UNIVERSITÁ DI UDINE

MASTER IN COMPUTER SCIENCE

ACADEMIC YEAR 2017-2018

Report of the Automated Reasoning project

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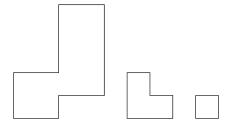
 ${\it Matriculation\ number:} \\ 116286$

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Problem: Filling a shape

Consider a figure whose outer edges are defined by a sequence of points (e.g., (0,0), (0,2), (2,2), (2,5), (4,5), (4,1), (2,1), (2,0)).

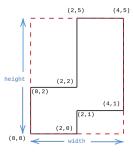


You have as many as you need "L" pieces (the biggest border is of size 2) plus unitary "U" pieces. "L" Pieces can rotate (in the same plane, they cannot be turned upside down — "U" pieces can rotate as well, but it doesn't matter too much).

The goal is to fill the internal shape of the figure, without overlapping the pieces minimizing the overall number of pieces used.

Solution

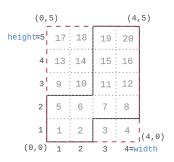
1. We obtain the minimum bounding box of the input points by calculating its width W and height H.



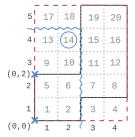
W = the maximum x coordinate of the input points H = the maximum y coordinate of the input points

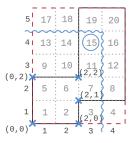
In this example, W = 4 and H = 5.

2. We think of the bounding box as if it is made up of cells. Each cell is uniquely identified by an index C, calculated from its coordinates (CX,CY) as follows: C = (CY - 1) * W + CX.



Then, we keep only the cells that belong to the shape: let C be a cell with coordinates CX and CY, and let N be the number of input points having coordinates X < CX and Y < CY. If N is even then C does *not* belong to the shape, else (N is odd) C belongs to the shape.





In this example, for the cell C = 14, N is equal to 2 and in fact it does

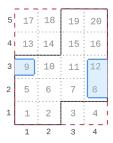
not belong to the shape, whereas for the cell C = 15, N is equal to 5 and it does belong to the shape.

- 3. A cell belonging to the shape must be filled with a "L" piece or a "U" piece.
- 4. To form "L" pieces: the index of the cell in the middle of the piece is 1 and W respectively distant from the index of the other two cells.



In this example, 11 is the cell in the middle of the piece and it is 1 from cell 12 and 4 from cell 15.

5. To prevent non-legal "L" pieces: a cell with multiple index of the width can *not* form a "L" piece with the next cell, because they are *not* adjacent in the bounding box.



In this example, cell 8 can not form a "L" piece with cell 9.

6. Objective function: minimize the number of "U" pieces.



In this example, the optimal solution is 3 "L" pieces and 3 "U" pieces.

Files

The solution includes the following files:

- \bullet filling_a_shape.mzn: MiniZinc encoding of the problem.
- \bullet filling_a_shape.lp: ASP encoding of the problem.
- instance_generator.lp: ASP encoding of the instance generator.
- instance_converter.cpp: C++ program to convert the raw instances generated by the instance generator into the .dzn and .lp format.

MiniZinc encoding

```
1 % Problem: Filling a shape
2 %
3 % Consider a figure whose outer edges are defined by
      a sequence of points (e.g.,
      (0,0), (0,2), (2,2), (2,5), (4,5), (4,1), (2,1), (2,0)).
4\, % You have as many as you need "L" pieces (the
      biggest border is of size 2) plus unitary "U"
      pieces. "L" pieces can rotate (in the
5 % same plane, they can NOT be turned upside down -
      "U" pieces can rotate as well, but it doesn't
      matter too much).
6 % The goal is to fill the internal shape of the
      figure, without overlapping the pieces minimizing
      the overall number of pieces used.
  % By Michele Collevati
9
10 % Parameters
11 int: num_points; % number of points
12 array[1..num_points,1..2] of int: seq_points; %
      sequence of points
  int: width = max([seq_points[p,1] | p in
      1..num_points]); % width of the input points
      bounding box
  int: height = max([seq_points[p,2] | p in
14
      1..num_points]); % height of the input points
      bounding box
15 int: num_cells = width * height; % number of cells
      (area of the bounding box)
16 set of int: cells = 1..num_cells;
17 int: L_size = 3; % size in cells of a L-piece
18 set of int: piece_components = 1..L_size;
19 int: digs = ceil(log(10.0, num_cells));  % digits for
      output
20
21 % Decision variables
22 array[cells, piece_components] of var 0..num_cells:
      pieces;
23
  var 0..num_cells: num_U_pieces;  % number of "U"
      pieces
   var 0..floor(num_cells div L_size): num_L_pieces;
      number of "L" pieces
25
26 % Constraints
27
```

```
28 % Cells that do NOT belong to the shape must be equal
      to 0.
29 % Cells that belong to the shape must be different
      from 0.
30 % Let C be a cell.
31 % Let N be the number of points having x and y
      coordinates lower than C's coordinates.
32 % IF N is even THEN C does NOT belong to the shape
33 % ELSE (N is odd) C belongs to the shape
  constraint
35
     forall(y in 1..height,x in 1..width)
36
       (if (length([p | p in 1..num_points where
          seq_points[p,1] < x /  seq_points[p,2] < y])
          mod 2) = 0
37
        then forall(pc in piece_components)
38
               (pieces[(y-1)*width+x,pc] = 0)
        else forall(pc in piece_components)
40
               (pieces[(y-1)*width+x,pc] != 0)
41
        endif);
42
  % Or all piece_components of a cell c are = 0 (the
      cell c does NOT belong to the shape)
   % or all piece_components of a cell c are = c ("U"
44
      piece)
   % or exactly one piece_component of a cell c is = c
      ("L" piece).
   % To prevent a cell from having piece_components of a
   % "L" piece it does NOT belong to: each cell belonging
48~\% to a "L" piece must have a piece_component equal to
49 % its index.
50 constraint
51
     forall(c in cells)
52
       (forall(pc in piece_components) (pieces[c,pc] =
53
        forall(pc in piece_components) (pieces[c,pc] =
           c) \/
54
        sum(pc in piece_components) (pieces[c,pc] = c) =
           1);
55
56 % To form "L" pieces: the index of the cell in the
      middle of the piece
   \% is 1 and W respectively distant from the index of
      the other two cells.
58 % Symmetry breaker:
59 % pieces[c,1] is the cell in the middle of the piece
60 % pieces[c,2] is the cell distant 1 from the cell in
      the middle of the piece
61 % pieces[c,3] is the cell distant width from the cell
      in the middle of the piece
62 constraint
```

```
63
     forall(c in cells)
64
       (sum(pc in piece_components) (pieces[c,pc] = c) =
65
         abs(pieces[c,1] - pieces[c,2]) = 1 / 
             abs(pieces[c,1] - pieces[c,3]) = width);
66
67
   % To prevent non-legal "L" pieces: a cell with
      multiple index
   % of the width can NOT form a "L" piece with the next
      cell,
   % because they are NOT adjacent in the bounding box.
69
70
   constraint
71
     forall(c in cells)
72
       (if (c mod width) = 0
73
        then sum(pc in piece_components) (pieces[c,pc] =
            c) = 1 ->
74
                 not exists(pc in piece_components)
                    (pieces[c,pc] = c+1)
        else if (c mod width) = 1
75
76
              then sum (pc in piece_components)
                 (pieces[c,pc] = c) = 1 \rightarrow
77
                      not exists(pc in piece_components)
                          (pieces[c,pc] = c-1)
78
              else true
79
              endif
80
        endif);
81
   \% To tie together the 3 cells that form a "L" piece:
82
83 % they must have the same piece_components.
84
   constraint
85
     forall(c in cells)
86
       (sum(pc in piece_components) (pieces[c,pc] = c) =
87
          forall(pcI in piece_components,pcJ in
              piece_components)
             (pieces[c,pcI] = pieces[pieces[c,pcJ],pcI]));
88
89
90
   % number of "U" pieces
   constraint
92
     num_U_pieces = sum(c in cells)
93
                        (forall(pc in piece_components)
                            (pieces[c,pc] = c));
94
95
   % number of "L" pieces
96
   constraint
97
     num_L_pieces = sum(c in cells)
98
                        (sum(pc in piece_components)
                            (pieces[c,pc] = c) = 1) div
                           L_size;
99
```

```
100 % Solve
101 solve :: int_search(pieces, input_order,
       indomain_max, complete) minimize num_U_pieces;
102 %solve :: int_search(pieces, first_fail,
       indomain_max, complete) minimize num_U_pieces;
103
104 % Output
105 output
106 [
107
     if x = 1 then "\n" else " " endif ++
108
        if fix(pieces[(height-y)*width+x,1]) = 0
109
        then format_justify_string(digs,"*")
110
        else show_int(digs,pieces[(height-y)*width+x,1])
111
        \verb"endif"
112
   | y in 1..height,x in 1..width
113
114 ++ ["\n\n"] ++
115 [
116
      "Number of \"L\" pieces = ", show(num_L_pieces),
         "\n",
117
      "Number of \"U\" pieces = ", show(num_U_pieces),
         "\n"
118 ];
```

ASP encoding

```
1 % Problem: Filling a shape
3 % Consider a figure whose outer edges are defined by
      a sequence of points (e.g.,
      (0,0),(0,2),(2,2),(2,5),(4,5),(4,1),(2,1),(2,0).
4\, % You have as many as you need "L" pieces (the
      biggest border is of size 2) plus unitary "U"
      pieces. "L" pieces can rotate (in the
5\, % same plane, they can NOT be turned upside down -
      "U" pieces can rotate as well, but it doesn't
      matter too much).
  % The goal is to fill the internal shape of the
      figure, without overlapping the pieces minimizing
      the overall number of pieces used.
7
  % By Michele Collevati
10\, % width of the input points bounding box
11 width(W) :- W = \#\max\{X : point(X,Y)\}.
13 % height of the input points bounding box
14 height(H) :- H = \max\{ Y : point(X,Y) \}.
15
16 % x coordinates
17
  coord_x(1..W) :- width(W).
18
19 % y coordinates
20 coord_y(1..H) :- height(H).
22 % Keep only the cells that belong to the shape.
23 % Let C be a cell.
   % Let N be the number of points having x and y
      coordinates lower than C's coordinates.
25\, % IF N is even THEN C does NOT belong to the shape
26 % ELSE (N is odd) C belongs to the shape
27 cell(C) :- coord_x(CX), coord_y(CY), width(W),
              PS = #count{ PX,PY : point(PX,PY), PX <
28
                  CX, PY < CY \},
29
              PS \setminus 2 == 1,
              C = (CY - 1) * W + CX.
30
31
32\, % A cell filled by a "L" piece can NOT even be filled
      by a "U" piece.
33 n_u_piece(A) := cell(A), cell(B), cell(C),
     l_piece(A,B,C).
```

```
34 \text{ n_u_piece(B)} := \text{cell(A)}, \text{cell(B)}, \text{cell(C)},
      l_piece(A,B,C).
35 \quad n_u_{piece}(C) := cell(A), cell(B), cell(C),
      l_piece(A,B,C).
36
37
  % A cell filled by a "U" piece can NOT even be filled
      by a "L" piece.
   n_1-piece(A,B,C):- cell(A), cell(B), cell(C),
       u_piece(A).
39 \quad n_1_piece(A,B,C) :- cell(A), cell(B), cell(C),
       u_piece(B).
   n_1_piece(A,B,C) := cell(A), cell(B), cell(C),
      u_piece(C).
41
42\, % To form "L" pieces: the index of the cell in the
      middle of the piece
   % is 1 and W respectively distant from the index of
      the other two cells.
44 % Symmetry breaker: in l_piece(A,B,C)
   % A is the cell in the middle of the piece
46\, % B is the cell distant 1 from the cell in the middle
      of the piece
  \% C is the cell distant width from the cell in the
      middle of the piece
   0 { l_piece(A,B,C) } 1 :- cell(A), cell(B), cell(C),
      width(W),
                               |A - B| == 1, |A - C| == W,
49
50
                               not n_l_piece(A,B,C).
51
   % "U" pieces
52
  u_piece(A) :- cell(A), not n_u_piece(A).
55 % A cell can be filled by at most 1 "L" piece.
   :- cell(C), cell(A), cell(B), cell(D), cell(E),
56
57
      l_piece(C,A,B), l_piece(C,D,E), A != D.
58
59
   :- cell(C), cell(A), cell(B), cell(D), cell(E),
60
      l_{piece}(C,A,B), l_{piece}(C,D,E), B != E.
61
62
   :- cell(C), cell(A), cell(B), cell(D), cell(E),
63
      l_piece(A,C,B), l_piece(D,C,E), A != D.
64
65
   :- cell(C), cell(A), cell(B), cell(D), cell(E),
66
      l_piece(A,C,B), l_piece(D,C,E), B != E.
67
68
   :- cell(C), cell(A), cell(B), cell(D), cell(E),
      l_piece(A,B,C), l_piece(D,E,C), A != D.
70
   :- cell(C), cell(A), cell(B), cell(D), cell(E),
71
72
      l_piece(A,B,C), l_piece(D,E,C), B != E.
```

```
74
   :- cell(C), cell(A), cell(B), cell(D), cell(E),
       l_piece(C,A,B), l_piece(D,C,E).
75
76
77
    :- cell(C), cell(A), cell(B), cell(D), cell(E),
78
       1_piece(C,A,B), 1_piece(D,E,C).
79
80
   :- cell(C), cell(A), cell(B), cell(D), cell(E),
81
       1_piece(A,C,B), 1_piece(D,E,C).
82
83 % To prevent non-legal "L" pieces: a cell with
       multiple index
    \% of the width can NOT form a "L" piece with the next
       cell,
85\, % because they are NOT adjacent in the bounding box.
86 :- cell(A), cell(B), cell(C), width(W),
      l_piece(A,B,C),
       A \setminus W == 0,
88
       B == A + 1.
89
90
91
   :- cell(A), cell(B), cell(C), width(W),
92
      l_piece(A,B,C),
93
       A \setminus W == 1,
94
       B == A - 1.
95
96\, % Number of "U" pieces
   num_u_pieces(NUP) :- NUP = #count{ A : cell(A),
97
       u_piece(A) }.
98
99~\% Number of "L" pieces
100 num_l_pieces(NLP) :- NLP = #count{ A,B,C : cell(A),
       cell(B), cell(C), l_piece(A,B,C) }.
101
102 % Minimize the number of "U" pieces
103 #minimize { NUP : num_u_pieces(NUP) }.
105\, % Maximize the number of "L" pieces
106 %#maximize { NLP : num_l_pieces(NLP) }.
107
108 #show l_piece/3.
109 #show u_piece/1.
110 #show num_u_pieces/1.
111 #show num_l_pieces/1.
```

Execution results

The benchmarks were made on the Asus N550JK laptop with an Intel Core i7-4700HQ CPU @ $2.40\mathrm{GHz} \times 8$, 16GB of DDR3L 1600MHz SDRAM and Antergos Linux 64-bit operating system.

Software used:

• MiniZinc

- 1. MiniZinc to FlatZinc converter, version: 2.2.2, build: 34369965, Copyright (C) 2014-2018 Monash University, NICTA, Data61
- 2. Gecode, version: 6.1.0, supported FlatZinc version: 1.6

• ASP

- 1. clingo, version: 5.3.0, address model: 64-bit, libelingo version: 5.3.0, configuration: with Python 3.7.0 and with Lua 5.3.5, license: The MIT License
- 2. gringo, version: 5.3.0, address model: 64-bit, libgringo version: 5.3.0, configuration: with Python 3.7.0 and with Lua 5.3.5, license: The MIT License
- 3. clasp, version: 3.3.4, address model: 64-bit, libclasp version 3.3.4 (libpotassco version 1.1.0), configuration: WITH_THREADS=1, Copyright (C) Benjamin Kaufmann, license: The MIT License

Time limit (t.l.): 5 min. opt. for optimal value

Search strategy used:

- on MiniZinc
 - 1. FF: solve :: int_search(pieces, first_fail, indomain_max, complete) minimize num_U_pieces
 - 2. IO: solve :: int_search(pieces, input_order, indomain_max, complete) minimize num_U_pieces

• on ASP

- 1. tweety (default): clingo --configuration=tweety
 It uses defaults geared towards typical ASP problems¹.
- 2. handy: clingo --configuration=handy
 It uses defaults geared towards large problems¹.

¹More info can be found here (page 75): https://github.com/potassco/guide/releases/download/v2.2.0/guide.pdf

| | MiniZinc | | | | | | | ASP | | | | | | |
|------------------------|-----------|-----------|---------|-----------|-----------|--------|-----------|-----------|---------|-----------|-----------|--------|--|--|
| | FF | | IO | | | tweety | | | handy | | | | | |
| Instances | L pieces | U pieces | time | L pieces | U pieces | time | L pieces | U pieces | time | L pieces | U pieces | time | | |
| benchmark_instance_4-1 | 3 (opt.) | 4 (opt.) | 0.06s | 3 (opt.) | 4 (opt.) | 0.06s | 3 (opt.) | 4 (opt.) | 0.011s | 3 (opt.) | 4 (opt.) | 0.014s | | |
| benchmark_instance_4-2 | 2 (opt.) | 2 (opt.) | 0.05s | 2 (opt.) | 2 (opt.) | 0.05s | 2 (opt.) | 2 (opt.) | 0.005s | 2 (opt.) | 2 (opt.) | 0.006s | | |
| benchmark_instance_4-3 | 1 (opt.) | 4 (opt.) | 0.05s | 1 (opt.) | 4 (opt.) | 0.05s | 1 (opt.) | 4 (opt.) | 0.004s | 1 (opt.) | 4 (opt.) | 0.005s | | |
| benchmark_instance_4-4 | 2 (opt.) | 2 (opt.) | 0.05s | 2 (opt.) | 2 (opt.) | 0.05s | 2 (opt.) | 2 (opt.) | 0.004s | 2 (opt.) | 2 (opt.) | 0.005s | | |
| benchmark_instance_4-5 | 2 (opt.) | 1 (opt.) | 0.05s | 2 (opt.) | 1 (opt.) | 0.04s | 2 (opt.) | 1 (opt.) | 0.005s | 2 (opt.) | 1 (opt.) | 0.004s | | |
| benchmark_instance_5-1 | 2 (opt.) | 4 (opt.) | 0.07s | 2 (opt.) | 4 (opt.) | 0.06s | 2 (opt.) | 4 (opt.) | 0.009 s | 2 (opt.) | 4 (opt.) | 0.006s | | |
| benchmark_instance_5-2 | 4 (opt.) | 5 (opt.) | 0.19s | 4 (opt.) | 5 (opt.) | 0.18s | 4 (opt.) | 5 (opt.) | 0.023s | 4 (opt.) | 5 (opt.) | 0.028s | | |
| benchmark_instance_5-3 | 3 (opt.) | 3 (opt.) | 0.07s | 3 (opt.) | 3 (opt.) | 0.07s | 3 (opt.) | 3 (opt.) | 0.008s | 3 (opt.) | 3 (opt.) | 0.009s | | |
| benchmark_instance_5-4 | 4 (opt.) | 4 (opt.) | 0.08s | 4 (opt.) | 4 (opt.) | 0.08s | 4 (opt.) | 4 (opt.) | 0.018s | 4 (opt.) | 4 (opt.) | 0.028s | | |
| benchmark_instance_5-5 | 2 (opt.) | 4 (opt.) | 0.07s | 2 (opt.) | 4 (opt.) | 0.06s | 2 (opt.) | 4 (opt.) | 0.006s | 2 (opt.) | 4 (opt.) | 0.006s | | |
| benchmark_instance_6-1 | 3 (opt.) | 4 (opt.) | 0.12s | 3 (opt.) | 4 (opt.) | 0.11s | 3 (opt.) | 4 (opt.) | 0.011s | 3 (opt.) | 4 (opt.) | 0.014s | | |
| benchmark_instance_6-2 | 4 (opt.) | 7 (opt.) | 0.30s | 4 (opt.) | 7 (opt.) | 0.30s | 4 (opt.) | 7 (opt.) | 0.027s | 4 (opt.) | 7 (opt.) | 0.028s | | |
| benchmark_instance_6-3 | 3 (opt.) | 10 (opt.) | 0.32s | 3 (opt.) | 10 (opt.) | 0.32s | 3 (opt.) | 10 (opt.) | 0.024s | 3 (opt.) | 10 (opt.) | 0.027s | | |
| benchmark_instance_6-4 | 7 (opt.) | 5 (opt.) | 4.62s | 7 (opt.) | 5 (opt.) | 4.79s | 7 (opt.) | 5 (opt.) | 0.109s | 7 (opt.) | 5 (opt.) | 0.084s | | |
| benchmark_instance_6-5 | 9 (opt.) | 1 (opt.) | 0.18s | 9 (opt.) | 1 (opt.) | 0.30s | 9 (opt.) | 1 (opt.) | 0.106s | 9 (opt.) | 1 (opt.) | 0.111s | | |
| benchmark_instance_7-1 | 12 (opt.) | 3 (opt.) | 157.85s | 12 (opt.) | 3 (opt.) | 64.25s | 12 (opt.) | 3 (opt.) | 1.156s | 12 (opt.) | 3 (opt.) | 0.370s | | |
| benchmark_instance_7-2 | 11 (opt.) | 3 (opt.) | 47.07s | 11 (opt.) | 3 (opt.) | 12.75s | 11 (opt.) | 3 (opt.) | 0.993s | 11 (opt.) | 3 (opt.) | 0.271s | | |
| benchmark_instance_7-3 | 7 (opt.) | 6 (opt.) | 2.63s | 7 (opt.) | 6 (opt.) | 2.61s | 7 (opt.) | 6 (opt.) | 0.130s | 7 (opt.) | 6 (opt.) | 0.093s | | |
| benchmark_instance_7-4 | 11 (opt.) | 3 (opt.) | 112.47s | 11 (opt.) | 3 (opt.) | 81.74s | 11 (opt.) | 3 (opt.) | 0.698s | 11 (opt.) | 3 (opt.) | 0.266s | | |
| benchmark_instance_7-5 | 6 (opt.) | 4 (opt.) | 0.24s | 6 (opt.) | 4 (opt.) | 0.24s | 6 (opt.) | 4 (opt.) | 0.041s | 6 (opt.) | 4 (opt.) | 0.045s | | |

| | | | iZinc | | ASP | | | | | | | |
|-------------------------|-----------|-----------|---------|-----------|-----------|---------|-----------|-----------|---------------------|-----------|-----------|--------|
| | FF | | | IO | | | tweety | | | handy | | |
| Instances | L pieces | U pieces | time | L pieces | U pieces | time | L pieces | U pieces | time | L pieces | U pieces | time |
| benchmark_instance_8-1 | 12 (opt.) | 6 (opt.) | 105.66s | 12 (opt.) | 6 (opt.) | 167.52s | 12 (opt.) | 6 (opt.) | 1.126s | 12 (opt.) | 6 (opt.) | 0.476s |
| benchmark_instance_8-2 | 10 (opt.) | 7 (opt.) | 11.74s | 10 (opt.) | 7 (opt.) | 11.65s | 10 (opt.) | 7 (opt.) | 0.424s | 10 (opt.) | 7 (opt.) | 0.282s |
| benchmark_instance_8-3 | 14 | 6 | t.l. | 14 | 6 | t.l. | 15 (opt.) | 3 (opt.) | 7.554s | 15 (opt.) | 3 (opt.) | 0.873s |
| benchmark_instance_8-4 | 8 (opt.) | 8 (opt.) | 4.77s | 8 (opt.) | 8 (opt.) | 4.67s | 8 (opt.) | 8 (opt.) | 0.123s | 8 (opt.) | 8 (opt.) | 0.138s |
| benchmark_instance_8-5 | 7 (opt.) | 12 (opt.) | 3.47s | 7 (opt.) | 12 (opt.) | 3.21s | 7 (opt.) | 12 (opt.) | 0.153s | 7 (opt.) | 12 (opt.) | 0.149s |
| benchmark_instance_9-1 | 7 (opt.) | 8 (opt.) | 2.81s | 7 (opt.) | 8 (opt.) | 2.78s | 7 (opt.) | 8 (opt.) | 0.095s | 7 (opt.) | 8 (opt.) | 0.103s |
| benchmark_instance_9-2 | 9 (opt.) | 7 (opt.) | 8.75s | 9 (opt.) | 7 (opt.) | 8.68s | 9 (opt.) | 7 (opt.) | 0.184s | 9 (opt.) | 7 (opt.) | 0.188s |
| benchmark_instance_9-3 | 12 (opt.) | 5 (opt.) | 12.61s | 12 (opt.) | 5 (opt.) | 12.57s | 12 (opt.) | 5 (opt.) | 0.685s | 12 (opt.) | 5 (opt.) | 0.369s |
| benchmark_instance_9-4 | 21 | 5 | t.l. | 21 | 5 | t.l. | 22 (opt.) | 2 (opt.) | 3.870s | 22 (opt.) | 2 (opt.) | 2.958s |
| benchmark_instance_9-5 | 15 (opt.) | 7 (opt.) | 28.37s | 15 (opt.) | 7 (opt.) | 10.72s | 15 (opt.) | 7 (opt.) | 1.438s | 15 (opt.) | 7 (opt.) | 0.937s |
| benchmark_instance_10-1 | 11 | 12 | t.l. | 11 | 12 | t.l. | 11 (opt.) | 12 (opt.) | 1.607s | 11 (opt.) | 12 (opt.) | 0.543s |
| benchmark_instance_10-2 | 23 | 6 | t.l. | 23 | 6 | t.l. | 24 (opt.) | 3 (opt.) | 66.635s | 24 (opt.) | 3 (opt.) | 4.298s |
| benchmark_instance_10-3 | 25 | 2 | t.l. | 25 | 2 | t.l. | 24 | 5 | t.1. | 25 (opt.) | 2 (opt.) | 6.555s |
| benchmark_instance_10-4 | 24 | 8 | t.l. | 24 | 8 | t.l. | 24 | 8 | t.l. | 25 (opt.) | 5 (opt.) | 7.648s |
| benchmark_instance_10-5 | 21 | 6 | t.l. | 21 | 6 | t.l. | 21 (opt.) | 6 (opt.) | $135.619\mathrm{s}$ | 21 (opt.) | 6 (opt.) | 2.849s |

| | MiniZinc | | | | | | | ASP | | | | | | |
|-------------------------|-----------|----------|---------|-----------|----------|---------|-----------|-----------|----------|-----------|-----------|---------|--|--|
| | FF | | IO | | | tweety | | | handy | | | | | |
| Instances | L pieces | U pieces | time | L pieces | U pieces | time | L pieces | U pieces | time | L pieces | U pieces | time | | |
| benchmark_instance_11-1 | 21 | 14 | t.l. | 21 | 14 | t.l. | 22 | 11 | t.l. | 22 (opt.) | 11 (opt.) | 6.477s | | |
| benchmark_instance_11-2 | 19 | 11 | t.l. | 19 | 11 | t.l. | 20 (opt.) | 8 (opt.) | 7.891s | 20 (opt.) | 8 (opt.) | 2.339s | | |
| benchmark_instance_11-3 | 24 | 9 | t.l. | 24 | 9 | t.l. | 25 (opt.) | 6 (opt.) | 43.940s | 25 (opt.) | 6 (opt.) | 5.379s | | |
| benchmark_instance_11-4 | 25 | 14 | t.l. | 25 | 14 | t.l. | 27 (opt.) | 8 (opt.) | 82.097s | 27 (opt.) | 8 (opt.) | 12.680s | | |
| benchmark_instance_11-5 | 14 | 20 | t.l. | 14 | 20 | t.l. | 14 (opt.) | 20 (opt.) | 15.499s | 14 (opt.) | 20 (opt.) | 1.451s | | |
| benchmark_instance_12-1 | 17 | 13 | t.l. | 17 | 13 | t.l. | 18 (opt.) | 10 (opt.) | 44.050s | 18 (opt.) | 10 (opt.) | 2.620s | | |
| benchmark_instance_12-2 | 32 | 14 | t.l. | 32 | 14 | t.l. | 34 | 8 | t.l. | 15 | 65 | t.l. | | |
| benchmark_instance_12-3 | 18 (opt.) | 5 (opt.) | 199.16s | 18 (opt.) | 5 (opt.) | 185.48s | 18 (opt.) | 5 (opt.) | 1.784s | 18 (opt.) | 5 (opt.) | 1.434s | | |
| benchmark_instance_12-4 | 36 | 11 | t.l. | 36 | 11 | t.l. | 36 | 11 | t.l. | 22 | 53 | t.l. | | |
| benchmark_instance_12-5 | 18 | 12 | t.l. | 18 | 12 | t.l. | 19 (opt.) | 9 (opt.) | 4.849s | 19 (opt.) | 9 (opt.) | 2.334s | | |
| benchmark_instance_13-1 | 20 | 22 | t.l. | 20 | 22 | t.l. | 22 (opt.) | 16 (opt.) | 182.870s | 22 (opt.) | 16 (opt.) | 4.238s | | |
| benchmark_instance_13-2 | 21 | 7 | t.l. | 21 | 7 | t.l. | 22 (opt.) | 4 (opt.) | 7.439s | 22 (opt.) | 4 (opt.) | 3.324s | | |
| benchmark_instance_13-3 | 27 | 23 | t.l. | 27 | 23 | t.l. | 30 (opt.) | 14 (opt.) | 225.564s | 30 (opt.) | 14 (opt.) | 19.180s | | |
| benchmark_instance_13-4 | 38 | 17 | t.l. | 38 | 17 | t.l. | 39 | 14 | t.1. | 16 | 83 | t.l. | | |
| benchmark_instance_13-5 | 20 | 17 | t.l. | 20 | 17 | t.l. | 21 (opt.) | 14 (opt.) | 7.810s | 21 (opt.) | 14 (opt.) | 3.304s | | |

As can be seen from the table, better results were obtained in ASP compared to MiniZinc. In particular, very low execution times were obtained using the handy configuration, but for instances that have gone in time limit it has behaved very badly.