

Fundamental Physics

So, I have to start by telling you, the syllabus for this course. Not the detailed one, just the big game plan. The game plan is that we will do electromagnetism. Electromagnetism is a new force that I will introduce to you and give you all the details, and I will do optics. Optics is part of electromagnetism. Then near the end, we will do quantum mechanics. Quantum mechanics is not like a new force, it's a whole different ball game. It's not about what forces are acting on this or that object that make it move or change its path, the question there is should we even be thinking about trajectories? Should we even be thinking about particles going on any trajectory or what the right trajectory is? And you will find out that most of the cherished ideas are destroyed but, the good news is that you need quantum mechanics only to study little tiny things, like atoms or molecules. Of course, the big question is where do you draw the line? How small is small. Some people even ask me do you need quantum mechanics to describe the human brain? The answer is yes, if it is small enough. So, I've gone to parties where a few minutes of talking to a person and I'm thinking, "okay this person's brain needs a fully quantum mechanical treatment." But most of the time everything macroscopic, you can describe the way you do in classical mechanics, you don't need the quantum theorem. Alright so now we'll start with the brand new force of electromagnetism. Before doing the force, I've gotta remind you people of certain things I expect you all to understand about the dynamics between force and acceleration that you must have learned last year. I don't want to take any chances so I'm going to start by reminding you how to use this famous equation of Newton. So we have seen this equation probably in high school, but it's a lot more subtle than you think, certainly a lot more subtle than I thought when I first learned it. So I will tell you what I have figured out all these years on ways different to look at $F=MA$. Otherwise, if you have the equation, what's it good for? The only thing everybody notices right away is A stands for acceleration and we all know how to measure it. Whether I write any symbol on the board, you should be able to tell me how you'd measure it. Otherwise, you don't know what you're talking about as a physicist. Acceleration I won't spend too much time on because you should know what instrument you'd need. I will remind you that if you have meter sticks and clocks, you can follow the body as it moves, you can find its position now and its position later, take the difference, divide by the time, and you get velocity. Then find the velocity now, find the velocity later, take the difference, divide by time, you get acceleration. So acceleration really requires three measurements, two for each velocity. But we talk about acceleration right now, because you can make those three measurements arbitrarily near each other and if the limiting time that goes between them to zero, you can talk about those velocities right now and acceleration right now. In your car, the needlepoint is at 60, that is the velocity right now, the instantaneous continuity. You step on the gas and feel this push, that's acceleration right now, that's property of that instant. So we know acceleration, but the question is can I use the equation to find the mass of something? Very often when I pose the question the answer given is you go to scale or weighing machine and find the mass, unless you know that's not the correct answer because the weight of an object is related to the earth by gravity, but the mass of an object is defined anywhere. So here's one way you can do it. You might say, "well, take a known force and find the acceleration it produces," but we haven't talked about how to measure the force either, all you have is this equation. The correct thing to do is to buy yourself a spring and go to the bureau of standards and tell them to loan you a block of

some material like this that's called a kilogram, that is a kilogram by definition. There is no God-given way to define mass, you pick a random entity and say that's a kilogram. So that's not right and that's not wrong, that's what a kilogram is. So if you bring that kilogram and hook it up to the spring, and you pull it by some amount, maybe to that position and you release, you notice the acceleration of the one kilogram and the mass of that thing is just 1. Then you detach that mass, then you ask, the other person says, "what's the something else?" I don't know what the something else is. Let's say potato, and you take the potato or anything, elephant, potato, you pull that guy by the same distance and release that and you find your acceleration. Since you pulled it by the same amount, the force is the same. Whatever it is, we don't know what it is but it is the same. Therefore we know, the acceleration of 1 kilogram times 1 kilogram is equal to the unknown mass times the acceleration of the unknown mass. That's how by measuring this you can measure what that mass is. So by principle you can find the mass of anything. So imagine the masses of all objects have been determined by this process. Then you can also use $F=MA$ to find out what forces are acting on bodies in different situations, because if you don't know what force is acting on a body, you cannot predict anything. So you can go back to the spring, and say, "I want to know what force the spring exerts when it's pulled by various amounts." Well you pull it by some amount X , you attach it to a known m and you find the acceleration and that's the force. And if you plot it, you'll find F as a function of x will be roughly straight lined, and it will take the form $F=-kx$, the k is called the force constant. So this is an example of you're finding on the left-hand side of Newton's law. You've got to understand the extinction of $F=-kx$ and $F=ma$, what's the difference? This says if you know the force, I can tell you the acceleration, but it is your job to go find out every time what forces might be acting on a body. If it is connected to the spring, and you pull the spring and it observes the forces, someone has got to make this measurement to find out what the force is. Alright, so that's one kind of force. Another force that you can find is if you need the surface of the area. If you drop something, it seems to accelerate towards the ground, and everything accelerates by the same amount g . Well according to Newton's laws, if anything was to accelerate it's because there is a force on it. For the force on any mass m , must be mg because if you divide by m , you gotta get g . So that force of masses near the earth is mg , that's another force. Something interesting about that force is that unlike the spring force, where the spring is touching the mass and you can see it's pulling it or when I touch this chair, you can see I'm doing it. The pull of gravity is a bit strange because there is no contact between the earth and the object that's falling. It was a great abstraction that things can reach out and pull things that are not touching them, and gravity was the first formally described force where that is true. And another excursion in the same theme is that if this object gets very far, like the moon over there, then the force is not given by mg , the force is given by this law of gravitation. If you're already near the surface of the earth, you would get a constant force that would be mg , but if you move far from the center of the earth, you got to take that into account. That's what Newton did and realized the force is $\frac{F}{r^2}$. So every time things accelerate, you've got to find the reason, and that reason is the force. Many forces can be acting on a body and with all the forces that are acting on a body and that explains the acceleration and you are done. But sometimes it won't, that's when you have a new force. And the final application of $F=ma$ is this one, if you knew the force, for example on a planet. Here's a planet, the sun is here, and you know the force acting up given by Newton's Law of gravity, you can find the acceleration that will help you find out where it would be. One second later, and you repeat the calculation, and you will get the trajectory. So $F=ma$ is good for three things and that's what I want you to understand. To define the mass, to calculate forces acting on bodies by seeing how they accelerate, and finally, to find the accelerations of bodies given the forces. See the cycle of Newtonian dynamics, and what

I'm going to do now is to add one more new force because I'm going to find out that there is another force that is not listed here. I'm going to demonstrate to you that new force. Here's my demonstration, the only demonstration you will see in my class because everything else I try generally fails, but this one always works. So, I have here a piece of paper, okay, and I take this trusty comb and I comb the part of my head that's suited for this experiment, then I bring it next to this, and you see I'm able to lift it. It's not the force of gravity because gravity doesn't care if you comb your hair or not. And also when I shake it, it falls down. So you are thinking, "okay maybe that it is a new force but it doesn't look awfully strong because it's not able to even overcome gravity because it eventually yielded to gravity and fell to the ground." That's actually a mistake to think so. In fact, this new force that I'm talking about is 10^{40} stronger than gravitational force. I will tell you by what metric I came up with that number. But it's an enormously strong force and you've got to know why I say it's such a strong force when I shook and the thing fell down. So the reason is that if you look at this experiment, here's the comb and here's the paper, the comb is trying to pull the paper, but what is trying to pull it down? So here is me, here is the comb, here's the paper, the entire planet is pulling it down. Himalaya's pulling it down, pacific ocean is pulling it down. Bin Laden sitting in his cave is pulling it down, everything is pulling it down. I am one of these people generally convinced the world is acting against me, but this time I am right. Everything is acting against me and I am able to try and focus all of that to this tiny comb. That is how you compare the electric force to the gravitational force. It takes the entire planet to compensate whatever tiny force it creates between the comb and the piece of paper. To really get a number out of this, I would have to do a little more, but I just want to point out to you to say new force much stronger than gravitational force. So I want to tell you a few other experiments people did without going into what the explanation is right now. Let me just tell you, if you go through history what all did people do? So, one experiment you can do, you take a piece of glass and you rub it on some animal that is passing by, water buffalo, that's why I cannot do all experiments in class. You rub it on that guy, then you do it to a second piece of glass and you find out that they repel each other, meaning that they tend to fly apart. Then you take a piece of hard rubber and you rub that on something else, silk, yeti, something, then you put that here so I'll draw a different shape to that thing, that's the rubber, and you find when you do that to this, these two attract each other. So sometimes they repel, sometimes they attract. Here's another thing you can do, buy some Nylon thread and you hang a small metallic sphere and you bring one of these rods next to it, it doesn't matter which one. Initially they're attracted, and suddenly when you touch it and you move it, they start repelling each other. What's going on? That's another thing you could do. The last thing I want to mention is if you take two of these things, which are repelling each other. Let's say they are attracting each other, then you connect them with a piece of nylon and you take it away, nothing happens. If you connect them with a piece of wire and take the wire, they no longer attract each other. So these are examples of different things. I'm just going to say you do this, you do that, then finally you need a theory that explains everything, so that's the theory that I'm going to give you now. That's the theory of electrostatics and I don't have time to go into the entire history of how people arrived at this final formula, so I'm just going to tell you one formula that really will explain everything that I've described so far, and that formula is called Coulomb's Law. Even though Mr. Coulomb's name is on it, he was not the first one to formulate parts of the law, but he gave the final inadaptation to the law that other people had contributed to. So Coulomb's law says that certain entities have a property called charge. You have charge and you don't have charge, but if you have charge, the charge that you have, meaning any of these objects is measured by Coulombs. But it wasn't Coulomb's idea to call it Coulombs, whenever you make a discovery, you're breathlessly waiting for

somebody to name it after you, but it's not in good taste to name it after yourself. So he didn't say, "Call it Coulombs," he certainly wrote down this law. The law says that if you have one entity, because some amount of charge called q_1 , and then there's another entity of the charge q_2 , they will exert a force on each other which is given by $q_1 q_2 \times$ the constant which is written as $\frac{1}{4\pi\epsilon_0} \cdot \frac{1}{r^2}$ where r is the distance between them. And you can in this picture, "What do you mean by distance? Is it from here to there? Or is it from center to center?" We're assuming here, that the distance between them is much bigger than the initial sizes. For example, you say how far am I from Los Angeles? Well 3,225 miles but you are you talking about your right hand or you left hand? Well I'm a particle for this purpose so it doesn't matter. So here we are assuming that either they are mathematically point charges, or they are charges with a finite size but are separated by a distance much bigger than the size. So r could stand if you like, for center to center, it doesn't matter too much. This is what Coulomb said, now if you look at this number here, $\frac{1}{4\pi\epsilon_0}$ it's value is $9 \cdot 10^9$. What that means is the following, if you take one Coulomb of body with one Coulomb of charge, and another body with one Coulomb of charge, and they're separated by one meter, then the force between them will be this number because everything else is a 1, so it would be $9 \cdot 10^9$. That's an enormous force, and normally you don't run into 1 Coulomb of charge, but the reason a Coulomb was picked as the particle, it has to do with currents and so on. Anyways, this is the definition of it. But if you want to be more precise, I should write a formula more carefully because force is a vector. Also I should say force on whom? And due to what? So let's say there are two charges. Let's say q_1 is sitting at the origin, and q_2 is sitting at a point where it's position is the vector r . Then the force r_2 due to one is given by $\frac{q_2 q_1}{4\pi\epsilon_0} \cdot \frac{1}{r^2}$ that's the magnetude force, but I want to set this to a force that pushes $q_1 q_2$ away. So I want to make this a vector, but I got the magnitude of the vector. As you know to make a real vector, you take its magnitude and multiply it by a vector length of that direction. The unit vector we can write in many ways, one should say e_r , which is a standard name for a vector of length 1 in the direction of r . But I'll give you another choice; you can also write it as $\frac{r}{\text{length of } r}$ that also would be a vector at length according to r . So there are many ways to write anything that makes it a vector. And $F_2 \cdot 1 = -F_1 \cdot 2$. Now, how do we get attraction and how do we get repulsion? We get it because if q_1 and q_2 are both vectors and if we use the formula, you'll find that they repel each other, but if they're opposite signs, the same calculation but you put a - sign through the whole thing, that will turn repulsion into attraction. So you must allow for the possibility that q can be of either sign. Q can also be 0, there are certain entities that don't have any electric charge, that if you put next to 1 million Coulombs nothing happens. So something's a plus charge, something's a minus charge, something's no charge. They're all contained in this Coulomb's law. Now again, skipping all the intermediate discoveries, I want to tell you a couple of things we know about charge. First thing is q is conserved. Conserved is a Physics term for saying, it does not change with time. For example, when energy is conserved, particles get collided to all kinds of things, but if you add that energy before, you get the same energy afterwards. When that happens when the quantity is observed, the claim is electrical charge is conserved. Electrical charge may migrate from A to B and B to C, but if you add up the total charge and say the chemical reaction of any process, including a big particle accelerators, things collide and all kinds of stuff starts flying out, the charge of the final products always equal the charge of the incoming products. But charge conservation needs to be amended with one extra term extra-qualification called (local). Suppose I say the number of students in the class is conserved, that means you count that many times and you've got to get the same number. Well here's a possibility suddenly one of you guys disappears and appears here in

the same instant. That's also consistent with conservation of charge numbers because the number didn't change, what disappeared there appeared here. But that is not a local disappearance of charge because it disappears in a world and appears in another one and it's not even a meaningful law to have in the presence of relativity. Can any of you guys think of why that might be true? Why charge disappearing some where and appearing somewhere else could not be of any profound principle? Yes? Yep. (student speaks) Well, we don't know that it was the same thing that even traveled, it may not have traveled, it may even be. Oh here's another thing, suppose an electron, suppose a proton disappears there and a neutron appears there and the concepts charge, but we don't think that it made a positron right? So it is not that it has traveled, I didn't think about that, it is a good point, but that's not the reason you object. Yep, (student speaks). That is the correct answer. The answer is that it is not simultaneous in every frame of reference and there's no special theory that if 2 things are simultaneous in one frame of reference, if you see the same two events in a moving plane or anything, they will not be simultaneous. Therefore, in any other frame of reference, either the charge would have been created first, then after a period of time reappear, or destroyed somewhere or then appear after delay. Or the appearance could take place before the destruction so suddenly you've got two charges. So, conservation of charge which is conserved non-locally cannot have a significance except in one frame of reference. If you believe that all observers are equivalent, and you write down loss that makes sense for everybody, it can only be local. So electrical charge is conserved and it is local, locally conserved. Stuff doesn't just disappear, stuff moves around. You can keep track of it, and if you add it up you get the same number. The second part of q, which is not necessary for any of this older phenomenon, is that q is quantized. That means the electrical charge that we run into, does not take a continuum of possible values. For example, the length of any object you might think of in the case of classical mechanics, you may think of any number you like. The electrical charge is not continuous. As far as we can tell, all the charges we have ever seen are all multiples of a certain basic unit of charge which turns out to be 1.6×10^{-19} Coulombs. Every charge is either that or some multiple that could be plus or minus the multiple. The charge is granular, not continuous. Okay so, I'm going to give you a little more knowledge we have had since the time of Coulomb that sort of explains these things. What's really going on microscopically? We don't have to pretend we don't know, we do. So we might as well use that information for now on. What we do know is that everything is made up of atoms and if we look at the discarded nucleus, a lot of guys sitting here, some are called protons and some are called neutrons, and then there are some guys running around called electrons. Of course, we will see at the end of the semester that this picture is wrong. But it is good enough for this purpose. It is certainly true that there are charges in an atom that are in the center and other light charges that are in the periphery. All things carrying electric charge in our world, in daily life, are they the protons or electrons? You can produce strange particles in an accelerator. They would also carry some charge. They would be that they are a multiple of this charge, but it is very long. So this stable things that you and I are made of, that just about everything in the room is made of, is made of proton, neutron, and electrons. The charge of the neutron as you can guess is 0, the charge of the electron by some strange convention, was given this minus sign by Franklin, and the charge of the proton is a plus $1.6 \cdot 10^{-19}$ coulombs. There are a lot of amazing things I find here, I don't know if you've thought about it. The first interesting thing is that any electron anywhere in the universe has exactly the same charge. It also has exactly the same mass. Now you might say, "look that's a totally because if it wasn't the same charge, if it wasn't the same mass, you would call it something else. But what makes it a non-empty statement is that there are many many many electrons that are absolutely identical. Look, you try to manufacture two cars, the chances they are identical is 0. I

got one of those cars so I know that. It doesn't work like it's supposed to. So, despite all the best efforts people make, things are not identical, but at the microscopic level of electrons and protons, every proton in the universe is identical. And they can be manufactured in a collision in another part of the universe, this can be manufactured in a collision in Geneva. The stuff that comes out identical, that is a mystery; at least in classical mechanics that is a mystery because the quantum theory gives you an answer of at least why electrons are identical, the way all protons are identical. The fact that they are absolutely identical particles is very very important. It also makes your life easy because if every particle was different from every other particle, you cannot make any predictions. We know that the Hydrogen atom in the receding galaxy is identical to the atom on the earth. That's why when the radiation coming from the atom has a shifted wavelength of frequency, we attribute it to the motion of the galaxy from that we would use the Doppler in space. Another explanation would be well that's a different height of an atom. Maybe that's where the answer's different, but we all believe that the same height of an atom means it is moving away from us. Therefore one of the remarkable things is that all electrons and all protons are equal. But a really big mystery is why is the charge of an electron are equal and opposite the charge of a proton? They are not the same particle, their masses are different, and their other interactions are different. In terms of electrical charge these two numbers are equal as far as anybody knows, that's the other mystery. Two different particles are not related by any manifest family relationship have the same charge, except in sight. There are theories called grand unified theories that try to explain this, but they certainly are not part of any standard established theory. But it's a key to everything you see in daily life because that's what makes the atom electrically neutral. Okay, now we can understand the quantization of charge because charge is carried by these guys, but these guys are either there or not there so you can only have so many electrons. You cannot have a particle with an electrical particle proton. Now, let's try to understand all these experiments in terms of what we know. First of all when you take this piece of glass and you rub it, the atoms in glass are neutral. They've got an equal number of protons and neutrons, but when you rub it the glass atom loses some electrons to whatever you rubbed it on. Therefore, it becomes positively charged because some negative have been taken out. In the case of the rubber stick, it gains the electrons and whatever animal you rub it on, it loses the electrons. So actually real charge transfer takes place only through electrons. Protons carry charge but you are never going to lose protons unless you lose an accelerator, it's really deeply bound to the nucleus. Electrons are the ones that do all of the business of electricity in daily life, like current flowing in the wire, it's all the motion of electrons. So, from this and Coulomb's law, can you understand the attraction between these two? How many people think you can understand the attraction from Coulomb's law between these two rods? Nobody thinks you can. Why do you think you cannot? (Student speaks). Okay any other reason why Coulomb's law is not enough? Well how would we apply Coulomb's law to understand the attraction between these two rods? What would we have to do? (Students speak). No, once you've got the F the A will follow. Can you compare the force between two rods, one is a lot of positive charge one is a lot of negative charge. Yes? (student speaks). Suppose I tell you how many charges there are, now we do know because the two positive charges move towards each other. Okay I'll tell you what it is. It's an assumption we all make, but we're not really supposed to make it. It's not a consequence of any logic. Coulomb's law talks about two charges, two point charges. What if there are three charges in the universe, what does the force this one in experience do to these two? This is q_1 , this is q_2 , this is q_3 . Coulomb's law doesn't tell you that, you only see two at a time but we make an extra assumption called superposition which says that if you want the force on 3 when there is q_1 and q_2 , you will find the force, do the q_2 , you find the force do the q_3 , and then add them up. The fact that

you can add these two vectors is not a logical requirement, in fact it's not even true at an extremely accurate level that the force between two charges, it's not effected by the presence of a third one. It is an excellent approximation, but you will realize it is something you have to find to be true experimentally, not something you can say as logical consequence. Logically there is no reason why the interaction between two entities should not be effected by the third one, but it seems to be good approximation for what we do and that's the reason why eventually we can find the extender object and another extended object by looking at the force of everyone of these to everyone of those and adding all the vectors. So superposition plus coulomb's law is what you need. Then you can certainly understand the attraction. How about the comb and the piece of paper? That's an interesting example and it's connected to this one. You see, the paper is electrically neutral. Let me do paper and comb instead of this one, it's got the same moral. Here's the piece of paper, here's the comb; the comb is positively charged, the paper is neutral. So in a way there is nothing to be attracted to the one, but if we bring it close enough that our equal positive and negative charges, what will happen is the negative charges will migrate near these positive charges from the other end, leaving positive charges at the back. So that the system will separate into a little bit of the negative and the leftover positive will be further away. Therefore, even though it is neutral, attraction or plus for this minus is stronger than the repulsion of this plus with this plus. That's called polarization, polarization is when a charge separates. Some materials cannot repolarize in this case, no matter how much you do this with a comb it won't work. Some materials can be polarized. A piece of paper is an example of what can be polarized, you can understand that too. And in this example, if you bring a lot of plus charges here and if you look at what is going here, the minus guys here will sit here and the plus will be left over in the back. Then this attraction would be plus and minus is bigger than this repulsion, so it would be attracted to it. But once it touches it, the rod touches that then what you have is a lot of plus charges here and they repel each other. They want to get out three, they couldn't get out, but if you make contact some of them will jump to that one. Then when you separate them, you will have a ball with plus charges and you will have a rod with plus charges and they will repel each other. And finally, I said if you take two of these spheres, suppose one was positively charged and one was negatively charged, they're attracting each other. If you connect them with a nylon wire and stick, nothing happens. But if you collect them with a wire, what happens is the extra negative charges here will go to that side, and when you are done they will both become electrically neutral. Okay so that's why, so the point of this one is electric charges can flow through some materials but not other materials. If it can flow through some materials, it's called a conductor. If it cannot flow through them it is called an insulator. So in real life, you got a pull. So, if you're changing a light bulb, if you didn't want to get an electric shock, then you have to take a piece of wood before you stick your finger in and unless you've got other intentions then you will find that you don't get a shock because the wood doesn't conduct electricity. But, if you stand on a metallic stool on a metallic floor and put your hand in the socket, you will be part of electrical circuit. Human conductors are a big part of electricity, but what saves you is that you can't go from your feet to the floor. Now there are also semiconductors are somewhere in between, in our course, either we will talk more about insulators which don't conduct electricity at all, or perfect conductors which conduct electricity. Okay so somebody, of what I've said so far is that there is a new force of nature, to be part of that game you have to have charge. If you have no charge then you cannot play that game. Like neutrons cannot play this game. Nothing is attracted or repelled to neutrons and neutrons cannot attract, so it's got to be measured in Coulombs. So let me ask you another question. Supposed I tell you this is coulomb's law, how are you going to test that this law is correct? I'm giving you a bonus you don't even have to discover

the law, I'm giving you the law. All you have to do is to verify it and don't use any other definition but this law itself. How will you know it depends on q_1 and q_2 in this fashion? How will you know that that's an r in that fashion? That's what I'm asking you. Can anybody think of some setup or some experiments you will do? Let me ask an easier question, how will you know it goes like $\frac{1}{r^2}$? Yep? (Student speaks). Well, you are right if you vary the distance between them and shows the force like that, but how do you know the force exists? Yes what is your plan? You're right both of you are right. You can maybe hold this guy fixed and let this go and see how it accelerates and if you know the mass of this guy, then you know the force. You can vary the distance to another distance and it would be half the distance so you verify it. The other one is with a spring. You can take a spring of two metal uncharged objects, then you would dump some charge on this and some charge on that, and then the spring will expand and you can see what force the spring exerts and see if it's proportional to $\frac{1}{r^2}$. That's how Newton used the $\frac{1}{r^2}$ force law, you find the acceleration of the apple is $\frac{1}{r^2}$ which is the acceleration of the moon towards the earth and the moon was 60 times older than the apple 60^2 is 3600, that's how you found $\frac{1}{r^2}$. I was very lucky it could have been $\frac{1}{r^2} \cdot 1.0$, but it happens to be $\frac{1}{r^2}$. Anyways even if it's not $\frac{1}{r^2}$, if it's $\frac{1}{r^3}$, $\frac{1}{r^4}$ whatever it is you can find it by taking 2 charges, see what don't have to know what they are, that's what I'm trying to emphasize here. If all your trying to see is does it vary $\frac{1}{r^2}$ except r , double the r and see what happens. The best way is what you said, watch the acceleration. If it falls to $\frac{1}{4}$ of the value for doubling the distance, it's $\frac{1}{r^2}$. Alright suppose I got $\frac{1}{r^2}$, I want to know if it depends on the charges as the first component in q_1 and then first component in q_2 . How should we do that? Don't say we have electrons left because you cannot see electrons that well. Back in the old days people didn't even know electrons existed, so how do you manage to vary the charge in a normal, yep? (student speaks.) Ah okay many identical spheres, (student speaks) very good let me repeat what she said. Suppose we have many identical spheres, I won't even try to draw identical spheres because I haven't learned how to draw spheres. Let's imagine you have a whole bunch of these guys, you put some charge on this but you don't know what it is. We don't know what q is because you don't have to know what q is, so let that be one of the objects, that's my q . For the other object keep a fixed object, containing some other q . This is our charge q , don't vary the r . The question is can you change q to $\frac{q}{2}$? And the answer was if it's got some charge maybe, plus, bring it in contact with the second identical sphere. If it really is identical you have to agree that when you separate them, they must exactly have half each. That's a symmetry argument because for any reason you give me for why one of them should have more, I would tell you why the other one should have more. You cannot, so they will split it evenly. Therefore, charges that split evenly to $\frac{q}{2}$ here and $\frac{q}{2}$ here. Then you can take this and put it there and you've got $\frac{q}{2}$. Then you can do other combinations. For example, you can do this $\frac{q}{2}$ and connect it around so it becomes neutral. So this has got 0 again. You can touch that to $\frac{q}{2}$ and separate them and each will have $\frac{q}{4}$. So in this way you can vary the charge in a known way, half of it, double it. I'll give you some homework problems where you want to get $\frac{5}{16}$ of a Coulomb. Well with enough spheres you can do that. Again, what I want you to notice is that you didn't know what q was, all you know is that you went to $\frac{q}{2}$ and you brought spheres and separated them. That's so you can find the difference linearly q_1 of course, but also so you can find

linearly q^2 because it's up to you to decide who you want to call q_1 and who you want to call q_2 . Okay so I want you people to understand all the time you that you should be able to tell me how to measure anything. Okay that's very important, it's what you should think about. If you think in those terms, you'll also find you're doing all the problems very well. You're thinking of pushing symbols and cancelling factors that apply, you won't get the feeling for what's happening. So, everything you write down, you should be able to measure. If you say, "Oh I want to measure the force," you've got to be sure how to measure it. In one way it's like you said, find m_{max} . If you know the m it's the force. But everything makes sure we can measure it. If I give you a sphere charged with something, then of course we've got to decide. Suppose I give you a sphere that is positively charged and I want you to find out how many charges are on this sphere. This time I give you how many Coulombs there are, what will you do? What process will you use? Well then you have a problem because you are not able to figure out, but if I tell you here's an object it is three meters long, you can test it because you can go bring the meter stick from the bureau of standards and measure it three times. I'm asking you if I give you a certain charge and say how much charge is there? By what process can be calibrated charges? Yep? (Student speaks). That is correct. If you knew one standard charge, somehow or the other, if you knew its value, you can bring the unknown one next to it, put it at a known distance, you know the r , you know the 4π , you know the ϵ_0 , you find the force, you can find this charge. So, what we need to know is how to get a reference charge. So how do I know something has a Coulomb? How do I get 1 Coulomb of charge just to be sure? You know what you could do? You haven't defined yet the reference, so you should think about how will I get a Coulomb of charge or any other charge. So I could take these two spheres that she talked about, each with the same charged cube. If you don't know what it is, I put them at the one meter distance and I measure the force and how hard the hold one without running to the other one. Once I got the force, the only thing unknown in the equation is $q_1 q_2$. I know r , I know $4\pi\epsilon_0$, all I need is q . So, every time you write something, think about how you measure it, because in that process you're learning how the physics is done. If you try to avoid that you will be just juggling equations and that doesn't work for you and that doesn't work for me. Anybody who wants to do good physics should be constantly be paying attention to the physical phenomenon and not to the symbols that stand for physical objects. Alright so final thing I want to do in this connection is to give this number I mentioned $\frac{F_{\text{gravity}}}{F_{\text{electric}}}$. I said gravity is 10^{-40} times weaker, but you have to be precise on how you got the number. See it's not like selling toothpaste where you can say the 7.2 times whiter. I don't know how those guys measure whiteness to two decimal places. You know that's a different game, but here you have to say how you got the number, in what context do you make the comparison? The internal answer does depend on. They'll be some variations, but those tiny variations are swamped by this enormous ratio. So what you could do is take any two bodies and find the ratio of gravity to electric force. One option is to take two elementary particle, whichever two you like, I will take an electron and a proton, but you can take an electron and a positron, or a proton or proton, it doesn't matter. These two guys attract each other gravitationally and electrically. So I would write the force of gravitation as $\frac{GMp}{r^2}$. Notice in $\frac{q_1 q_2}{4\pi\epsilon_0 r^2}$. In this experiment, in this calculation r^2 does not matter. So you don't have to decide how far you want to keep them, because they both go like $\frac{r}{n}$ so you can pick anything. So whatever you pick, you're gonna cancel, and you will be left with this number $q_1 q_2$ and the $\frac{1}{4\epsilon_0}$ is $9 \cdot 10^9$. So now we put in some numbers. So G is 10^{-11} with some tree factors but the mass is 10^{-27} kg and the mass of the

electron 10^{-30} kg. You will say how come they have these nice round numbers? They are not, they are rounded, I'm not putting them because I'm just counting powers of 10. Q_1 is $1.6 \cdot 9 \cdot 10^9$, so two of those q 's is 10^{98} , which would equal 10^{-39} and $9 \cdot 10^9$ is roughly 10^{10} . So if you do all that, you will find that this is 10^{-40} . That is some typical situation and he found his ratios for forces. If there are two elementary particles which are building blocks of matter and you brought them to any distance you like, you compare the electrical attraction to the gravitational attraction. So one question is if gravity is so weak, how did anyone discover the force of gravity? If all you had were protons and electrons, you would have to measure the force between them. Suppose you only knew the force of electricity, you didn't know about gravitation. One way to find out if there is an extra force is to measure the force to accuracy good to 40 decimal places. The 40 decimal places you find if something is wrong. You plug it in and figure it out, correction comes from M_{1n} , but that's not how it was done, you guys know that. How did anyone discover force of gravity when it was fully developed? Yes. (Student speaks). Yes, most things are electrically neutral. In other words, electric force, even though it is very strong, comes with opposite charges. It can come with a plus sign or a minus sign. Therefore, if you take the planet earth that's got lots and lots of charges in every atom, but every atom is neutral. You've got the moon that's got lots and lots of atoms but they're all neutral. But the mass of the electron does not cancel the mass of the proton, so mass can never be hidden. Electric charge can be hidden, mass never cancels. That's the reason why in spite of the incredible electrical forces they are potentially capable of exerting, they are present to each other in neutral entities. Therefore, this remaining force which is not shielded is what you see and has a dramatic role in the structure of the universe, the force of gravity, the most cosmological calculation, you can forget mainly electric forces all around gravitational force because it can be neutralized. So you cannot hide gravity, everything has mass, even protons which have no mass, their energy will attract them by gravitation forces. So gravity cannot be hidden and that's the origin of something called dark matter. So how many of you guys have heard about dark matter? Okay anyone want to volunteer something? Someone whose name begins with T. Anybody whose name begins with T and also knows the answer to this? The trouble is you people are plagued with one quality not good in physics mainly are modest. So you don't want to tell me the answers so I have to excuse for the person who gives the answer. If your seat is the number 142, anybody in seat 142? Maybe they're not even numbered. Look, ok anybody with a red piece of clothing that knows the answer to this? Go ahead. (Student speaks), pardon me, right. Basically there's no way you can see it, and there's dark matter right in this room. Okay and there's dark matter everywhere, but the reason, the way people found out it was dark matter. Do you know how that was determined? Yep? (Student speaks). Yes. So maybe one example I can talk about on our own galaxy. So here's our visible galaxy, with its own spiral. Now if something is orbiting this galaxy, just by using Newtonian gravity and by knowing the velocity of the object as it goes around, you can calculate how much mass is enclosed by the orbit as property of gravitation. From the orbit you can find out how much mass is enclosed. So what you will find is if you find something orbiting the center of the galaxy at that radius, with some mass. If you take the objects with bigger radius, you will enclose more and more mass until you find orbits as big as the galaxy, then the mass enclosed as a function of the radius, should come in stock because after that the orbit's getting bigger but not enclosing anymore mass. But what people found is that even after you cross the normal size of the galaxy, you still keep picking up mass. That is the dark matter halo of our galaxy, it is dark to everything but you cannot escape gravity, you cannot avoid gravitational force. People are trying to find dark matter, people at Yale are trying to find dark matter, but the thing is you don't know exactly what it is. It's not any of the usual suspects because then they would be attracted very

strongly. So you're trying to find something not knowing exactly what it is. You've got detectors that will detect something and you go through it everyday in your lab, and you're hoping that one of these dark matter pickles will collide with the stars from your detector and trigger a reaction. Of course there will be lots of reactions everyday, but most of them are due to other things, that's called background. Control the background out and whatever is left has got to be due to dark matter and again, how do you know it's dark matter? How do you know it's not something else? Well you can see that if you're drifting through dark matter in a moving earth, you would be rushing and running into more of them in the direction and less from the other direction because you're running into the wind. So by looking at the direction dependence, you can try to see if it's dark matter. Anyway, dark matter was discovered by simple Newtonian gravitation. The particles that form dark matter are very interesting to particle physicists. There are many candidates to the particle theory. The origin of the discrepancy came from this being Newtonian gravity. The final thing today before we break is that there's one variation of Coulomb's law. By the way, I do not know your mathematical training of how much math you know, so you have to be on the lookout. If I write something that looks very alien to you, you've got to go take care of that. In particular, how to do integrals in more than one dimension. What I wanted to discuss today is the following, we know how to do Coulomb's law due to any number of point charges. So if you put enough charge Q here, you want the force on this guy, you draw these lines, you take the r^2 to that, you get the r^2 of that, add all the vectors, it's very simple. We will also take problems where the charges are continuous. So here's an example, here's a ring of charge. The ring has some radius, you pick your radius r , and the charge on it is continuous not discrete. In real life everything is discrete, but to a force observer, it will look like it's continuous. So, you can draw some pictures here of charges all over and λ is the number of Coulombs over a meter. So if you snip one meter of the wire, you'll have λ Coulombs and you want to find the electric force of some of the charge q due to this wire. So you cannot do a sum, you have to do an integral. That's what I'm driving at. We're going to do one interval, then we'll do more complicated ones later. So I want to find, so what I will do is I will divide this into segments each of length that say dl . Then, I will find the force of the charge here, df . I will add the forces due to all the segments. The force of this segment will be the charge, this segment is so small, you can treat it as a point charge. The amount of charge there to here is $\lambda \cdot dl$, that's the q_1 . The q_2 is the q I put there and then there's a $4\pi\epsilon_0$ where r^2 would be the distance and $z \cdot R$, $R^2 + z^2$, that's the distance. But now that force is a vector pointing in that direction, but I know that the total force is going to point at this direction because for every guy I find at this side, I can find one in the opposite direction pointing that way. They will always cancel horizontally, the only remaining force would be in the Z direction. So I'm going to keep only the compliment of the force in the z direction. I did the z in the z direction. For that, you have to take this force and multiply by the cosin of the data. I hope you know how to find the compliment of the force in that direction if the cosin of the angle in between them. That angle is in this and the cosin of this is $\frac{z}{\sqrt{r^2+z^2}}$ and the root. That is the df due to the segment and total force in the z direction is integral of this. And what does that integrate? λ , q , all these are constant, r , everything is a constant, you have to add all the dl 's. If you add all the dl 's you will get the circumference. In other words, this is going to be $\frac{\lambda q z}{4\pi\epsilon_0 (R^2 + z^2)^{\frac{3}{2}}} \int dl$, the integral of dl is just $2\pi r$. In other words, every one of them is making equal contribution so the integral doesn't depend on where you are in the circle, it's measuring the length of the circle. That's the answer. The force looks like $\lambda \cdot 2\pi R$ what is that? λ is the charge per unit length. That times the length of the loop is the charge

of the loop is the charge you are putting there divided by $4\pi\epsilon_0$ divided by $(R^2 + z^2)^{\frac{3}{2}}$. That's an example of calculating the force which would be in this direction. Now, once you've done this calculation, you may think, "maybe I missed a factor of π or a factor of e or something." Can you think of a way to test this? What test would you like to apply to this result? Yep? (Student Speaks). Very good. What he said is if you put $z=0$, you're sitting in the middle of the circle and you're getting pushed equally from all sides and you better not have a force and that's certainly correct. Anything else? Any other test? Yep? (Student speaks). Yes. It will point down and be negative that is correct, but how about the magnitude of the force itself rather than just the direction? Yep? (Student speaks). Yes. If you go very very far, and someone is holding a loop, you cannot even see that it is a loop, it's some tiny speck and it should produce the feel. So what feel should it produce? It should produce the Coulomb force that q^2 or $4\pi\epsilon_0 \cdot d^2$ and when z is much much much bigger than r , this is 1 kilometer, this is inches, you forget that this. You get $z^2 - \frac{3}{2}$ and z^3 , that means the whole thing here reduces to $\frac{1}{z^2}$ and it will look like the force between 2 charges. So I would ask you, after you do the calculations, to test your result. Okay, before going I've got to tell you something about those who come late. I realize that you guys come from near and far, so the doors I want you to use. That's door 1, that's the least problematic. Door number 2 is this one, because at the beginning of the lecture I'm usually at this side of the board so you guys can come in. Door number 3 is by the picture, but do not stand in front of the camera and contemplate your future. If you do, I will make sure you don't have a future so don't do that. If you come fashionably late, never come through that door, maybe this one. In fact if you come through that door, and I'm at the board, you're very very late. So I think you should take the day off and start fresh next time. Okay thank you.