

Recurrent Neural Networks

Week 3: Teaching Computers to Remember

NLP Course 2025

Professional Template Edition

September 29, 2025

- 1 Part 1: Motivation & Fundamentals
- 2 Part 2: Architecture & Mathematics
- 3 Part 3: The Vanishing Gradient Problem
- 4 Part 4: Implementation & Applications

Learning Path: From feedforward to recurrent networks. Master how RNNs maintain memory through time, solve the vanishing gradient problem with LSTMs, and build powerful sequence models that power modern applications.

Part 1: Motivation & Fundamentals

Why Sequential Processing Matters

Why Your Voice Assistant Sometimes Fails

The Conversation:

You: "Set a timer for 10 minutes"

Alexa: "Timer set for 10 minutes"

You: "Actually, make it 15"

Alexa: "I'm not sure what you want me to make"

What Went Wrong?

- "it" refers to "timer" from 2 turns ago
- Static embeddings can't track conversation
- No memory of previous utterances
- Each word processed independently

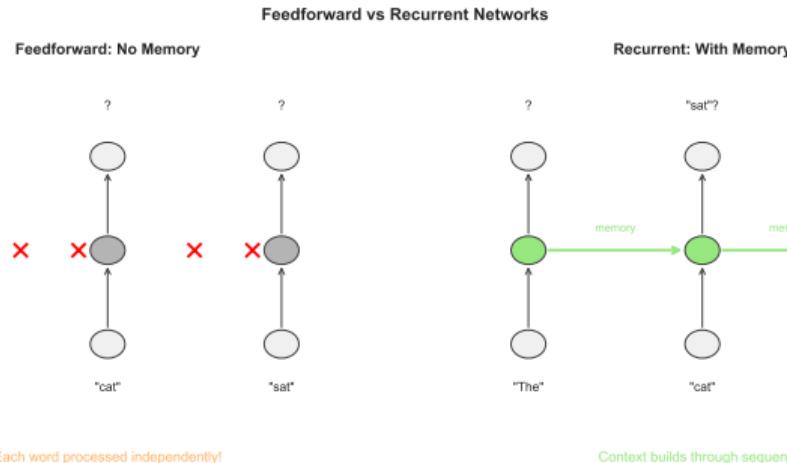
The Problem: Word embeddings alone don't remember context!

The Solution:

Networks that maintain memory of past inputs - **Recurrent Neural Networks**

Key Insight: Language understanding requires memory - words depend on what came before

Same Words, Different Order, Different Meaning



Example 1:

- “Dog bites man” → Not news
- “Man bites dog” → Front page!

Word Embeddings See: {dog, bites, man} - Same vectors!

Fundamental Challenge: Language is inherently sequential - we need models that process it that way

Example 2:

- “not bad” → Positive
- “bad, not good” → Negative

Order Creates Meaning: Sequential processing essential

How Humans Read: The Inspiration for RNNs

Reading “The movie was really...” step by step:

Human Process:

1. Read “The” → remember it
2. Read “movie” → remember “The movie”
3. Read “was” → remember “The movie was”
4. Read “really” → expect adjective next

RNN Process:

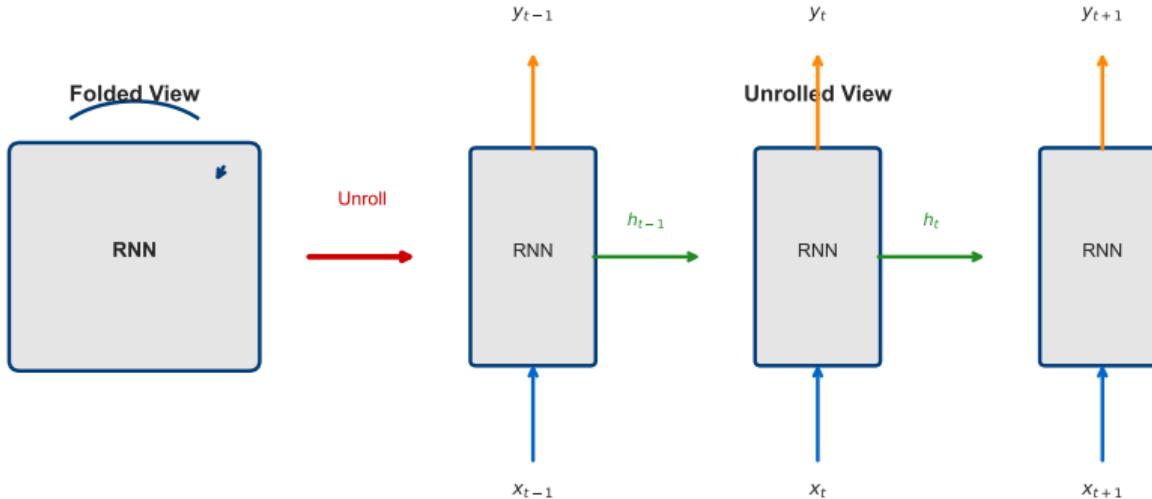
1. Process one word at a time
2. Maintain “hidden state” (memory)
3. Update memory with each word
4. Use memory to predict next

We naturally maintain a running memory!

Hidden state = What the network remembers

Core Concept: RNNs process sequences like humans do - one step at a time with memory

Visualizing RNNs: Unfolding Through Time



Key Insights:

- Same weights W used at every step
- Can handle any sequence length
- Memory flows forward in time
- Each step sees current input + previous memory
- Output depends on entire history
- Training updates all time steps

Unfolding Insight: One network becomes many - but all share the same parameters

Interactive Exercise: Trace the Hidden State

Task: Manually compute RNN hidden states for “cat sat”

Given: $h_t = \tanh(W_h h_{t-1} + W_x x_t)$ where $W_h = 0.5$, $W_x = 0.8$

Step 1: Process “cat”

- $x_1 = 1.0$ (embedding for “cat”)
- $h_0 = 0$ (initial state)
- $h_1 = \tanh(0.5 \cdot 0 + 0.8 \cdot 1.0)$
- $h_1 = \tanh(0.8) = ?$

Your answer: $h_1 = \underline{\hspace{2cm}}$

Step 2: Process “sat”

- $x_2 = 0.5$ (embedding for “sat”)
- $h_1 = 0.664$ (from step 1)
- $h_2 = \tanh(0.5 \cdot 0.664 + 0.8 \cdot 0.5)$
- $h_2 = \tanh(0.732) = ?$

Your answer: $h_2 = \underline{\hspace{2cm}}$

Learning by Doing: Computing by hand builds intuition for how memory accumulates

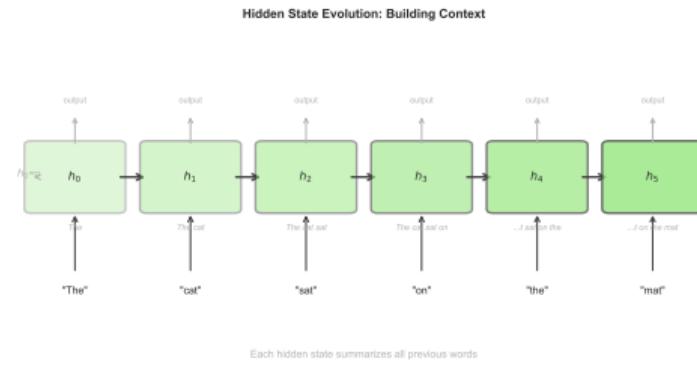
Part 1 Summary: Why We Need RNNs

What We Learned:

- Language is fundamentally sequential
- Order changes meaning dramatically
- Static embeddings lack memory
- Context spans multiple time steps
- Humans read with running memory

RNN Solution:

- Process sequences step-by-step
- Maintain hidden state (memory)
- Share weights across time
- Handle variable lengths



Key Takeaway:
RNNs = Neural Networks + Memory

Next: Dive deep into the mathematics and architecture of RNNs

Part 2: Architecture & Mathematics

The Elegant Mathematics of Memory

The Complete RNN:

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$$
$$y_t = W_{hy}h_t + b_y$$

Components:

- x_t : Input at time t (word embedding)
- h_t : Hidden state at time t (memory)
- y_t : Output at time t (predictions)
- W_{hh} : Hidden-to-hidden weights
- W_{xh} : Input-to-hidden weights
- W_{hy} : Hidden-to-output weights

In Plain English:

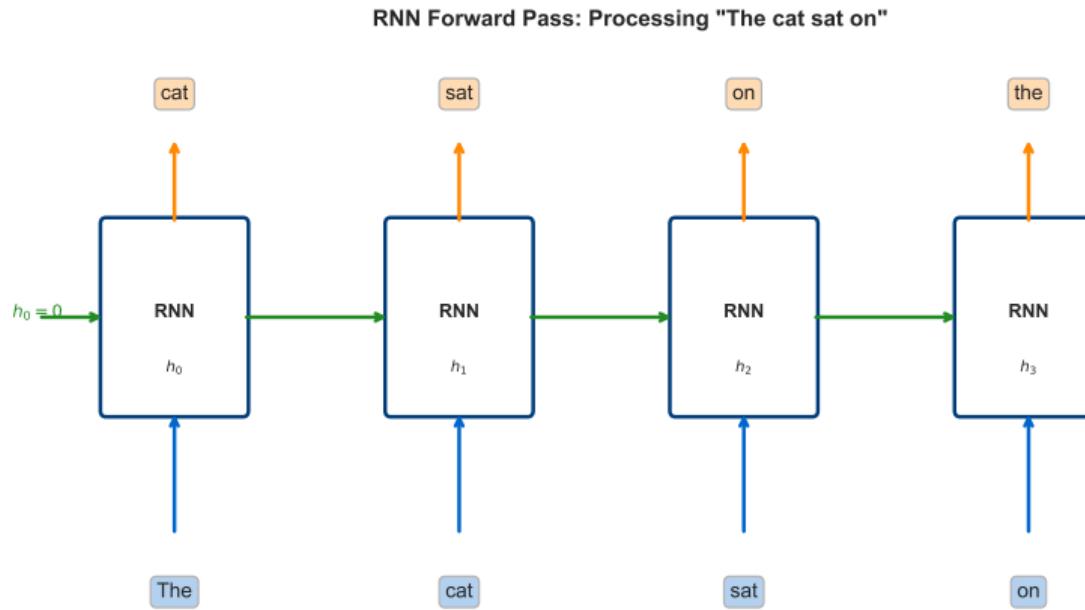
- **Line 1:** Combine old memory with new input
- **tanh:** Squash to [-1, 1] range
- **Line 2:** Generate output from memory
- **Shared W :** Same transformation always

New memory = $f(\text{old memory}, \text{new input})$

Mathematical Elegance: Two equations capture the essence of sequential processing

Forward Pass: Concrete Example with Real Numbers

Processing “I love NLP” step by step:



Time $t = 1$ (“I”):



Time $t = 2$ (“love”):



Time $t = 3$ (“NLP”):



Why Share Weights?

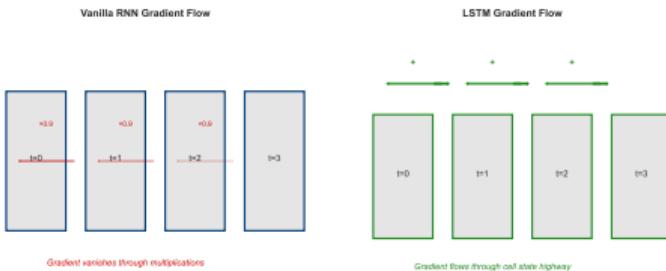
- **Generalization:** Learn patterns once
- **Efficiency:** Fewer parameters
- **Flexibility:** Any sequence length
- **Consistency:** Same processing always

Example Benefits:

- “not good” at position 1 or 10
- Learn “ing” ending anywhere
- Transfer learning across positions

The Dark Side:

- **Gradient flow:** Through same W repeatedly
- **Vanishing:** Gradients $\rightarrow 0$
- **Exploding:** Gradients $\rightarrow \infty$
- **Long-term memory:** Difficult



Trade-off: Weight sharing enables generalization but creates gradient challenges

Complete RNN Implementation in PyTorch

```
1 import torch
2 import torch.nn as nn
3
4 class SimpleRNN(nn.Module):
5     def __init__(self, input_size,
6                  hidden_size, output_size):
7         super().__init__()
8         self.hidden_size = hidden_size
9
10        # Define layers
11        self.i2h = nn.Linear(
12            input_size + hidden_size,
13            hidden_size
14        )
15        self.h2o = nn.Linear(
16            hidden_size, output_size
17        )
18
19    def forward(self, input, hidden):
20        # Combine input and hidden
21        combined = torch.cat(
22            (input, hidden), 1
23        )
24
25        # Update hidden state
26        hidden = torch.tanh(
27            self.i2h(combined)
28        )
29
30        # Generate output
31
```

Usage Example:

```
1 # Initialize
2 rnn = SimpleRNN(
3     input_size=100,
4     hidden_size=256,
5     output_size=100
6 )
7
8 # Process sequence
9 hidden = torch.zeros(1, 256)
10 for word in sentence:
11     output, hidden = rnn(
12         word, hidden
13     )
```

Key Points:

- Hidden size typically 128-512
- Input/output = vocabulary size
- Hidden state persists
- Same weights all steps

Hands-On: Debug This RNN Code

Find and fix the 3 bugs in this RNN implementation:

```
1 Bug 1: What's wrong here? self.embedding = nn.Embedding(  
    vocab_size, vocab_size)  
2 Bug 2: What's missing? self.rnn = nn.RNN(  
    input_size = 128, hidden_size = 256)  
3 def forward(self, x): Bug 3: What's the issue? embedded =  
    self.embedding(x) output = self.rnn(embedded)  
    return output
```

Your Fixes:

Bug 1: _____

Bug 2: _____

Bug 3: _____

Hints:

- Check dimensions
- RNN needs 2 outputs
- Embedding size \neq vocab size

Debugging Practice: Common RNN mistakes - dimension mismatches and missing components

Checkpoint Quiz: Test Your Understanding

Questions:

Q1: Why do RNNs use tanh activation?

- a) To make training faster
- b) To keep values in [-1, 1]
- c) To add non-linearity
- d) Both b and c

Q2: What makes RNNs different from feedforward networks?

- a) More parameters
- b) Recurrent connections
- c) Deeper architecture
- d) Faster training

Q3: Hidden state size 256 means:

- a) 256 words vocabulary
- b) 256-dimensional memory

Answers:

A1: d) Both b and c

- tanh bounds outputs
- Adds essential non-linearity
- Prevents value explosion

A2: b) Recurrent connections

- Hidden state feeds back
- Creates memory mechanism
- Key architectural difference

A3: b) 256-dimensional memory

- Size of h_t vector
- Network's memory capacity
- Independent of vocab/length

Common Misconceptions About RNNs

Misconception 1: “RNNs can remember everything”

Reality:

- Memory degrades over time
- Typically 5-10 steps effective
- Vanishing gradients limit range

Misconception 2: “Bigger hidden state = better”

Reality:

- Overfitting risk increases
- Training becomes harder
- Sweet spot: 128-512

Misconception 3: “RNNs process in parallel”

Reality:

- Strictly sequential
- Can't parallelize time steps
- This limits speed

Misconception 4: “RNNs are obsolete”

Reality:

- Still best for some tasks
- Efficient for short sequences
- Active research area

Clarity Check: Addressing these misconceptions prevents future confusion

Key Equations to Remember:

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$$

$$y_t = W_{hy}h_t + b_y$$

What We Covered:

- Mathematical foundation
- Forward pass computation
- Weight sharing benefits/problems
- PyTorch implementation
- Common bugs and fixes

Key Insights:

- Simple equations, complex behavior
- Memory through recurrence
- Same weights everywhere
- Sequential processing only
- Foundation for LSTMs/GRUs

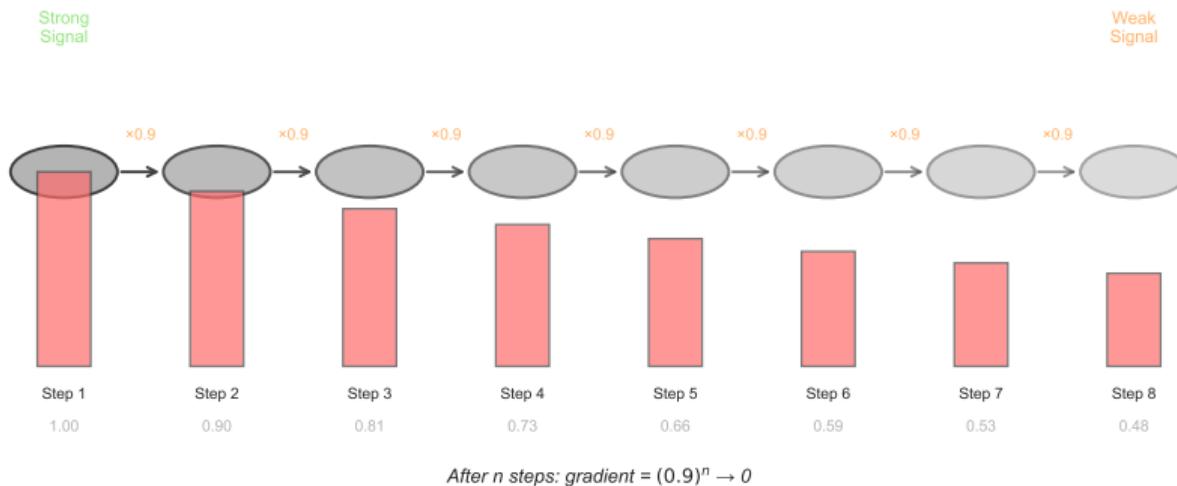
Next: The critical problem that almost killed RNNs - vanishing gradients

Part 3: The Vanishing Gradient Problem

Why RNNs Forget and How LSTMs Remember

The Telephone Game: Understanding Gradient Degradation

Gradient Vanishing: Like a Telephone Game



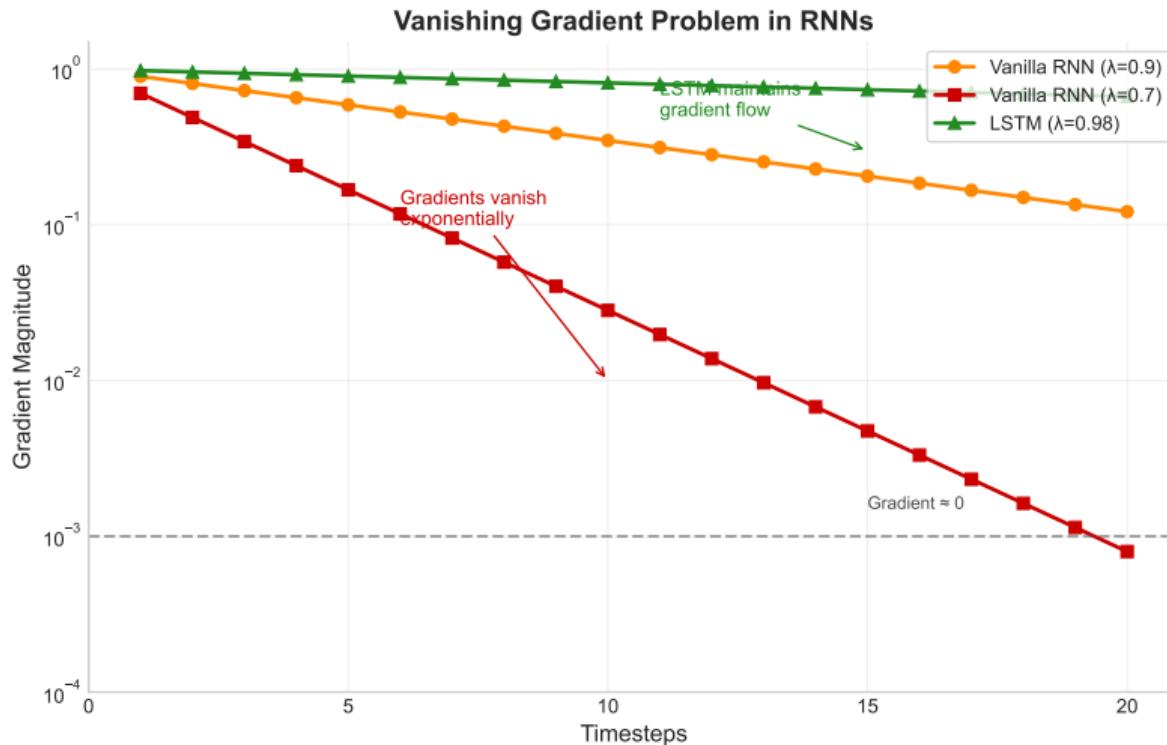
The Game:

- Person 1: "Buy milk and bread"
- Person 5: "Buy...milk and ...?"

Mathematical Reality: Gradient after t steps:

The Mathematics of Vanishing Gradients

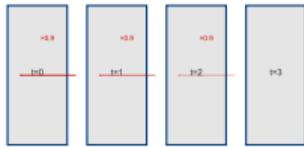
Backpropagation Through Time (BPTT):



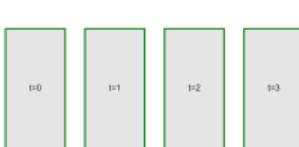
Vanishing vs Exploding: Two Sides of the Same Coin

Vanishing Gradients:

Vanilla RNN Gradient Flow

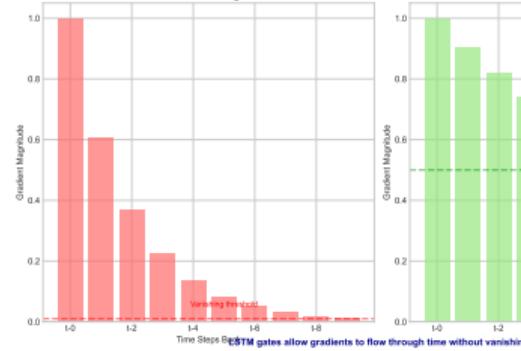


LSTM Gradient Flow

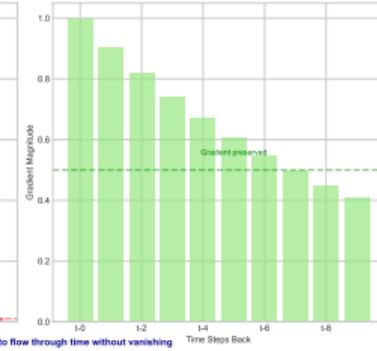


Exploding Gradients:

Vanilla RNN: Vanishing Gradients



LSTM: Preserved Gradients



- Weights < 1 : Signal dies
- Can't learn long dependencies
- Network "forgets" early inputs
- Most common problem

- Weights > 1 : Signal explodes
- NaN values crash training
- Gradient clipping helps
- Easier to detect/fix

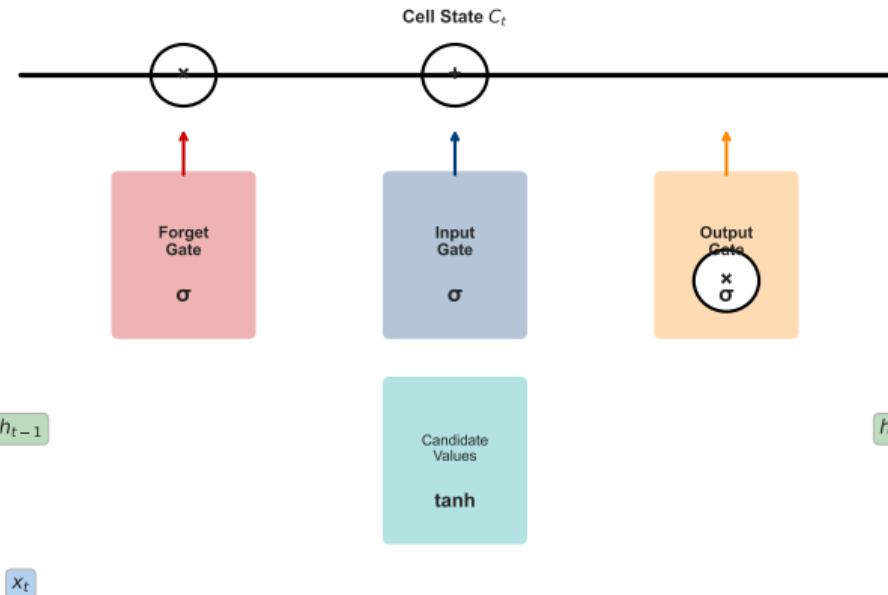
Critical Issue: Standard RNNs can effectively use only 5-10 steps of context

The Dilemma: Too small weights = vanishing, too large = exploding, just right = impossible

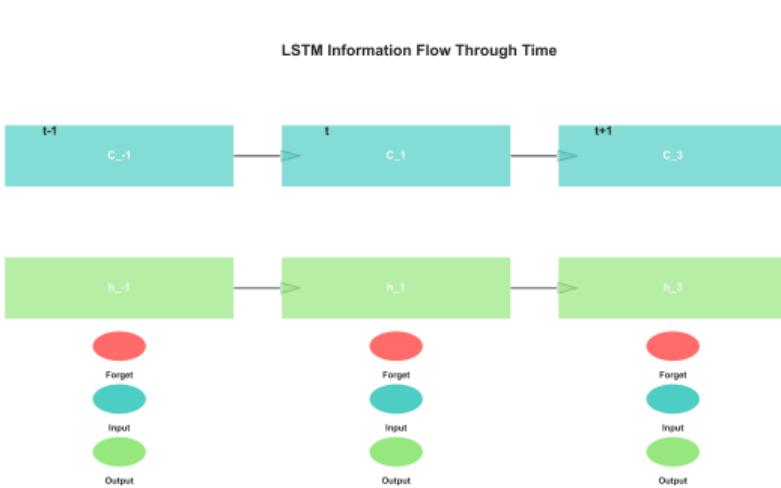
LSTM: The Elegant Solution to Memory

Long Short-Term Memory (LSTM) - Hochreiter & Schmidhuber, 1997

LSTM Architecture: Information Flow Through Gates



LSTM Architecture: Three Gates to Rule Them All



What Each Gate Does:

Forget Gate (f_t): "Forget that we saw 'not' "

Input Gate (i_t): "Store 'important' strongly"

Output Gate (o_t): "Output positive sentiment"

Key: Line 5

$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

Addition creates gradient highway!

Gate Equations:

$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f) \quad (1)$$

$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i) \quad (2)$$

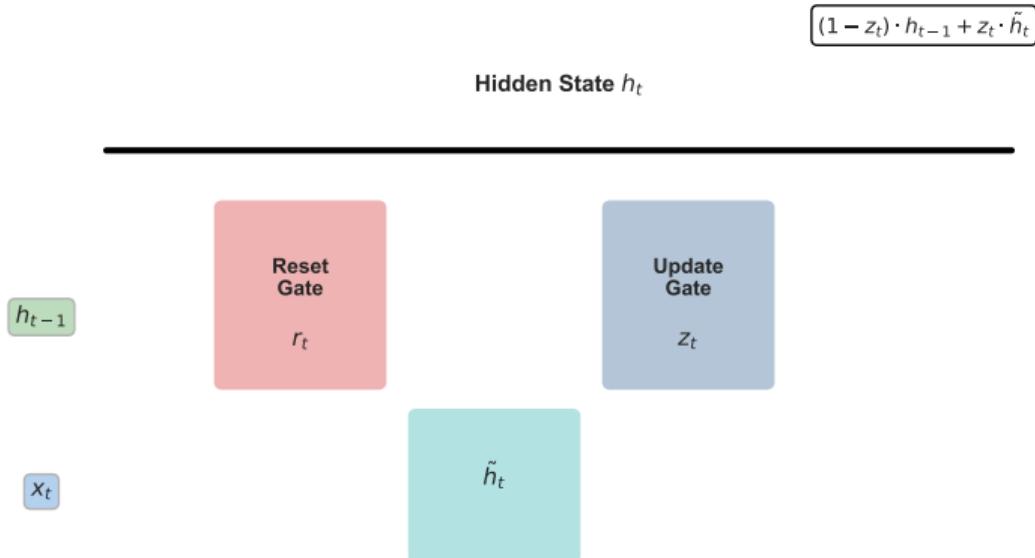
$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o) \quad (3)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C) \quad (4)$$

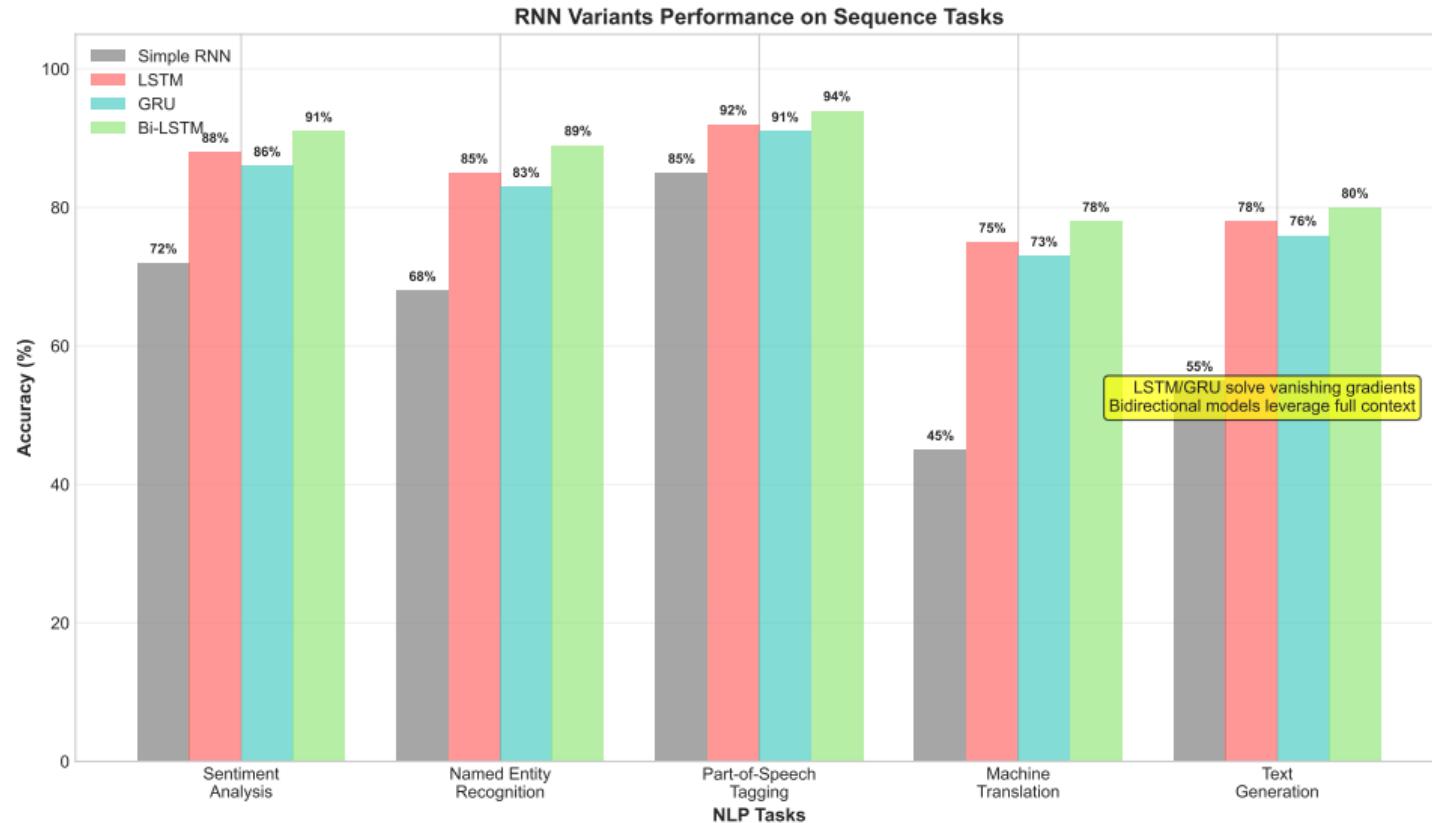
$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t \quad (5)$$

GRU: LSTM's Streamlined Sibling

GRU Architecture: Simplified Gating



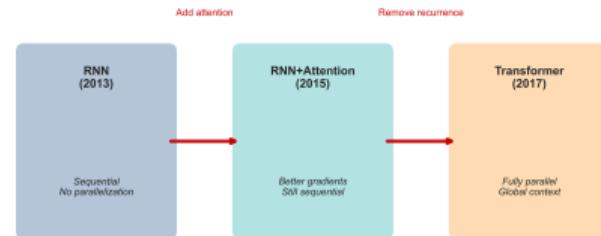
RNN vs LSTM vs GRU: Head-to-Head Comparison



The Problem:

- Gradients vanish exponentially
- Can't learn long dependencies
- Telephone game effect
- 5-10 step practical limit
- Made RNNs nearly useless

Evolution of Sequence Models



The Solution:

- LSTM: Gated architecture
- Addition instead of multiplication
- Gradient highway preserved
- 100+ step dependencies
- Revolutionized sequence modeling

Historical Impact:
LSTMs enabled modern NLP (1997-2017 dominance)

Next: Implement these concepts in code and explore real applications

Part 4: Implementation & Applications

From Theory to Practice

Complete LSTM Implementation in PyTorch

```
1 import torch
2 import torch.nn as nn
3
4 class TextGenerator(nn.Module):
5     def __init__(self, vocab_size,
6                  embed_size=128,
7                  hidden_size=256,
8                  num_layers=2):
9         super().__init__()
10
11         # Embedding layer
12         self.embed = nn.Embedding(
13             vocab_size, embed_size
14         )
15
16         # LSTM layers
17         self.lstm = nn.LSTM(
18             embed_size, hidden_size,
19             num_layers, batch_first=True,
20             dropout=0.2
21         )
22
23         # Output layer
24         self.linear = nn.Linear(
25             hidden_size, vocab_size
26         )
27
28     def forward(self, x, hidden=None):
29         # Embed input
30         embeds = self.embed(x)
```

Training Loop:

```
1 model = TextGenerator(
2     vocab_size=10000
3 )
4 optimizer = torch.optim.Adam(
5     model.parameters()
6 )
7 criterion = nn.CrossEntropyLoss()
8
9 for epoch in range(10):
10     hidden = None
11     for batch in dataloader:
12         # Forward pass
13         output, hidden = model(
14             batch, hidden
15         )
16
17         # Detach hidden
18         hidden = tuple(
19             h.detach()
20             for h in hidden
21         )
22
23         # Loss & backward
24         loss = criterion(
25             output, targets
26         )
27         loss.backward()
28
29         # Gradient clipping
```

Training Progression: Watch the Network Learn

`.../figures/sequence_analysis.pdf`

Still Dominant:

- **Speech Recognition:**
 - Google's RNN-T on phones
 - Apple's on-device Siri
 - 450MB models, real-time
- **Time Series:**
 - Stock price prediction
 - Weather forecasting
 - Energy demand modeling
- **Music Generation:**
 - MuseNet's LSTM backbone
 - Real-time synthesis
 - Latency-critical tasks

Hybrid Approaches:

- **Transformer + LSTM:**
 - Global + local context
 - Best of both worlds
 - State-of-the-art results
- **Efficient Models:**
 - Mobile deployment
 - Edge computing
 - Battery-powered devices
- **Streaming Applications:**
 - Live transcription
 - Real-time translation
 - Online learning systems

Key Insight: RNNs excel where sequential processing and low latency matter

Industry Reality: Despite transformers, RNNs remain crucial for specific use cases

Lab Preview: Build Your Own Text Generator

Week 3 Lab: Shakespeare Sonnet Generator with LSTM

What You'll Build:

1. Load Shakespeare sonnets
2. Preprocess and tokenize
3. Implement LSTM from scratch
4. Train character-level model
5. Generate new sonnets
6. Visualize hidden states

Sample Output:

*"Shall I compare thee to a summer's day?
Thou art more lovely and more temperate:
Rough winds do shake the darling buds..."*

Key Learning Objectives:

- Understand sequence modeling
- Debug vanishing gradients
- Implement teacher forcing
- Use gradient clipping
- Tune hyperparameters
- Compare RNN vs LSTM

Bonus Challenges:

- Bidirectional LSTM
- Attention mechanism
- Beam search decoding
- Style transfer

Hands-On Learning: Theory becomes practice - build what you've learned

Interactive Exercise: Debug Training Problems

Your LSTM is training poorly. Diagnose the issue:

Symptoms:

- Loss = NaN after epoch 2
- Gradients $\gtrsim 1000$
- Outputs all same token
- Accuracy stuck at 10%

Your Diagnosis:

Problem: _____

Solution: _____

Prevention: _____

Common Issues & Fixes:

1. Exploding Gradients

- Use gradient clipping
- Reduce learning rate
- Check weight initialization

2. Poor Initialization

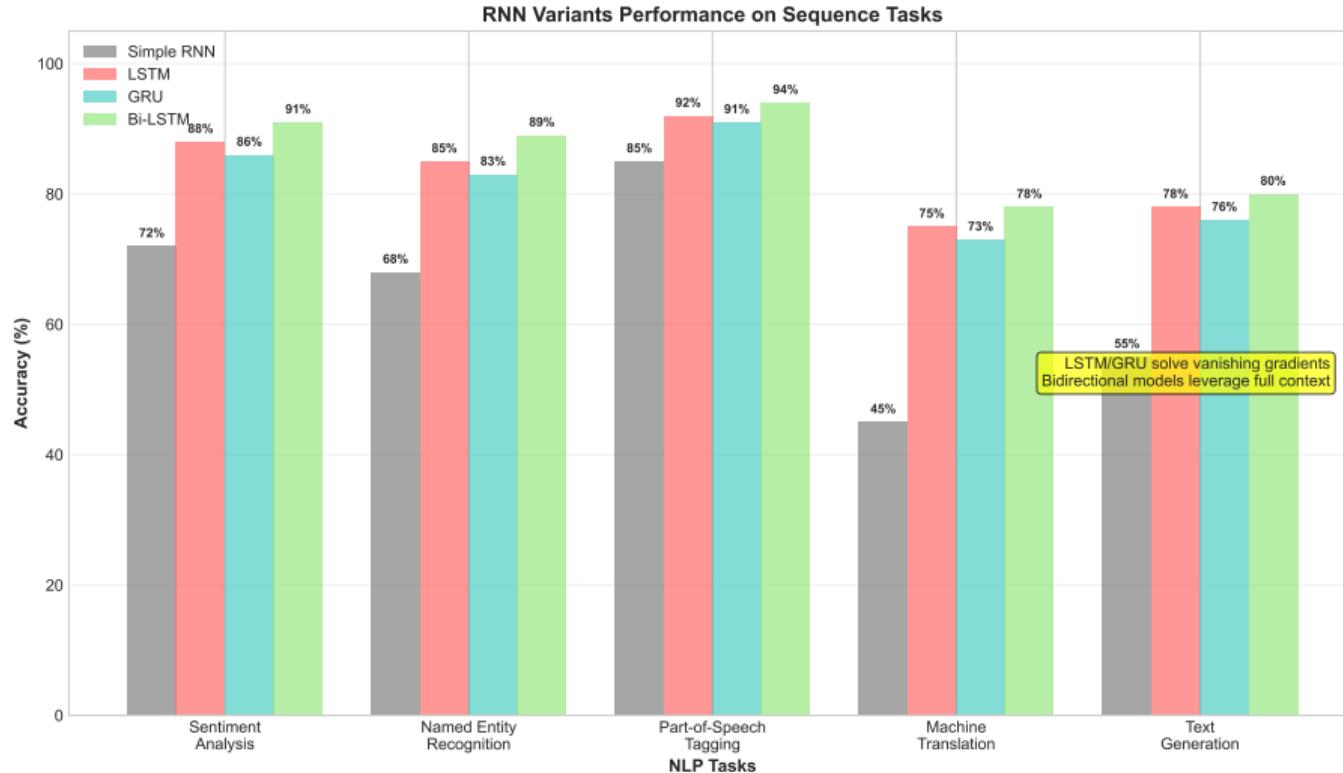
- Use Xavier/He init
- Initialize forget gate bias to 1

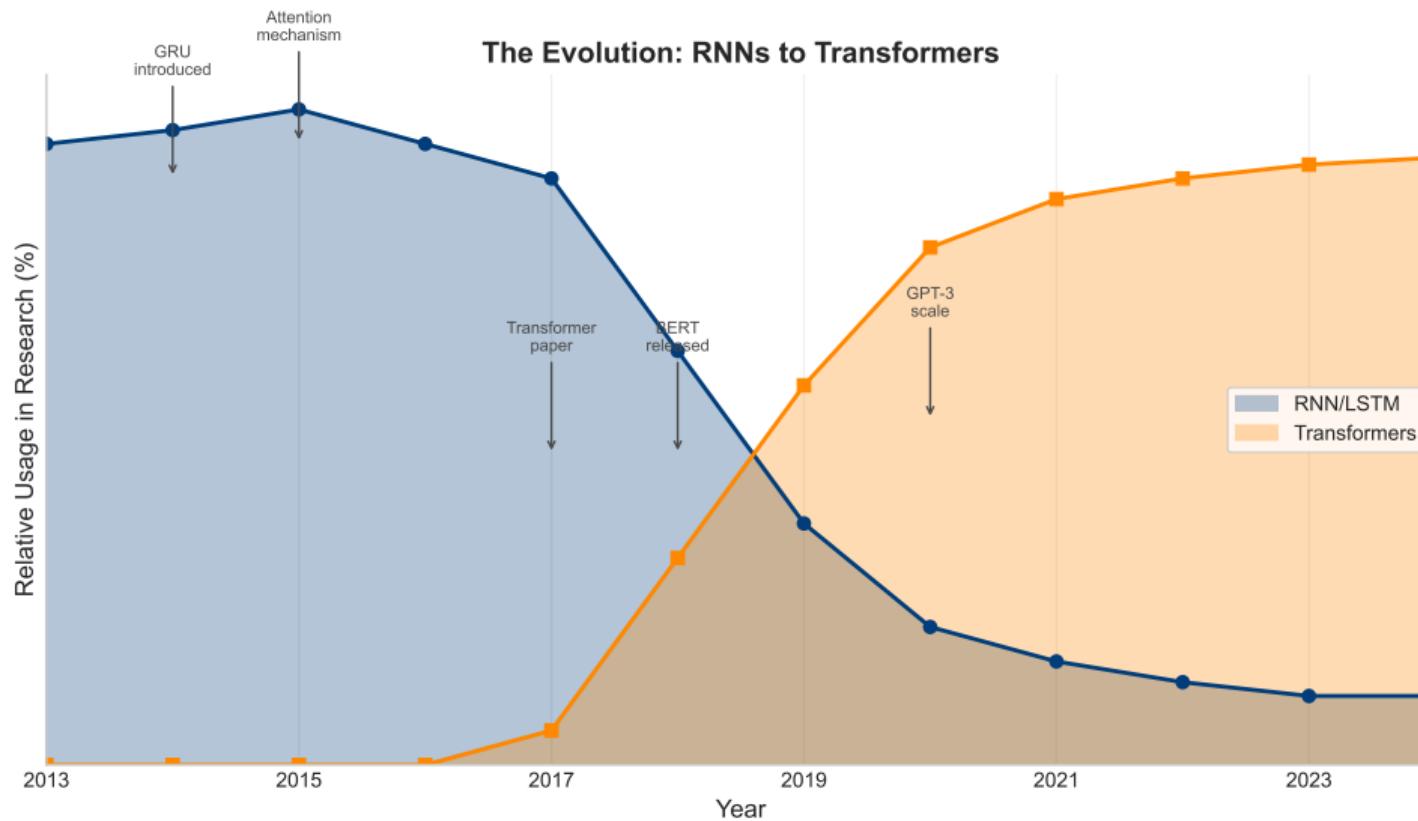
3. Wrong Loss Function

- Use CrossEntropyLoss
- Check label dimensions

Debugging Skills: Recognizing symptoms leads to faster fixes

Performance Shootout: When to Use What





Part 1: Motivation

- Language is sequential
- Order creates meaning
- Need memory mechanisms

Part 2: Architecture

- Hidden state = memory
- Weight sharing across time
- Simple equations, complex behavior

Part 3: Problems & Solutions

- Vanishing gradients limit RNNs
- LSTMs use gates for control
- Addition creates gradient highway

Part 4: Implementation

- PyTorch makes it easy
- Gradient clipping essential
- Still used in production

Key Takeaways:

1. RNNs = Neural nets + Memory
2. LSTMs solve vanishing gradients
3. Gates control information flow
4. Foundation for modern NLP

Next Week: Sequence-to-Sequence Models and the Attention Revolution!

Congratulations: You've mastered the fundamentals of sequential neural networks!