Prep Book Bundle

Digital SAT

Math

Algebra
Advanced
Geometry and Trigonometry
Problem Solving and Data Analysis

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MATH for the SAT

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Digital SAT

Math

Algebra

SAT Math Algebra

1. For the linear function f, the graph of $y = f(x)$ in the xy-plane has a slope of 5 and passes through the point $(0, -45)$. Which equation defines f?

A. $f(x) = 5x - 45$

B. $f(x) = 5x + 45$

C. $f(x) = 45x - 5$

D. $f(x) = -5x - 45$

2. A charity organization is tracking the number of meals it can provide for the homeless each week based on the amount of funds raised. The equation that models this relationship is given by the function $f(w) = -4.04w + 98.7$, where $f(w)$ represents the number of meals provided and w is the amount of money raised in hundreds of dollars. Based on this model, approximately how many meals can be provided for each additional hundred dollars raised?

A. 1 meal

B. 4.04 meals

C. 98.7 meals

D. 100 meals

3. In the xy-plane, which of the following regions does NOT contain any points that are part of the solution set to $2x + 5y < 10, x < 0$?

A. The region where $x < 0$ and $y > 2$

B. The region where $x < 0$ and $y < 0$

C. The region where $x > 0$ and $y < 2$

D. The y-axis

4. A certain school has a total of 46 students enrolled in a club. The number of students in the club is at least 32 more than the number of students who have signed up for a volunteer activity. If we let x represent the number of students signed up for the volunteer activity, which inequality represents this situation?

A. $x + 32 \leq 46$

B. $x + 46 \geq 32$

C. $x + 32 \geq 46$

D. $x + 46 \leq 32$

5. In the xy-plane, line m has a slope of 4 and a y-intercept of (0, -54). What is the x-coordinate of the x-intercept of line m?

6. A company produces eco-friendly products. The total revenue of the company in the year 2023 is represented by the equation $831 = 71 + 76(x - 8)$, where x represents the number of years since 2015. If the revenue is expected to reach 831 dollars in 2023, how many years since 2015 has the company been operating?

A. 10 years

B. 15 years

C. 18 years

D. 20 years

7. If $86(x + 9) = 36(x + 9) + 150$, what is the value of $x + 9$?

A. -9

B. -6

C. 3

D. 6

8. What is the y-coordinate of the y-intercept of the graph of $y = g(x)$ in the xy-plane, given that $g(x) = f(x) - 5$ and $f(x) = 6(31x - 13)$?

A. -83

B. -78

C. -73

D. -68

9. For each real number r, which of the following points lies on the graph of each equation in the xy-plane for the given system? $-35x - 29y = 45$,
$-280x - 232y = 360$

A. $(r, -\frac{35}{29}r - \frac{45}{29})$

B. $(r, \frac{35}{29}r + \frac{45}{29})$

C. $(r, \frac{45}{35}r + \frac{29}{35})$

D. $(r, -\frac{29}{35}r - \frac{45}{35})$

10. A research study shows that the average number of hours students spend on community service per year, x, can be estimated by the function $f(x) = 5x + 89$. Which statement best interprets the value 89 in this context?

A. Students will spend a total of 89 hours on community service after 5 years.

B. The average number of community service hours is expected to grow by 5 hours each year.

C. The average hours of community service increased from 89 hours after each year.

D. The estimated number of community service hours was 89 hours when no additional hours are counted.

SAT Math Algebra Solutions

1. For the linear function f, the graph of $y = f(x)$ in the xy-plane has a slope of 5 and passes through the point $(0, -45)$. Which equation defines f?

A. $f(x) = 5x - 45$

B. $f(x) = 5x + 45$

C. $f(x) = 45x - 5$

D. $f(x) =- 5x - 45$

Answer

A

Solution

Concept Check : The question intends for the student to understand the concept of linear functions, specifically how to use the slope-intercept form of a linear equation ($y = mx + b$) to find the equation of a line. The student is expected to know how to identify the slope and y-intercept of a linear function.

Solution Strategy : To solve this problem, the student should recognize that the slope-intercept form of a line is given by $y = mx + b$, where m represents the slope and b represents the y-intercept. Given the slope of 5 and the point (0, -45), the student should substitute these values into the equation to find the linear function f.

Quick Wins : Remember that the slope-intercept form is very helpful in these types of problems. The slope (m) is the coefficient of x, and the y-intercept (b) is the constant term. If a point is given where x equals 0, it directly gives you the y-intercept. Be sure to correctly substitute the values and simplify the equation properly.

Mistake Alert : Be careful not to confuse the point (0, -45) with the slope. It is easy to mistakenly use the y-coordinate as the slope or vice versa. Additionally, double-check your arithmetic when substituting values into the equation to avoid simple calculation errors.

SAT Know-How : This problem falls under the category of Algebra, specifically focused on linear equations and functions. It assesses the student's ability to apply the slope-intercept form of a linear equation to find a function. Mastering this skill is crucial for solving various types of algebraic problems on the SAT, showcasing the importance of understanding the relationship between slope, intercepts, and linear functions.

1. Understand the General Form of a Linear Function: The equation of a line in slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept.
2. Identify the Given Slope: The problem states that the slope m is 5.
3. Determine the Y-Intercept: Since the line passes through the point (0, -45), the y-intercept b is -45.
4. Express the function $f(x)$: Substitute the values of m and b into the slope-intercept form: $f(x) = 5x - 45$.

2. A charity organization is tracking the number of meals it can provide for the homeless each week based on the amount of funds raised. The equation that models this relationship is given by the function $f(w) = -4.04w + 98.7$, where $f(w)$ represents the number of meals provided and w is the amount of money raised in hundreds of dollars. Based on this model, approximately how many meals can be provided for each additional hundred dollars raised?

A. 1 meal

B. 4.04 meals

C. 98.7 meals

D. 100 meals

Answer

B

Solution

Concept Check : The intent of the question is to assess the student's understanding of linear equations and how they relate to real-world scenarios. Specifically, it requires knowledge of interpreting the slope of a linear function to determine the change in the output (number of meals) based on a change in the input (amount of money raised). The student is expected to recognize that the slope of the function indicates how many additional meals are provided for each unit increase in the independent variable.

Solution Strategy : To solve this problem, the student should focus on the given linear equation f(w) = -4.04w + 98.7. They need to identify the slope of the function, which represents the rate of change of meals provided with respect to the amount of money raised. In this case, the slope is the coefficient of w, which is -4.04. The student should interpret this slope in the context of the problem to find out how many meals can be provided for each additional hundred dollars raised.

Quick Wins : Remember that in a linear equation of the form y = mx + b, 'm' represents the slope and indicates how much y changes for a one-unit increase in x. In this context, since w is in hundreds of dollars, the value of the slope directly tells you how many meals change per one hundred dollars raised. Pay close attention to the sign of the slope; a negative slope indicates a decrease in the number of meals with an increase in funds raised, which might seem counterintuitive at first.

Mistake Alert : Be careful not to confuse the slope with the y-intercept. The y-intercept (98.7) tells you the starting point (the number of meals provided when no funds are raised), while the slope (-4.04) tells you how meals change with each additional hundred dollars raised. Also, ensure you correctly interpret the context of the problem; in this case, a negative slope means that raising more funds leads to fewer meals provided, which is an important aspect to recognize.

SAT Know-How : This problem falls under the category of algebra, specifically linear equation word problems. It assesses the student's ability to interpret the slope of a linear function in a real-world context. By understanding how to analyze the slope, students can effectively determine the impact of changes in one variable on another. Approaching SAT math problems with a clear understanding of linear relationships and careful interpretation of their components is key to success.

Identify the meaning of the slope in the function $f(w) = -4.04w + 98.7$.
The slope (m) is -4.04, which indicates the number of additional meals provided decreases by 4.04 for each additional hundred dollars raised.
Since the question asks for the number of meals provided, we consider the magnitude of the slope, which is 4.04.

3. In the xy-plane, which of the following regions does NOT contain any points that are part of the solution set to $2x + 5y < 10, x < 0$?

A. The region where $x < 0$ and $y > 2$

B. The region where $x < 0$ and $y < 0$

C. The region where $x > 0$ and $y < 2$

D. The y-axis

Answer

A

Solution

Concept Check : The intent of the question is to assess the student's understanding of linear inequalities, specifically how to graph them and identify solution sets in the xy-plane. The student is expected to know how to manipulate and interpret inequalities and recognize the graphical representation of these inequalities.

Solution Strategy : To approach this problem, the student should first convert the inequality into an equation ($2x + 5y = 10$) to find the boundary line. Then, they should determine the region that satisfies the inequality ($2x + 5y < 10$) by testing a point not on the line (often the origin is a convenient choice). After identifying the solution region, the student can analyze the given options to see which region does not include any points from this solution set.

Quick Wins : To efficiently solve the problem, remember to always graph the boundary line of the inequality. Use a dashed line for '<' or '>' inequalities to indicate that points on the line are not included in the solution set. After identifying the solution region, test points in the regions to confirm if they satisfy the inequality. Keep in mind that the inequality can be rewritten in slope-intercept form ($y = mx + b$) to easily identify the y-intercept and slope for graphing.

Mistake Alert : Be cautious when determining the type of line to draw (dashed vs. solid) based on the inequality symbol. Also, when testing points, ensure that you select points that are definitely in the regions you are evaluating. It's easy to mistakenly assume a region is part of the solution set without checking a point from that region against the inequality.

SAT Know-How : This question falls under the category of Algebra, focusing on solving linear inequalities and understanding the graphical representation of their solutions. It assesses the student's ability to manipulate inequalities and interpret their solution sets. Mastery of these concepts is crucial for performing well on the

SAT, as it involves critical thinking and graphical analysis skills.

Step 1: Write the inequality $2x + 5y < 10$ in slope-intercept form.
$2x + 5y < 10$ can be rearranged by subtracting 2x from both sides:
$5y < -2x + 10$.
Step 2: Divide every term by 5 to solve for y: $y < -\frac{2}{5}x + 2$.
This inequality tells us that the solution set is the region below the line $y = -\frac{2}{5}x + 2$ in the xy-plane.
Step 3: Analyze each option against this inequality:
Option A: The region where $x < 0$ and $y > 2$. The line $y = -\frac{2}{5}x + 2$ has a y-intercept of 2, so $y > 2$ is above the line, meaning this region does NOT contain any points of the solution set.
Option B: The region where $x < 0$ and $y < 0$. A negative y-value can satisfy the inequality because it is below the line, so this region may contain points of the solution set.
Option C: The region where $x > 0$ and $y < 2$. A y-value less than 2, especially for positive x, could satisfy the inequality as it is below the line.
Option D: The y-axis ($x = 0$). Substituting x = 0 into the inequality gives $5y < 10$, or $y < 2$, which can be satisfied for y-values less than 2 on the y-axis.

4. A certain school has a total of 46 students enrolled in a club. The number of students in the club is at least 32 more than the number of students who have signed up for a volunteer activity. If we let x represent the number of students signed up for the volunteer activity, which inequality represents this situation?

A. $x + 32 \leq 46$

B. $x + 46 \geq 32$

C. $x + 32 \geq 46$

D. $x + 46 \leq 32$

Answer

A

Solution

Concept Check : The intent of the question is to assess the student's understanding of linear inequalities, particularly in the context of a real-world scenario. The student is expected to be familiar with how to translate verbal descriptions into mathematical inequalities and understand the relationship between the total number of students in the club and those signed up for a volunteer activity.

Solution Strategy : To approach this problem, the student should focus on identifying the key components of the situation presented. The total number of students in the club is 46, and there is a relationship described between the number of students in the club and those signed up for the volunteer activity. The student should express this relationship as an inequality, taking into account that the number of students in the club (which is 46) is at least 32 more than the number of students signed up (represented by x). This leads to the formulation of an inequality that captures this relationship.

Quick Wins : When translating word problems to inequalities, it is helpful to break down the problem into smaller parts. Identify what you know (in this case, the total number of students and the relationship described) and define your variable clearly. Use phrases like 'at least' to recognize that you will use greater than or equal to (≥) in your inequality. Remember to carefully consider the direction of the inequality based on the wording of the problem.

Mistake Alert : Be careful with the mathematical symbols and terms used in the problem. Phrases like 'at least' indicate that you should use a greater than or equal to symbol (≥), while 'more than' would use a greater than symbol (>). Additionally, ensure you correctly interpret the relationship between the total number of students and those signed up for the volunteer activity to avoid misformulating the inequality.

SAT Know-How : This problem belongs to the Algebra category, specifically focusing on linear inequality word problems. It assesses the student's ability to interpret a real-world scenario and convert it into a mathematical inequality. Understanding how to extract relevant information and translate it accurately into inequalities is a critical skill for the SAT, and practicing this will aid in developing problem-solving proficiency.

Identify the given relationship: The number of students in the club (46) is at least 32 more than x.
Express this relationship mathematically: 46 is at least $x + 32$.
This can be transformed into an inequality: $46 \geq x + 32$.
Rearrange the terms to find the correct inequality: $x + 32 \leq 46$.
Therefore, the inequality representing the situation is $x + 32 \leq 46$.

5. In the xy-plane, line m has a slope of 4 and a y-intercept of (0, -54). What is the x-coordinate of the x-intercept of line m?

Answer

13.5

Solution

Concept Check : The intent of the question is to assess the student's understanding of linear equations, specifically how to identify the slope and y-intercept of a line. The student is expected to know how to use the slope-intercept form of a linear equation, which is $y = mx + b$, where m is the slope and b is the y-intercept. The goal is to find the x-coordinate of the x-intercept, where the line crosses the x-axis ($y = 0$).

Solution Strategy : To approach this problem, the student should start by writing the equation of the line using the given slope and y-intercept. The slope (m) is 4, and the y-intercept (b) is -54, so the equation will take the form $y = 4x - 54$. Next, the student needs to find the x-coordinate of the x-intercept by setting y to 0 and solving for x. This involves rearranging the equation to isolate x and determine its value.

Quick Wins : A useful tip is to remember that the x-intercept occurs where $y = 0$. To find this, simply substitute 0 for y in the linear equation. After substituting, be careful to simplify correctly. Also, it can help to visualize the graph of the line to understand where it crosses the x-axis.

Mistake Alert : Students should be cautious not to misinterpret the slope and y-intercept, ensuring they are correctly inputting these values into the slope-intercept form. Additionally, they should not forget to isolate x correctly when solving for the x-intercept, as small arithmetic mistakes can lead to incorrect answers.

SAT Know-How : This problem falls under the category of Algebra, specifically focusing on the graphs of linear equations and functions. It assesses the student's ability to apply the slope-intercept form to find the x-intercept of a line. Mastery of this concept is essential for SAT math, as it tests both understanding and application of linear relationships.

1. Write the equation of line m using the slope-intercept form of a line equation, which is $y = mx + b$.
2. Substitute the given slope and y-intercept into the equation: $y = 4x - 54$.
3. To find the x-coordinate of the x-intercept, set $y = 0$ and solve for x:
$0 = 4x - 54$

4. Rearrange the equation to solve for x:
$54 = 4x$
5. Divide both sides by 4 to isolate x:
$x = \frac{54}{4}$
6. Simplify the fraction:
$x = 13.5$
7. Therefore, the x-coordinate of the x-intercept of line m is 13.5.

6. A company produces eco-friendly products. The total revenue of the company in the year 2023 is represented by the equation $831 = 71 + 76(x - 8)$, where x represents the number of years since 2015. If the revenue is expected to reach 831 dollars in 2023, how many years since 2015 has the company been operating?

A. 10 years

B. 15 years

C. 18 years

D. 20 years

Answer

C

Solution

Concept Check : The intent of the question is to assess the student's ability to interpret a linear equation in the context of a real-life scenario. Students are expected to understand how to manipulate and solve linear equations, and they should be familiar with the concepts of revenue, time variables, and how to relate them in the context of the problem.

Solution Strategy : To approach this problem, students should first identify what the variable 'x' represents in the context of the problem, which is the number of years since 2015. Then, they should recognize that the equation given can be rearranged to isolate 'x'. This will require applying algebraic principles, such as distributing terms, combining like terms, and solving for the variable. It's essential to keep track of the relationships between the years and the revenue stated in the equation.

Quick Wins : A helpful tip is to break down the equation step-by-step. Start by simplifying the right side and isolating 'x' on one side of the equation. It can also be beneficial to convert the equation into a more familiar linear form. Additionally, double-check your calculations at each step to ensure accuracy. Finally, remember to interpret the final value of 'x' in the context of the problem to ensure it makes sense.

Mistake Alert : Students should be careful not to make common mistakes such as misreading the equation or mistakenly adding or subtracting terms incorrectly. Additionally, pay attention to the meaning of the variable 'x' to avoid misinterpreting what the solution represents. It's also important to check that the context of the problem aligns with the solution you arrive at, ensuring that it is a reasonable answer in terms of the years since the company began operating.

SAT Know-How : This problem falls under the category of Algebra, specifically focusing on linear equation word problems. It assesses the student's skills in interpreting, manipulating, and solving linear equations in real-world contexts. Mastery of these skills is essential for success on the SAT, as it demonstrates the ability to connect mathematical concepts to practical situations.

1. Begin by simplifying the equation: $831 = 71 + 76(x - 8)$.
2. Distribute 76 in the term $76(x - 8)$: $76x - 608$.
3. The equation becomes: $831 = 71 + 76x - 608$.
4. Combine like terms on the right side: $71 - 608 =- 537$.
5. The equation now is: $831 = 76x - 537$.
6. Add 537 to both sides to isolate the term with x: $831 + 537 = 76x$.
7. This simplifies to: $1368 = 76x$.
8. Divide both sides by 76 to solve for x: $x = \frac{1368}{76}$.
9. Calculate the division: $x = 18$.
10. Therefore, the company has been operating for 18 years since 2015.

7. If $86(x + 9) = 36(x + 9) + 150$, what is the value of $x + 9$?

A. -9

B. -6

C. 3

D. 6

Answer

C

Solution

Concept Check : The intent of this question is to assess the student's ability to solve a linear equation using the substitution method. Students are expected to understand how to isolate variables and perform algebraic operations to find the value of 'x + 9.'

Solution Strategy : To approach this problem, the student should start by recognizing that both sides of the equation contain the term '(x + 9).' The first step is to distribute the coefficients (86 and 36) to the term '(x + 9)' on both sides. After simplifying both sides, the student should then rearrange the equation to isolate 'x' or directly solve for 'x + 9' by manipulating the equation appropriately.

Quick Wins : A helpful tip is to combine like terms after distributing the coefficients. This will make it easier to isolate the variable. Also, consider rewriting the equation in terms of 'x + 9' directly, which could simplify your calculations. Always double-check your calculations after each step to ensure accuracy.

Mistake Alert : Students should be cautious about making errors in distribution and combining like terms. It's easy to miscalculate coefficients or to accidentally drop a term when rearranging the equation. Double-check your work to avoid these common mistakes, especially when dealing with negative numbers or when moving terms from one side of the equation to the other.

SAT Know-How : This problem falls under the category of Algebra, specifically focusing on solving linear equations and inequalities through substitution. It assesses the student's skills in distributing terms, combining like terms, and isolating variables. Mastering these techniques is essential for success in SAT math, as it reinforces the foundational skills needed for more complex algebraic concepts.

Step 1: Simplify both sides by subtracting $36(x + 9)$ from both sides of the equation.
$86(x + 9) - 36(x + 9) = 150$

Step 2: Combine like terms on the left side.
$50(x + 9) = 150$
Step 3: Isolate $(x + 9)$ by dividing both sides by 50.
$x + 9 = \frac{150}{50}$
Step 4: Simplify the fraction.
$x + 9 = 3$

8. What is the y-coordinate of the y-intercept of the graph of $y = g(x)$ in the xy-plane, given that $g(x) = f(x) - 5$ and $f(x) = 6(31x - 13)$?

A. -83

B. -78

C. -73

D. -68

Answer

A

Solution

Concept Check : The question intends for the student to understand the concept of y-intercepts, particularly in the context of composite functions. Students should know how to find the y-intercept of a function and how transformations like vertical shifts (subtracting a constant) affect it. Additionally, familiarity with linear functions and their properties is essential.

Solution Strategy : To solve the problem, the student should first determine the y-intercept of the function $f(x) = 6(31x - 13)$. This is achieved by substituting $x = 0$ into the function to find $f(0)$. Once $f(0)$ is found, the student can then calculate $g(0)$ using the relationship $g(x) = f(x) - 5$, which will give the y-coordinate of the y-intercept for $g(x)$.

Quick Wins : Remember that the y-intercept occurs where $x = 0$. When dealing with composite functions, it can be helpful to break the problem into smaller parts—first find the y-intercept of the inner function ($f(x)$) and then apply any transformations to find the final y-intercept of the outer function ($g(x)$). Also, keep track of the signs when performing operations on constants.

Mistake Alert : Students may forget to substitute $x = 0$ in both functions or may miscalculate when applying the vertical shift (subtracting 5). It is also easy to lose track of negative signs or to confuse the order of operations, especially when handling the composite functions.

SAT Know-How : This problem falls under the category of Algebra, specifically focusing on graphs of linear equations and functions. It assesses the student's ability to find y-intercepts and understand the effects of transformations on functions. This type of question tests critical thinking and problem-solving skills, which are essential for success on the SAT.

First, determine $f(x)$ at $x = 0$: $f(0) = 6(31(0) - 13)$
Calculate: $f(0) = 6(0 - 13)$
Simplify: $f(0) = 6(- 13)$
Calculate: $f(0) =- 78$
Substitute $f(0)$: $g(0) =- 78 - 5$
Simplify: $g(0) =- 83$
Thus, the y-coordinate of the y-intercept of the graph $y = g(x)$ is -83.

9. For each real number r, which of the following points lies on the graph of each equation in the xy-plane for the given system? $-35x - 29y = 45$, $-280x - 232y = 360$

A. $(r, -\frac{35}{29}r - \frac{45}{29})$

B. $(r, \frac{35}{29}r + \frac{45}{29})$

C. $(r, \frac{45}{35}r + \frac{29}{35})$

D. $(r, -\frac{29}{35}r - \frac{45}{35})$

Answer

A

Solution

Concept Check : The question aims to assess the student's understanding of systems of linear equations, specifically focusing on recognizing equivalent equations and identifying points that satisfy both equations simultaneously.

Solution Strategy : To approach this problem, students should first recognize that the given system of equations may represent the same line. This means that any point on one line will also be on the other line. Students should simplify or manipulate one of the equations to see if it can be transformed into the other form, or check if they are scalar multiples of each other. After confirming that the equations are equivalent, students can substitute the provided options for (x, y) into either equation to see if they satisfy it.

Quick Wins : When working with systems of equations, it's helpful to look for ways to manipulate the equations. You can multiply or divide entire equations by the same non-zero constant, or rearrange them to slope-intercept form (y = mx + b) to visually assess their relationship. Additionally, checking potential solutions can be done by substituting points back into the original equations, which allows you to confirm if they are valid solutions quickly.

Mistake Alert : Be careful not to confuse equivalent equations with independent ones. Ensure that you check if both equations actually represent the same line before concluding that they have infinite solutions. Also, pay attention to arithmetic errors when manipulating the equations or substituting points. Miscalculations here can lead to incorrect conclusions about the solutions.

SAT Know-How : This problem falls under the category of algebra, specifically in the unit of solving systems of linear equations. It assesses the student's ability to recognize equivalent equations and find common solutions. Mastering this type of problem is crucial for the SAT, as it tests problem-solving skills and the understanding of linear relationships, which are key concepts in algebra.

Step 1: Check for equivalence between given equations.
Multiply the entire first equation by 8 to compare with the second equation:
$8(-35x - 29y) = 8(45)$.
This simplifies to: $-280x - 232y = 360$, which is exactly the same as the second equation.
Step 2: Since the equations are equivalent, they represent the same line.
Step 3: Express y in terms of x for the simplified equation $-35x - 29y = 45$.
Rearrange to solve for y: $-29y = 35x + 45$.
Divide the entire equation by -29 to solve for y: $y = -\frac{35}{29}x - \frac{45}{29}$.
Therefore, for any real number $x = r$, the corresponding y is $y = -\frac{35}{29}r - \frac{45}{29}$.
Step 4: Match this result with the given options to find which point (r, y) corresponds to this expression.

10. A research study shows that the average number of hours students spend on community service per year, x, can be estimated by the function $f(x) = 5x + 89$. Which statement best interprets the value 89 in this context?

A. Students will spend a total of 89 hours on community service after 5 years.

B. The average number of community service hours is expected to grow by 5 hours each year.

C. The average hours of community service increased from 89 hours after each year.

D. The estimated number of community service hours was 89 hours when no additional hours are counted.

Answer

D

Solution

Concept Check : The intent of this question is to assess the student's understanding of function interpretation, specifically the concept of the y-intercept in the context of a linear equation. The student is expected to recognize what the constant term in the linear equation represents regarding the average number of hours spent on community service.

Solution Strategy : To approach this problem, the student should first identify the components of the linear equation provided, $f(x) = 5x + 89$. The student needs to recognize that the term 'x' represents the number of years, and the function f(x) represents the estimated average hours spent on community service. The constant term '89' should then be interpreted in relation to what it signifies in this context.

Quick Wins : Consider breaking down the function into its parts: the slope (5) and the y-intercept (89). Remember that the y-intercept indicates the value of the function when x is 0. Think about what it means for students who have not yet spent any time on community service, and how that relates to the average hours in this scenario. This perspective can clarify the meaning of the constant term.

Mistake Alert : Be cautious not to confuse the slope with the y-intercept. The slope (5) indicates how much the average number of hours increases with each additional year, while the y-intercept (89) has a specific meaning that should not be overlooked. Additionally, ensure you are interpreting '89' correctly in the context of the problem—this is not just a number but has relevance to the average hours spent on community service.

SAT Know-How : This problem falls under the category of algebra, specifically focusing on interpreting linear equations and their components. It assesses the student's ability to read and understand the context of mathematical functions. The ability to interpret the y-intercept in a real-world scenario is a valuable skill in SAT problem-solving, highlighting the importance of grasping how mathematical concepts apply beyond mere calculations.

Step 1: Understand the Function $f(x) = 5x + 89$.
The function is in the form $y = mx + b$, where m is the slope and b is the y-intercept.
Step 2: Identify the meaning of the constant term, 89.
In a linear equation, the y-intercept (89 in this case) represents the starting value - the value of the function when $x = 0$.
Step 3: Analyze the context provided by the problem.
In this context, the value 89 represents the estimated initial average hours of community service before any additional time ($x = 0$) is counted.
Step 4: Check each option for alignment with the interpretation of 89.
Option A is incorrect because it confuses the function's application over 5 years.
Option B is incorrect because it describes the slope (5) rather than the y-intercept (89).
Option C is incorrect because it suggests 89 is a change rather than an initial value.
Option D correctly identifies 89 as the initial estimate when no additional hours are factored in.

2

Digital SAT

Math

Advanced

SAT Math Advanced

1. For a polynomial function, the graph of $y = f(x)$ in the xy-plane contains the points (7, 0), (0, 0), (6, 0), (5, 0), (3, 0), and (4, 0). Which of the following must be a factor of $f(x)$?

A. $x^2 + 9x - 18$

B. $x^3 - 18x^2 + 107x - 210$

C. $x^2 + 7x$

D. $x^3 - 15x^2 - 74x - 120$

2. The function f is defined by $f(x) = 29 + 44\sqrt{x}$. What is the value of $f(25)$?

A. 29

B. 125

C. 249

D. 300

3. A physicist is studying a certain radioactive substance whose quantity decreases over time. The equation representing the quantity of this substance in grams after x years is given by: $f(x) = 1100(0.41)^x$. Which of the following is the best interpretation of 1100 in this context?

A. The estimated amount of substance remaining after 1 year.

B. The estimated amount of substance remaining after 2 years.

C. The estimated initial quantity of the substance at time 0.

D. The estimated quantity of substance after 6 years.

4. The function f is a quadratic function. In the xy-plane, the graph of $y = f(x)$ has a vertex at (6, -4) and passes through the points (-3, 239) and (-5, 359). What is the value of $f(7) - f(4)$?

A. -15

B. -12

C. -9

D. -5

5. A diplomatic cable containing sensitive information is sent to a foreign country. The function given by $h(t) = 925 + (975 - 925)(8.52)^{-0.317t}$ describes the rate of decay of information secrecy, in some units, t hours after the cable is sent. What was the initial rate of information secrecy when the cable was sent?

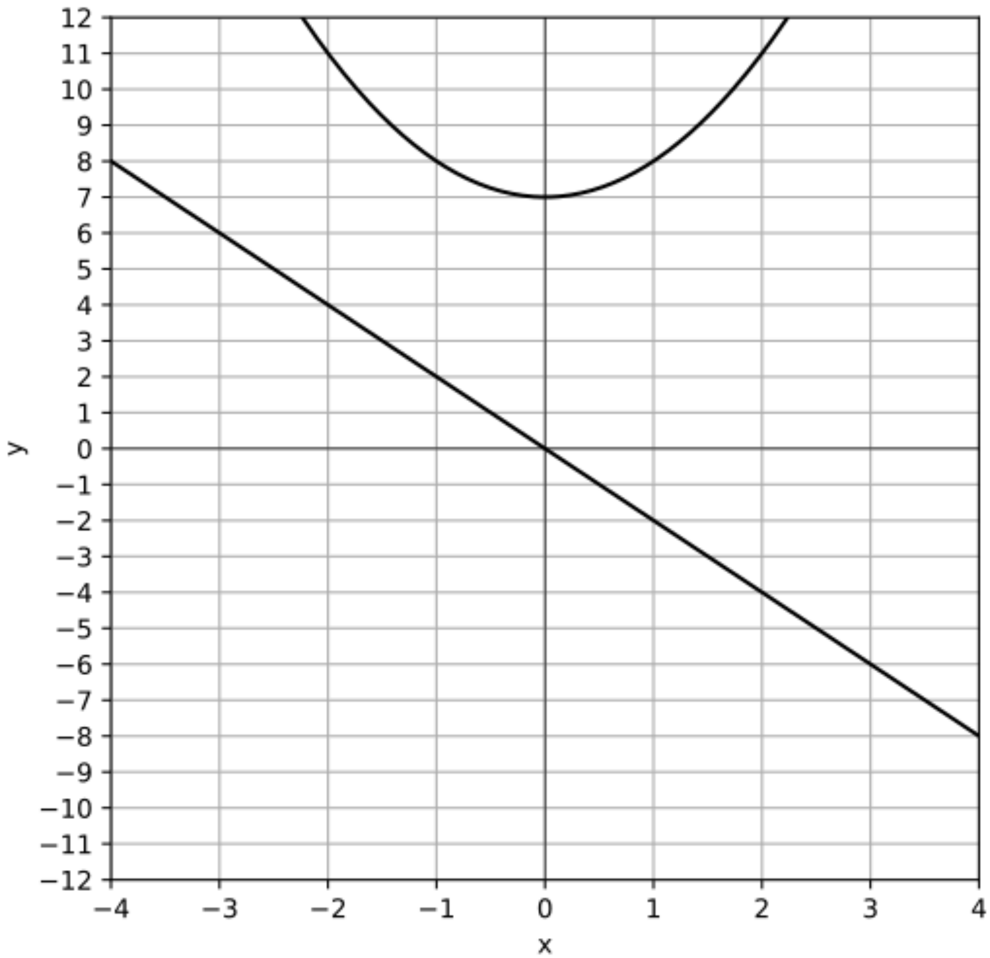
A. 825

B. 925

C. 975

D. 1025

6. The graph of a system of a linear and a quadratic equation is shown. Which system of equations is represented by the graph?



A. $y = x^2 + 7$ and y = -2x

B. $y = x^2 - 7$ and y = -2x

C. $y = x^2 - 7$ and y = 2x

D. $y = x^2 + 7$ and y = 2x

7. A financial analyst is modeling the projected economic activity in a city using the equation for active jobs: $y = -0.3x^2 - 3x + 86$, where y represents the number of active jobs and x is the number of months since January 2020. What is the best interpretation of the y-intercept of the graph of this equation in the xy-plane?

A. At the end of January 2020, the projected number of active jobs was 0.

B. At the end of January 2020, the projected number of active jobs was 86, suggesting a lack of job activity or data errors.

C. At the end of July 2020, the projected number of active jobs was 86.

D. At the end of July 2020, the projected number of active jobs was 0.

8. What is the y-intercept of the function defined by $f(x) = 2^x + 15$ in the xy-plane?

A. (0, 15)

B. (0, 16)

C. (0, 17)

D. (0, 18)

9. Which expression is equivalent to $-6x^5y^7(7x^7 + 4x^4 + 84)$?

A. $-42x^{12}y^6 - 24x^9y^7 - 504x^5y^7$

B. $-42x^{12}y^7 - 24x^6y^7 - 504x^5y^6$

C. $-42x^{12}y^7 - 24x^9y^6 - 504x^5y^7$

D. $-42x^{12}y^7 - 24x^9y^7 - 504x^5y^7$

10. The functions f and g are defined by the equations shown, where a is an integer constant, and a <0. If y = f(x) and y = g(x) are graphed in the xy-plane, which of the following equations displays, as a constant or coefficient, the y-coordinate of the y-intercept of the graph of the corresponding function? $f(x) = a(-4.2)^{x+29}$
$g(x) = a(-4.2)^{x} + 29$

A. f(x) only

B. g(x) only

C. f(x) and g(x)

D. Neither f(x) nor g(x)

SAT Math Advanced Solutions

1. For a polynomial function, the graph of $y = f(x)$ in the xy-plane contains the points (7, 0), (0, 0), (6, 0), (5, 0), (3, 0), and (4, 0). Which of the following must be a factor of $f(x)$?

A. $x^2 + 9x - 18$

B. $x^3 - 18x^2 + 107x - 210$

C. $x^2 + 7x$

D. $x^3 - 15x^2 - 74x - 120$

Answer

B

Solution

Concept Check : The intent of this question is to assess the student's understanding of polynomial functions and their roots. The student is expected to know that if a polynomial has a root at a certain x-value, then (x - that value) is a factor of the polynomial. The roots provided in the problem indicate where the polynomial crosses the x-axis, which directly relates to identifying factors.

Solution Strategy : To approach this problem, the student should first recognize that the given points where the polynomial crosses the x-axis (the x-values of the points) are the roots of the polynomial function. For each root, a corresponding factor can be formed. The student should consider each x-coordinate of the roots provided and develop the factors based on the form (x - root). The task is to identify which of these factors must be part of the polynomial function.

Quick Wins : A helpful tip is to list out all the roots provided in the problem and write down their corresponding factors. For example, if one of the roots is x = 0, then the factor is (x - 0) or simply x. Additionally, recognizing that any factor can be multiplied by a constant does not affect the roots, so focus on the linear factors derived from the roots directly. This can help in quickly identifying the correct factors without getting distracted by the polynomial's leading coefficient.

Mistake Alert : Be careful not to confuse the roots with the factors. Each root (x-value) corresponds to a factor, but it's important to ensure you are setting them up correctly as (x - root). Also, pay attention to whether a root is repeated, as this might suggest a squared factor, but for this question, you only need to identify which factors must exist, not their multiplicities.

SAT Know-How : This problem falls under the category of Advanced Math, specifically focusing on polynomials and their factors related to their roots. The skills assessed include the ability to connect roots of polynomial functions to their corresponding factors and understand the fundamental relationship between these concepts in polynomial graphing and function analysis. Mastering this problem type is crucial for success in the SAT, as it tests understanding of polynomial behavior and algebraic manipulation.

The polynomial function f(x) can be expressed in terms of its factors: f(x) = x(x - 3)(x - 4)(x - 5)(x - 6)(x - 7).
Examine each option to determine if it contains these factors:
Option A: $x^2 + 9x - 18$. Attempt to factor:
- $x^2 + 9x - 18$ = (x - 3)(x + 6), which doesn't match any pair of zeros.
Option B: $x^3 - 18x^2 + 107x - 210$. Attempt to factor by identifying roots or using synthetic division:
- Using synthetic division, try x = 3, x = 4, x = 5, x = 6, and x = 7. Upon testing, x = 5 is a root.
- Factoring out (x - 5) from $x^3 - 18x^2 + 107x - 210$ leaves a quadratic factor: ($x^2 - 13x + 42$).
- Further factor ($x^2 - 13x + 42$) = (x - 6)(x - 7), confirming x = 6 and x = 7 are roots.
- Thus, $x^3 - 18x^2 + 107x - 210$ = (x - 5)(x - 6)(x - 7), which includes factors for zeros.
Option C: $x^2 + 7x$. Attempt to factor:
- $x^2 + 7x$ = x(x + 7), which doesn't match the zeros.
Option D: $x^3 - 15x^2 - 74x - 120$. Attempt to factor by identifying roots or using synthetic division:
- Using synthetic division, try x = 3, x = 4, x = 5, x = 6, and x = 7. None are roots.
- This polynomial does not factor to match the zeros.

2. The function f is defined by $f(x) = 29 + 44\sqrt{x}$. What is the value of $f(25)$?

A. 29

B. 125

C. 249

D. 300

Answer

C

Solution

Concept Check : The question is aimed at assessing the student's understanding of evaluating functions, particularly those involving irrational numbers such as square roots. The student is expected to know how to substitute a value into the function and perform the calculation accurately.

Solution Strategy : To solve the problem, the student should follow these steps: First, identify the given function, which is $f(x) = 29 + 44\sqrt{x}$. Then, substitute x with the value 25. This means calculating 44 times the square root of 25, and then adding 29 to the result. It's important to clearly follow the order of operations during this process.

Quick Wins : Remember to calculate the square root first before multiplying by 44. The square root of 25 is a common value (5), so it may be easier to recall. After finding 44 times the square root, don't forget to add 29 at the end to find the final value of $f(25)$. Keeping track of each step will help prevent errors.

Mistake Alert : Be careful with the square root calculation; it's easy to miscalculate if you're unsure of the value. Also, ensure you remember to perform the addition after the multiplication. Double-check your substitution to make sure the correct number is used in place of x.

SAT Know-How : This problem falls under the category of Advanced Math, focusing on nonlinear functions that include irrational components. It assesses the student's ability to evaluate functions accurately and apply basic operations. Mastering this type of problem is key for SAT success, as it reinforces the importance of function evaluation and precision in calculations.

Substitute $x = 25$ into the function: $f(x) = 29 + 44\sqrt{x}$.
Calculate $\sqrt{25}$, which is 5.

Substitute $\sqrt{25} = 5$ into the function: $f(25) = 29 + 44 \times 5$.
Perform the multiplication: $44 \times 5 = 220$.
Add the results: $29 + 220 = 249$.

3. A physicist is studying a certain radioactive substance whose quantity decreases over time. The equation representing the quantity of this substance in grams after x years is given by: $f(x) = 1100(0.41)^x$. Which of the following is the best interpretation of 1100 in this context?

A. The estimated amount of substance remaining after 1 year.

B. The estimated amount of substance remaining after 2 years.

C. The estimated initial quantity of the substance at time 0.

D. The estimated quantity of substance after 6 years.

Answer

C

Solution

Concept Check : The question is designed to test the student's understanding of exponential decay, particularly in the context of real-world applications such as radioactive decay. The student is expected to know how to interpret the parameters of an exponential function, specifically the initial quantity and the decay factor.

Solution Strategy : To approach this problem, the student should identify the components of the given exponential function. They should recognize that in the equation of the form $f(x) = a(b)^x$, 'a' represents the initial value of the quantity when $x = 0$. The student should think through the meaning of the parameters in relation to the problem context to determine what 1100 signifies.

Quick Wins : A good strategy is to substitute $x = 0$ into the function to see what value you get. This will help clarify what the initial quantity is. Also, it can be helpful to visualize the process of radioactive decay and how it relates to the parameters in the equation. Remember that the base of the exponent (0.41 in this case) indicates the rate of decay, whereas the coefficient provides the starting amount.

Mistake Alert : Be careful not to confuse the initial quantity with the decay factor. The number 1100 is significant at the start of the observation (when $x = 0$), while the base of the exponent, 0.41, tells you how much of the substance remains after each year. Misinterpreting these can lead to incorrect conclusions.

SAT Know-How : This problem falls under the category of advanced math, specifically focusing on exponential word problems related to real-life scenarios like radioactive decay. It assesses the student's ability to interpret mathematical models and understand the significance of parameters. Mastering this type of problem enhances your skills in applying mathematical concepts to analyze and interpret real-world data effectively.

Step 1: Understand the general form of an exponential decay function. It is typically given by $f(x) = a(b)^x$, where 'a' is the initial quantity, 'b' is the decay factor, and 'x' is the time elapsed.
Step 2: Compare the given function $f(x) = 1100(0.41)^x$ with the general form. Here, $'a' = 1100$ and $'b' = 0.41$.
Step 3: Interpret the meaning of $'a' = 1100$. This value represents the initial quantity of the substance before any decay has occurred, i.e., at time $x = 0$.
Step 4: Verify by setting $x = 0$ in $f(x) = 1100(0.41)^x$. The equation becomes $f(0) = 1100(0.41)^0 = 1100 \times 1 = 1100$.
Step 5: Therefore, 1100 represents the initial amount of the substance, confirming the interpretation that it is the quantity present at time $x = 0$.

4. The function f is a quadratic function. In the xy-plane, the graph of $y = f(x)$ has a vertex at (6, -4) and passes through the points (-3, 239) and (-5, 359). What is the value of $f(7) - f(4)$?

A. -15

B. -12

C. -9

D. -5

Answer

C

Solution

Concept Check : The intent of this question is to assess the student's understanding of quadratic functions, particularly in determining the function's equation when given its vertex and specific points on the graph. Students are expected to know how to use the vertex form of a quadratic function and how to manipulate it based on known points.

Solution Strategy : To solve the problem, the student should start by recalling the vertex form of a quadratic function, which is given by the equation: $f(x) = a(x - h)^2 + k$, where (h, k) is the vertex of the parabola. Here, the vertex is given as (6, -4), so h = 6 and k = -4. The next step is to substitute the coordinates of the points (-3, 239) and (-5, 359) into the equation to create two equations that can be solved to find the value of 'a'. Once 'a' is determined, the full function can be established, and then f(7) and f(4) can be calculated to find their difference.

Quick Wins : 1. Remember the vertex form of a quadratic function is essential in problems like this. 2. Substitute points carefully and ensure you maintain correct signs. 3. When substituting the points into the equation, simplify carefully to avoid arithmetic errors. 4. After finding the function, evaluate it at the required points to find the difference.

Mistake Alert : 1. Be cautious when substituting the points; double-check that you plug the correct x and y values into the equation. 2. Pay special attention to the negative signs when working with the vertex's coordinates and the points; it's easy to make minor mistakes here. 3. Ensure that you correctly calculate the values of f(7) and f(4) before subtracting them.

SAT Know-How : This problem belongs to the Advanced Math category and focuses on quadratic functions and their graphs. It assesses skills such as understanding the vertex form of a quadratic equation, substituting values correctly, and performing arithmetic operations accurately. Mastering these skills is crucial for solving quadratic-related problems on the SAT, as it enhances your problem-solving efficiency and reduces the chances of mistakes.

Start by writing the quadratic function in vertex form: $f(x) = a(x - 6)^2 - 4$.
Substitute the point (-3, 239) into the equation to solve for a:
$239 = a(-3 - 6)^2 - 4$.
Simplify: 239 = a(81) - 4.
Add 4 to both sides: 243 = 81a.
Solve for a: $a = \frac{243}{81} = 3$.
Verify a with the point (-5, 359):
$359 = 3(-5 - 6)^2 - 4$.
Simplify: 359 = 3(121) - 4.
Calculate: 359 = 363 - 4.
This confirms a = 3 is correct.
Now, calculate f(7) and f(4):
$f(x) = 3(x - 6)^2 - 4$.
$f(7) = 3(7 - 6)^2 - 4 = 3(1)^2 - 4 = 3 - 4 = -1$.
$f(4) = 3(4 - 6)^2 - 4 = 3(-2)^2 - 4 = 3(4) - 4 = 12 - 4 = 8$.
Calculate f(7) - f(4): -1 - 8 = -9.

5. A diplomatic cable containing sensitive information is sent to a foreign country. The function given by $h(t) = 925 + (975 - 925)(8.52)^{-0.317t}$ describes the rate of decay of information secrecy, in some units, t hours after the cable is sent. What was the initial rate of information secrecy when the cable was sent?

A. 825

B. 925

C. 975

D. 1025

Answer

C

Solution

Concept Check : The intent of the question is to assess the student's understanding of exponential decay functions, specifically how to evaluate such functions at a given point in time. Students should know how to interpret the parameters in the function and calculate the initial value when time is zero.

Solution Strategy : To solve the problem, the student should recognize that the initial rate of information secrecy corresponds to the value of the function when time, t, equals 0. This requires substituting 0 for t in the given function h(t) and simplifying the expression to find the initial rate.

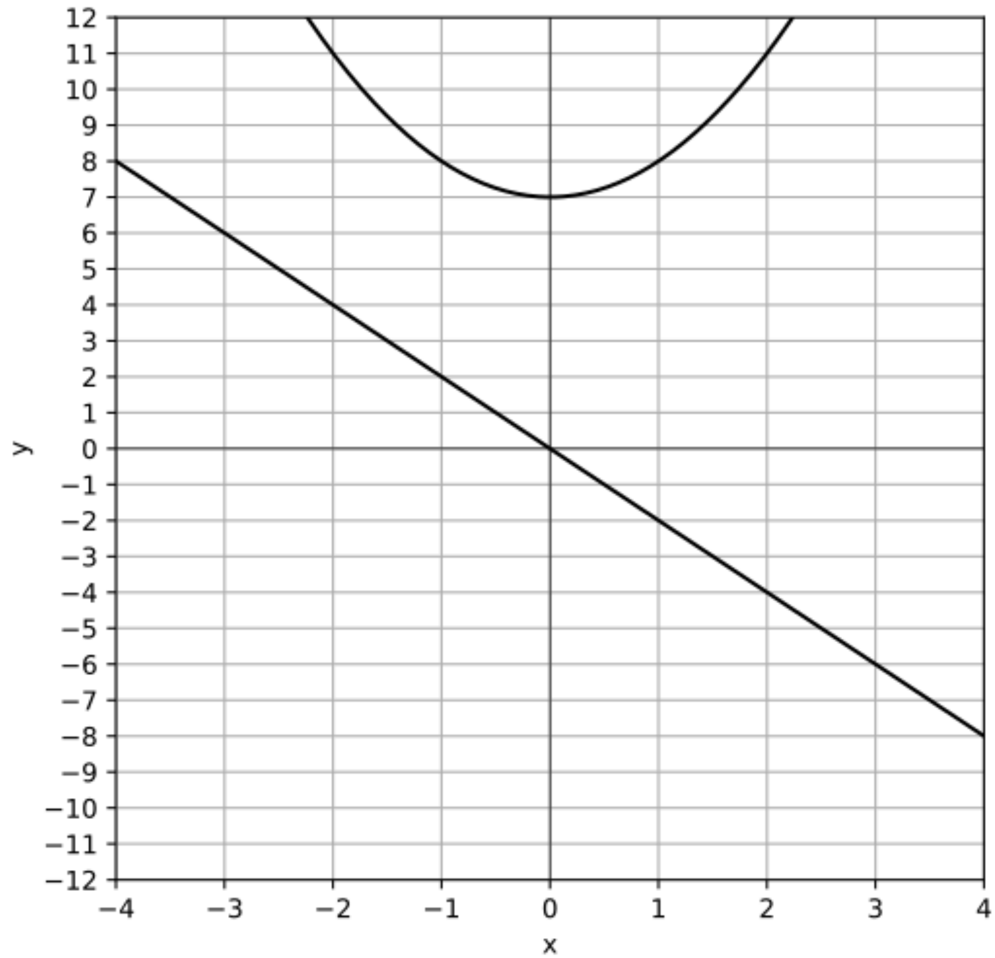
Quick Wins : A good approach to similar problems is to always identify what value of t corresponds to the initial condition. In this case, $t = 0$ gives the initial rate. Remember to carefully substitute the value into the function and simplify correctly. It can be helpful to break down the function into parts to see how each term contributes to the overall value.

Mistake Alert : Be cautious to ensure you substitute the value of t correctly and watch for any negative signs or operations that might affect the outcome. Pay close attention to the order of operations, especially when dealing with exponentials and constants in the function.

SAT Know-How : This problem falls under 'Advanced Math' and specifically focuses on exponential word problems. It assesses the student's ability to evaluate exponential functions and understand the implications of the parameters within the context of decay. This type of problem helps students develop the necessary skills for reasoning through mathematical functions and applying them in practical scenarios, essential for success in the SAT.

To find the initial rate of secrecy, we evaluate h(0) using the given function.
Substitute $t = 0$ into the function: $h(0) = 925 + (975 - 925)(8.52)^{0}$.
Therefore, $h(0) = 925 + (975 - 925) \times 1$.
Since any non-zero number raised to the power of 0 is 1, $8.(52)^{0} = 1$.
Now calculate: $h(0) = 925 + (975 - 925) = 975$.
Thus, The initial rate of information secrecy when the cable was sent is 975.

6. The graph of a system of a linear and a quadratic equation is shown. Which system of equations is represented by the graph?



A. $y = x^2 + 7$ and y = -2x

B. $y = x^2 - 7$ and y = -2x

C. $y = x^2 - 7$ and y = 2x

D. $y = x^2 + 7$ and y = 2x

Answer

C

Solution

Concept Check : The intent of this question is to assess the student's understanding of the relationship between linear and quadratic equations as represented graphically. Students are expected to identify the equations that correspond to the features of the graph, such as intersections, vertex, and the general shape of the quadratic function.

Solution Strategy : To approach this problem, students should carefully analyze the graph provided. They should look for key features such as the vertex of the quadratic, the slope of the linear equation, and points of intersection. By identifying the characteristics of the lines and curves, they can set up equations that represent these features. Additionally, they might consider the standard forms of both equations: y = mx + b for the linear equation and $y = ax^2 + bx + c$ for the quadratic equation.

Quick Wins : One effective strategy is to focus on the intersection points of the graph, as these will help you identify solutions to the system of equations. Additionally, remember that the vertex of the quadratic will give you information about the maximum or minimum point of the parabola. If the graph is labeled with coordinates, use those points to substitute into your equations to find the correct coefficients.

Mistake Alert : Be careful not to confuse the orientation of the quadratic (whether it opens upwards or downwards) and the slope of the linear function. Pay attention to the curves and lines; minor visual misinterpretations can lead to incorrect equations. Also, ensure that you check all possible answer choices against the graph rather than relying solely on one characteristic.

SAT Know-How : This problem falls under the category of Advanced Math, specifically focusing on quadratic graphs and their systems with linear equations. It assesses skills in analyzing graphical representations, deriving equations, and understanding the interaction between different types of functions. Mastering these concepts is crucial for effective problem-solving on the SAT.

1. The quadratic equations provided are of the form $y = x^2 \pm c$. The graph of the parabola should open upwards (standard $y = x^2$) and be shifted vertically by c units.
2. The linear equations provided are of the form y = mx. A positive slope (m >0) indicates a line that increases as x increases, while a negative slope (m <0) indicates a line that decreases as x increases.
3. Examine each pair of equations in the options:
- Option A: $y = x^2 + 7$ and y = -2x

The parabola opens upwards and is shifted up by 7 units, and the line has a negative slope so it decreases.
- Option B: $y = x^2 - 7$ and y = -2x
The parabola opens upwards and is shifted down by 7 units, and the line has a negative slope so it decreases.
- Option C: $y = x^2 - 7$ and y = 2x
The parabola opens upwards and is shifted down by 7 units, and the line has a positive slope so it increases.
- Option D: $y = x^2 + 7$ and y = 2x
The parabola opens upwards and is shifted up by 7 units, and the line has a positive slope so it increases.
4. Based solely on the descriptions given, the intersection of the line and parabola in the graph may help identify the correct pair. The parabola at $y = x^2 + 7$ intersects differently depending on whether the line increases or decreases.
5. The correct option will show two points of intersection between the parabola and the line. The intersection pattern can help determine the system of equations.

7. A financial analyst is modeling the projected economic activity in a city using the equation for active jobs: $y = -0.3x^2 - 3x + 86$, where y represents the number of active jobs and x is the number of months since January 2020. What is the best interpretation of the y-intercept of the graph of this equation in the xy-plane?

A. At the end of January 2020, the projected number of active jobs was 0.

B. At the end of January 2020, the projected number of active jobs was 86, suggesting a lack of job activity or data errors.

C. At the end of July 2020, the projected number of active jobs was 86.

D. At the end of July 2020, the projected number of active jobs was 0.

Answer

B

Solution

Concept Check : The intent of this question is to assess the student's understanding of quadratic functions, specifically the interpretation of the y-intercept in the context of a real-world application. Students should recognize that the y-intercept represents the value of y when x is zero, which corresponds to a specific point in time (January 2020 in this case).

Solution Strategy : To approach this problem, students should first identify what the y-intercept represents in the given equation. This involves substituting $x = 0$ into the equation to find the value of y. Then, they should interpret this value in the context of the problem, considering what it means in terms of active jobs at the starting point of the timeframe given (January 2020).

Quick Wins : When interpreting the y-intercept, remember that it gives you the starting value of the dependent variable when the independent variable is zero. In this case, think about what 'months since January 2020' means—$x = 0$ will indicate January 2020, so the y-intercept will tell you the number of active jobs at that time. Always read the problem carefully to ensure you understand the real-world connection.

Mistake Alert : Be careful not to confuse the y-intercept with other points on the graph. The y-intercept is specifically the point where the graph crosses the y-axis (x = 0). Additionally, ensure that you correctly substitute x = 0 into the equation; miscalculating the equation may lead to incorrect interpretations. Remember that negative values can have significant implications in real-world contexts.

SAT Know-How : This problem falls under the category of Advanced Math, focusing on quadratic and exponential word problems. It tests the student's ability to interpret mathematical models in real-world contexts, specifically through the lens of the y-intercept. Mastering this concept is crucial for SAT problem-solving, as it reinforces the connection between algebraic equations and their applications in various scenarios.

1. Identify the y-intercept in the given equation $y = -0.3x^2 - 3x + 86$.
2. The y-intercept occurs when $x = 0$.
3. Substitute $x = 0$ into the equation: $y = -0.3(0)^2 - 3(0)6 + 86$.
4. Simplify: $y = 86$.
5. The y-intercept of 86 means that at the start of January 2020, the projected number of active jobs is 86.

8. What is the y-intercept of the function defined by $f(x) = 2^x + 15$ in the xy-plane?

A. (0, 15)

B. (0, 16)

C. (0, 17)

D. (0, 18)

Answer

B

Solution

Concept Check : The question aims to assess the student's understanding of exponential functions and their properties, particularly how to find the y-intercept of a function. The student should know that the y-intercept occurs where the value of x is zero.

Solution Strategy : To find the y-intercept of the function $f(x) = 2^x + 15$, the student should substitute x with 0 in the function. This will involve evaluating the expression at that specific point. The thought process should involve recognizing that for any function, the y-intercept is found by determining the function's value when x equals zero.

Quick Wins : Remember that the y-intercept is simply the value of the function when x = 0. For exponential functions like $f(x) = 2^x$, it's important to know that 2^0 equals 1. Therefore, when you're substituting, you can quickly compute the value without needing to graph the function. It's also helpful to remember the general form of an exponential function and how it behaves as x changes.

Mistake Alert : Be careful not to confuse the y-intercept with the x-intercept. The y-intercept is found by setting x to 0, while the x-intercept involves setting $f(x)$ to 0 and solving for x. Also, ensure you do not overlook the constant term in the function, as it significantly affects the final result when calculating the y-intercept.

SAT Know-How : This problem falls under the 'Advanced Math' category focusing on nonlinear functions, specifically exponential functions. It tests the student's ability to evaluate a function at a specific point, which is a crucial skill in understanding function behavior. Mastering this concept is essential for success on the SAT, as it lays the groundwork for more complex mathematical reasoning.

Step 1: Understand that the y-intercept is found by setting $x = 0$.
Step 2: Substitute 0 for x in the function: $f(x) = 2^{x} + 15$ becomes $f(0) = 2^{0} + 15$.
Step 3: Calculate 2^{0}. Any number raised to the power of 0 is 1, so $2^{0} = 1$.
Step 4: Calculate $f(0) = 1 + 15$.
Step 5: Simplify the expression: $f(0) = 16$.
Step 6: The y-intercept of the function is the point where $x = 0$and $y = 16$, so the y-intercept is (0, 16).

9. Which expression is equivalent to $-6x^5y^7(7x^7+4x^4+84)$?

A. $-42x^{12}y^6-24x^9y^7-504x^5y^7$

B. $-42x^{12}y^7-24x^6y^7-504x^5y^6$

C. $-42x^{12}y^7-24x^9y^6-504x^5y^7$

D. $-42x^{12}y^7-24x^9y^7-504x^5y^7$

Answer

D

Solution

Concept Check : The question tests the student's understanding of polynomial operations, specifically multiplication. Students should know how to distribute the term $-6x^5y^7$ across each term in the polynomial $(7x^7+4x^4+84)$ and apply the rules of exponents when multiplying terms with the same base.

Solution Strategy : To solve this problem, the student should first recognize that they need to distribute $-6x^5y^7$ to each term inside the parentheses. This means multiplying $-6x^5y^7\ by\ 7x^7$, then by $4x^4$, and finally by 84. The student should keep in mind how to handle the coefficients and the variables while applying the laws of exponents, specifically that when multiplying like bases, you add the exponents.

Quick Wins : Start by rewriting the expression clearly; it helps to see each part. Remember to multiply the coefficients (the numbers in front) together and then handle the variable parts separately. When multiplying the variable parts, add the exponents of like bases. It can be helpful to write down intermediate steps to avoid confusion, ensuring that each term is calculated correctly before moving on to the next.

Mistake Alert : Be careful with the signs—multiplying by -6 means that the sign of each resulting term will flip. Also, double-check the exponents; it's easy to make a mistake when adding exponents, especially with higher-degree polynomials. Pay attention to the arrangement of your final expression to ensure all terms are included and properly simplified.

SAT Know-How : This problem is an example of operations with higher-degree polynomials, specifically focusing on polynomial multiplication. It assesses skills such as distribution, combining like terms, and the laws of exponents. Mastering these concepts is crucial for success in the SAT's math section, particularly in advanced math questions.

Step 1: Distribute $-6x^5y^7$ across each term in the parenthesis: $-6x^5y^7 \times 7x^7$, $-6x^5y^7 \times 4x^4$, and $-6x^5y^7 \times 84$.
Step 2: Calculate $-6 \times 7 = -42$, $x^5 \times x^7 = x^{(5+7)} = x^{12}$, y^7 remains unchanged. So the first term becomes $-42x^{12}y^7$.
Step 3: Calculate $-6 \times 4 = -24$, $x^5 \times x^4 = x^9$, y^7 remains unchanged. So the second term becomes $-24x^9y^7$.
Step 4: Calculate $-6 \times 84 = -504$, x^5 remains unchanged, y^7 remains unchanged. So the third term becomes $-504x^5y^7$.
Step 5: Combine all terms to form the expression: $-42x^{12}y^7 - 24x^9y^7 - 504x^5y^7$.

10. The functions f and g are defined by the equations shown, where a is an integer constant, and a <0. If y = f(x) and y = g(x) are graphed in the xy-plane, which of the following equations displays, as a constant or coefficient, the y-coordinate of the y-intercept of the graph of the corresponding function? $f(x) = a(-4.2)^{x+29}$
$g(x) = a(-4.2)^{x} + 29$

A. f(x) only

B. g(x) only

C. f(x) and g(x)

D. Neither f(x) nor g(x)

Answer

B

Solution

Concept Check : The intent of the question is to assess the student's understanding of exponential functions, specifically how to determine the y-intercept of these functions based on their equations. Students are expected to apply their knowledge of function properties and intercepts in the context of exponential graphs.

Solution Strategy : To approach this problem, the student should first recall the definition of the y-intercept. The y-intercept is found by evaluating the function at x = 0. Therefore, the student needs to substitute x = 0 into both equations for f(x) and g(x) and simplify to find the y-intercept of each function. This will involve understanding how the constant 'a' affects the output of the functions and considering what happens when x is replaced by 0.

Quick Wins : Remember that for any function f(x), the y-intercept is found at f(0). Substitute x = 0 into both f(x) and g(x) to find their respective y-values at the intercept. Pay attention to the role of the constant 'a' and how it interacts with the base of the exponential function. Additionally, make sure to evaluate any constants or coefficients accurately after substitution.

Mistake Alert : Be cautious when dealing with negative bases in exponential functions, as this can lead to complex behavior depending on the exponent. In this case, ensure that you carefully follow the algebra when substituting x = 0. Also, double-check that you are correctly interpreting the outputs, especially since 'a' is negative, which can affect the sign of the y-intercept. Lastly, ensure you are not confusing the y-intercept with other characteristics of the graph.

SAT Know-How : This problem falls under the category of Advanced Math, specifically focusing on exponential graphs and their properties. It assesses the student's ability to analyze and interpret the y-intercept of exponential functions. Mastering this type of problem requires a solid grasp of function evaluation and the behavior of exponential functions, which are critical skills in the SAT math section.

To find the y-intercept of f(x), substitute x = 0: $f(0) = a(-4.2)^{0+29} = a(-4.2)^{29}$.
To find the y-intercept of g(x), substitute x = 0:
$g(0) = a(-4.2)^{0} + 29 = a(1) + 29 = a + 29$.
The y-coordinate of the y-intercept of f(x) is $a(-4.2)^{29}$, which is not explicitly displayed as a constant or coefficient in its equation.
The y-coordinate of the y-intercept of g(x) is a + 29. The value 29 is explicitly shown as a constant in the equation $g(x) = a(-4.2)^{x} + 29$.
Thus, the y-coordinate of the y-intercept is displayed as a constant in the function g(x).

Digital SAT

Math

3

Geometry and Trigonometry

SAT Math Geometry and Trigonometry

1. A circle in the xy-plane has its center at (-4, -7) and has a radius of 3. An equation of this circle is represented by the standard form equation $x^2 + y^2 + ax + by + c = 0$, where a, b, and c are constants. What is the value of c?

2. In the xy-plane, circle N is the graph of the equation $(x + 9)^2 + (y + 4)^2 = 16$. Circle Q has the same center as circle N but has a radius that is three times the radius of circle N. Which equation represents circle Q?

A. $(x + 9)^2 + (y + 4)^2 = 64$

B. $(x + 9)^2 + (y + 4)^2 = 128$

C. $(x + 9)^2 + (y + 4)^2 = 144$

D. $(x - 9)^2 + (y - 4)^2 = 16$

3. Circle A has a radius of 3x and circle B has a radius of 63x. The area of circle B is how many times the area of circle A?

4. In triangle XYZ, the measure of angle X is 53°, the measure of angle Y is 90°, and the measure of angle Z is $\frac{m}{2}$°. What is the value of m?

A. 60

B. 70

C. 74

D. 76

5. Triangles PQR and STU are similar. Each side length of triangle PQR is 7 times the corresponding side length of triangle STU. The area of triangle PQR is 3479 square centimeters. What is the area, in square centimeters, of triangle STU?

A. 50

B. 71

C. 90

D. 100

6. Triangles GHI and JKL are congruent, where G corresponds to J, H corresponds to K, and both angles H and K are right angles. The measure of angle G is 48°. What is the measure, in degrees, of angle L?

A. Angle L = 42°

B. Angle L = 48°

C. Angle L = 60°

D. Angle L = 90°

7. What is the equation of the circle in the xy-plane that has a center at (8, -2) and a radius of 8?

A. $(x - 8)^2 + (y + 2)^2 = 64$

B. $(x + 8)^2 + (y - 2)^2 = 64$

C. $x^2 + y^2 - 16x + 4y = 0$

D. $x^2 + y^2 - 16x - 4y - 64 = 0$

8. A hemisphere is half of a sphere. If a hemisphere has a radius of 21 inches, which of the following is closest to the volume, in cubic inches, of this hemisphere?

A. 4,800

B. 8,400

C. 15,600

D. 19,385

9. For two angles, $\angle$A and $\angle$B, it is given that cos(A) = sin(B). The measures, in degrees, of $\angle$A and $\angle$B are 4x + 6 and 2x + 30, respectively. What is the value of x?

A. 5

B. 7

C. 9

D. 11

10. Circle A has a radius of 96 millimeters (mm). Circle B has an area of $784\pi mm^2$. What is the total area, in mm^2, of circles A and B?

A. 9400π

B. 9800π

C. 10000π

D. 10200π

SAT Math Geometry and Trigonometry Solutions

1. A circle in the xy-plane has its center at (-4, -7) and has a radius of 3. An equation of this circle is represented by the standard form equation $x^2 + y^2 + ax + by + c = 0$, where a, b, and c are constants. What is the value of c?

Answer

56

Solution

Concept Check : The intent of the question is to assess the student's understanding of the equation of a circle in the standard form and their ability to manipulate it to find specific constants. The student is expected to know how to derive the equation from the center and radius of the circle, and how to rearrange it into a specific form to isolate the constant c.

Solution Strategy : To solve this problem, the student should start with the standard form of the equation of a circle, which is given by $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius. In this case, the center is (-4, -7) and the radius is 3. The student should substitute these values into the standard equation, expand it, and then rearrange it to match the provided form $x^2 + y^2 + ax + by + c = 0$ to identify the value of c.

Quick Wins : A helpful approach is to first write down the standard equation of the circle using the given center and radius. After substituting the values, carefully expand the squared terms. Keep in mind that when expanding, you'll have to distribute correctly and combine like terms. To find c, ensure that you clearly identify all the terms after rearranging the equation.

Mistake Alert : Students should be cautious while expanding the squared terms and combining constants. It's easy to make sign errors, especially when dealing with negative values. Also, remember to carefully isolate the constant c at the end of the rearrangement. Double-check your arithmetic to avoid small mistakes that could lead to an incorrect answer.

SAT Know-How : This problem falls under the category of Geometry and Trigonometry, specifically focusing on circle equations and their representation. It assesses skills related to understanding the geometry of circles, manipulating algebraic expressions, and applying formulas correctly. Mastering these concepts is crucial for efficient SAT problem-solving, as it helps in recognizing patterns and applying relevant mathematical principles.

Start with the standard form equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius.
Substitute the given center $(-4, -7)$ and radius 3: $(x - (-4))^2 + (y - (-7))^2 = 3^2$.
This simplifies to $(x + 4)^2 + (y + 7)^2 = 9$.
Expand $(x + 4)^2$ and $(y + 7)^2$:
$(x + 4)^2 = x^2 + 8x + 16$
$(y + 7)^2 = y^2 + 14y + 49$
Combine these expansions: $x^2 + 8x + 16 + y^2 + 14y + 49 = 9$.
Rearrange to the general form: $x^2 + y^2 + 8x + 14y + 16 + 49 - 9 = 0$.
Combine constants: $16 + 49 - 9 = 56$.
Thus, the equation becomes: $x^2 + y^2 + 8x + 14y + 56 = 0$.
Therefore, the value of c is 56.

2. In the xy-plane, circle N is the graph of the equation $(x + 9)^2 + (y + 4)^2 = 16$. Circle Q has the same center as circle N but has a radius that is three times the radius of circle N. Which equation represents circle Q?

A. $(x + 9)^2 + (y + 4)^2 = 64$

B. $(x + 9)^2 + (y + 4)^2 = 128$

C. $(x + 9)^2 + (y + 4)^2 = 144$

D. $(x - 9)^2 + (y - 4)^2 = 16$

Answer

C

Solution

Concept Check : The question intends to assess the student's understanding of circle equations in the Cartesian plane, specifically how to represent a circle using its standard equation format. Students should know that the standard form of a circle's equation is given by $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius. The problem also tests the ability to manipulate and scale the radius accordingly.

Solution Strategy : To approach this problem, students should first identify the center and radius of circle N from the provided equation. The center can be determined from the values inside the parentheses, while the radius can be found by taking the square root of the constant on the right side of the equation. Once the radius of circle N is established, the student should then calculate the radius of circle Q, which is three times that of circle N. Finally, the student needs to write the equation for circle Q using the same center and the new radius.

Quick Wins : Remember that the center of the circle is derived from the equation's components, which are given in the form (x + 9) and (y + 4). The center will be (-9, -4). When scaling the radius, simply multiply the original radius by 3. After finding the new radius, substitute both the center coordinates and the new radius into the standard circle equation format.

Mistake Alert : Be careful with the signs when identifying the center from the equation; they can sometimes cause confusion. Double-check the calculations when scaling the radius to ensure you multiply correctly. Additionally, when rewriting the equation, ensure that you maintain the correct format of $(x - h)^2 + (y - k)^2 = r^2$ and that you accurately substitute the center and the new radius squared.

SAT Know-How : This problem falls under the category of Geometry and Trigonometry, specifically focusing on circle equations. It assesses the student's ability to manipulate equations and understand the relationship between the center and radius of a circle. Mastery of these concepts is crucial for success in SAT problem-solving, as it demonstrates a student's proficiency in working with geometric figures and their equations.

Step 1: Determine the center of circle N from its equation $(x + 9)^2 + (y + 4)^2 = 16$, which is (-9, -4).
Step 2: Calculate the radius of circle N: $\sqrt{16} = 4$.
Step 3: Since circle Q has a radius three times that of circle N, calculate the radius of circle Q: $3 \times 4 = 12$.
Step 4: Use the standard form equation for a circle to find circle Q's equation with center (-9, -4) and radius 12: $(x + 9)^2 + (y + 4)^2 = 12^2 = 144$.

3. Circle A has a radius of 3x and circle B has a radius of 63x. The area of circle B is how many times the area of circle A?

Answer

441

Solution

Concept Check : The intent of this question is to assess the student's understanding of the formula for the area of a circle and their ability to apply it to find the ratio of the areas of two circles based on their radii. The student is expected to know the formula for the area of a circle, which is $A = \pi r^2$, and how to compute ratios.

Solution Strategy : To solve this problem, the student should first calculate the area of both circles using the given radii. This involves substituting the values of the radii (3x for circle A and 63x for circle B) into the area formula. After finding the areas, the next step is to set up a ratio of the area of circle B to the area of circle A. This will involve dividing the area of circle B by the area of circle A.

Quick Wins : Remember that when calculating the area of a circle, the radius must be squared. Be careful with your calculations to avoid errors in squaring the radius. Once you have both areas, simplify the ratio as much as possible. It can also be helpful to keep track of the π terms, as they will cancel out when forming the ratio.

Mistake Alert : A common mistake is to forget to square the radius when calculating the area. Additionally, ensure that you correctly set up the ratio and simplify it properly. Double-check to make sure you are dividing the correct quantities (area B over area A) and that you handle the variables correctly.

SAT Know-How : This problem falls under the category of Geometry and Trigonometry, specifically focusing on the area of circles and the concept of ratios. It assesses the student's ability to apply formulas and manipulate algebraic expressions. Mastering this type of problem requires a solid understanding of geometric concepts and the ability to perform algebraic operations accurately, which is an essential skill for success on the SAT.

Step 1: Find the area of circle A using the formula: $Area = \pi \times radius^2$.
$Area\ of\ circle\ A = \pi \times (3x)^2 = \pi \times 9x^2$.
Step 2: Find the area of circle B using the same formula.
$Area\ of\ circle\ B = \pi \times (63x)^2 = \pi \times 3969x^2$.
Step 3: Find the ratio of the area of circle B to circle A.
$Area\ ratio = (\pi \times 3969x^2)/(\pi \times 9x^2)$.

Step 4: Cancel out common factors (π and x^2) in the ratio.
$Area\ ratio = \frac{3969}{9}$.
Step 5: Simplify the ratio:
$\frac{3969}{9} = 441$.
Thus, the area of circle B is 441 times the area of circle A.

4. In triangle XYZ, the measure of angle X is 53°, the measure of angle Y is 90°, and the measure of angle Z is $\frac{m}{2}$°. What is the value of m?

A. 60

B. 70

C. 74

D. 76

Answer

C

Solution

Concept Check : The intent of the question is to assess the student's understanding of the properties of triangles, specifically that the sum of the interior angles in a triangle is always 180 degrees. The student is expected to know how to set up an equation based on this property to solve for the unknown angle, which is expressed in terms of m.

Solution Strategy : To approach this problem, the student should first recall that the sum of the angles in any triangle is 180 degrees. Given the measures of angles X and Y, the student should set up an equation that incorporates the measure of angle Z, which is expressed as $\frac{m}{2}$. The equation can be formed as: $53° + 90° + \frac{m}{2}° = 180°$. From here, the student can simplify the equation to isolate m and solve for its value.

Quick Wins : A good tip is to always remember the triangle angle sum property (the sum of angles in a triangle is 180°). When you have an angle expressed as a fraction or variable, substitute it into the equation carefully. It may also help to rewrite the equation step by step to avoid confusion. Keep track of your calculations and double-check your work to ensure accuracy.

Mistake Alert : Students should be cautious not to miscalculate the sum of the known angles. Adding angles incorrectly can lead to an incorrect equation. Also, when solving for m, make sure to clearly isolate the variable and not confuse it with the angles themselves. Be careful with units and ensure all angles are in degrees.

SAT Know-How : This problem falls under the category of Geometry and Trigonometry, specifically focusing on congruence, similarity, and angle relationships in triangles. It is designed to assess a student's understanding of triangle properties and their ability to set up and solve equations based on given information. Mastering these skills is crucial for success in SAT mathematics.

Step 1: Recall the triangle angle sum property: $\angle X + \angle Y + \angle Z = 180°$.
Step 2: Substitute the known values into the equation: $53° + 90° + \frac{m}{2}° = 180°$.
Step 3: Simplify the equation: $143° + \frac{m}{2}° = 180°$.
Step 4: Isolate $\frac{m}{2}°$ by subtracting 143° from both sides: $\frac{m}{2}° = 180° - 143°$.
Step 5: Calculate: $\frac{m}{2}° = 37°$.
Step 6: Solve for m by multiplying both sides by 2: $m° = 37° \times 2$.
Step 7: Calculate: $m = 74$.

5. Triangles PQR and STU are similar. Each side length of triangle PQR is 7 times the corresponding side length of triangle STU. The area of triangle PQR is 3479 square centimeters. What is the area, in square centimeters, of triangle STU?

A. 50

B. 71

C. 90

D. 100

Answer

B

Solution

Concept Check : The question tests the student's understanding of the properties of similar triangles, particularly how the ratio of side lengths relates to the ratio of areas. Students should know that if two triangles are similar, the ratio of their areas is equal to the square of the ratio of their corresponding side lengths.

Solution Strategy : To solve the problem, the student should first recognize that if triangle PQR is 7 times larger in side lengths than triangle STU, the ratio of the side lengths is 7:1. Consequently, the ratio of the areas will be the square of the side length ratio, which is 7^2 or 49:1. Then, the student will need to set up an equation to find the area of triangle STU by dividing the area of triangle PQR by 49.

Quick Wins : Remember the key property that the ratio of the areas of similar triangles is the square of the ratio of their corresponding side lengths. When calculating the area of triangle STU, ensure you perform the division carefully, and it might help to write down the relationship between the areas clearly before calculating.

Mistake Alert : Be cautious with squaring the ratio of the side lengths; it's easy to mistakenly use the side length ratio directly instead of squaring it. Also, double-check calculations when dividing the area of triangle PQR by 49 to ensure that no arithmetic errors occur.

SAT Know-How : This problem falls under geometry, specifically focusing on the concepts of similarity and area relationships in triangles. It assesses the student's ability to apply knowledge of ratios and areas, which is an essential skill in geometry. Mastering these concepts will not only help in SAT problem-solving but also in understanding broader mathematical principles.

1. Determine the linear scale factor between the triangles: The scale factor k between corresponding side lengths is 7.
2. Understand how area scales with the linear dimensions: Since the triangles are similar, the ratio of their areas is the square of the scale factor.
3. Apply the area scaling formula: $Area\ of\ STU\ =\ Area\ of\ PQR$ divided by k^2.
4. Plug in the values: $Area\ of\ STU\ =\frac{3479}{7^2}$.
5. Calculate 7^2: $7\ \times\ 7\ =\ 49$.
6. Divide 3479 by 49 to find the area of triangle STU.
7. Perform the division: $\frac{3479}{49}=\ 71$.
8. Conclude that the area of triangle STU is 71 square centimeters.

6. Triangles GHI and JKL are congruent, where G corresponds to J, H corresponds to K, and both angles H and K are right angles. The measure of angle G is 48°. What is the measure, in degrees, of angle L?

A. Angle L = 42°

B. Angle L = 48°

C. Angle L = 60°

D. Angle L = 90°

Answer

A

Solution

Concept Check : The intent of the question is to assess the student's understanding of congruent triangles and angle relationships. The student is expected to know that corresponding angles in congruent triangles are equal and that the sum of angles in any triangle is 180 degrees.

Solution Strategy : To solve this problem, the student should first recognize that since triangles GHI and JKL are congruent, the corresponding angles are equal. Given that angle H and angle K are right angles, both measure 90°. The student should then use the fact that the sum of the angles in a triangle is 180° to find the measure of angle L by calculating the remaining angle in triangle JKL after accounting for angles J and K.

Quick Wins : Remember that in congruent triangles, corresponding angles are equal. Make sure to recall that the sum of the angles in any triangle is always 180°. This can help you find missing angles when you already know some of the angles. Also, clearly label which angles correspond to which triangles to avoid confusion.

Mistake Alert : Be careful not to confuse the angles when identifying which angles correspond to one another. It's easy to mistakenly assign the wrong angle values, especially if you're writing them down. Additionally, remember that a right angle measures 90°; make sure to apply this correctly when calculating the remaining angles.

SAT Know-How : This problem falls under the category of geometry, specifically focusing on congruence and angle relationships in triangles. It assesses the student's ability to apply properties of congruent triangles and use the sum of angles in a triangle to find missing angle measures. Mastering these concepts is crucial for solving similar SAT problems effectively and efficiently.

Triangular Angle Sum Property: The sum of angles in a triangle is 180°.
Given that triangles GHI and JKL are congruent, and H corresponds to K being right angles, both measure 90°.
Since angle G corresponds to angle J, both are 48°.
Apply the triangle angle sum property to triangle JKL: J + K + L = 180°.
Substitute known values: 48° + 90° + L = 180°.
Simplify: L = 180° - 48° - 90°.
Calculate: L = 42°.
Thus, the measure of angle L is 42°.

7. What is the equation of the circle in the xy-plane that has a center at (8, -2) and a radius of 8?

A. $(x - 8)^2 + (y + 2)^2 = 64$

B. $(x + 8)^2 + (y - 2)^2 = 64$

C. $x^2 + y^2 - 16x + 4y = 0$

D. $x^2 + y^2 - 16x - 4y - 64 = 0$

Answer

A

Solution

Concept Check : The intent of the question is to assess the student's understanding of the standard form of the equation of a circle. The student is expected to know how to apply the formula for the equation of a circle given its center and radius, which is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius.

Solution Strategy : To solve the problem, the student should start by identifying the center coordinates (h, k) from the given point (8, -2) and recognize that the radius r is given as 8. Then, they should substitute these values into the standard form of the circle's equation and simplify it to present the final equation.

Quick Wins : Remember the standard form of a circle's equation: $(x - h)^2 + (y - k)^2 = r^2$. Identify the center (h, k) and radius r clearly before substituting them into the formula. It might help to write the formula down first and then fill in the values step by step to avoid confusion.

Mistake Alert : Be careful when substituting the center values; the signs are important (use a minus sign for h and k in the formula). Also, ensure that you square the radius correctly, as missing this step could lead to an incorrect equation.

SAT Know-How : This problem falls under the category of Geometry and Trigonometry, specifically focusing on circle equations. It assesses the student's ability to apply the formula for a circle based on its center and radius. Mastering this type of problem requires familiarity with the geometric concepts involved and careful attention to detail when substituting values into the equation.

1. Recall the standard form of a circle's equation: $(x-h)^2 + (y-k)^2 = r^2$.
2. Identify the values of h and k from the center (8, -2), giving h = 8 and k = -2.
3. Substitute h = 8, k = -2, and r = 8 into the standard equation.

4. The equation becomes $(x-8)^2 + (y+2)^2 = 8^2$.
5. Calculate $8^2 = 64$.
6. Therefore, the equation of the circle is $(x-8)^2 + (y+2)^2 = 64$.

8. A hemisphere is half of a sphere. If a hemisphere has a radius of 21 inches, which of the following is closest to the volume, in cubic inches, of this hemisphere?

A. 4,800

B. 8,400

C. 15,600

D. 19,385

Answer

D

Solution

Concept Check : The question tests the student's understanding of the formula for the volume of a sphere and how to apply it to find the volume of a hemisphere. Students are expected to know that the volume of a sphere is found using the formula $V = \frac{4}{3}\pi r^3$, and since a hemisphere is half of a sphere, they will need to divide the result by 2.

Solution Strategy : To approach this problem, students should first recall the formula for the volume of a sphere. Then, they should substitute the given radius (21 inches) into the formula to calculate the volume of the full sphere. After finding that volume, they will divide the result by 2 to find the volume of the hemisphere. It is important to ensure that they are using the correct value of π, which is often approximated as 3.14 or $\frac{22}{7}$, depending on the level of precision required by the problem.

Quick Wins : A useful tip is to remember that the volume of a hemisphere is half the volume of a sphere, so you can quickly find the volume of the sphere and then divide by 2. Also, keep in mind that when working with the radius, you will be cubing it, which can sometimes lead to larger numbers, so be careful with your calculations. Using a calculator can help avoid arithmetic mistakes, especially with the π value.

Mistake Alert : Students should be careful not to confuse the formulas for different shapes. Ensure that you correctly apply the formula for the volume of a sphere and remember to divide by 2 for the hemisphere. Also, double-check the calculations when cubing the radius, as this is a common area for errors. Lastly, be mindful of rounding; if the problem asks for the closest estimate, ensure your final answer is rounded appropriately.

SAT Know-How : This problem is a geometry and trigonometry question focused on finding the volume of a hemisphere. It assesses the student's ability to apply formulas for volume accurately and understand the relationship between a sphere and a hemisphere. Mastering these concepts is crucial for solving similar SAT problems efficiently and accurately.

Step 1: Calculate the volume of the sphere using $V = \frac{4}{3}\pi r^3$.

With $r = 21$, we have $V = \frac{4}{3}\pi(21)^3$.

Step 2: Calculate 21^3.
$21 \times 21 = 441$
$441 \times 21 = 9261$
Step 3: Substitute $21^3 = 9261$ into the volume formula.
$V = \frac{4}{3}\pi \times 9261$
Simplify: $V = \frac{37044}{3}\pi$
Step 4: Calculate the volume of the hemisphere.
Hemisphere volume = $\frac{1}{2} \times \frac{37044}{3}\pi$
Simplify: $Hemisphere\ volume = \frac{18522}{3}\pi$
Step 5: Approximate π to 3.14.
Hemisphere volume $\approx \frac{18522}{3} \times 3.14$
Calculate: $\frac{18522}{3} = 6174$
$6174 \times 3.14 = 19385.16$
Step 6: Round 19385.16 to the nearest integer.
$Hemisphere\ volume \approx 19385 cubic\ inches$.

9. For two angles, ∠A and ∠B, it is given that cos(A) = sin(B). The measures, in degrees, of ∠A and ∠B are 4x + 6 and 2x + 30, respectively. What is the value of x?

A. 5

B. 7

C. 9

D. 11

Answer

C

Solution

Concept Check : The question is designed to assess the student's understanding of the relationship between cosine and sine functions, particularly how they relate angles. Students should know that cos(A) = sin(B) implies that A and B are complementary angles, meaning A + B = 90 degrees. Additionally, students should be able to set up and solve an equation based on the expressions given for angles A and B.

Solution Strategy : To approach this problem, students should first recognize that if cos(A) = sin(B), then the angles A and B are complementary. This means that the sum of the measures of angles A and B should equal 90 degrees. Therefore, the student should set up the equation: (4x + 6) + (2x + 30) = 90. Next, the student would combine like terms and solve for x.

Quick Wins : When solving problems involving angle relationships, always remember to look for complementary or supplementary angles. In this case, knowing that cos(A) = sin(B) leads to the conclusion about the angles summing to 90 degrees is crucial. Additionally, when setting up the equation, make sure to combine like terms carefully and double-check your arithmetic before solving for x.

Mistake Alert : Be cautious when combining like terms and ensure that the equation is set up correctly. It's easy to make a mistake in adding the constants or coefficients, which can lead to an incorrect solution. Also, ensure that the final value for x is reasonable within the context of angle measures, as angles must fall within the range of 0 to 180 degrees.

SAT Know-How : This problem falls under the category of Geometry and Trigonometry, specifically focusing on the concepts of angle relationships and trigonometric functions. It assesses the student's ability to apply knowledge of complementary angles and how to translate that into algebraic equations. Mastery of these skills is essential for SAT problem-solving, as it demonstrates the ability to connect geometric principles with algebraic manipulation.

Start with the identity cos(A) = sin(B), which implies A = 90° - B.
Substitute the given expressions: 4x + 6 = 90 - (2x + 30)
Simplify the equation: 4x + 6 = 90 - 2x - 30
Combine like terms: 4x + 6 = 60 - 2x
Add 2x to both sides: 4x + 2x + 6 = 60
Simplify: 6x + 6 = 60
Subtract 6 from both sides: 6x = 54
Divide by 6: x = 9
Thus, the value of x is 9.

10. Circle A has a radius of 96 millimeters (mm). Circle B has an area of $784\pi mm^2$. What is the total area, in mm^2, of circles A and B?

A. 9400π

B. 9800π

C. 10000π

D. 10200π

Answer

C

Solution

Concept Check : The intent of this question is to assess the student's understanding of the concept of area, specifically the area of circles. Students are expected to know the formula for the area of a circle, which is $A = \pi r^2$, and how to apply it to find the area of Circle A using its radius and Circle B using its given area.

Solution Strategy : To solve this problem, the student should first calculate the area of Circle A using its radius of 96 mm. They will apply the formula $A = \pi r^2$, substituting r with 96 mm. Next, they will consider Circle B, which already provides its area as $784\pi\ mm^2$. The final step will involve adding the areas of Circle A and Circle B together to find the total area.

Quick Wins : Remember the formula for the area of a circle. When calculating the area of Circle A, carefully square the radius before multiplying by π. For Circle B, since the area is already given in terms of π, you can directly add it to the area of Circle A once you compute it. It might be helpful to express both areas in a similar format before adding them.

Mistake Alert : Be cautious while squaring the radius for Circle A; it's easy to make a mistake in calculations. Double-check your multiplication and ensure that you are adding like terms (both areas should be in mm^2). Also, be careful not to confuse the radius with the area when dealing with Circle B.

SAT Know-How : This problem falls under the category of Geometry and Trigonometry, specifically focusing on the area of circles. It assesses the student's ability to apply the area formula effectively and perform basic arithmetic operations. Mastery of these skills is crucial for solving SAT problems efficiently, as they often require quick calculations and a clear understanding of geometric concepts.

1. Calculate the area of Circle A using the formula for the area of a circle, which is $A = \pi r^2$.
2. Given that the radius of Circle A is 96 mm, substitute it into the formula: $A = \pi(96)^2$.
3. Calculate 96 squared: $96 \times 96 = 9216$.
4. Thus, the area of Circle A is 9216π square millimeters.
5. Add the area of Circle A and Circle B: $9216\pi + 784\pi$.
6. Perform the addition: $9216\pi + 784\pi = 10000\pi$.
7. Therefore, the total area of circles A and B is 10000π square millimeters.

4

Digital SAT

Math

Problem Solving and Data Analysis

SAT Math Problem Solving and Data Analysis

1. In a community consisting of 190 members, the members were surveyed about their ethical views, and their opinions were categorized into three groups as shown in the frequency table: Group A had 40 members, Group B had 70 members, and Group C had 80 members. If one member is selected at random, what is the probability that the selected member belongs to Group A?

name	frequency
A	40
B	70
C	80

A. $\frac{4}{19}$

B. $\frac{2}{9}$

C. $\frac{1}{4}$

D. $\frac{1}{2}$

2. The population of Town B increased by 17% from 2020 to 2021. If the population of Town B in 2021 is represented as k times the population in 2020, what is the value of k?

A. 0.17

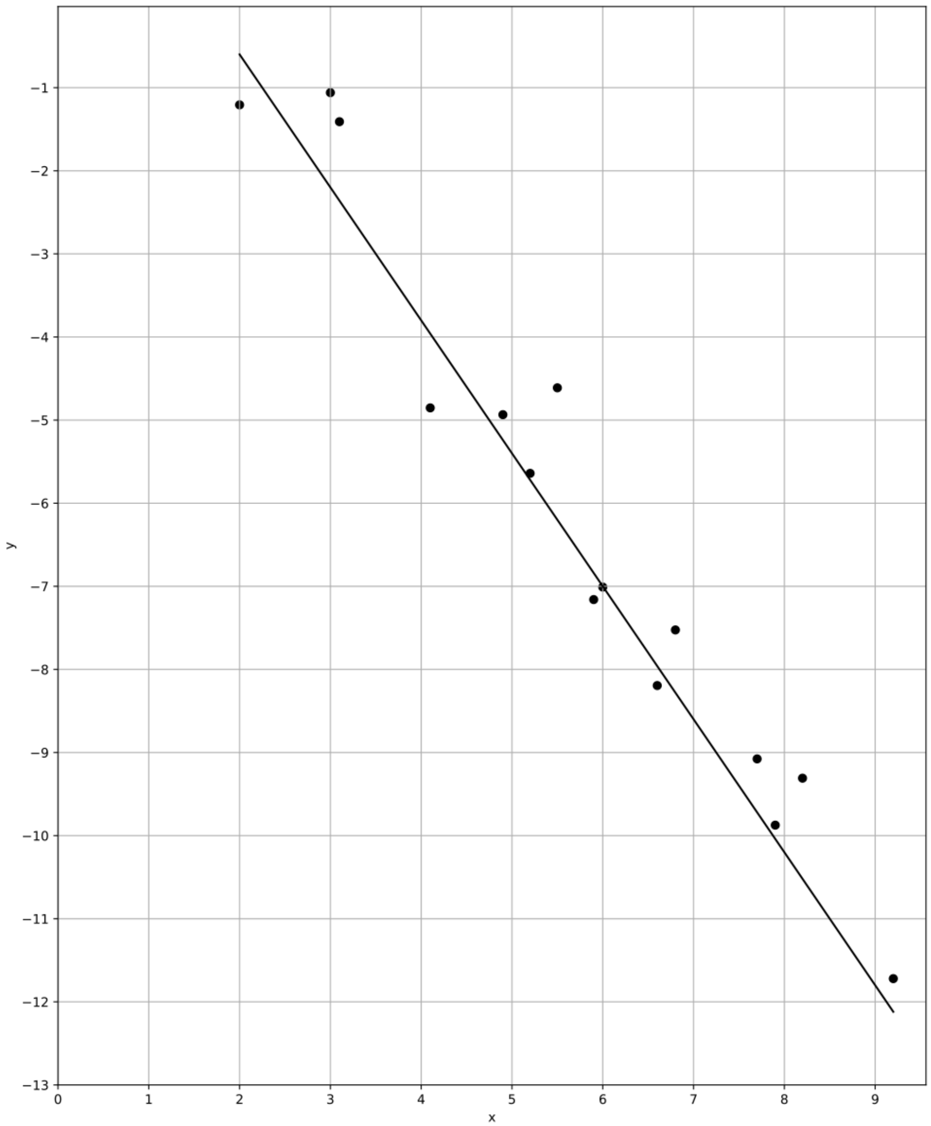
B. 0.83

C. 1.17

D. 1.34

3. What is the median of the data set shown? data set = [64, 15, 54, 92, 90, 31]

4. Which of the following equations best represents the line of best fit shown in the scatter plot?



A. y = 2.6 - 1.6x

B. y = 2.6 + 1.6x

C. y = -2.6 + 1.6x

D. y = -2.6 - 1.6x

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5. The positive number a is 6600% of the number c, and c is 62% of the number b. If a - b = wc where w is a constant, what is the value of w?

6. The ratio x to y is equivalent to the ratio 5 to 7. If y = 50t, what is the value of x in terms of t?

A. $\frac{1}{10}t$

B. 25t

C. $\frac{250}{7}t$

D. $\frac{7}{250}t$

7. The table shows the distribution of voters for two candidates, A and B. If a voter is selected at random, what is the probability that the voter supports candidate A, given that the voter is from Type A? (Express your answer as a decimal or fraction, not as a percent.)

name	Type A	Type B	Total
A	72	77	149
B	29	37	66
Total	101	114	215

8. In a smart home system, 45% of the devices are connected to the Internet. If 900 devices are currently connected, what is the total number of devices in the system?

A. 1800

B. 2000

C. 2200

D. 2500

9. A city has a total wealth of 3,299,940 dollars. After accounting for the wealth of 3,036,600 dollars in the top 10% of earners, the remaining wealth is distributed evenly among the other residents. If there are 63 remaining residents, how much wealth does each of these residents receive on average?

A. $4, 000

B. $4, 180

C. $4, 200

D. $4, 500

10. At a political rally, there are a total of 780 attendees. Each attendee can be categorized as either a voter supporting Candidate X, a voter supporting Candidate Y, or undecided. If the probability of randomly selecting a voter supporting Candidate X is 0.55 and the probability of selecting a voter supporting Candidate Y is 0.25, how many attendees are classified as undecided?

A. 120

B. 135

C. 156

D. 180

SAT Math Problem Solving and Data Analysis Solutions

1. In a community consisting of 190 members, the members were surveyed about their ethical views, and their opinions were categorized into three groups as shown in the frequency table: Group A had 40 members, Group B had 70 members, and Group C had 80 members. If one member is selected at random, what is the probability that the selected member belongs to Group A?

name	frequency
A	40
B	70
C	80

A. $\frac{4}{19}$

B. $\frac{2}{9}$

C. $\frac{1}{4}$

D. $\frac{1}{2}$

Answer

A

Solution

Concept Check : The intent of the question is to assess the student's understanding of basic probability concepts, specifically how to calculate the probability of an event occurring based on the relative frequency of that event in a given population. Students should know how to use the formula for probability, which is the number of favorable outcomes divided by the total number of outcomes.

Solution Strategy : To approach this problem, the student should first recognize that they need to find the probability of selecting a member from Group A. The key steps involve identifying the number of members in Group A and the total number of members in the community. The student should then set up the probability formula: $P(Group\ A) \ = \frac{Number\ of\ members\ in\ Group\ A}{Total\ number\ of\ members}$.

Quick Wins : When calculating probability, always ensure that you correctly identify the total number of outcomes. In this case, it's the total number of community members. Also, make sure to double-check that you have the correct number for Group A. It may help to write down the formula and plug the numbers in step-by-step to avoid confusion.

Mistake Alert : Students should be cautious not to confuse the sizes of the groups. Ensure you do not mistakenly add the numbers of the groups together, as this could lead to an incorrect total. Also, be careful with the arithmetic; double-check your calculations to avoid simple errors that could change the outcome.

SAT Know-How : This problem falls under the category of Problem Solving and Data Analysis, specifically focusing on probability and relative frequency. It assesses the student's ability to calculate the probability of an event based on given data, which is a fundamental skill in statistics. Mastering such problems is essential for success on the SAT, as it helps students develop critical thinking and analytical skills necessary for interpreting data.

Identify the total number of members in the community, which is 190.
Determine the number of members in Group A, which is 40.
The probability of selecting a member from Group A is the ratio of Group A members to the total members.
Calculate the probability: $Probability = \frac{Number\ of\ Group\ A\ members}{Total\ number\ of\ members} = \frac{40}{190}$.
Simplify the fraction $\frac{40}{190}$ by dividing both the numerator and the denominator by their greatest common divisor, which is 10.
Simplified, $\frac{40}{190}$ becomes $\frac{4}{19}$.

2. The population of Town B increased by 17% from 2020 to 2021. If the population of Town B in 2021 is represented as k times the population in 2020, what is the value of k?

A. 0.17

B. 0.83

C. 1.17

D. 1.34

Answer

C

Solution

Concept Check : The question aims to assess the student's understanding of exponential growth and percentage increase. Students should know how to express growth as a factor of the original amount and apply the concept of percentages in the context of population growth.

Solution Strategy : To solve the problem, students should start by understanding that a 17% increase can be represented as a multiplication factor. They need to express the relationship between the population in 2020 and 2021 using the percentage increase formula. This will involve converting the percentage to a decimal and applying it to the original population figure.

Quick Wins : Remember that a 17% increase means that the new population is 117% of the original population. To find the factor 'k', convert 17% to a decimal (0.17) and add it to 1. This will give you the multiplier that represents the population in 2021 relative to 2020. Practice converting percentages to decimal form, as it is a common step in many problems.

Mistake Alert : Be careful with the conversion between percentage and decimal. A common mistake is to forget to add 1 when calculating the total percentage after an increase. Ensure you clearly distinguish between the original population and the increased population to avoid confusion.

SAT Know-How : This problem belongs to the category of Problem Solving and Data Analysis, specifically focusing on linear and exponential growth. It tests the student's ability to apply knowledge of percentage increases in real-world contexts. Mastering such problems will enhance your skills in interpreting data trends and applying mathematical concepts effectively.

Step 1: Express the percentage increase as a decimal: 17% = 0.17.
Step 2: Calculate the population in 2021 in terms of the population in 2020:
Population in 2021 = P + 0.17P = 1.17P.
Step 3: Express the population in 2021 as k times the population in 2020:
$k = \frac{Population\ in\ 2021}{Population\ in\ 2020} = \frac{1.17P}{P}$.
Step 4: Simplify the expression:
k = 1.17.

3. What is the median of the data set shown? data set = [64, 15, 54, 92, 90, 31]

Answer

59

Solution

Concept Check : The intent of this question is to assess the student's understanding of how to find the median of a given data set. The student is expected to know the concept of median as a measure of central tendency, which requires them to arrange the data in order and identify the middle value(s).

Solution Strategy : To solve this problem, the student should first arrange the data set in ascending order. Once the data is sorted, the student will need to determine the median by identifying the middle value. If there is an odd number of values, the median is the middle number. If there is an even number of values, the median is the average of the two middle numbers.

Quick Wins : 1. Always start by sorting the data in increasing order. This will help you clearly see the middle value(s). 2. Remember that the median is the middle value: for an odd number of data points, it's the single middle value, and for an even number, it's the average of the two middle values. 3. Use a pencil and paper or a calculator to avoid mistakes when calculating averages.

Mistake Alert : 1. Be careful to sort the numbers correctly; mixing them up will lead to the wrong median. 2. Don't forget to check whether the number of data points is odd or even, as this affects how you calculate the median. 3. Ensure that you calculate the average correctly if needed, especially when dealing with two middle values.

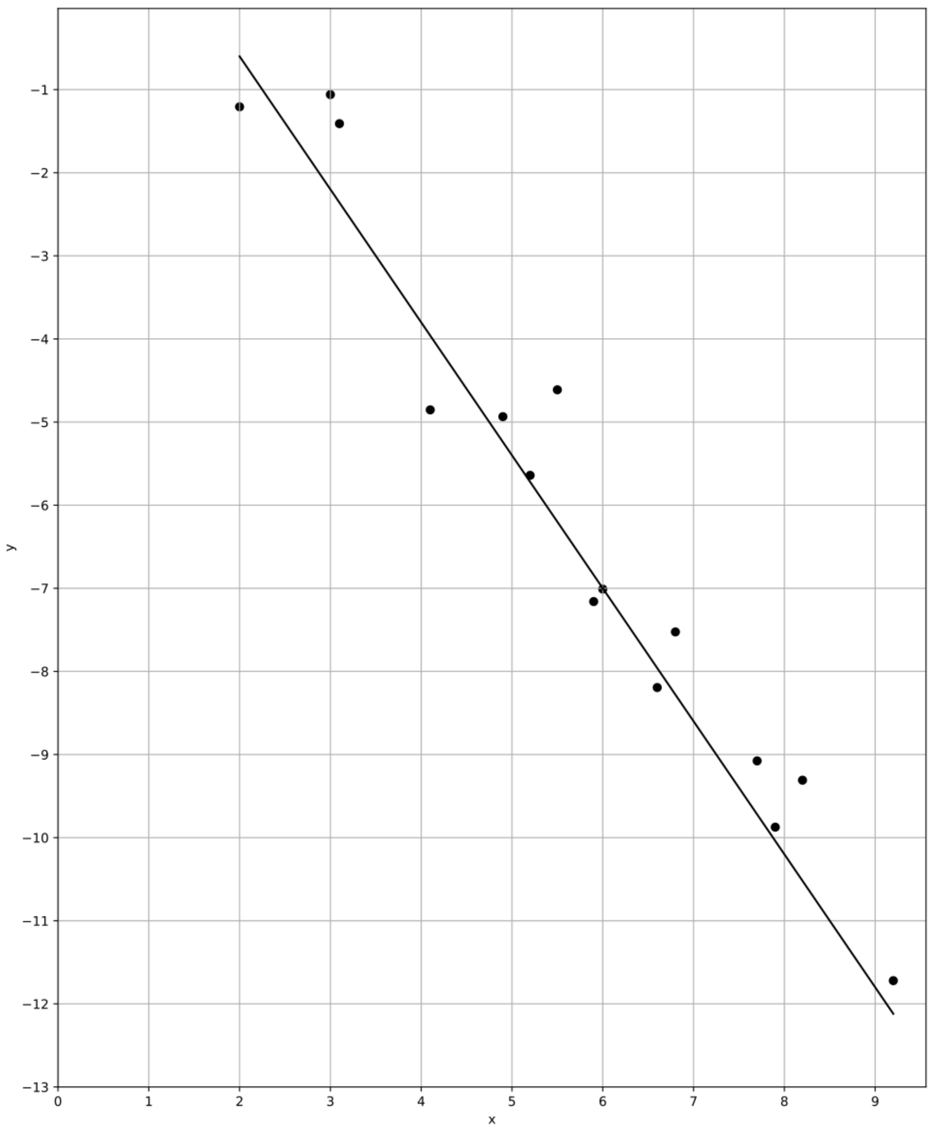
SAT Know-How : This problem falls under the category of Problem Solving and Data Analysis, specifically focusing on understanding the center, spread, and shape of distribution. It tests the student's skills in organizing data and calculating the median, a fundamental concept in statistics. Mastering this skill is essential for effectively analyzing data sets, which is a key component of the SAT math section.

Step 1: Arrange the data set in ascending order: [15, 31, 54, 64, 90, 92].
Step 2: Identify the middle numbers of the data set since it has an even number of points.
Step 3: The middle numbers are the 3rd and 4th numbers in the ordered list, which are 54 and 64.
Step 4: Calculate the median by finding the average of these two middle numbers:

$\frac{54+64}{2} = \frac{118}{2} = 59.$

Conclusion: The median of the data set is 59.

4. Which of the following equations best represents the line of best fit shown in the scatter plot?



A. y = 2.6 - 1.6x

B. y = 2.6 + 1.6x

C. y = -2.6 + 1.6x

D. y = -2.6 - 1.6x

Answer

A

Solution

Concept Check : The intent of this question is to assess the student's understanding of linear regression and their ability to interpret scatter plots. Students are expected to recognize the characteristics of a line of best fit and how various linear equations can represent the data points visually displayed in the scatter plot.

Solution Strategy : To approach this problem, students should first analyze the scatter plot for the overall trend of the data points. They should consider whether the points are clustering around a particular line and determine the slope and y-intercept of that line. Additionally, students may want to evaluate each provided equation to see which one aligns most closely with the observed data trend.

Quick Wins : When examining the scatter plot, look for patterns such as positive or negative correlation. Pay attention to how steep the line should be, and note the points where the line crosses the y-axis. Using the coordinates of any two points from the line, students can calculate the slope (rise/run) to help confirm which equation might be correct. If given multiple options, graphing them can also help visualize which best fits the scatter plot.

Mistake Alert : Students should be cautious of misinterpreting the scatter plot, especially by focusing only on outlier points that may not follow the overall trend. Additionally, they should avoid making assumptions about the slope or intercept without checking their calculations against multiple points. Be careful not to rush into selecting an answer without verifying how well it fits with the data points.

SAT Know-How : This problem falls under the 'Problem Solving and Data Analysis' category, specifically focusing on scatter plots and finding the line of best fit. It assesses skills such as interpreting visual data, understanding linear relationships, and applying concepts of slope and intercept. Mastering this type of question can enhance a student's ability to analyze data effectively on the SAT.

1. Analyze the line of best fit on the scatter plot and determine its slope.
- From the provided graph, the plot shows a line that appears to rise from left to right, indicating a positive slope.
2. Determine the y-intercept where the line crosses the y-axis.
- Based on the information, the y-intercept is positive as the line crosses above the origin on the graph.
3. Evaluate each option to see which equation fits the observed characteristics of the line:
- Option A: $y = 2.6 - 1.6x$ (positive y-intercept, negative slope)
- Option B: $y = 2.6 + 1.6x$ (positive y-intercept, positive slope)
- Option C: $y = -2.6 + 1.6x$ (negative y-intercept, positive slope)
- Option D: $y = -2.6 - 1.6x$ (negative y-intercept, negative slope)

4. Based on the negative slope and positive y-intercept observed from the graph, Option A is the correct equation.

5. The positive number a is 6600% of the number c, and c is 62% of the number b. If a - b = wc where w is a constant, what is the value of w?

Answer

64.3871

Solution

Concept Check : The intent of this question is to assess the student's understanding of percentages and their ability to manipulate relationships between different variables. The student is expected to know how to express one quantity as a percentage of another and how to set up equations based on these relationships.

Solution Strategy : To solve this problem, the student should start by converting the percentage statements into equations. For instance, since a is 6600% of c, this can be expressed as a = 66c (because 6600% means 66 times c). Similarly, since c is 62% of b, this can be expressed as c = 0.62b. Once these relationships are established, the student can substitute and rearrange the equations to find the value of w in the equation a - b = wc.

Quick Wins : When working with percentages, remember to convert them into decimal form for calculations. Also, keep track of the relationships between the variables step by step. Writing down each conversion clearly will help you avoid confusion. It may also be helpful to substitute the expressions derived from the percentage relationships into each other to simplify the problem.

Mistake Alert : Be cautious with the conversion of percentages to decimal form; a common mistake is misinterpreting the percentage value (e.g., confusing 6600% with 66.0). Ensure that you apply the percentage correctly and double-check the algebraic manipulations to avoid errors in signs or values.

SAT Know-How : This problem is categorized under Problem Solving and Data Analysis, specifically focusing on basic percentages. It tests the student's ability to understand and apply relationships among variables through percentages and to manipulate these relationships algebraically. Mastery of such problems requires a good grasp of percentage calculations and algebraic skills, which are essential for success on the SAT.

1. Express the given relationships as equations:
- Since a is 6600% of c, a = 66c (because 6600% $= \frac{6600}{100} = 66$).
- Since c is 62% of b, c = 0.62b or equivalently, $b = \frac{c}{0.62}$.
2. Substitute these equations into the equation a - b = wc:

- Substitute a = 66c and $b = \frac{c}{0.62}$ into a - b = wc.
- Therefore, 66c - (c/0.62) = wc.
3. Simplify the equation:
- To combine terms, express b in terms of c by converting the division to a multiplication: $b = c \times \frac{1}{0.62} = c \times 1.6129032258$.
- Replace b in the equation: 66c - 1.6129032258c = wc.
- Combine like terms: 66c - 1.6129032258c = 64.3870967742c.
4. Solve for w:
- Since 64.3870967742c = wc, then w = 64.3870967742.
5. Round w to four decimal places if necessary:
- w ≈ 64.3871.

6. The ratio x to y is equivalent to the ratio 5 to 7. If y = 50t, what is the value of x in terms of t?

A. $\frac{1}{10}$t

B. 25t

C. $\frac{250}{7}$t

D. $\frac{7}{250}$t

Answer

C

Solution

Concept Check : The intent of the question is to assess the student's understanding of ratios and their ability to set up and manipulate equations based on proportional relationships. The student should know how to express one variable in terms of another using given ratios.

Solution Strategy : To approach this problem, the student should first recognize that the statement 'the ratio x to y is equivalent to the ratio 5 to 7' means that $\frac{x}{y} = \frac{5}{7}$. From there, the student can substitute the given value of y (which is 50t) into the equation. The goal is to solve for x in terms of the variable t.

Quick Wins : When dealing with ratios, it can be helpful to cross-multiply to eliminate the fractions. Remember to maintain the equality of the ratios throughout your calculations. Additionally, breaking down the problem step-by-step can make it easier to follow the logic and avoid mistakes.

Mistake Alert : Be careful not to confuse the ratio components when substituting values. It's also important to double-check the arithmetic when multiplying or dividing, as small errors can lead to incorrect final answers. Ensure you maintain the correct variable relationships throughout the calculations.

SAT Know-How : This problem falls under the category of Problem Solving and Data Analysis, specifically focusing on ratios, rates, and proportions. It assesses the student's ability to manipulate equations and express one variable in terms of another. Mastering these skills is crucial for success in the SAT, particularly in the math sections where proportional reasoning is frequently tested.

1. Express the given ratio as an equation: x : y = 5 : 7, which means 7x = 5y.

2. Substitute the given expression for y (y = 50t) into the equation: $7x = 5(50t)$.
3. Perform the multiplication: $7x = 250t$.
4. Solve for x by dividing both sides by 7: $x = \frac{250}{7}t$.
5. Therefore, the value of x in terms of t is $x = \frac{250}{7}t$.

7. The table shows the distribution of voters for two candidates, A and B. If a voter is selected at random, what is the probability that the voter supports candidate A, given that the voter is from Type A? (Express your answer as a decimal or fraction, not as a percent.)

name	Type A	Type B	Total
A	72	77	149
B	29	37	66
Total	101	114	215

Answer

$\frac{72}{101}$

Solution

Concept Check : The intent is to assess the student's understanding of conditional probability and the ability to interpret data from a table. Students should be familiar with the concept of conditional probability, which is the probability of an event occurring given that another event has already occurred.

Solution Strategy : To solve this problem, the student should first identify the relevant values from the table related to candidate A and the voters from Type A. The calculation will involve determining the number of voters that support candidate A among those voters from Type A, and then dividing this number by the total number of voters from Type A to find the conditional probability.

Quick Wins : When dealing with tables, it's crucial to carefully read the labels and understand what each row and column represents. To find the conditional probability, remember the formula $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$, where P(A|B) is the probability of A given B, P(A and B) is the number of favorable outcomes, and P(B) is the total number of outcomes for B. Make sure to simplify your fraction if necessary.

Mistake Alert : Be careful not to confuse the total number of voters with the number of voters from Type A when calculating the probabilities. Double-check the values you extract from the table to avoid simple arithmetic errors. Ensure that you are using the correct numerator and denominator when applying the conditional probability formula.

SAT Know-How : This problem falls under the category of Problem Solving and Data Analysis, specifically focusing on conditional probability. It assesses the student's ability to interpret data from a table and apply the concept of conditional probability. Understanding how to extract relevant data and apply the appropriate formulas is essential for success in SAT math problem-solving.

Identify the relevant quantities for the probability calculation. We need the number of voters supporting candidate A from Type A and the total number of Type A voters. The probability that a voter supports candidate A, given that the voter is from Type A, is calculated as the ratio of the number of Type A voters supporting candidate A to the total number of Type A voters.
Number of voters supporting candidate A from Type A = 72
Total number of Type A voters = 101
Probability that a voter supports candidate A given $Type\ A\ = \frac{72}{101}$
Since 72 and 101 have no common factors other than 1, the fraction $\frac{72}{101}$ is already in its simplest form.

8. In a smart home system, 45% of the devices are connected to the Internet. If 900 devices are currently connected, what is the total number of devices in the system?

A. 1800

B. 2000

C. 2200

D. 2500

Answer

B

Solution

Concept Check : The question intends to assess the student's understanding of percentages and their ability to translate a word problem into a mathematical equation. The student is expected to know how to work with percentages and apply the concept to find the total number of devices based on the given percentage and quantity.

Solution Strategy : To solve this problem, the student should first identify the relationship between the percentage of connected devices and the total number of devices. The equation can be formed by utilizing the known percentage (45%) and the number of connected devices (900). The student should think about how to express the total devices in terms of the known quantity and the percentage.

Quick Wins : Start by converting the percentage into a decimal form (for 45%, this would be 0.45). Then, set up the equation where 0.45 of the total number of devices equals 900. This can be expressed as: Total Devices × 0.45 = 900. From there, you can solve for the total number of devices by isolating the variable.

Mistake Alert : Be careful not to confuse the percentage with the actual number of devices. It's important to remember that 45% does not mean 45 devices; it means 45 out of every 100 devices. Additionally, make sure to perform the arithmetic operations correctly when isolating the total number of devices to avoid errors.

SAT Know-How : This problem is a classic example of a percentage word problem, which falls under the category of Problem Solving and Data Analysis in the SAT. It assesses the student's ability to interpret percentages and apply them in real-world contexts. Mastering this type of problem requires a solid understanding of percentages and the ability to set up and solve equations, which are essential skills for success on the SAT.

Let x represent the total number of devices in the system.
45% of x is equal to 900 devices. This can be expressed as the equation:
$0.45 \times x = 900.$
To find x, divide both sides of the equation by 0.45: $x = \frac{900}{0.45}$.
Calculate 900 divided by 0.45:
$x = \frac{900}{0.45} = 2000.$
Thus, the total number of devices in the system is 2000.

9. A city has a total wealth of 3,299,940 dollars. After accounting for the wealth of 3,036,600 dollars in the top 10% of earners, the remaining wealth is distributed evenly among the other residents. If there are 63 remaining residents, how much wealth does each of these residents receive on average?

A. $\$4,000$

B. $\$4,180$

C. $\$4,200$

D. $\$4,500$

Answer

B

Solution

Concept Check : The intent of the question is to assess the student's understanding of ratios, rates, and proportions, particularly in the context of distributing a total amount of wealth among a group of individuals. The student should be familiar with basic arithmetic operations, particularly subtraction and division, and understand how to set up a problem involving averages.

Solution Strategy : To solve this problem, the student should first determine the total wealth that is not held by the top 10% of earners by subtracting the wealth of the top earners from the total wealth of the city. Then, the student should divide this remaining wealth by the number of remaining residents to find the average wealth per resident. It is important to keep track of the calculations step by step to avoid confusion.

Quick Wins : Start by clearly identifying the total wealth and the amount held by the top earners. Use clear notation for each step: label the total wealth, the wealth of the top earners, and the remaining wealth. When dividing, ensure you double-check the division for accuracy. If you have a calculator, it can help to quickly verify your results.

Mistake Alert : Be careful with the arithmetic operations, especially when subtracting and dividing numbers. It is easy to make a mistake in the calculations, so double-check each step. Additionally, ensure that you are dividing the remaining wealth by the correct number of residents, as miscounting this can lead to incorrect results.

SAT Know-How : This problem falls under the category of Problem Solving and Data Analysis, specifically focusing on ratios, rates, and proportions. It assesses the student’s ability to perform basic arithmetic operations and apply them to real-world scenarios. By understanding how to break down the problem into manageable steps and checking their work, students can develop their problem-solving skills effectively, which is crucial for success in the SAT.

1. Calculate the remaining wealth after accounting for the top 10%.
Remaining wealth = Total wealth - Wealth of top 10%
Remaining wealth = 3,299,940 - 3,036,600
Remaining wealth = 263,340 dollars
2. Determine the average wealth received by each of the 63 remaining residents.

$$Average\ wealth\ per\ remaining\ resident\ = \frac{Remaining\ wealth}{Number\ of\ remaining\ residents}$$

$$Average\ wealth\ per\ remaining\ resident\ = \frac{263{,}340}{63}$$

Divide 263,340 by 63 to find the average.
Average wealth per remaining resident = 4,180 dollars

10. At a political rally, there are a total of 780 attendees. Each attendee can be categorized as either a voter supporting Candidate X, a voter supporting Candidate Y, or undecided. If the probability of randomly selecting a voter supporting Candidate X is 0.55 and the probability of selecting a voter supporting Candidate Y is 0.25, how many attendees are classified as undecided?

A. 120

B. 135

C. 156

D. 180

Answer

C

Solution

Concept Check : The intent of this question is to assess the student's understanding of basic probability concepts, particularly how to apply probabilities to a total population to find unknown quantities. The student is expected to know how to calculate the proportion of a group based on given probabilities and how to perform simple arithmetic operations.

Solution Strategy : To solve the problem, the student should first identify the total number of attendees and the probabilities given for Candidates X and Y. They will need to calculate the total number of attendees who support each candidate by multiplying the total number of attendees by the respective probabilities. After finding those numbers, the student will subtract the sum of these values from the total number of attendees to determine the number of undecided attendees.

Quick Wins : Start by clearly defining the total number of attendees and the probabilities for each candidate. Remember to convert the probabilities into actual counts by using multiplication. Keep your calculations organized, and double-check your work as you go along. It may help to write down the equation you are using to find the number of undecided voters, as this can clarify your thought process.

Mistake Alert : Be careful with the arithmetic calculations, especially when multiplying the total number of attendees by the probabilities. Also, ensure that you are correctly accounting for all parts of the problem—after finding the number of supporters for both candidates, remember to correctly subtract from the total number of attendees to find the undecided group. Watch out for rounding errors or misinterpretation of the probability values.

SAT Know-How : This problem falls under the category of Problem Solving and Data Analysis, specifically focusing on probability and relative frequency. It tests the student's ability to apply probability concepts to real-world scenarios and perform basic arithmetic operations. Mastering such problems is essential for developing skills in logical reasoning and quantitative analysis, which are critical for success on the SAT.

First, calculate the probability of selecting an undecided voter. This is given by subtracting the sum of the probabilities of the voters for Candidate X and Candidate Y from 1.
Probability of undecided = 1 - 0.55 - 0.25 = 0.20
Now, calculate the number of undecided attendees by multiplying the probability of an undecided voter with the total number of attendees.
Number of undecided attendees = Probability of undecided × Total attendees
Number of undecided attendees = 0.20 × 780 = 156

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