Math

Digital SAT

Advanced

SAT Math Advanced

1. The equation relates the quantities a, x, and z. Which equation correctly expresses x in terms of a and z? $a + 25 = \frac{x}{z}$

A. $x = \frac{a+25}{z}$

B. $x = \frac{z}{a+25}$

C. $x = a + 25z$

D. $x = az + 25z$

2. A renewable energy company is analyzing its solar panel installations, which are designed to cover a square area. If the side length of the square area is represented by 's' meters and the total energy output is modeled by the function $E(s) = 5s^2 + 3s + 2$, where E(s) is the energy output in kilowatts, what is the energy output of the solar panels if the side length of the installation area is 4 meters?

A. 88

B. 92

C. 94

D. 98

3. A solution to the given system of equations is (x, y). What is a possible value of x? $y = \frac{1}{2}(x - 4)^2 + 7\,, y = 2x + 5$

A. 4

B. 6

C. 8

D. 10

4. How many times does the graph of the given equation in the xy-plane cross the x-axis, where a, b, and c are positive constants such that $a > 4$? $y = 5\left(\frac{a}{4}\right)^{x+c} - b$

A. 0

B. 1

C. 2

D. 3

5. One solution to the given equation can be written as $x = \frac{-7+\sqrt{k}}{2}$, where k is a constant. What is the value of k? $x^2 + 7x + 10 = 0$

A. 9

B. 16

C. 25

D. 36

6. The function g is defined by $g(x) = 3x^2 - 5x + 8$. What is the value of $g(1)$?

A. 2

B. 4

C. 6

D. 8

7. A city implemented a new public policy aiming to reduce air pollution. The estimated reduction in air pollution levels, measured in tons, in the first five years after the policy is modeled by the function $f(x) = 500(0.90)^x$, where x is the number of years since the policy was enacted. What does the value 500 represent in this context?

A. The amount of air pollution measured in tons after 5 years

B. The estimated air pollution level in tons during the baseline year before the policy was enacted

C. The percentage decrease in air pollution level each year

D. The total reduction in air pollution expected after 5 years

8. The function F models the future value of an investment in thousands, t years after 2020. According to the model, the investment is expected to grow by a rate of k% every year. What is the value of k if $F(t) = 50(1.05)^t$?

A. 4

B. 5

C. 6

D. 7

9. Which expression is equivalent to $3x^3 + 12x^2y + 6xy^2 + 24y^3$?

A. $(3x + 6y)(x^2 + 4y)$

B. $(3x^2 + 6y^2)(x + 4y)$

C. $(3x^3 + 2y^2)(x^2 + 4y)$

D. $(3x^2 + 2y^2)(x + 4y)$

10. In the given system of equations, d is a constant. The system has two distinct real solutions. Which of the following could be the value of d?

$y = 2x + d, y = - 3(x - 4)^2$

A. -8

B. -6

C. -4

D. 0

SAT Math Advanced Solutions

1. The equation relates the quantities a, x, and z. Which equation correctly expresses x in terms of a and z? $a + 25 = \frac{x}{z}$

A. $x = \frac{a+25}{z}$

B. $x = \frac{z}{a+25}$

C. $x = a + 25z$

D. $x = az + 25z$

Answer

D

Solution

This problem tests the student's ability to manipulate and isolate a variable in an algebraic equation, specifically involving operations with polynomials and fractions. To solve for x, you need to express x in terms of a and z. Start by isolating the fraction on one side of the equation by subtracting 25 from both sides, then multiply both sides by z to solve for x.
Remember that solving for a variable often involves reversing operations. In this case, you need to handle both addition and division to isolate x. Keep your operations clear and systematic.
A common mistake is forgetting to multiply the entire expression by z. Ensure that you apply operations to both sides of the equation correctly. Also, be cautious with the signs when subtracting and multiplying.
This problem is a classic example of isolating a variable within an algebraic equation. It assesses algebraic manipulation skills, which are crucial for advanced mathematics. Mastering these skills is essential for solving more complex equations efficiently on the SAT.

Given the equation: $a + 25 = \frac{x}{z}$., Step 1: Multiply both sides by z to eliminate the fraction: $(a + 25)z = x$., Step 2: Express x in terms of a and z: $x = az + 25z$.

2. A renewable energy company is analyzing its solar panel installations, which are designed to cover a square area. If the side length of the square area is represented by ’s’ meters and the total energy output is modeled by the function $E(s) = 5s^2 + 3s + 2$, where E(s) is the energy output in kilowatts, what is the energy output of the solar panels if the side length of the installation area is 4 meters?

A. 88

B. 92

C. 94

D. 98

Answer

C

Solution

This problem tests the student’s understanding of polynomial functions, specifically higher-degree polynomials, and their ability to evaluate these functions given a specific value for the variable. To solve this problem, the student needs to substitute the given side length ’s’ into the polynomial function $E(s)$ and perform the arithmetic operations to find the energy output. Firstly, plug the given side length ($s = 4$) into the function $E(s)$. Make sure to follow the order of operations ($\frac{PEMDAS}{BODMAS}$) carefully: calculate the square term first, then the linear term, and finally add the constant term. Simplify step by step to avoid mistakes. Be careful with the operations, especially squaring the side length and adding the terms in the correct order. Common mistakes include forgetting to square the side length or misplacing the decimal points. Double-check your calculations to ensure accuracy. This type of problem is common in the SAT Advanced Math section and aims to evaluate the student’s ability to work with polynomial functions and apply them to real-world contexts. Developing proficiency in these types of problems requires practice in substituting values into polynomial expressions and performing arithmetic operations accurately. Being meticulous and systematic in your approach will help minimize errors and improve efficiency.

Substitute $s = 4$ into the function $E(s)$., Calculate $E(4) = 5(4)^2 + 3(4) + 2$., First, calculate $4^2 = 16$., Next, calculate $5 \times 16 = 80$., Then, calculate $3 \times 4 = 12$., Now, sum these results: $80 + 12 + 2 = 94$., So, the energy output when the side length is 4 meters is 94 kilowatts.

3. A solution to the given system of equations is (x, y). What is a possible value of x?
$y = \frac{1}{2}(x - 4)^2 + 7, y = 2x + 5$

A. 4

B. 6

C. 8

D. 10

Answer

D

Solution

This problem is designed to test the student's ability to solve a system of equations involving a quadratic and a linear equation. It checks the understanding of graph intersections and algebraic solutions.
To solve this problem, students should set the equations equal to each other since both are equal to y, and then solve for x. This involves expanding the quadratic equation, setting up a quadratic equation in standard form, and then using methods such as factoring, completing the square, or the quadratic formula to find the possible values of x.
When dealing with quadratic and linear systems, remember that solutions correspond to the intersection points of a parabola and a line. It might be helpful to sketch the graphs to visualize potential solutions before solving algebraically. Also, check if the quadratic equation can be easily factored after expansion to simplify calculations.
Be careful when expanding the quadratic expression and ensure that all terms are correctly simplified. Additionally, check all potential solutions in the original equations to verify they are valid, as extraneous solutions can sometimes arise.
This type of problem is common in SAT's advanced math section, as it assesses not only algebraic manipulation skills but also conceptual understanding of graph intersections and their physical interpretations. Mastery of these concepts can be beneficial, as it combines different branches of mathematics into a single problem, which is a frequent characteristic of SAT questions.

Set $\frac{1}{2}(x - 4(\ )^2 + 7$ equal to $2x + 5$:, $\frac{1}{2}(x - 4)^2 + 7 = 2x + 5$, Subtract 7 from both sides:, $\frac{1}{2}(x - 4)^2 = 2x - 2$, Multiply every term by 2 to eliminate the fraction:, $(x - 4)^2 = 4x - 4$, Expand $(x - 4)^2$:, $x^2 - 8x + 16 = 4x - 4$, Rearrange all terms to one side:, $x^2 - 8x + 16 - 4x + 4 = 0$, Combine like terms:, $x^2 - 12x + 20 = 0$, Use the quadratic formula to solve for x, where

$a = 1, b =- 12, andc = 20:, x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, x = \frac{-(-12) \pm \sqrt{(-12(\ )^2 - 4 \cdot 1 \cdot 20}}{2 \cdot 1}$,
$x = \frac{12 \pm \sqrt{144-80}}{2}, x = \frac{12 \pm \sqrt{64}}{2}, x = \frac{12 \pm 8}{2}$, Calculate the possible values:,
$x = \frac{12+8}{2} = 10, x = \frac{12-8}{2} = 2$, Possible values of x are 10 and 2.

4. How many times does the graph of the given equation in the xy-plane cross the x-axis, where a, b, and c are positive constants such that $a > 4$? $y = 5\left(\frac{a}{4}\right)^{x+c} - b$

A. 0

B. 1

C. 2

D. 3

Answer

B

Solution

This problem tests the student's understanding of exponential functions and their graphical behavior, particularly how to determine the number of x-intercepts of an exponential graph. To solve this problem, the student needs to identify the x-intercepts of the given equation. This involves setting $y = 0$ and solving for x. Specifically, they must solve the equation $0 = 5\left(\frac{a}{4}\right)^{x+c} - b$ for x. Remember that an exponential function of the form $y = ka^x + c$ has a horizontal asymptote. For the given equation, as x approaches infinity or negative infinity, the term $\left(\frac{a}{4}\right)^{(x+c)}$ will either grow or decay exponentially depending on the value of $\frac{a}{4}$. This can help you determine whether the function crosses the x-axis. Be careful with the base of the exponential function. Since $a > 4$, $\frac{a}{4}$ is greater than 1, meaning the function grows exponentially. Also, check the signs and values of b and how it affects the crossing points. Ensure not to confuse the behavior of the function based on whether the base is greater than or less than 1. This type of problem assesses the student's ability to analyze the behavior of exponential functions and their graphs. It requires understanding how to manipulate exponential equations and determine their intercepts. The primary skill tested is the ability to discern the number of times a given exponential graph will intersect the x-axis, considering the transformations applied to the function. Understanding these concepts is crucial for success in advanced math sections of the SAT.

Set $y = 0$ in the equation: $0 = 5\left(\frac{a}{4}\right)^{x+c} - b$., This simplifies to: $5\left(\frac{a}{4}\right)^{x+c} = b$., Divide both sides by 5: $\left(\frac{a}{4}\right)^{x+c} = \frac{b}{5}$., Since $a > 4$, $\frac{a}{4} > 1$, indicating an increasing exponential function., The equation $\left(\frac{a}{4}\right)^{(x+c)} = \frac{b}{5}$ has a solution for x if and only if $\frac{b}{5} > 0$. Thus, the equation has one solution for x, meaning the graph crosses the x-axis once.

5. One solution to the given equation can be written as $x = \frac{-7+\sqrt{k}}{2}$, where k is a constant. What is the value of k? $x^2 + 7x + 10 = 0$

A. 9

B. 16

C. 25

D. 36

Answer

A

Solution

The problem aims to test the student's understanding of solving quadratic equations using the quadratic formula. It specifically evaluates the student's ability to identify and manipulate the components of the formula, and to recognize the relationship between the given solution and the quadratic equation. To solve the problem, follow these steps: 1. Recognize that the given equation is in the standard quadratic form, $ax^2 + bx + c = 0$. 2. Use the quadratic formula $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve for x. 3. Compare the given solution form with the quadratic formula to identify the discriminant $\sqrt{k}$ and solve for the value of k. Remember that the quadratic formula is derived from completing the square of a quadratic equation. The discriminant $b^2 - 4ac$ under the square root sign determines the nature of the roots. In this case, equate the discriminant to k and solve for it. Be careful with signs when working with the quadratic formula. It's easy to make mistakes with negative signs, especially when dealing with the term $-b$ and the discriminant. Also, ensure you correctly identify the values of a, b, and c from the quadratic equation. This type of problem is common on the SAT and tests a student's ability to apply the quadratic formula accurately. The key skills evaluated include recognizing the standard form of a quadratic equation, correctly applying the quadratic formula, and manipulating algebraic expressions. Mastery of these skills is essential for success in advanced mathematics topics on the SAT.

The standard form of the quadratic equation is $ax^2 + bx + c = 0$ where a=1, b=7, c=10., Using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$., Substitute values: $x = \frac{-7 \pm \sqrt{7^2 - 4 \times 1 \times 10}}{2 \times 1}$., Calculate the discriminant: $b^2 - 4ac = 7^2 - 4 \times 1 \times 10 = 49 - 40 = 9$., Complete the formula: x = $\frac{-7 \pm \sqrt{9}}{2}$. Thus, k = 9.

6. The function g is defined by $g(x) = 3x^2 - 5x + 8$. What is the value of $g(1)$?

A. 2

B. 4

C. 6

D. 8

Answer

C

Solution

This problem tests the student's ability to evaluate a quadratic function by substituting a specific value for the variable x. It assesses the understanding of basic algebraic manipulation and function evaluation. To solve this problem, the student should substitute the given value of x (which is 1) into the quadratic function $g(x)$ and then simplify the resulting expression. Carefully substitute $x = 1$ into the function. Make sure to follow the order of operations: first square the value, then multiply by coefficients, and finally add or subtract the constant terms. Be attentive to arithmetic errors, especially during multiplication and addition/subtraction steps. Also, ensure that you do not skip any steps in the order of operations. This problem is a straightforward evaluation of a quadratic function, a fundamental skill in algebra. It assesses the student's ability to correctly substitute values and perform basic arithmetic operations. Mastery of this type of problem is crucial for more advanced topics in mathematics, and accuracy is essential. Practicing similar problems can help improve speed and reduce errors in function evaluation tasks commonly found in the SAT.

Substitute $x = 1$ into the function $g(x)$., Calculate $g(1) = 3(1)^2 - 5(1) + 8$., Simplify: $g(1) = 3(1) - 5 + 8$., Further simplify: $g(1) = 3 - 5 + 8$., Finally calculate: $g(1) = 6$.

7. A city implemented a new public policy aiming to reduce air pollution. The estimated reduction in air pollution levels, measured in tons, in the first five years after the policy is modeled by the function $f(x) = 500(0.90)^x$, where x is the number of years since the policy was enacted. What does the value 500 represent in this context?

A. The amount of air pollution measured in tons after 5 years

B. The estimated air pollution level in tons during the baseline year before the policy was enacted

C. The percentage decrease in air pollution level each year

D. The total reduction in air pollution expected after 5 years

Answer

B

Solution

This problem aims to test students' understanding of exponential functions and their ability to interpret parameters in real-world contexts. Specifically, it evaluates whether students can identify what the initial value in an exponential decay function represents. To solve this problem, you should focus on understanding the components of the exponential function given. Recognize that in the context of the function, the term '500' represents the initial value or starting amount at year zero (when the policy was first enacted). Remember that in an exponential function of the form $y = a(b)^x$, the 'a' term represents the initial value before any changes occur as a result of the exponential process. In this problem, identify what the situation was at the start (year 0). Be careful not to confuse the initial value with the rate of change or the decay factor. The initial value is the amount present at the beginning ($x = 0$), while the decay factor in this problem is 0.90. This problem is a good example of how SAT math questions often require both an understanding of mathematical concepts and the ability to apply these concepts to real-world scenarios. Recognizing the meaning of parameters in an exponential function is crucial. To excel in such problems, practice identifying and interpreting each component of the function accurately.

The initial value of the function is 500. In exponential decay functions, the initial value represents the starting amount before any decay has occurred., In this context, 500 represents the air pollution level measured in tons during the baseline year before the policy was enacted., As the function models a reduction, and 500 is the amount without decay applied, it is the amount before any reduction.

8. The function F models the future value of an investment in thousands, t years after 2020. According to the model, the investment is expected to grow by a rate of k% every year. What is the value of k if $F(t) = 50(1.05)^t$?

A. 4

B. 5

C. 6

D. 7

Answer

B

Solution

This problem aims to test the student's understanding of exponential growth and their ability to interpret and manipulate exponential functions. Specifically, it assesses their ability to identify the growth rate from an exponential equation. To solve this problem, identify the form of the exponential function $F(t) = P(1 + r)^t$, where P is the initial value, r is the growth rate, and t is time. In this case, compare $F(t) = 50(1.05)^t$ with the general form to find the growth rate r. Here, 1.05 represents $1 + \frac{k}{100}$, so set up the equation $1 + \frac{k}{100} = 1.05$ and solve for k.

Remember that in an exponential function of the form $P(1 + r)^t$, the term (1 + r) directly gives you the growth factor. Subtract 1 from this factor and multiply by 100 to get the percentage growth rate. Be careful not to confuse the exponent t with the base of the exponential expression. Ensure you correctly identify the growth factor and convert it to a percentage. Also, double-check your algebra when solving for k. This problem is a typical example of exponential growth questions frequently seen on the SAT. It evaluates your ability to interpret exponential models, which is crucial for understanding real-world applications in finance and natural sciences. Mastering this type of problem will improve your overall performance in the Advanced Math section.

The given function is $F(t) = 50(1.05)^{t}$., In the general exponential growth form $F(t) = P(1 + r)^{t}$, P is the initial investment and (1 + r) is the growth multiplier., Comparing this with the given model, we see that $1 + r = 1.05$., Therefore, $r = 1.05 - 1 = 0.05$., To convert the growth rate r to a percentage, multiply by 100., $k\% = 0.05 \times 100 = 5\%$.

9. Which expression is equivalent to $3x^{3} + 12x^{2}y + 6xy^{2} + 24y^{3}$?

A. $(3x + 6y)(x^{2} + 4y)$

B. $(3x^{2} + 6y^{2})(x + 4y)$

C. $(3x^{3} + 2y^{2})(x^{2} + 4y)$

D. $(3x^{2} + 2y^{2})(x + 4y)$

Answer

B

Solution

This problem tests the student's ability to factor polynomial expressions, specifically recognizing and applying the greatest common factor and factoring by grouping.
The student should first identify the greatest common factor (GCF) of all the terms in the polynomial. In this case, the GCF is 3. Then, factor out the GCF from each term. Next, the student should look for patterns or group terms to factor further if possible. Grouping terms and factoring each group can lead to a fully factored expression.
Look for the greatest common factor first, as this simplifies the expression and makes further factoring easier. After factoring out the GCF, group terms in pairs and factor each pair separately. Always double-check by expanding to ensure the factored expression matches the original polynomial.
Be careful with signs when factoring. Make sure not to overlook common factors, especially if they involve variables. Additionally, ensure all terms are accounted for and correctly grouped when factoring by grouping.
Factoring polynomial expressions is a crucial skill in algebra, often used as a step in solving equations or simplifying expressions. This problem type assesses the student's ability to recognize patterns and apply factoring techniques. Mastery of these skills is essential for success in more advanced math topics, particularly in calculus and higher-level algebra.

Step 1: Identify the GCF of all terms: $3x^3$, $12x^2y$, $6xy^2$, $24y^3$., The GCF is 3., Factor out the GCF: $3(x^3 + 4x^2y + 2xy^2 + 8y^3)$., Step 2: Factor the expression $x^3 + 4x^2y + 2xy^2 + 8y^3$., Recognize it can be grouped: $(x^3 + 4x^2y) + (2xy^2 + 8y^3)$., Factor each group separately: $x^2(x + 4y) + 2y^2(x + 4y)$., Factor out the common term (x + 4y): $(x + 4y)(x^2 + 2y^2)$., Combine with the GCF: $3(x + 4y)(x^2 + 2y^2)$., Step 3: Match this expression with the given options., Option 2 is $(3x^2 + 6y^2)(x + 4y)$, which upon expansion gives $3x^3 + 12x^2y + 6xy^2 + 24y^3$, matches the original expression.

10. In the given system of equations, d is a constant. The system has two distinct real solutions. Which of the following could be the value of d?

$y = 2x + d, y =- 3(x - 4)^2$

A. -8

B. -6

C. -4

D. 0

Answer

A

Solution

The problem aims to test the student’s understanding of solving systems involving a linear and a quadratic equation. Specifically, it evaluates the student’s ability to determine the conditions under which the system has two distinct real solutions. First, equate the two given equations to find the intersection points, which would involve solving the equation $2x + d =- 3(x - 4)^2$. Rearrange and solve this quadratic equation in terms of x. For the system to have two distinct real solutions, the quadratic equation should have two distinct roots, which means its discriminant must be greater than zero. Calculate the discriminant and find the range of values for d that satisfy this condition. Remember that the discriminant of a quadratic equation $ax^2 + bx + c = 0$ is given by $b^2 - 4ac$. For two distinct real solutions, this discriminant should be positive. Additionally, carefully expand and simplify the quadratic equation to ensure all terms are correctly combined. Be cautious when expanding the quadratic term $-3(x-4)^2$. Mistakes in expansion and simplification can lead to incorrect discriminant calculations. Also, ensure that the quadratic equation is correctly converted to the standard form before applying the discriminant condition. This problem assesses the student's ability to handle systems involving both linear and quadratic equations, a crucial skill in advanced mathematics. It specifically tests their understanding of the discriminant's role in determining the nature of the roots of a quadratic equation. Mastery of these concepts is essential for success in the SAT Math section, particularly in advanced algebra topics.

Set the equations equal: $2x + d =- 3(x-4)^2$, Rearrange and simplify: $2x + d =- 3(x^2 - 8x + 16)$, $2x + d =- 3x^2 + 24x - 48$, Rearrange terms: $3x^2 - 22x + (48 + d) = 0$, Calculate the discriminant: $\Delta = b^2 - 4ac$, where $a = 3, b =- 22, c = 48 + d$, Discriminant: $\Delta = (-22)^2 - 4(3)(48 + d)$, $\Delta = 484 - 12(48 + d)$, $\Delta = 484 - 576 - 12d$, $\Delta =- 92 - 12d$, For the system to have two distinct real solutions, $\Delta > 0$, $-92 - 12d > 0$, $-12d > 92$, $d <- \frac{46}{6}$, $d <- 7.6667$

