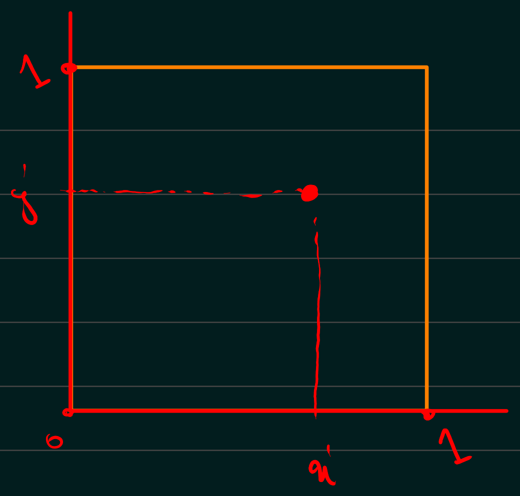
$$\nabla^2 G = -4\pi\,\delta(x-x')\,\delta(y-y')$$



$$\begin{cases} f(x) = f(x+2L) \\ f\left(\frac{\pi}{L}x\right) = f\left(\frac{\pi}{L}(x+2L)\right) = f\left(\frac{\pi}{L}x + 2\pi\right) \end{cases}$$

$$G = \sum_{n=-\infty}^{+\infty} A_n(y; x', y')\, e^{i2\pi n x}$$

$$\delta(x-x') = \sum_{n=-\infty}^{+\infty} B_n\, e^{i2\pi n x} \rightsquigarrow B_n = \int_0^1 \delta(x-x')\, e^{-i2\pi n x}\, dx = e^{-i2\pi n x'}$$

$$\delta(x-x') = \sum_{n=-\infty}^{+\infty} e^{i2\pi n (x-x')}$$

$$\nabla^2 G = (\partial_x^2 + \partial_y^2)\, G = \sum_{n=-\infty}^{+\infty} \left(A_n'' - (2\pi n)^2 A_n \right) e^{i2\pi n x} = -4\pi\,\delta(x-x')\,\delta(y-y')$$
$$= \sum_{n=-\infty}^{+\infty} -4\pi\,\delta(y-y')\, e^{-i2\pi n x'}\, e^{i2\pi n x}$$

$$A_n'' - (2\pi n)^2 A_n = -4\pi\,\delta(y-y')\, e^{-i2\pi n x'}$$

Homo: $A_n'' = (2\pi n)^2 A_n \rightsquigarrow A_n(y) = c_1 \sinh(2\pi n y) + c_2 \cosh(2\pi n y)$

non-Homo: $\displaystyle\int_{y'-\epsilon}^{y'+\epsilon} A_n'' - (2\pi n)^2 A_n = -4\pi\,\delta(y-y')\, e^{-i2\pi n x'}\, dy = A_n'\Big|_{y'-\epsilon}^{y'+\epsilon} \underbrace{- (2\pi n)^2 \tilde{A}_n * 2\epsilon}_{0} = -4\pi\, e^{-i2\pi n x'}$

$$A_n'\Big|_{y'-\epsilon}^{y'+\epsilon} = -4\pi\, e^{-i2\pi n x'} \qquad (*1)$$

شروط مرزی :

۱- پیوستگی پتانسیل ⟵ پیوستگی A_n در کل بازه

۲- ارضاء $(*1)$

۳- ارضاء شرط مرزی

$A_n(0)=0$

$A_n(1)=0$

$$A_n(y)=\begin{cases} C_1\sinh(2\pi n y)+C_2\cosh(2\pi n y) & y<y' \\ C_3\sinh(2\pi n y)+C_4\cosh(2\pi n y) & y>y' \end{cases}$$

$$A_n(0)=0 \longrightarrow \boxed{C_2=0}$$

$$A_n(1)=0 \longrightarrow C_3\sinh(2\pi n)=-C_4\cosh(2\pi n) \Rightarrow \boxed{C_3=-C_4\coth(2\pi n)}$$

$$\alpha_1 C_4$$

پیوستگی $\quad A_n^+(y')=A_n^-(y') \rightsquigarrow C_1\sinh(2\pi n y')=C_4\big(-\coth(2\pi n)\sinh(2\pi n y')+\cosh(2\pi n y')\big)$

$$\boxed{C_1=C_4\{\coth(2\pi n y')-\coth(2\pi n)\}} \rightsquigarrow \alpha_2 C_4$$

$$A_n(y)=C_4\begin{cases} \alpha_1\sinh(2\pi n y) & y<y' \\ \alpha_2\sinh(2\pi n y)+\cosh(2\pi n y) & y>y' \end{cases}$$

$(*1)\quad A'_n\Big|_{y'-\varepsilon}^{y'+\varepsilon}=-4\pi e^{-i2\pi n x'} \longrightarrow C_4 \times \cancel{2}\pi n\{\alpha_2\cosh(2\pi n y')+\sinh(2\pi n y')-\alpha_1\cosh(2\pi n y')\}$

$$=\cancel{-4}\pi e^{-i2\pi n x'}$$

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$$C_4=\frac{-2}{n}e^{-i2\pi n x'}\{(\alpha_2-\alpha_1)\cosh(2\pi n y')+\sinh(2\pi n y')\}^{-1}$$

$$\alpha_2-\alpha_1=\{\coth(2\pi n y')-\cancel{\coth(2\pi n)}\}\cancel{+}\{\cancel{+}\cancel{\coth}(2\pi n)\}=\coth(2\pi n y')$$

$$C_4 = \frac{-2}{n} e^{-i2\pi n x'} \left\{(\alpha_2-\alpha_1)\cosh(2\pi n y') + \sinh(2\pi n y')\right\}^{-1}$$

$$\alpha_2-\alpha_1 = \left\{\coth(2\pi n y') - \cancel{\coth(2\pi n)}\right\} \cancel{+} \left\{\cancel{+\coth(2\pi n)}\right\} = \coth(2\pi n y')$$

$$C_4 = \frac{-2e^{-i2\pi n x'}}{n}\left\{\coth(2\pi n y')\cancel{\cosh(2\pi n y')} + \cancel{\sinh}(2\pi n y')\right\}^{-1}$$

$$\frac{\cosh^2}{\sinh} + \sinh = \frac{\cosh^2+\sinh^2}{\sinh}$$

$$\Rightarrow C_4 = \frac{-2e^{-i2\pi n x'}}{n}\left\{\frac{\sinh(2\pi n y')}{\sinh^2(2\pi n y') + \cosh^2(2\pi n y')}\right\}$$

$$\Longrightarrow G(x,y;x',y') = \sum_{n=-\infty}^{+\infty} \frac{-2}{n} e^{i\pi 2n(x-x')}\left\{\frac{\sinh(2\pi n y')}{\sinh^2(2\pi n y')+\cosh^2(2\pi n y')}\right\}\begin{cases}\alpha_1\sinh(2\pi n y) & y<y' \\ \alpha_2\sinh(2\pi n y) + \cosh(2\pi n y) & y>y'\end{cases}$$

$$\alpha_1 = \left\{\coth(2\pi n y') - \coth(2\pi n)\right\}$$

$$\alpha_2 = -\coth(2\pi n)$$