Introduction of Particle Physics
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Contents

Elementary Particle Kinematic and Dynamics

第 1 章

1.1 Scale

- $$\left.\begin{aligned} Atom &: \sim 10^{-8}\text{cm} \\ Nucleon &: \sim 10^{-12}\text{cm} \\ Proton &: 0.8 \times 10^{-14}\text{cm} \end{aligned}\right\} \textbf{Structure}$$

- $$Electron :< 10^{-16}\text{cm} \leftarrow \textbf{point particle}$$

1.2 Units

Natural units:

$$\hbar = c = 1 \Leftarrow \left\{\begin{aligned} &\hbar = 6.58211 \times 10^{-15}\text{Gev} \cdot \text{Sec} \\ &c = 3 \times 10^{10}\text{cm/Sec} \end{aligned}\right\} \Rightarrow \begin{aligned} &1\text{Gev} = (6.6 \times 10^{-25}\text{Sec})^{-1} \approx 1.52 \times 10^{24}\text{Sec}^{-1} \\ &1\text{Gev}^{-1} \approx 6.6 \times 10^{-25}\text{s} \\ &1\text{Sec} = 3 \times 10^{10}\text{cm} \end{aligned}$$

Dimension:

Dimension		**Type**	
M^0	velocity	angular momentum	
M^1	mass	energy	momentum
M^{-1}	length	time	
M^{-2}		cross section	

$$\begin{aligned} m_e = m_e c = m_e c^2 &\approx \frac{1}{2}\text{MeV} \\ &= \frac{1}{\hbar / m_e c} \approx (4 \times 10^{-11}\text{cm})^{-1} \\ &= \frac{1}{\hbar / m_e c^2} \approx (1.3 \times 10^{-21}\text{Sec})^{-1} \end{aligned}$$

Physical constants

$$G_F{}^{①} = 1.16637(2) \times 10^{-5}\text{GeV}^{-2}$$

① *Fermi constant*

$$M_{pl}{}^{②} = G_N^{-1/2} = M_{pl} = 1.2 \times 10^{19}\text{GeV}$$

② *Planck mass*

$$M_W = 80.41 \pm 0.10\text{GeV}$$

$$M_Z = 91.187 \pm 0.007\text{GeV}$$

$$M_e = 0.51099906(15)\text{MeV}$$

$$M_p{}^{③} = 938.2723(3)\text{MeV}$$

③ *proton mass*

1.3 Kinematics

Mass

$$E = \frac{m}{\sqrt{1-v^2}}, \mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1-v^2}} \Rightarrow E^2 - \mathbf{p}^2 = m^2, \tag{1.3.1}$$

here E is energy ,$\mathbf{P}$ is momentum,$\mathbf{v}$ is velocity,and m is rest mass.

Experiment unstable particle

Probability density is given by

$$\boxed{\rho(M) = \frac{\Gamma}{2\pi\left[(M-m)^2 + \frac{\Gamma^2}{4}\right]}} \tag{1.3.2}$$

Fig. 1.3.1 shows Breit-Wigner distribution,where m is the mass, Γ is the width.

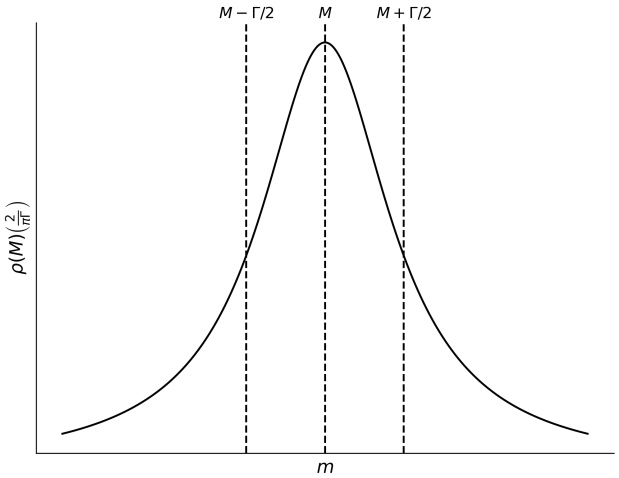

Figure 1.3.1. Breit-Wigner distribution

Lifetime,unstable particles——most of particles are unstable.

$$t \to t + \mathrm{d}t, N \to N - \mathrm{d}N$$
$$\mathrm{d}N = -\tau^{-1}N(t)\mathrm{d}t \Rightarrow N(t) = N(0)e^{-\frac{t}{\tau}},$$

here τ is lifetime and $\boxed{\tau = \frac{1}{\Gamma}}$.

Proof. **QM**

Consider a free particle with its wave function being $\psi(x,t)$

$$i\frac{\partial\psi(x,t)}{\partial t} = H\psi(x,t) \quad Schrödinger eq \tag{1.3.3}$$

Here H is Hamiltionian,which has eigenvalue E ——Energy.

The solution of $Schrödinger$ eq is

$$\psi(x,t) = e^{-iEt}\psi(x) \tag{1.3.4}$$

Normalization condition is

$$\int |\psi(x,t)|^2 \mathrm{d}^3x = 1, \tag{1.3.5}$$

and $|\psi(x,t)|$ has the physical meaning of the probability density of the particle at time t and space x.
If E is real ,the probability fnding the particle is independent of time $|\psi(x,t)|^2 = |\psi(x)|$.
To introduce an exponential decay of a state ,a small imaginary part is added to the energy

$$E = m - \frac{1}{2} i\Gamma, \tag{1.3.6}$$

here m, Γ is real ,and the factor is chosen for convenience.Then we have

$$\psi(x,t) = e^{-i(m-\frac{1}{2}i\Gamma)t}\psi(x) = \psi(x)e^{-imt}e^{-\frac{\Gamma}{2}t} \Rightarrow |\psi(x,t)|^2 = |\psi(x)|^2 e^{-\Gamma t}. \tag{1.3.7}$$

At the same time ,we have

$$N(t) = N(0)e^{-\frac{t}{\tau}}, \tag{1.3.8}$$

then $\Gamma\tau = 1$.
Use Fourier transformation to descerbe the wave function in terms of energy instead of time.
Consider a function $f(x,t)$

$$f(x,t) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \mathrm{d}M g(x,M) \exp(-iMt) \tag{1.3.9}$$

$$g(x,M) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \mathrm{d}t f(x,t) \exp(+iMt) \xlongequal[t<0,\psi(x,t)=0]{f(x,t)=\psi(x,t)} \tag{1.3.10}$$

$$\frac{1}{(2\pi)^{1/2}} \int_{0}^{\infty} \mathrm{d}t \psi(x) \exp\left(i(M - m + \frac{1}{2}i\Gamma)t\right) \tag{1.3.11}$$

$$= \frac{\psi(x)}{(2\pi)^{1/2}} \frac{i}{M - m + i\Gamma/2}. \tag{1.3.12}$$

The probability density $\rho(M)$ of finding the particle with mass M

$$\rho(M) = c\int \mathrm{d}^3x |g(g,M)|^2 \xlongequal{\int \mathrm{d}^3x\psi(x)=1} \frac{c}{2\pi} \frac{1}{(M-m)^2} + \frac{\Gamma^2}{4} = \frac{\Gamma}{2\pi} \frac{1}{(M-m)^2 + \Gamma^2/4}, \tag{1.3.13}$$

where we use the formula

$$\int_{\infty}^{\infty} f(M) dM = 1 \Rightarrow C = \Gamma. \tag{1.3.14}$$

The measurement on the mass of unstable parcels is uncertain

$$\tau\Gamma = 1 \longrightarrow \tau\Gamma = h \quad (\Delta t \Delta E \geqslant \hbar). \tag{1.3.15}$$

. . . . □

Some common particles have the following masses,

- photon: $\tau_\gamma = \infty$

- electron: $\tau > 2 \times 10^{23}$yr
- muon: $\tau = 2.19703(4) \times 10^{-6}$ s
- $\pi^{\pm} : \tau = 26029(23) \times 10^{-8}$ s
- $\pi^{0} : \tau = (84 \pm 06) \times 10^{-17}$ s
- N : $\tau = (896 \pm 10)$s
- Proton: $\tau > 10^{32} y_r$

Electric charge

$$e = 1.60217733(49) \times 10^{-11}\text{C}$$

$$\frac{|q_p| - |q_e|}{|q_e|} < 10^{-21} \qquad \leftarrow |q_n| = |q_p| = |q_e| \Rightarrow q_n = 0$$

Why the electric charge of particle is quantized?

Spin

- $Fermon : 1/2, 3/2, 5/2, \cdots$
- $Boson : 0, 1, 2, \cdots$

The spins of common particles are as follows:

Example	H	γ	e	μ	π	p	n	W	Z
Spin	0	1	1/2	1/2	0	1/2	1/2	1	1

Table 1.1. The spins of common particles

1.4 Particle classification

In the standard model

(1). Gauge bosons: $\gamma, W^{\pm}, Z$,g(8 gluons) ——Spin 1
(2). Scale boson: Higgs boson H ——Spin=0
(3). Fermions: leptons + quarks ——Spin =1/2

Leptons:

l	Q	l_e	l_μ	l_τ
e	-1	1	0	0
ν_e	0	1	0	0
μ	-1	0	1	0
ν_μ	0	0	1	0
τ	-1	0	0	1
ν_τ	0	0	0	1

Table 1.2. Lepton Table

There are also six antileptons with all the signs reversed position $Q = 1, l_e = -1$.
Lepton are conserved!

$$\underset{1}{e^-} + \underset{1}{e^+} \not\to \underset{0}{\pi^+} + \underset{0}{\pi^-}$$

Quarks:

q	Q	B
d	$-1/3$	1/3
u	$2/3$	1/3
s	-1/3	1/3
c	2/3	1/3
b	-1/3	1/3
t	2/3	1/3

Table 1.3. Quark Charges and Baryon Numbers

Here B=Baryon,which is also conserved.

$$\underset{1}{p} \not\to \underset{0}{e^+} + \underset{0}{\pi^0}$$

Observed particles

(1). Gauge bosons: $\gamma, W^{\pm}, Z$
(2). Leptons:$e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$
(3). Hadrons (强子,strong interaction)

- Mesons(介子):spin=$0, 1, \cdots$, B=0
- Bargons(重子):spin=$1/2, 3/2$, B=1

1.5 Interactions

1.5.1 The fow forces

Force	Strength	Theory	Mediator
Strong	$\alpha_s \sim 1$,large r;$\alpha_s < 1$,small r	Chromodynamics	Gluon
EM	$\alpha = 1/137$	Electrodynamics	Photon
Weak	$G_F m_p^2 \sim 10^{-5}$	Flavordynamics	$W + Z$
Gravitational	$G_N m_p^2 \sim 10^{-38}$	Geometrodynamics(General relativity)	Graviton

Note. 1. *No completely satisfactory quantum theory of Gravity yet.*
2. *Gravity is too weak to play a significant role in elementary particle physics.*
3. *this chapter——qualitative;subsequent chapters——quantitative*

Elementary Particle Dynamics

第 2 章

2.1 Quatum Electrodynamics(QED)

2.1.1 Examples

Elementary process All electromagnetic phenomena are ultimately reducible to the following elementary process.Here f is q,e,but ν_e is not allowed(neutral).

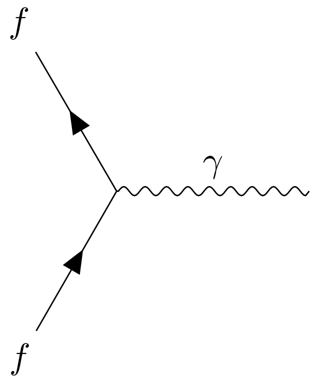

Figure 2.1.1. Charged particle f enters,emits(or absorbs) a photon γ and exits.

Møller scattering

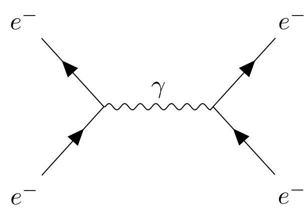

Figure 2.1.2. $e^- + e^- \to e^- + e^-$.The interaction is “mediated”by the exange of photon.

Bhabha scattering

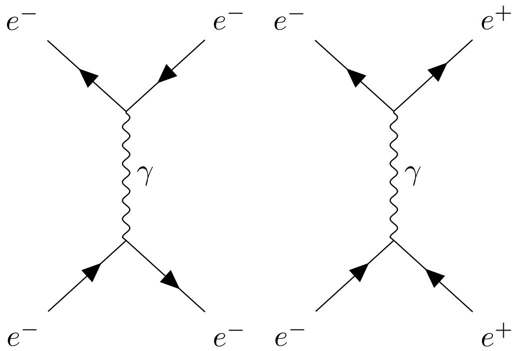

Figure 2.1.3. $e^+ + e^- \to e^+ + e^-$.A particle line running “backward” in time is interpreted as the corresponding antiparticle going forward.The photon is its own antiparticle ,so no arrows on the line.

Pair antnihilation

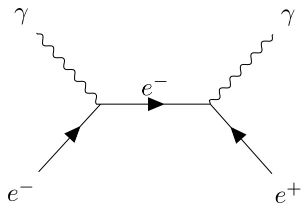

Figure 2.1.4. $e^+ + e^- \to \gamma + \gamma$.

Pair production

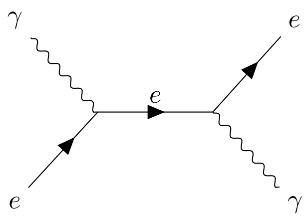

Figure 2.1.5. $\gamma + \gamma \to e^+ + e^-$.

Compton scattering

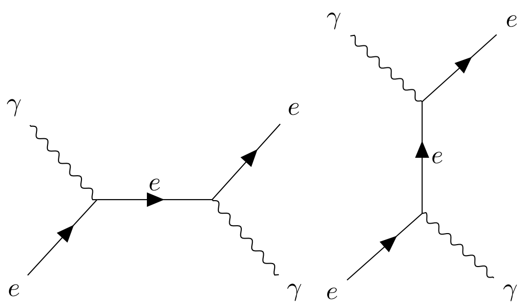

Figure 2.1.6. $e^- + \gamma \to e^- + \gamma$.

If we allow more vertices ,the possibilities rapidly proliferate .For example.with four vertices

- Internal lines represent particle that are not observed——called “virtual” particles.

- External lines represent “real” (observed) particles.

Note.
- *Feynman diagrams are purely symbolic, they donot represent particle trajectones.*
- *Feynman hagrams are calculated by using so called feynman rules (chapter 6).*
- *Each vertex withe a diagram introduces a factor $\alpha = 1/137$ - fine structure constant. Diagrams with more and more vertices contributes less and less to the final result and may be ignored.*
- *Feynman rules enforce conservation of energy and momentum at each vertex, and hence for the diagram as a whole.*

Exmples

- $e^- \not\to e^- + \gamma$,which violates conservation of energy.
  In the center-of-mass frame, the electron is initially at rest :
  $E_i = m_e$,but $E_f \geqslant m_e + E_\gamma, E_i \neq E_f$.
- $e^+ + e^- \not\to \gamma$,which violates conservation of momentum .
  $\mathbf{P}_i = \mathbf{P}_e + \mathbf{P}_e = 0$ in the CM frame ,but the final momentum canot be zero, since photon always travels at speed of light.

2.2 Quantum Chromodynamices(QCD)

2.2.1 Examples

Fundamental process

Leptons do not carry color,they do not participate in the strong interaction.

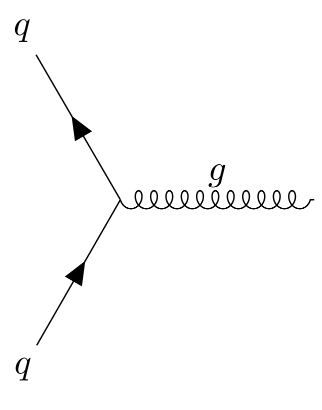

Figure 2.2.1. q——quark,g——gluon

$\mathbf{q} + \mathbf{q} \to \mathbf{q} + \mathbf{q}$

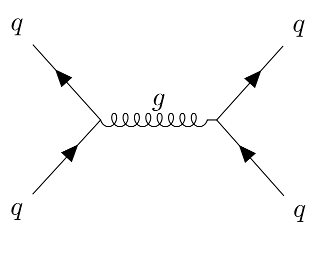

Figure 2.2.2. $q + q \to q + q$

- Quark contains three kinds of color ——red, green, blue.

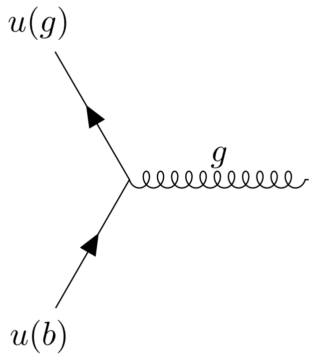

Figure 2.2.3. $q \to q + g$, the color of quark may change.

- Gluons carry 8 kinds color: gluon-gluon vertices

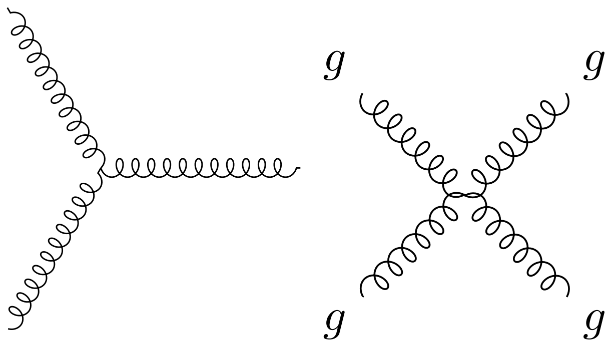

Figure 2.2.4. Three-gluon vertex and four-gluon vertex

Unlike QED $\left(\alpha \sim 10^{-2}\right)$, $\alpha_s \sim 1$ .Does Feynman's procedures have a problem for QED?

QED Screening effect

QED the vacuum itself behaves like a dielectric————it sprouts positron-election pairs.

The virtual electron in each bubble is attracted toward of and the vital positron 4 repelled away, the resulting vacuum polarization partially screens the charges and reduces its field coupling varies $\Longrightarrow$ the EH coupling varies with distance

$$\alpha(r) \approx \frac{\alpha}{1 - \frac{\alpha}{3\pi} \ln\left(\frac{r_e}{r}\right)}.$$

QCD

$$\alpha_s\left(|q^2|\right) = \frac{\alpha_s(\mu')}{1 - \frac{\alpha_s\left(\mu'\right)}{12\pi} a \ln\left(\frac{|q^2|}{\mu^2}\right)}.$$

Unlike QED, for QCD

$$\gamma \to 0, \alpha_s(r) \to 0(\textbf{Asgmptotic freedom})$$
$$\gamma \to R_c, \alpha_s(\gamma) \to \infty$$

$\Rightarrow$ quarks and gluons are confined inside hadrons whose sijes are of order R_c.

2.3 Weak interactions

QED	electric charge——charged particles
QCD	g,color——quarks
QFD	$W^{\pm}$, Z^0,weak charge——all quarks and leptons

2.3.1 Leptons

The fundamental process

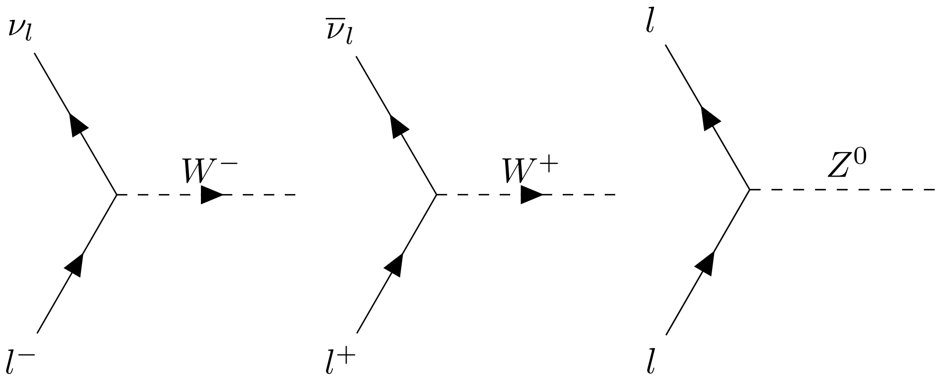

Figure 2.3.1. The fundamental process

$$l = \begin{cases} e, \mu, \tau \\ \nu_e, \nu_\mu, \nu_\tau \end{cases}$$

Example

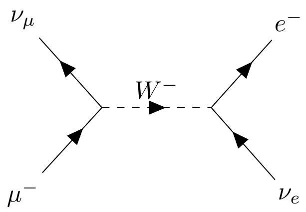

Figure 2.3.2. $\mu^- + \nu_e \to e^- + \overline{\nu}_e$

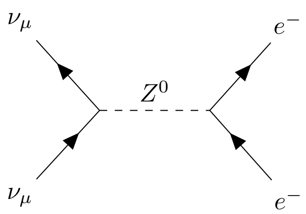

Figure 2.3.3. $\nu_\mu + e^- \to \nu_\mu + e^-$

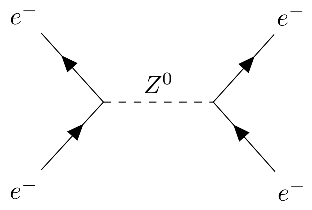

Figure 2.3.4. $e^- + e^- \to e^- + e^-$

References