Digital

SAT Math

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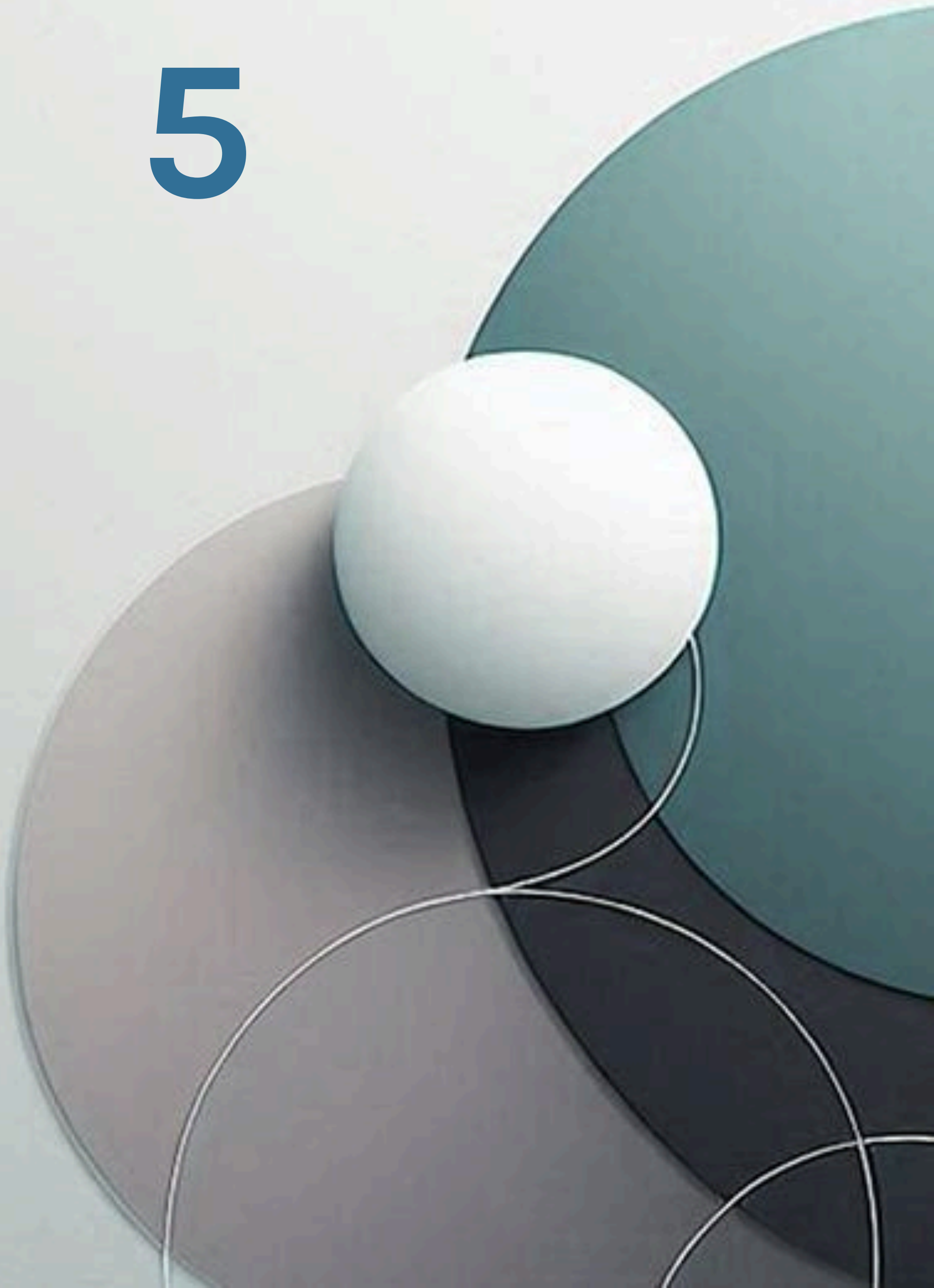

SAT Math Problems

1. The table shows two values of x and their corresponding values of y. The graph of the linear equation representing this relationship passes through the point $(\frac{1}{2}, b)$. What is the value of b?

x	y
-5	20
10	-25

2. If $5(y - 2) = 2(y - 2) + 27$, what is the value of y - 2?

3. A scholarly journal plans to implement an annual increase in its publication fees following a model where the cost in year x is given by the function $f(x) = 300(1.33)^x$, where x represents the number of years after 2024. What is the interpretation of 300 in this context?

A. The total publication fee after 3 years of increases.

B. The publication fee at the start of 2024.

C. The projected increase in publication fees each year after 2024.

D. The average publication fee across all years.

4. What is the center of the circle in the xy-plane defined by the equation $(x + 5)^2 + (y - 3)^2 = 16$?

5. The table shows the distribution of different big data technologies adopted by two technology companies. If a technology represented in the table is selected at random, what is the probability of selecting a technology related to Company A, given that the technology is related to Data Storage? (Express your answer as a decimal or fraction, not as a percent.)

Technologies	Company A	Company B	Total
Data Storage	40	30	70
Data Mining	25	35	60
Data Analytics	20	20	40
Data Visualization	15	15	30
Total	100	100	200

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. $\frac{3}{5}$

D. $\frac{4}{7}$

6. The table shows two values of x and their corresponding values of y. The graph of the linear equation representing this relationship passes through the point $(\frac{5}{2}, b)$. What is the value of b?

x	y
0	16
8	72

A. $\frac{67}{2}$

B. 34

C. $\frac{69}{2}$

D. 35

7. A circle in the xy-plane has its center at $(4, -2)$ and a radius of 5. An equation of this circle is $x^2 + y^2 + ax + by + c = 0$, where a, b, and c are constants. What is the value of c?

A. 0

B. -5

C. -10

D. 5

8. For the linear function g, the graph of $y = g(x)$ in the xy-plane has a slope of 12 and passes through the point (0, -4). Which equation defines g?

A. $y = -12x + 4$

B. $y = -12x - 4$

C. $y = 12x + 4$

D. $y = 12x - 4$

9. The function g is defined by $g(x) = 3x^2 - 5x + 12$. What is the value of $g(1)$?

10. A research team developed a new drug delivery system for pediatric brain tumors modeled by the quadratic equation: $f(x) = -2(x - 5)^2 + 120$, where $f(x)$ represents the estimated effectiveness of the drug delivery system after x weeks. What is the maximum effectiveness of the drug delivery system, and at what time does this maximum effectiveness occur?

A. Maximum effectiveness: 120, Time: 5 weeks

B. Maximum effectiveness: 110, Time: 6 weeks

C. Maximum effectiveness: 100, Time: 5 weeks

D. Maximum effectiveness: 120, Time: 6 weeks

SAT Math Solutions

1. The table shows two values of x and their corresponding values of y. The graph of the linear equation representing this relationship passes through the point $(\frac{1}{2}, b)$. What is the value of b?

x	y
-5	20
10	-25

Answer

$\frac{7}{2}$

Solution

This problem tests the student's ability to understand linear equations and their graphs, specifically in determining the y-coordinate of a given x-value using a table of values. It assesses knowledge of the slope-intercept form of linear equations and the ability to derive or interpret the equation from a given set of data points.
To solve this problem, first determine the linear equation using the table values. Identify two pairs of (x, y) from the table and calculate the slope (m). Use one of the points to solve for the y-intercept (c) using the equation $y = mx + c$. With the full equation, substitute $x = \frac{1}{2}$ to find the value of b.
When calculating the slope, remember it's the change in y divided by the change in x (rise over run). Ensure all calculations are accurate, and recheck the equation before substituting values. Use clear and organized steps to avoid confusion.
Be cautious with fractions and ensure that arithmetic operations are performed correctly. When calculating the slope, make sure the order of subtraction is consistent for both x and y values to avoid sign errors. Double-check your substitution into the linear equation to ensure accuracy.
This type of problem is common in testing algebraic understanding and application of linear equations. It not only evaluates the ability to form equations from given data but also tests the skill of accurately substituting and calculating unknowns. Mastery of such problems aids in strengthening foundational algebra skills, crucial for SAT success.

Step 1: Calculate the slope of the line using the formula $\frac{y2-y1}{x2-x1}$.
Slope $m = \frac{-25-20}{10-(-5)} = \frac{-45}{15} =- 3$
Step 2: Use the point-slope form of the equation $y - y1 = m(x - x1)$ to find the

line equation.
Using point (10, -25): $y + 25 =- 3(x - 10)$
Expand: $y + 25 =- 3x + 30$
Solve for y: $y =- 3x + 5$
Step 3: Substitute $x = \frac{1}{2}$ into the equation to find 'b'.

$b =- 3 \times \frac{1}{2} + 5$

$b =- \frac{3}{2} + 5$

Convert 5 to improper fraction: $b =- \frac{3}{2} + \frac{10}{2}$

$b = \frac{7}{2}$

Thus, the value of 'b' is $\frac{7}{2}$.

2. If $5(y - 2) = 2(y - 2) + 27$, what is the value of y - 2?

Answer

9

Solution

This problem is designed to assess the student's ability to solve linear equations using the substitution method, and to understand the concept of isolating variables to find the solution.
The student should start by recognizing that the equation involves a common expression, (y - 2), on both sides. By simplifying the equation, students can isolate y - 2 and solve for its value.
Notice that both sides of the equation have the expression (y - 2). Begin by simplifying the equation: distribute the constants and then combine like terms. This will allow you to solve directly for y - 2.
Be careful with the distribution of numbers and ensure that you maintain equality by performing the same operation on both sides of the equation. Avoid common errors like incorrect distribution or combining terms incorrectly.
This type of problem tests a fundamental skill in algebra: simplifying and solving linear equations. By practicing this method, students improve their ability to handle more complex algebraic expressions and equations. Mastery of these techniques is essential for success in more advanced math courses, making it an invaluable component of SAT preparation.

Start with the given equation: $5(y - 2) = 2(y - 2) + 27$.
Subtract 2(y - 2) from both sides to isolate terms involving y - 2:
$5(y - 2) - 2(y - 2) = 27$.
This simplifies to: $(5 - 2)(y - 2) = 27$.

Which further simplifies to: $3(y - 2) = 27$.
Divide both sides by 3 to solve for y - 2:
$y - 2 = \frac{27}{3}$.
Simplifying gives: $y - 2 = 9$.

3. A scholarly journal plans to implement an annual increase in its publication fees following a model where the cost in year x is given by the function $f(x) = 300(1.33)^x$, where x represents the number of years after 2024. What is the interpretation of 300 in this context?

A. The total publication fee after 3 years of increases.

B. The publication fee at the start of 2024.

C. The projected increase in publication fees each year after 2024.

D. The average publication fee across all years.

Answer

B

Solution

This problem is designed to test the student's understanding of exponential functions, specifically how to interpret the components of an exponential equation in a real-world context. It assesses the ability to connect mathematical models to practical scenarios.
To approach this problem, you should recognize that the function given, $f(x) = 300(1.33)^x$, is in the form of an exponential growth model. In this model, the number 300 represents the initial value or the starting cost of publication fees in the year 2024 (when $x = 0$).

Remember that in an exponential function of the form $f(x) = a(b)^x$, 'a' represents the initial amount or the value when x equals zero. This is a key concept in understanding and interpreting exponential functions. Always identify the role of each component in the function.
Be careful not to confuse the initial value with the rate of increase. The number 300 is not the rate of increase; it is the starting value. Also, ensure you understand that x represents years after 2024, so x = 0 corresponds to the year 2024.
This problem evaluates your ability to interpret exponential functions in real-world situations, a crucial skill for solving advanced math problems on the SAT. Understanding how to decipher each part of an exponential equation is essential, and this type of problem often appears in various contexts. Practice recognizing initial values and growth rates within exponential models to excel in this section.

The function $f(x) = 300(1.33)^{x}$ represents the cost in year x in an exponential growth model.
When x = 0, the cost is $f(0) = 300(1.33)^{0} = 300$.
Therefore, 300 represents the initial publication fee at the start of 2024, before any increases.

4. What is the center of the circle in the xy-plane defined by the equation $(x + 5)^2 + (y - 3)^2 = 16$?

Answer

(-5, 3)

Solution

This problem tests the student's understanding of the standard form of a circle's equation and their ability to identify the center and radius from this form. Students should recognize that the equation $(x - h)^2 + (y - k)^2 = r^2$ represents a circle centered at (h, k) with radius r.
To solve this problem, students need to identify the form of the given equation $(x + 5)^2 + (y - 3)^2 = 16$ and compare it to the standard form of a circle's equation $(x - h)^2 + (y - k)^2 = r^2$. Recognize that the equation can be rewritten as $(x - (-5))^2 + (y - 3)^2 = 16$, indicating that the center of the circle (h, k) is (-5, 3).
When dealing with circle equations, always rewrite the equation in the form $(x - h)^2 + (y - k)^2 = r^2$ to easily identify the center (h, k) and the radius r. Remember that the signs in the equation are opposite to those in the center coordinates.
Be careful with the signs when determining the center of the circle. In the equation $(x - h)^2 + (y - k)^2 = r^2$, the center is at (h, k), so you must pay attention to the minus signs in the equation to correctly identify the positive or negative values of h and k. Additionally, make sure not to confuse the squared term 16 with the radius; the radius is the square root of 16, which is 4.
This type of problem is common in SAT math sections and assesses a student's ability to work with the standard equation of a circle. Recognizing the structure of the equation and understanding how to manipulate it to extract the center and radius is crucial. Mastery of these concepts is essential for success in geometry and trigonometry problems on the SAT. Practice with a variety of circle equations to become comfortable with quickly identifying the center and radius.

Identify the standard form from the given equation: $(x + 5)^2 + (y - 3)^2 = 16$

compared to $(x - h)^2 + (y - k)^2 = r^2$.
Rewrite $(x + 5)^2$ as $(x - (-5))^2$ to match the standard form.
Similarly, rewrite $(y - 3)^2$., By comparing, we see that $h =- 5$ and $k = 3$, and $r^2 = 16$.
Thus, the center of the circle is (-5, 3).

5. The table shows the distribution of different big data technologies adopted by two technology companies. If a technology represented in the table is selected at random, what is the probability of selecting a technology related to Company A, given that the technology is related to Data Storage? (Express your answer as a decimal or fraction, not as a percent.)

Technologies	Company A	Company B	Total
Data Storage	40	30	70
Data Mining	25	35	60
Data Analytics	20	20	40
Data Visualization	15	15	30
Total	100	100	200

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. $\frac{3}{5}$

D. $\frac{4}{7}$

Answer

D

Solution

This problem tests the student's understanding of conditional probability, particularly how to calculate the probability of an event given a specific condition using a table of data. It checks if the student can interpret data in a tabular format and apply probability formulas correctly.
To solve this problem, students need to identify the relevant data in the table concerning technologies related to Data Storage and then focus only on those entries. They must then calculate the probability that, given a technology is related to Data Storage, it is related to Company A. This involves using the conditional

probability formula: $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$.
First, isolate the rows or columns that pertain to Data Storage. Focus on these entries, ignoring all other technologies. Then count the total number of Data Storage entries, and specifically those associated with Company A. Use these counts to set up your fraction for conditional probability.
Ensure that you are only considering the technologies related to Data Storage when calculating probabilities. A common mistake is to include unrelated categories, which can lead to incorrect answers. Also, remember to express your final answer as a decimal or fraction as instructed.
This type of SAT problem is designed to assess the student's capability in handling conditional probabilities in a real-world context using tables. Mastery of this problem involves attentiveness to detail and the ability to filter relevant data from a larger dataset. It highlights the importance of methodical data handling and precise calculation, critical skills in both academic and professional data analysis contexts.

First, find the total number of technologies related to Data Storage: 70.
Next, find the number of Data Storage technologies related to Company A: 40.
Calculate the probability $P(A|B)$ as the ratio of Data Storage technologies related to Company A to the total Data Storage technologies:
$$P(A|B) = \frac{Number\ of\ Data\ Storage\ technologies\ related\ to\ Company\ A}{Total\ number\ of\ Data\ Storage\ technologies} = \frac{40}{70} = \frac{4}{7}$$

6. The table shows two values of x and their corresponding values of y. The graph of the linear equation representing this relationship passes through the point $(\frac{5}{2}, b)$. What is the value of b?

x	y
0	16
8	72

A. $\frac{67}{2}$

B. 34

C. $\frac{69}{2}$

D. 35

Answer

A

Solution

This problem tests the student's ability to understand linear equations and their representation on a graph. Specifically, it focuses on recognizing the relationship between variables in a linear equation and using given points to find unknown values.
To approach this problem, first identify the linear equation from the given data table by finding the slope (m) and using one of the points to determine the y-intercept (c). Then, use the linear equation in the form y = mx + c to find the value of b when x is $\frac{5}{2}$.
Remember that the slope (m) of a line can be calculated using the formula $m = \frac{(y2-y1)}{(x2-x1)}$. Once the slope is found, use it with one of the points to find the equation of the line. After that, substitute $x = \frac{5}{2}$ into the equation to solve for b.
Be careful with fractions when calculating the slope and substituting values. It's easy to make mistakes with negative signs or when simplifying fractions. Make sure to double-check your calculations.
This type of problem is common in the SAT and is designed to evaluate your understanding of linear equations and their graphs. It requires careful calculation and attention to detail, especially with fractions. Mastering this type of question will help you in algebra and function graph questions on the SAT.

1. Calculate the slope (m) of the line using the points (0, 16) and (8, 72): - Slope

formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$,

$m = \frac{72-16}{8-0} = \frac{56}{8} = 7$

2. Use the slope-point form to find the equation of the line:
Point-slope form: $y - y_1 = m(x - x_1)$
Substitute: $y - 16 = 7(x - 0)$
Simplify: $y = 7x + 16$
3. Substitute $x = \frac{5}{2}$ to find b: $y = 7 \cdot \frac{5}{2} + 16, y = \frac{35}{2} + 16$
Convert 16 to a fraction: $16 = \frac{32}{2}$
Combine: $y = \frac{35}{2} + \frac{32}{2} = \frac{67}{2}$
Therefore, the value of b is $\frac{67}{2}$.

7. A circle in the xy-plane has its center at $(4, -2)$ and a radius of 5. An equation of this circle is $x^2 + y^2 + ax + by + c = 0$, where a, b, and c are constants. What is the value of c?

A. 0

B. -5

C. -10

D. 5

Answer

B

Solution

This problem is designed to assess whether students understand how to represent the equation of a circle in the standard form and convert it into the general form. It tests students' knowledge of circle equations and their ability to apply algebraic manipulation.

First, recall the standard form of the equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius. Substitute the given center $(4, -2)$ and radius 5 into this form to get: $(x - 4)^2 + (y + 2)^2 = 25$.

Next, expand this equation to convert it into the form $x^2 + y^2 + ax + by + c = 0$. Expand the squares and combine like terms to identify the values of a, b, and c. When expanding the standard form equation, be careful with the signs, especially since $y + 2$ will become $(y + 2)^2$. Also, remember to expand $(x - 4)^2$ and

$(y + 2)^2$ fully before combining like terms.
A common mistake is to forget to expand the squares correctly or to incorrectly combine like terms. Ensure each step is double-checked for accuracy. Specifically, ensure that you do not lose or incorrectly handle the negative signs when expanding and combining terms.
This problem is a classic test of understanding the equations of circles and algebraic manipulation. It evaluates the ability to convert between different forms of circle equations, which is a fundamental skill in geometry and algebra. Mastery in this area is essential for solving more complex geometric problems on the SAT.

Start with the general equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$.
Substitute the given center $(4, - 2)$ and radius 5 into this equation:
$(x - 4)^2 + (y - 2)^2 = 5^2$.
Expand $(x - 4)^2$: $x^2 - 8x + 16$.
Expand $(y + 2)^2$: $y^2 + 4y + 4$.
Write the expanded equation: $x^2 - 8x + 16 + y^2 + 4y + 4 = 25$.
Combine like terms: $x^2 + y^2 - 8x + 4y + 20 = 25$.
Subtract 25 from both sides to match the form $x^2 + y^2 + ax + by + c = 0$:
$x^2 + y^2 - 8x + 4y + 20 - 25 = 0$.
Simplify to get: $x^2 + y^2 - 8x + 4y - 5 = 0$.
The constant term c is therefore -5.

8. For the linear function g, the graph of $y = g(x)$ in the xy-plane has a slope of 12 and passes through the point (0, -4). Which equation defines g?

A. $y = -12x + 4$

B. $y = -12x - 4$

C. $y = 12x + 4$

D. $y = 12x - 4$

Answer

D

Solution

The problem aims to test the student's understanding of the equation of a line in slope-intercept form and their ability to apply the given slope and y-intercept to find the correct linear function.
To solve this problem, the student should recall the slope-intercept form of a linear equation, which is $y = mx + b$, where m is the slope and b is the y-intercept. Given the slope (m) is 12 and the y-intercept (b) is -4, the student should substitute these values into the equation to find $y = 12x - 4$.
Remember that the slope-intercept form of a linear equation is $y = mx + b$. The slope (m) tells you how steep the line is, and the y-intercept (b) is the point where the line crosses the y-axis.
Substituting the given values directly into this form will quickly give you the correct equation.
Be careful not to confuse the slope and the y-intercept. Always ensure that you substitute the correct values into the appropriate places in the equation.
Additionally, remember that the y-intercept is the value of y when x is zero.
This type of problem is fundamental in algebra and tests the student's ability to work with linear equations. Mastery of the slope-intercept form is essential for solving more complex problems involving linear functions. By practicing this type of problem, students can become more adept at quickly identifying and applying the necessary components to find the correct linear equation. This skill is crucial for the SAT and other standardized tests.

The slope-intercept form of a line is given by $y = mx + b$.
Substitute the slope $m = 12$ into the equation: $y = 12x + b$.
Since the line passes through the point (0, -4), the y-intercept $b = -4$.
Thus, the equation becomes $y = 12x - 4$.

9. The function g is defined by $g(x) = 3x^2 - 5x + 12$. What is the value of $g(1)$?

Answer

10

Solution

This problem tests the student's ability to evaluate a quadratic function by substituting a given value for the variable. Students should demonstrate understanding of function notation and basic arithmetic operations.
To solve this problem, substitute $x = 1$ into the quadratic function $g(x) = 3x^2 - 5x + 12$. Calculate the value step by step by following the order of operations: first square the value of x, then multiply by the coefficients, and finally perform the addition or subtraction.
Carefully substitute the value into the function and ensure each step follows the correct order of operations: parentheses, exponents, multiplication/division (from left to right), and addition/subtraction (from left to right).
A common mistake is to forget the order of operations or to make an arithmetic error. Be attentive to negative signs and ensure each multiplication and addition is performed correctly.
This type of problem is a straightforward example of evaluating functions, a fundamental skill in algebra. It checks the student's ability to handle basic arithmetic and function notation, which are essential for more complex problems. Careful calculation and understanding of the order of operations are key to solving these problems efficiently.

Substitute $x = 1$ into the function: $g(1) = 3(1)^2 - 5(1) + 12$.
Calculate each term: $3(1)^2 = 3, - 5(1) =- 5$, and 12 is constant.
Compute the value: $g(1) = 3 - 5 + 12$.
Simplify the expression: $3 - 5 =- 2$, then $- 2 + 12 = 10$.
Thus, the value of $g(1)$ is 10.

10. A research team developed a new drug delivery system for pediatric brain tumors modeled by the quadratic equation: $f(x) =- 2(x - 5)^2 + 120$, where $f(x)$ represents the estimated effectiveness of the drug delivery system after x weeks. What is the maximum effectiveness of the drug delivery system, and at what time does this maximum effectiveness occur?

A. Maximum effectiveness: 120, Time: 5 weeks

B. Maximum effectiveness: 110, Time: 6 weeks

C. Maximum effectiveness: 100, Time: 5 weeks

D. Maximum effectiveness: 120, Time: 6 weeks

Answer

A

Solution

This problem tests the student's ability to understand and analyze quadratic equations, particularly by identifying the maximum value and the vertex of a parabola. The student needs to apply knowledge of the standard form of a quadratic equation and its properties.
To solve this problem, recognize that the given equation is in vertex form, which is helpful to find the maximum or minimum point of a quadratic function. The vertex form of a quadratic equation is given by: $f(x) = a(x - h)^2 + k$ where (h, k) is the vertex of the parabola. In this problem, the vertex form is:
$f(x) =- 2(x - 5)^2 + 120$
Here, the vertex (h, k) is $(5, 120)$. Thus, the maximum effectiveness of the drug delivery system is 120, and it occurs at 5 weeks.
When dealing with quadratic functions, particularly those related to maximum or minimum values, it is often helpful to rewrite the equation in vertex form if it is not already provided. This makes it easier to identify the vertex directly. Remember that if the coefficient of the squared term (a) is negative, the parabola opens downwards, indicating a maximum point at the vertex.
Make sure to correctly identify the vertex form and not confuse it with the standard form of a quadratic equation $(ax^2 + bx + c)$. Also, be cautious about the sign of the coefficient $'a'$; a negative $'a'$ confirms a maximum point, while a positive $'a'$ indicates a minimum point. Double-check your calculations to ensure accuracy.
This problem is a classic example of quadratic word problems that appear on SAT exams. It evaluates the student's ability to interpret and manipulate quadratic functions, which is a key skill in advanced math. Understanding the properties of quadratic functions, especially the vertex, is crucial for solving these types of problems efficiently. As long as students remember the significance of the vertex form and the impact of the coefficient $'a'$, they should be able to solve such problems

with confidence.

The quadratic equation given is $f(x) = -2(x - 5)^2 + 120$.
The vertex form of a parabola is $f(x) = a(x - h)^2 + k$, where (h, k) is the vertex.
Here, $h = 5$ and $k = 120$
so the vertex is (5, 120).
Since the parabola opens downwards, the vertex represents the maximum point.
This means the maximum effectiveness of the drug delivery system is 120 and it occurs after 5 weeks.