Advanced Paper

1. If A be the area bounded by the curve $y = x^2 + 4x + 5$, the axes of co-ordinates and the minimum ordinate, then $[A]$ is equal to : ($[x]$ is the greatest integer less than equal to x)

2. The latus rectum of the conic passing through origin and having the property that normal at point (x,y) intersects x-axis at $(x + 1, 0)$ is :

3. Let circle 'C' with two diameters intersecting at angle of 30^0. A circle 'S' of radius 1 unit is inscribed in between given diameters and circle C. If r be the radius of circle 'C', then the least degree of $f(r)$ is : (where $f(r)$ be a polynomial of r with integral co-efficient)

4. If $I = \int_{-1/2}^{+1/2}\left(\sin^{-1}\left(3x-4x^3\right)-\cos^{-1}\left(4x^3-3x\right)\right)dx$, then value of $\left[-3I\right]$ (where [ ] is G.I.F) is

5. If $I = \int_0^1\left(\prod_{r=1}^{6}(x+r)\right)\left(\sum_{k=1}^{6}\frac{1}{x+k}\right)dx$, and P is the number of divisors of I, then value of $\left(\frac{P}{8}\right)$ is equal to

6. A function $f(x) = \begin{cases} x^m \sin\frac{1}{x}, x \neq 0 \,\&\, m \in N \\ 0 \qquad\quad , \text{if } x = 0 \end{cases}$. The least value of m for which $f'(x)$ is continous at $x = 0$ is

7. For a certain value of c, if $l = \lim_{x\to-\infty}\left[\left(x^5+7x^4+2\right)^c - x\right]$ is finite and non zero, then value of $(2l + c)$ is equal to

8. Let a, b, c be the roots of equation $x^3 + x^2 - 333x - 1002 = 0$ then vlaue of $a^3 + b^3 + c^3 - 2000$ is equal to

MCQ

1. Let $p(x)$ be a function on R such that $p'(x) = p'(5-x)$ for all $x \in [0,5]$. If $p(0) = 1, p(5) = 7$, then $\int_1^4 p(x)\,dx$ is equal to

   (A) 4 (B) 6 (C) 8 (D) 12

2. If a and b are two positive co-prime integer such that $\lim_{n\to\infty}\left(\frac{{}^{3n}C_n}{{}^{2n}C_n}\right)^{\frac{1}{n}} = \frac{a}{b}$, then

   (A) $a - b = 11$ (B) $2a - 3b = 13$ (C) $2b - a = 5$ (D) $a + b = 29$

3. If $\sum_{r=1}^{15}\frac{r\times 2^r}{(r+2)!} = 1 - \frac{2^k}{n!}$, then

   (A) $n - k = 1$ (B) $n - k = 2$ (C) $n + k = 35$ (D) $n + k = 34$

4. If $20^{f(x)} + 11^x - 2011 = 0$ and $f(x)$ is defined, then possible integral value(s) of x is / are

   (A) – 8 (B) 0 (C) 2 (D) 6

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5. In a ΔABC, if $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ b & c & a \end{vmatrix} = 0$ and $\alpha = \sum \sin^2 A, \beta = \sum \cos A, \gamma = \sum \tan A$ then

(A) $\alpha > \beta$ (B) $\alpha + \beta^2 = \dfrac{9}{2}$ (C) $\alpha + \beta < \gamma$ (D) $\alpha - \beta^2 = 0$

6. Let ABC be a scalene triangle and the curve
$y = \left[\sin B - \sin(B+C)\right]x^2 + \left[\sin A - \sin(A+B)\right]x + \left[\sin C - \sin(C+A)\right]$ touches x-axis, then

(A) a, b, c are in A.P.
(B) a^2, b^2, c^2 are in A.P.
(C) $\cot\dfrac{A}{2}, \cot\dfrac{B}{2}, \cot\dfrac{C}{2}$ are in A.P.
(D) $\dfrac{\cos\dfrac{A-C}{2}}{\sin\dfrac{B}{2}} > \dfrac{3}{2}$

7. If O be the origin and tangent to the curve $\left(\dfrac{x}{a}\right)^{2/3} + \left(\dfrac{y}{b}\right)^{2/3} = 1$ meet the axes in A and B respectively so that $OA = \alpha$ and $OB = \beta$, then

(A) (α, β) lies on ellipse
(B) $\dfrac{\beta}{b} + i\dfrac{\alpha}{a}$ is unimodular
(C) locus of $\left(\dfrac{\alpha}{a}, \dfrac{\beta}{b}\right)$ is a straight line
(D) $\alpha^2 + \beta^2 = a^2 + b^2$

8. If $A = \sum_{r=0}^{\infty} \ln 3^{2^{-r}}$, $B = \sum_{r=1}^{\infty} \dfrac{(-1)^{r+1}}{2r}\left(2^{-r} + 3^{-r}\right)$ and $C = \sum_{r=1}^{\infty} \dfrac{(-1)^{r+1}}{r \times 2^{r-1}}$, then

(A) $4B + A = C$
(B) $4B + C = A$
(C) $2C, A, 8B$ are in A.P.
(D) $8B, C, 2A$ are in A.P.

9. If A and B are symmetric matrices of same order, then

(A) $A + B$ is symmetric
(B) $AB + BA$ is symmetric
(C) $AB - BA$ is symmetric
(D) AB is an orthogonal matrix

10. In a ΔABC. Let $\vec{a} = \overrightarrow{BC}, \vec{b} = \overrightarrow{CA}$ & $\vec{c} = \overrightarrow{AB}$. If $|\vec{a}| = 12, |\vec{b}| = 4\sqrt{3}, \vec{b}.\vec{c} = 24$, then which of the following is / are true ?

(A) $\vec{a}.\vec{b} + 72 = 0$
(B) $\left|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}\right| = 48\sqrt{3}$
(C) $\left|\vec{c}\right|^2 = 20$
(D) $\dfrac{\left|\vec{c}\right|^2}{2} = |12 + |a||$

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1. Match the column

Column I	Column II
(A) Number of solution(s) of the equation $9^{\log_3(\log_2 x)} = \log_2 x - (\log_2 x)^2 + 1$ is	(p) 1
(B) If P be the maximum distance of the centre of the conic $\frac{x^2}{16}+\frac{y^2}{9}=1$ from the chord of contact of mutally perpendicular tangents of the conic then [P] (where [ ] is G.I.F.) is	(q) 2
(C) Let $x+\frac{1}{x}=1$ & $a=x^{2018}+\frac{1}{x^{2018}}$ and 'b' is the unit place digit of the number $2^{2^n}+1, n \in N$ & $n>10$ then value of $(b-2a)$ is equal to:	(r) 3
(D) If the graph of $f(x=)2x^3 + ax^2 + bx, a, b, \in N$ cut the x-axis at three distinct points then minimum value of $(2a-b)$ is :	(s) 5

2.

Column I	Column II
(A) If positive numbers x, y, z are in AP then minimum value of $\frac{x+y}{2y-x}+\frac{y+z}{2y-z}$ is	(p) 1
(B) Let $\|z\|-1\|=1$ and $z-2=kz\ \tan(\arg z)$ then $\|k\|$ is equal to	(q) 2
(C) Let $ABCDA_1B_1C_1D_1$ is a cube of edge 1 unit. P and Q are the mid points of the edges $B_1\ A_1$ and and B_1C_1 respectively. If d be the distance from vertex D to the plane PBQ, then $3d$ is equal to	(r) 3
(D) The direction cosines of a line satisfy the relation $(\lambda l + m =)n$ and $lm + mn + nl = 0$. The value of λ for which lines are perpendicular to each other is	(s) 4