Digital SAT Math

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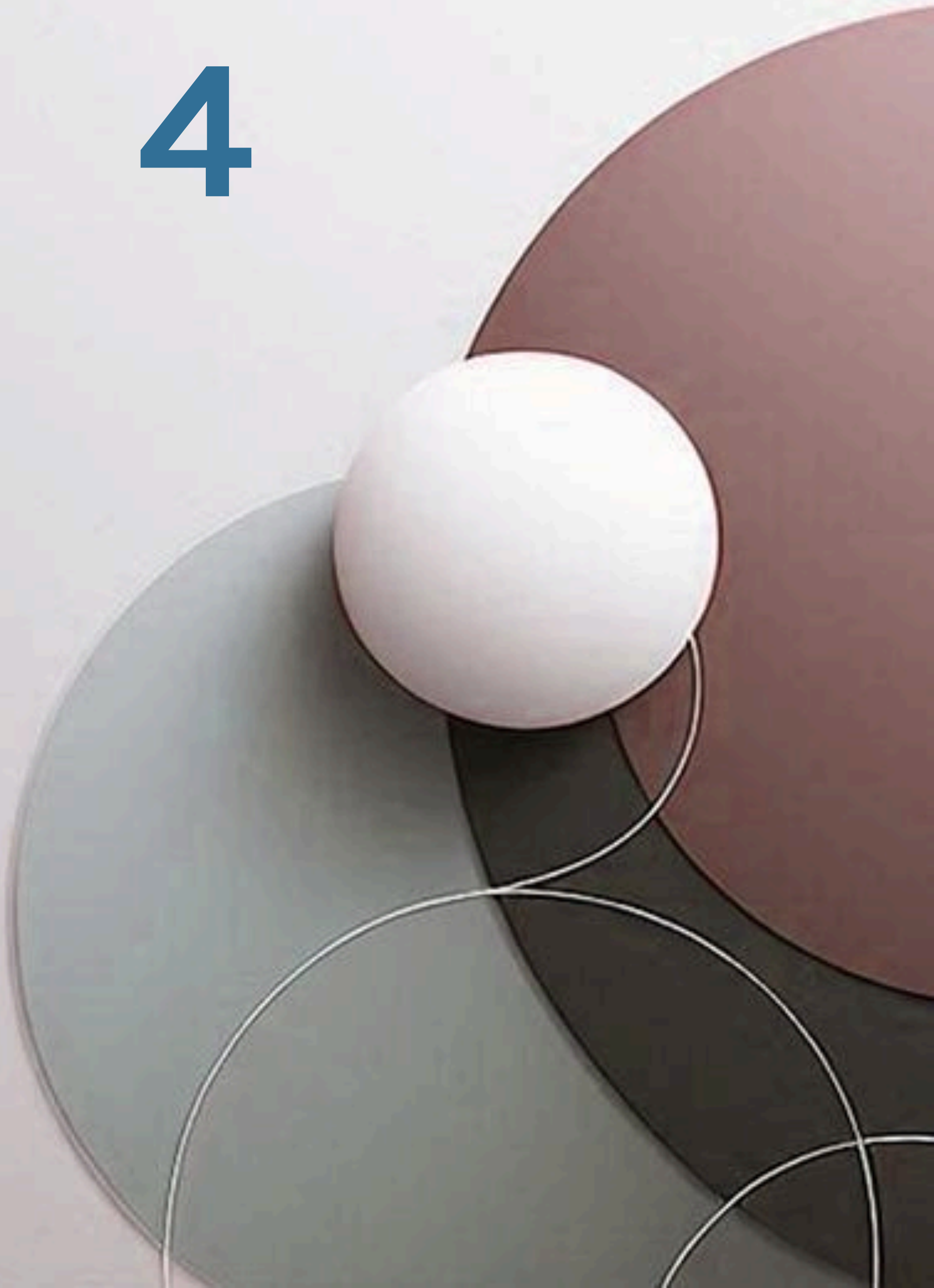

SAT Math Problems

1. For the polynomial function defined by $f(x) = 3x^4 - 5x^3 + 2x - 8$, what is the value of b if the graph of $y = f(x)$ passes through the point $(0, b)$?

2. If $\frac{3x}{y} = 9$ and $\frac{x}{zy} = 15$, what is the value of z?

A. 0.1

B. 0.2

C. 0.3

D. 0.4

3. How many times does the graph of the given equation in the xy-plane cross the x-axis, where a, b, and c are positive constants such that $a > 4$? $y = 5\left(\frac{a}{4}\right)^{x+c} - b$

A. 0

B. 1

C. 2

D. 3

4. A local art company is planning to sell two types of cultural products: handmade crafts and digital artwork. The revenue from handmade crafts is \$50 per item, and the revenue from digital artwork is \$30 per item. If the company sells a total of 200 items for a total revenue of \$7, 000, which of the following systems of equations represents this situation, where x is the number of handmade crafts and y is the number of digital artworks sold?

A. $x + y = 200, 50x + 30y = 7000$

B. $x + y = 200, 30x + 50y = 7000$

C. $50x + y = 200, 30x + y = 7000$

D. $x + 50y = 200, y + 30x = 7000$

5. A wooden cube is cut from a tree trunk for a woodworking project. If the edge of the cube measures 4 centimeters, and it has a mass of 10.24 grams, what is the density of the cube, in grams per cubic centimeter?

6. A non-profit organization is distributing funds to two different community projects aimed at improving healthcare access. Project A receives \$1, 200, and Project B receives \$1, 800. Considering the total budget of \$6, 000 for these projects, what ratio of funding is allocated to Project A compared to Project B?

A. 2:3

B. 1:2

C. 1:1.5

D. 3:4

7. For the given polynomial function $f(x) = 2x^3 - 5x^2 + 3x - 7$, the graph of $y = f(x)$ in the xy-plane passes through the point $(0, b)$, where b is a constant. What is the value of b?

A. -7

B. -5

C. 0

D. 3

8. Line p is defined by the equation $3x - 5y = 12$. Line q is parallel to line p in the xy-plane. What is the slope of line q?

9. In a survey of 70 individuals regarding their support for international policies, the results indicated that 12 supported Country A, 25 supported Country B, and 33 supported Country C. If one individual is selected at random, what is the probability that the individual supports Country B?

Type	Frequency
Country A	12
Country B	25
Country C	33

A. $\frac{1}{2}$

B. $\frac{3}{5}$

C. $\frac{5}{14}$

D. $\frac{33}{70}$

10. Circle C has a radius of $4y$ and circle D has a radius of $120y$. The area of circle D is how many times the area of circle C?

SAT Math Solutions

1. For the polynomial function defined by $f(x) = 3x^4 - 5x^3 + 2x - 8$, what is the value of b if the graph of $y = f(x)$ passes through the point $(0, b)$?

Answer

-8

Solution

This problem aims to test the student's understanding of polynomial functions, specifically how to find the y-intercept of a higher-degree polynomial function. The student should recognize that the y-intercept occurs where the input, x, is zero.
To find the y-intercept of the polynomial function, substitute $x = 0$ into the function $f(x) = 3x^4 - 5x^3 + 2x - 8$. This will simplify the expression, leaving only the constant term, which represents the y-intercept.
Remember that the y-intercept of a function is simply the value of the function when x is 0. Therefore, for any polynomial function, you just need to evaluate the constant term.
Be careful not to overlook the fact that higher-degree terms disappear when x = 0. Only the constant term remains, which is the y-intercept. Also, ensure calculations are accurate to avoid simple arithmetic mistakes.
This type of problem is straightforward if you understand the concept of the y-intercept for polynomial functions. It assesses the student's ability to recognize and apply the concept of evaluating a polynomial at x = 0. In the context of the SAT, practicing these types of problems can help improve speed and accuracy, as recognizing patterns and applying basic concepts quickly is often rewarded.

To find b, we substitute x = 0 into the polynomial f(x)., $f(0) = 3(0)^4 - 5(0)^4 + 2(0) - 8$, Simplifying the expression: f(0) = 0 - 0 + 0 - 8, f(0) = -8, Therefore, the value of b is -8.

2. If $\frac{3x}{y} = 9$ and $\frac{x}{zy} = 15$, what is the value of z?

A. 0.1

B. 0.2

C. 0.3

D. 0.4

Answer

B

Solution

This problem tests the student's understanding of working with ratios and proportions. The student needs to understand how to manipulate and solve equations involving ratios to find the value of an unknown variable.
First, simplify the given equations to find the value of x in terms of y. Then, substitute this value into the second equation to solve for z.
Isolate variables step-by-step and keep your work organized. Double-check each step to ensure that the algebraic manipulations are correct
Be careful with the algebraic manipulations and make sure not to lose or incorrectly handle any variables. Also, ensure that the value you find for z is consistent with the given equations.
This problem is a good example of how SAT questions often require students to apply multiple algebraic steps to find a solution. It tests a student's ability to understand and manipulate ratios and proportions, which are essential skills for the SAT Math section. By practicing problems like this, students can improve their problem-solving efficiency and accuracy.

From Equation 1, $\frac{3x}{y} = 9$.
Multiply both sides by y to solve for x: $3x = 9y$.
Divide both sides by 3 to get $x = 3y$.
Substitute $x = 3y$ into Equation 2: $\frac{3y}{zy} = 15$.
Simplifying, $\frac{3}{z} = 15$.
Multiply both sides by z to get $3 = 15z$.
Divide both sides by 15 to solve for z: $z = \frac{3}{15}$.
Simplify $\frac{3}{15}$ to $\frac{1}{5}$.
The value of z is $\frac{1}{5}$.

3. How many times does the graph of the given equation in the xy-plane cross the x-axis, where a, b, and c are positive constants such that $a > 4$? $y = 5\left(\frac{a}{4}\right)^{x+c} - b$

A. 0

B. 1

C. 2

D. 3

Answer

B

Solution

This problem tests the student's understanding of exponential functions and their graphical behavior, particularly how to determine the number of x-intercepts of an exponential graph.
To solve this problem, the student needs to identify the x-intercepts of the given equation. This involves setting $y = 0$ and solving for x. Specifically, they must solve the equation $0 = 5\left(\frac{a}{4}\right)^{x+c} - b$ for x.
Remember that an exponential function of the form $y = ka^{x} + c$ has a horizontal asymptote. For the given equation, as x approaches infinity or negative infinity, the term $\left(\frac{a}{4}\right)^{(x+c)}$ will either grow or decay exponentially depending on the value of $\frac{a}{4}$. This can help you determine whether the function crosses the x-axis.
Be careful with the base of the exponential function. Since $a > 4$, $\frac{a}{4}$ is greater than 1, meaning the function grows exponentially. Also, check the signs and values of b and how it affects the crossing points. Ensure not to confuse the behavior of the function based on whether the base is greater than or less than 1.
This type of problem assesses the student's ability to analyze the behavior of exponential functions and their graphs. It requires understanding how to manipulate exponential equations and determine their intercepts. The primary skill tested is the ability to discern the number of times a given exponential graph will intersect the x-axis, considering the transformations applied to the function. Understanding these concepts is crucial for success in advanced math sections of the SAT.

Set $y = 0$ in the equation: $0 = 5\left(\frac{a}{4}\right)^{x+c} - b$.

This simplifies to: $5\left(\frac{a}{4}\right)^{x+c} = b$.

Divide both sides by 5: $\left(\frac{a}{4}\right)^{x+c} = \frac{b}{5}$.

Since $a > 4, \frac{a}{4} > 1$, indicating an increasing exponential function.

The equation $\left(\frac{a}{4}\right)^{(x+c)} = \frac{b}{5}$ has a solution for x if and only if $\frac{b}{5} > 0$.
Thus, the equation has one solution for x, meaning the graph crosses the x-axis once.

4. A local art company is planning to sell two types of cultural products: handmade crafts and digital artwork. The revenue from handmade crafts is \$50 per item, and the revenue from digital artwork is \$30 per item. If the company sells a total of 200 items for a total revenue of \$7, 000, which of the following systems of equations represents this situation, where x is the number of handmade crafts and y is the number of digital artworks sold?

A. $x + y = 200, 50x + 30y = 7000$

B. $x + y = 200, 30x + 50y = 7000$

C. $50x + y = 200, 30x + y = 7000$

D. $x + 50y = 200, y + 30x = 7000$

Answer

A

Solution

The problem is designed to assess the student's ability to interpret and formulate a real-world scenario into a system of linear equations. It tests understanding of how to create equations from word problems, particularly in the context of revenue and quantity.
To solve this problem, the student needs to identify the key information given: the price per item for handmade crafts and digital artwork, the total number of items sold, and the total revenue. The student should then set up two equations: one for the total number of items and one for the total revenue. The first equation will be $x + y = 200$, representing the total number of items sold, and the second equation will be $50x + 30y = 7000$, representing the total revenue.
Identify what x and y represent early on, and make sure to carefully translate the word problem into mathematical equations. Double-check that the coefficients in your equations correctly match the context of the problem, such as the price per item and total counts.
Be cautious about mixing up the coefficients for x and y in the revenue equation. Sometimes students mistakenly reverse the values or misunderstand the relationship between the total number of items and the individual prices. Also, ensure that you correctly interpret the total revenue as the sum of revenues from both products.
This type of problem is common in SAT Algebra sections, where translating word

problems into equations is crucial. It tests a student's ability to understand and apply linear equations in practical contexts. Successfully solving these problems typically requires careful reading and precise translation of the scenario into mathematical terms. It is an essential skill for SAT success, as it demonstrates the ability to apply algebraic concepts to real-world situations.

Set up the first equation for the total number of items: $x + y = 200$
Set up the second equation for the total revenue: $50x + 30y = 7000$
Thus, the system of equations is: $x + y = 200$ and $50x + 30y = 7000$

5. A wooden cube is cut from a tree trunk for a woodworking project. If the edge of the cube measures 4 centimeters, and it has a mass of 10.24 grams, what is the density of the cube, in grams per cubic centimeter?

Answer

0.16

Solution

This problem tests the student's understanding of geometric concepts related to volume and their ability to apply the formula for density. The student needs to connect the physical property of density with geometric measurements.
To solve this problem, the student should first calculate the volume of the cube using the formula for the volume of a cube ($V = (side)^3$). With the volume determined, the next step is to apply the formula for density ($Density = \frac{mass}{Volume}$) to find the density of the cube.
Remember that the volume of a cube can be calculated by cubing the length of one of its edges. After finding the volume, use the given mass to find the density by dividing the mass by the volume. Keep units consistent to avoid any confusion.
A common mistake is to forget to cube the edge length when calculating the volume. Ensure that you correctly use the units of measure throughout the calculation to avoid errors in the final density calculation.
This problem is a classic example of integrating geometric and physical concepts, which is often seen in SAT problems. It not only assesses the student's ability to perform basic geometric calculations but also their understanding of physical properties and their application in real-world scenarios. Such problems are designed to test a student's analytical skills and ability to connect different areas of mathematics.

Step 1: Calculate the Volume of the Cube, The formula for the volume of a cube is: $Volume = (edge)^3$, For a cube with an edge of 4 centimeters, $Volume = 4^3 = 64$ cubic centimeters.

Step 2: Calculate the Density, Density is given by the formula: $Density = \frac{Mass}{Volume}$,
Substitute the given values: $Density = \frac{10.24\ grams}{64\ cubic\ centimeters}$.
Calculate the density: Density = 0.16 grams per cubic centimeter.

6. A non-profit organization is distributing funds to two different community projects aimed at improving healthcare access. Project A receives $1, 200, and Project B receives $1, 800. Considering the total budget of $6, 000 for these projects, what ratio of funding is allocated to Project A compared to Project B?

A. 2:3

B. 1:2

C. 1:1.5

D. 3:4

Answer

A

Solution

This problem tests the student's ability to understand and apply the concept of ratios in a real-world context, specifically in the distribution of funds. It assesses the student's ability to calculate ratios and understand their meaning in terms of proportions.
To solve this problem, first identify the amount of money allocated to each project. Then, express this allocation as a ratio of Project A's funds to Project B's funds. The ratio can be found by dividing the amount allocated to Project A by the amount allocated to Project B, and simplifying the result.
Remember that a ratio is a way to compare two quantities. To find the ratio of funding for Project A to Project B, divide the amount for Project A ($1, 200) by the amount for Project B ($1, 800). Simplify the fraction to get the simplest form, which represents the ratio.
Be careful not to confuse the ratio of funding with the total budget amount. The problem asks for the ratio between the funds allocated to the two projects, not their relation to the total budget. Additionally, ensure that the ratio is simplified to its smallest form.
This type of problem is common in SAT as it tests not only mathematical skills but also the ability to apply these skills to real-world scenarios. Understanding ratios and their simplification is crucial as it is a fundamental concept in problem-solving and data analysis. Pay attention to the details in the problem statement and practice simplifying fractions to avoid common pitfalls.

To find the ratio of funding for Project A to Project B, we use the formula:
$Ratio = \frac{Funding\ for\ Project\ A}{Funding\ for\ Project\ B}$
Substituting the given values, we have: $Ratio = \frac{\$1,200}{\$1,800}$
Simplifying the fraction: $\frac{\$1,200}{\$1,800} = \frac{12}{18}$
Divide both the numerator and denominator by their greatest common divisor, which is 6: $\frac{\frac{12}{6}}{\frac{18}{6}} = \frac{2}{3}$
Thus, the simplified ratio of funding allocated to Project A compared to Project B is 2:3.

7. For the given polynomial function $f(x) = 2x^3 - 5x^2 + 3x - 7$, the graph of $y = f(x)$ in the xy-plane passes through the point $(0, b)$, where b is a constant. What is the value of b?

A. -7

B. -5

C. 0

D. 3

Answer

A

Solution

This problem aims to test the student's understanding of how to find the y-intercept of a polynomial function. It assesses the ability to substitute specific values into polynomial functions and understand the resulting outputs.
To find the y-intercept of the function, students need to substitute $x = 0$ into the polynomial function and simplify the expression to find the value of y. In this case, substitute $x = 0$ into $f(x) = 2x^3 - 5x^2 + 3x - 7$.
Remember that the y-intercept of any function is found by evaluating the function at $x = 0$. By substituting $x = 0$ into the polynomial, all terms containing x will be zero, simplifying the calculation.
Be careful with the signs when substituting and simplifying the expression. Ensure that all terms are correctly evaluated and combined. It's common to make mistakes with negative signs or arithmetic during simplification.
This problem is a straightforward application of finding the y-intercept in polynomial functions, a common concept in advanced math. It tests basic algebraic manipulation skills and the understanding of function properties. Such problems are standard in SAT to ensure students can handle polynomial functions and their

graphical representations.

Substitute $x = 0$ into $f(x)$: $f(0) = 2(0)^3 - 5(0)^2 + 3(0) - 7$
Calculate each term: $2(0)^3 = 0, - 5(0)^2 = 0, 3(0) = 0$, and the constant term is -7.
Add these results: $0 + 0 + 0 - 7 =- 7$.
Thus, $b = f(0) =- 7$.

8. Line p is defined by the equation $3x - 5y = 12$. Line q is parallel to line p in the xy-plane. What is the slope of line q?

Answer

$\frac{3}{5}$

Solution

This problem tests the student's understanding of the concept of parallel lines and their slopes. Specifically, it assesses whether the student can identify the slope of a line from a given linear equation and apply this knowledge to find the slope of another line that is parallel to it.
To solve this problem, the student should first convert the given equation of line p into the slope-intercept form ($y = mx + b$), where m represents the slope. This can be done by isolating y on one side of the equation. Once the equation is in the proper form, the slope (m) can be identified. Since line q is parallel to line p, it will have the same slope as line p.
Remember that parallel lines have identical slopes. Focus on rearranging the equation into the slope-intercept form, as this is the quickest way to identify the slope. Practice converting equations between different forms to become more efficient.
Be careful with algebraic manipulation; simple arithmetic errors can lead to incorrect results. Ensure that you correctly isolate y to find the slope accurately. Double-check your rearrangement of the equation to avoid any mistakes.
This type of problem is common in SAT algebra sections and evaluates the student's ability to manipulate linear equations and understand the properties of parallel lines. Mastery of converting equations to the slope-intercept form and recognizing that parallel lines share the same slope is crucial for success in these questions. Practice and familiarity with these concepts will aid in quickly and accurately solving similar problems.

To find the slope of line p, let's rewrite its equation in slope-intercept form, which is $y = mx + b$, where m is the slope.
Start with the equation: $3x - 5y = 12$.
Subtract 3x from both sides to get: $- 5y =- 3x + 12$.

Divide every term by -5 to solve for y: $y = \frac{3}{5}x - \frac{12}{5}$.

From this equation in slope-intercept form, we see that the slope of line p is $\frac{3}{5}$.

Since line q is parallel to line p, it will have the same slope.

Therefore, the slope of line q is also $\frac{3}{5}$.

9. In a survey of 70 individuals regarding their support for international policies, the results indicated that 12 supported Country A, 25 supported Country B, and 33 supported Country C. If one individual is selected at random, what is the probability that the individual supports Country B?

Type	Frequency
Country A	12
Country B	25
Country C	33

A. $\frac{1}{2}$

B. $\frac{3}{5}$

C. $\frac{5}{14}$

D. $\frac{33}{70}$

Answer

C

Solution

This problem tests the student's understanding of basic probability concepts and their ability to interpret and use frequency data to calculate probabilities.
To approach this problem, students need to identify the total number of individuals surveyed, which is 70, and the number of individuals who support Country B, which is 25. The probability that a randomly selected individual supports Country B is the ratio of the number of supporters of Country B to the total number of individuals surveyed.
Remember that probability is calculated as the number of favorable outcomes divided by the total number of possible outcomes. Always double-check the numbers given in the problem to ensure accuracy.
A common mistake is to incorrectly sum the frequencies or misinterpret the

question, such as finding the probability for the wrong country. Pay close attention to the details provided in the problem.
This type of problem is straightforward and requires a solid understanding of basic probability concepts. It evaluates the student's ability to interpret data and apply probability formulas accurately. In SAT exams, being adept at these kinds of problems can lead to quick wins, freeing up time for more complex questions.

To find the probability that an individual supports Country B, we use the formula:
$Probability\ = \frac{Number\ of\ individuals\ supporting\ Country\ B}{Total\ number\ of\ individuals\ surveyed}$
Substitute the given values into the formula: $Probability\ = \frac{25}{70}$
Simplify the fraction: $Probability\ = \frac{5}{14}$

10. Circle C has a radius of $4y$ and circle D has a radius of $120y$. The area of circle D is how many times the area of circle C?

Answer

900

Solution

This question tests the student's understanding of the relationship between the radius and area of a circle, and how to apply this understanding to find ratios.
To solve this problem, students should use the formula for the area of a circle, $A\ =\ \pi r^2$, to find the areas of both circles. Then, they should calculate the ratio of the area of circle D to the area of circle C by dividing the area of D by the area of C.
Remember that the area of a circle is proportional to the square of its radius. So, if you know the ratio of the radii, you can square that ratio to find the ratio of the areas directly.
Be careful to correctly square the radius values to avoid calculation mistakes. Also, ensure you are comparing the areas in the correct order as per the question.
This type of problem is common in SAT geometry sections, where understanding the relationship between dimensions and derived measures like area or volume is crucial. It tests your ability to manipulate formulas and understand proportional relationships, which are essential skills for the SAT.

1. Calculate the area of circle C using the formula $A\ =\ \pi r^2$.
- The radius of circle C is 4y., - $Area\ of\ circle\ C\ =\ \pi(4y)^2\ =\ \pi(16y^2)$.
2. Calculate the area of circle D using the formula $A\ =\ \pi r^2$.
- The radius of circle D is $120y$.
- $Area\ of\ circle\ D\ =\ \pi(120y)^2\ =\ \pi(14400y^2)$.

3. Find the ratio of the area of circle D to circle C.

- $Ratio = \frac{Area\ of\ circle\ D}{Area\ of\ circle\ C} = \frac{\pi(14400y^2)}{\pi(16y^2)}$.

4. Simplify the expression by canceling π and y^2.

- $Ratio = \frac{14400}{16} = 900$.

5. The area of circle D is 900 times the area of circle C.