precision |
|---|---|---|---|
| 1-gram | The, cat, on, the, mat | 6 | $5/6 = 0.83$ |
| 2-gram | "The cat", "on the", "the mat" | 5 | $3/5 = 0.60$ |
| 3-gram | "on the mat" | 4 | $1/4 = 0.25$ |
| 4-gram | None | 3 | $0/3 = 0.00$ |

**Step-by-step calculation:**

- Brevity penalty: $\text{BP} = e^{1-6/6} = 1.0$ (same length)
- Geometric mean: $\sqrt[4]{0.83 \times 0.60 \times 0.25 \times 0.01} = 0.22$
- Final BLEU: $1.0 \times 0.22 = 0.22$

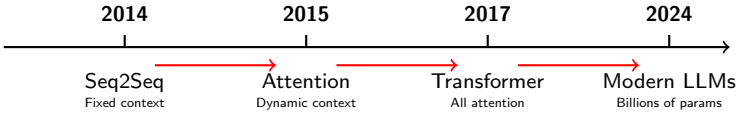
**Interpretation:** 0.22 means "Understandable but needs improvement"

# Seq2Seq in Production Today



# From Seq2Seq to Transformers

**The Evolution Timeline:**




**Key Innovations:**

1. **Seq2Seq (2014):** Separate encoding and decoding
2. **Attention (2015):** Solve the bottleneck problem
3. **Transformer (2017):** Remove RNNs entirely, use only attention
4. **GPT/BERT (2018+):** Pre-training on massive data

**Key Insight:** Everything you learned today is the foundation of modern LLMs!
*ChatGPT, Claude, and Gemini all build on these seq2seq concepts.*

# Week 4 Summary: Key Takeaways

**Problems Solved:**

1. Variable-length I/O
2. Information bottleneck
3. Long-range dependencies
4. Translation alignment

**Key Concepts:**

- Encoder-Decoder separation
- Context vectors
- Teacher forcing
- Attention mechanism
- Beam search

**You Can Now:**

- Build a seq2seq model
- Implement attention
- Diagnose bottleneck issues
- Choose attention types
- Evaluate with BLEU

**Next Week: Transformers**

- "Attention is All You Need"
- Self-attention
- Multi-head attention
- Positional encoding

**Quick Review:** Can you explain why we need TWO networks for translation?
*Answer: Because input length $\neq$ output length!*