LSTM - Long Short-Term Memory

Understanding Through a Complete Example

First: What Are “Gates”?

Three Gates Control Memory Like Volume Knobs (0 to 1)

FORGET

$\times$

REMOVES
old information

Value 0-1:

- 0.0 = erase all
- 0.5 = keep half
- 1.0 = keep all

Example:
0.1 at period
$\longrightarrow$ Erase 90%!

INPUT

$+$

ADDS
new information

Value 0-1:

- 0.0 = add nothing
- 0.5 = add half
- 1.0 = add all

Example:
0.9 on “cat”
$\longrightarrow$ Store lots!

OUTPUT

$\longrightarrow$

REVEALS
stored information

Value 0-1:

- 0.0 = hide all
- 0.5 = show half
- 1.0 = show all

Example:
0.9 at “was”
$\longrightarrow$ Use memory!

How They Work Together:

New Memory = (Forget $\times$ Old Memory) + (Input $\times$ New Info)
Output = Output Gate $\times$ Memory

Now let’s see how these three gates work together...

The 4-Step Process: Concrete Example

Updating Memory From “cat” to “dog”

Starting Point:
Old Memory: [0.8, 0.6, 0.4] ← contains “cat” info

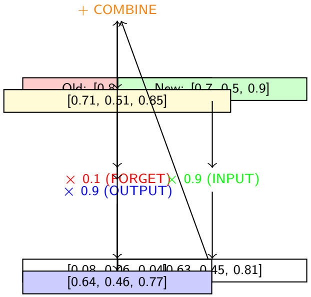
Step 1: FORGET Gate = 0.1
Multiply old memory by 0.1:
, 0.6, 0.4
× 0.1 = [0.08, 0.06, 0.04]
Result: 90% erased! “cat” mostly removed.

Step 2: INPUT Gate = 0.9
Create new candidate: [0.7, 0.5, 0.9]
Multiply by 0.9:
, 0.5, 0.9
× 0.9 = [0.63, 0.45, 0.81]
Result: 90% of “dog” info added.

Step 3: COMBINE (Addition!)
Old (erased) + New (filtered):
, 0.06, 0.04
+ [0.63, 0.45, 0.81]
= [0.71, 0.51, 0.85]
Result: Updated memory = “dog” info

Step 4: OUTPUT Gate = 0.9
Filter what network sees:
, 0.51, 0.85
× 0.9 = [0.64, 0.46, 0.77]
Result: 90% revealed to next layer

Visual Flow:



Checkpoint: Critical Point

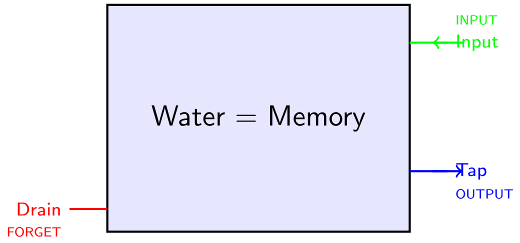
Notice Step 3 uses ADDITION!

- RNN: Multiplication only
- LSTM: Addition path
- This prevents vanishing!

Why This Works: The Water Tank Analogy

Think of Memory as a Water Tank with Three Valves

The Tank System:



How Each Valve Works:

FORGET = Drain Valve
Controls how much water flows OUT
0.1 = Open 10% → 90% drains away
Removes old water (old memory)

INPUT = Input Valve
Controls how much new water flows IN
0.9 = Open 90% → lots added
Adds fresh water (new memory)

OUTPUT = Output Tap

The Key Insight:

Intuition: Why This Design?

Three INDEPENDENT valves on ONE tank!

Each valve controls a different aspect:

- Drain: How much OLD to remove
- Input: How much NEW to add
- Tap: How much to USE now

This is EXACTLY what LSTM does with memory!

Real Example:

At period "." in sentence:

- Drain: 90% (0.1 forget)
- Input: 40% (0.4 input)
- Tap: 30% (0.3 output)

→ Tank mostly empties, little added, little used!

At noun "dog":

- Drain: 30% (0.7 forget)
- Input: 90% (0.9 input)
- Tap: 90% (0.9 output)

→ Tank fills up, lots available to use!

Why Three Separate Gates? Real Scenarios

Reading: "The cat sat. The dog..."

Scenario 1: At "cat"

Gate Values:

- F = 0.8 (keep)
- I = 0.9 (STORE!)
- O = 0.8 (show)

What Happens:

- Keep previous context
- STORE subject strongly
- Show it to network

Goal:
Remember "cat" for rest of sentence

Memory:
$\rightarrow$ [cat, context]

Scenario 2: At "."

Gate Values:

- F = 0.1 (ERASE!)
- I = 0.4 (small)
- O = 0.3 (HIDE)

What Happens:

- ERASE old sentence
- Small punctuation add
- HIDE memory

Goal:
Clean slate for new sentence

Memory:
$\rightarrow$ [mostly empty]

Scenario 3: At "dog"

Gate Values:

- F = 0.7 (keep some)
- I = 0.9 (NEW!)
- O = 0.9 (REVEAL!)

What Happens:

- Keep some context
- STORE new subject
- REVEAL all info

Goal:
New focus, need it NOW

Memory:
$\rightarrow$ [dog, some context]

Key Insight: Each situation needs DIFFERENT gate values!
That's why LSTM has three independent gates, not just one.
The network LEARNS which values to use for each word.

Now let's watch these gates in action on a real sentence...

Watch LSTM Process a Sentence

Sentence: “The cat was hungry. The dog was sleeping.”

Word	Forget	Input	Output	Memory State
The	0.9	0.3	0.2	*article*
cat	0.8	**0.9**	0.8	*subject: cat*
was	0.9	0.7	0.9	*cat + verb*
hungry	0.8	0.8	0.7	*cat is hungry*
.	**0.1**	0.4	0.3	*sentence ends*
The	0.1	0.8	0.2	*new article*
dog	0.7	**0.9**	0.9	*subject: dog*
was	0.9	0.8	**0.9**	*using dog info*

0.1 = Forget

0.9 = Store/Use

Period → Reset

Just observe for now. Notice any patterns? We'll explain HOW in a moment...

What Did You Notice?

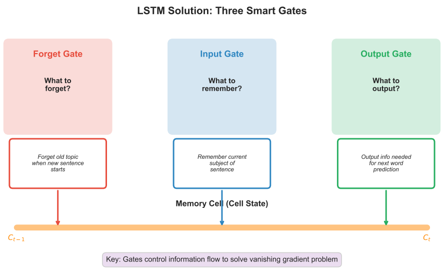
Common Observations:

Students usually notice:

- **"It drops to 0.1 at the period!"**
- **"It's 0.9 on important words (cat, dog)"**
- "The memory changes from cat to dog"
- "It resets between sentences"
- "Three different columns of numbers"

Key Questions:

1. HOW does it know to forget at period?
2. HOW does it know cat and dog are important?
3. HOW does it decide when to use memory?



LSTM Solution: Three Smart Gates
Forget Gate
What to forget?
Input Gate
What to remember?
Output Gate
What to output?
Forget old topic when new sentence starts
Remember current subject of sentence
Output info needed for next word prediction
Memory Cell (Cell State)
C_{t-1}
C_t
Key: Gates control information flow to solve vanishing gradient problem

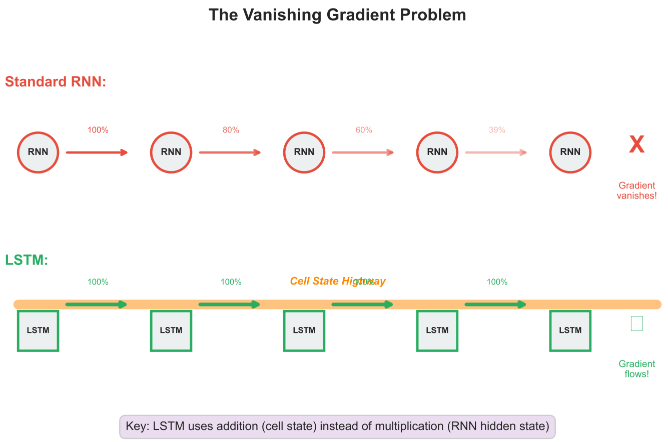
Checkpoint: The Big Reveal

Those three columns are called **GATES**:

- Forget Gate: Controls what to erase
- Input Gate: Controls what to store
- Output Gate: Controls what to use

But WHY do we need gates?

Why Do We Need Controlled Memory?



RNN Problem:

- Gradients vanish ($0.5^{50} \approx 0$)
- Forgets early information
- Can't handle long dependencies
- Would lose "cat" by "dog"

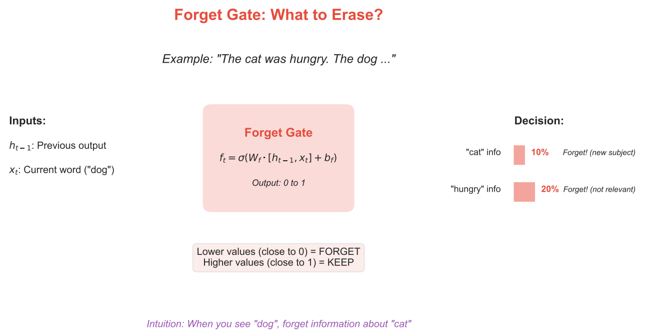
Example: "I grew up in Paris. I speak fluent ___"

LSTM Solution:

- Cell state highway (addition not multiplication)
- Three gates for CONTROL
- Can preserve info for 100+ steps
- Then ERASE when sentence ends

Remember Our Table?

Forget Gate: How We Get That 0.1


Forget Gate: What to Erase?
Example: "The cat was hungry. The dog ..."
Inputs:
h_{t-1}: Previous output
x_t: Current word ("dog")
Forget Gate
$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$
Output: 0 to 1
Decision:
"cat" info
10%
Forget! (new subject)
"hungry" info
20%
Forget! (not relevant)
Lower values (close to 0) = FORGET
Higher values (close to 1) = KEEP
Intuition: When you see "dog", forget information about "cat"

Back to Our Table - Row 5:

Word	Forget
"."	0.1

What This 0.1 Means:

- 0.0 = forget everything
- 1.0 = keep everything
- 0.1 = forget 90% (keep only 10%)

Why at period?

The Formula That Produces 0.1:

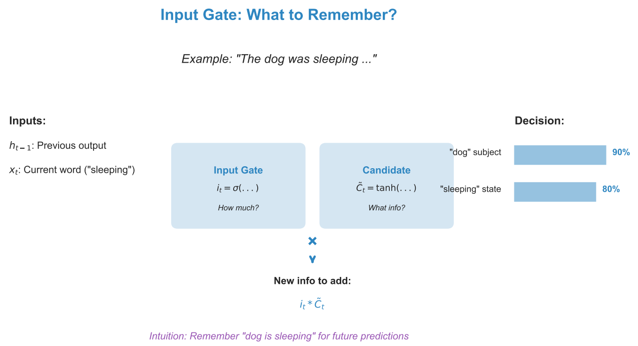
$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$$

How It Decides:

1. Look at current word (".")
2. Look at previous hidden state
3. Compute weighted sum
4. Apply sigmoid $\longrightarrow$ output 0 to 1

Cell State Update:

Input Gate: How We Get That 0.9


Input Gate: What to Remember?
Example: "The dog was sleeping ..."
Inputs:
h_{t-1}: Previous output
x_t: Current word ("sleeping")
Input Gate
$i_t = \sigma(...)$
How much?
Candidate
$\tilde{C}_t = \tanh(...)$
What info?
×
New info to add:
$i_t * \tilde{C}_t$
Decision:
"dog" subject
90%
"sleeping" state
80%
Intuition: Remember "dog is sleeping" for future predictions

Back to Our Table - Row 7:

Word	Input
"dog"	**0.9**

What This 0.9 Means:

- 0.0 = add nothing
- 1.0 = add everything
- 0.9 = add 90% of candidate

Why at "dog"?

The Formulas (Two Parts):

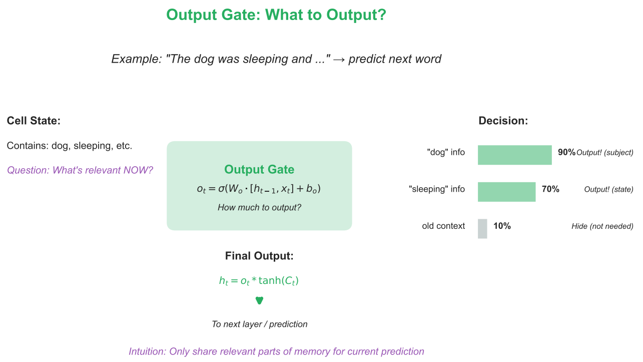
$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$

How It Works:

1. Create candidate info ($\tilde{C}_t$) with tanh
2. Decide how much to use ($i_t = 0.9$)
3. Multiply: 0.9 × candidate
4. Add to cell state

Output Gate: When to USE Memory



Output Gate: What to Output?
Example: "The dog was sleeping and ..." → predict next word
Cell State:
Contains: dog, sleeping, etc.
Question: What's relevant NOW?
Output Gate
$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o)$
How much to output?
Decision:
"dog" info
90%
Output! (subject)
"sleeping" info
70%
Output! (state)
old context
10%
Hide (not needed)
Final Output:
$h_t = o_t * \tanh(C_t)$
To next layer / prediction
Intuition: Only share relevant parts of memory for current prediction

Back to Our Table - Row 8:

Word	Output
"was"	**0.9**

What This 0.9 Means:

- 0.0 = hide everything
- 1.0 = reveal everything
- 0.9 = output 90% of memory

Why at "was"?

The Formulas:

$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o)$$

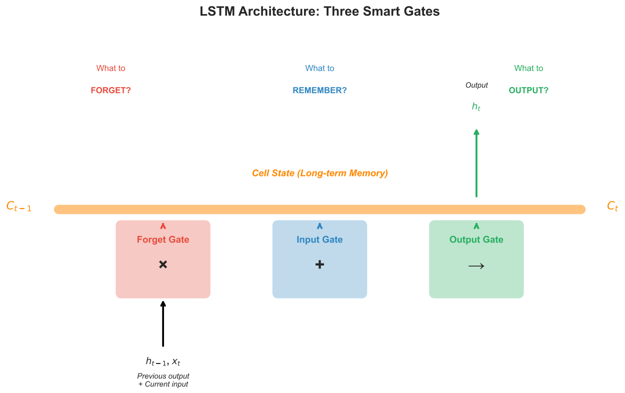
$$h_t = o_t \odot \tanh(C_t)$$

How It Works:

1. Look at cell state (has "dog" info)
2. Decide what's relevant NOW
3. Filter memory through gate (0.9)
4. Send h_t to prediction layer

Key Insight:

The Big Picture: Three Gates Working Together



The Cell State Highway:

- Protected memory channel
- Information flows easily
- Gates control entry/exit
- Gradients don't vanish!

At Each Time Step:

1. **Forget:** Erase old (0.1 $\longrightarrow$ erase "cat")
2. **Input:** Add new (0.9 $\longrightarrow$ add "dog")

Intuition: Visual Analogy

Think of LSTM like a notebook:

- Forget Gate = Eraser
  (Clear old notes at period)
- Input Gate = Pen
  (Write important info like "dog")

Now Look Again - You Understand EVERYTHING!

Sentence: "The cat was hungry. The dog was sleeping."

Word	Forget	Input	Output	What LSTM "Thinks"
The	0.9 (keep)	0.3 (weak)	0.2 (hide)	Article seen, nothing special yet
cat	0.8 (keep)	**0.9** (STORE!)	0.8 (show)	Subject! Important noun!
was	0.9 (keep)	0.7 (add)	0.9 (need!)	Verb connects to cat
hungry	0.8 (keep)	0.8 (add)	0.7 (show)	Describes the cat's state
.	**0.1** (ERASE!)	0.4 (end)	0.3 (hide)	Sentence over! Clear memory!
The	0.1 (clear)	0.8 (new!)	0.2 (hide)	NEW sentence starts fresh
dog	0.7 (keep)	**0.9** (NEW!)	0.9 (use!)	NEW subject! (forgot cat)
was	0.9 (keep)	0.8 (add)	**0.9** (USE!)	Using DOG info for prediction

Checkpoint: The Magic Transition

Watch rows 4→5→6→7: **hungry** → **.** → **The** → **dog**
Memory Evolution: [cat, hungry] → FORGET (0.1) → [end] → ADD (0.9) → [dog]
This intelligent memory control is what RNNs cannot do! LSTM uses gates to:

- Preserve important info (0.9 on subject nouns)
- Erase when context changes (0.1 at sentence boundaries)
- Reveal info when needed (0.9 output for predictions)

Summary: From Table to Understanding

Your Learning Journey:

1. **Observed:** Patterns in the table (0.1 at period, 0.9 on important words)
2. **Understood WHY:** Vanishing gradients (RNNs can't remember long-term)
3. **Learned HOW:** Gate equations (Sigmoid produces those 0.1 and 0.9 values)
4. **Mastered:** Complete picture (Gates control memory intelligently)

Key Equations:

$$
\begin{aligned}
f_t &= \sigma(W_f[h_{t-1}, x_t] + b_f) \\
i_t &= \sigma(W_i[h_{t-1}, x_t] + b_i) \\
o_t &= \sigma(W_o[h_{t-1}, x_t] + b_o) \\
C_t &= f_t \odot C_{t-1} + i_t \odot \tilde{C}_t \\
h_t &= o_t \odot \tanh(C_t)
\end{aligned}
$$

Real World: Where LSTMs Excel

Applications (2015-2020):

- Machine Translation (Google Translate)
- Speech Recognition (Siri, Alexa)
- Text Generation (early GPT)
- Video Analysis
- Music Generation
- Handwriting Recognition

Modern Context (2024):
Transformers now dominate NLP, but LSTMs:

- Still used in time series
- Efficient for streaming data
- Foundation for understanding attention

The Core Insight:
That table showed you *exactly* how gates work. Every 0.1 and 0.9 has a purpose. That's the real magic of LSTMs!

Questions?