Digital

SAT Math

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SAT Math Problems

1. What is the y-coordinate of the y-intercept of the graph of $y = f(x) + 3$ in the xy-plane? $f(x) = 4(2x - \frac{5}{3})$

A. $-\frac{11}{3}$

B. $-\frac{10}{3}$

C. -3

D. $-\frac{8}{3}$

2. Which expression is equivalent to $3x^4 + 9x^3y + 6x^2y^2 + 18y^4$?

A. $(3x^2 + 6y)(x^2 + 3y^2)$

B. $(3x^2 + 2y)(x^2 + 9y^2)$

C. $(3x^3 + 3y^2)(x + 6y)$

D. $(3x^2 + 3y)(x^2 + 6y^2)$

3. A tech startup is designing a new app. The initial cost of development is represented by the equation $y = 15000 + 300x$, where y is the total cost in dollars and x is the number of app features added. If the startup plans to include 10 features, what is the total projected cost of the app?

A. $16000

B. $17000

C. $18000

D. $19000

4. A community decided to implement a new policy to sustainably manage their water supply, reducing the withdrawal rate by 25%. After this reduction, they found that 300,000 gallons of water were available for use. How much water was being withdrawn before the reduction?

A. 200,000

B. 250,000

C. 300,000

D. 400,000

5. A school is considering investing in a new virtual reality (VR) system for its classrooms. The initial cost for the VR equipment is $\$1,200$. Additionally, the school plans to spend \$300 for each training session needed to teach teachers how to use the equipment. If the total amount spent on the VR system after training sessions is $\$2,400$, how many training sessions did the school schedule?

A. 3

B. 4

C. 5

D. 6

6. The function g is defined by $g(x) = \frac{3}{5}x + 45$. What is the value of g(25)?

7. A local activist group organized a rally to protest social justice issues. They initially planned to have 720 participants. After a week of outreach, they found that 15% of the planned participants had to withdraw due to scheduling conflicts. How many participants remained for the rally?

A. 612

B. 620

C. 630

D. 640

8. The function g is defined by $g(x) = 3x^2 - 5x + 8$. What is the value of $g(1)$?

A. 2

B. 4

C. 6

D. 8

9. What is the median of the data set shown? data set = [4, 50, 8, 23, 15, 42, 16]

10. In ΔABC, $\angle B$ is a right angle and the length of BC is 180 millimeters. If cos(A) = $\frac{4}{5}$, what is the length, in millimeters, of AB?

SAT Math Solutions

1. What is the y-coordinate of the y-intercept of the graph of $y = f(x) + 3$ in the xy -plane? $f(x) = 4(2x - \frac{5}{3})$

A. $-\frac{11}{3}$

B. $-\frac{10}{3}$

C. -3

D. $-\frac{8}{3}$

Answer

A

Solution

This problem tests a student's understanding of linear functions and how transformations affect their graphs, specifically focusing on the y-intercept. It checks if the student can manipulate and evaluate functions to find specific points on the graph.
First, simplify the given function $f(x) = 4(2x - \frac{5}{3})$ to find its expression. Then, determine the y-intercept of the graph $y = f(x) + 3$ by evaluating the function at $x = 0$. Simplify the resulting expression to find the y-coordinate.
Remember that the y-intercept occurs where x equals zero. Substitute $x = 0$ into the function to find the initial y-value, then account for any vertical shifts due to additional constants such as +3.
Be careful with the arithmetic when simplifying expressions. Pay attention to distributing and combining terms, especially with fractions and constants. It's easy to make mistakes with signs and arithmetic involving fractions.
This type of problem is typical for SAT algebra questions, focusing on understanding function transformations and intercepts. It requires students to be comfortable with function notation and basic algebraic manipulation. Successfully solving this problem demonstrates a good grasp of fundamental algebraic concepts and transformations.

First, calculate $f(0)$: $f(0) = 4(2(0) - \frac{5}{3}) = 4(0 - \frac{5}{3}) = 4(-\frac{5}{3})$, Multiply: $4 \times -\frac{5}{3} = -\frac{20}{3}$

The y-coordinate of the y-intercept of $y = f(x)$ is $-\frac{20}{3}$.

To find the y-intercept of $y = f(x) + 3$, add 3 to $-\frac{20}{3}$: $y =- \frac{20}{3} + 3$, Convert 3 to a fraction: $3 = \frac{9}{3}$, Add the fractions: $-\frac{20}{3} + \frac{9}{3} =- \frac{11}{3}$
Therefore, the y-coordinate of the y-intercept is $-\frac{11}{3}$.

2. Which expression is equivalent to $3x^4 + 9x^3y + 6x^2y^2 + 18y^4$?

A. $(3x^2 + 6y)(x^2 + 3y^2)$

B. $(3x^2 + 2y)(x^2 + 9y^2)$

C. $(3x^3 + 3y^2)(x + 6y)$

D. $(3x^2 + 3y)(x^2 + 6y^2)$

Answer

A

Solution

The problem aims to test the student's ability to factor polynomial expressions, specifically focusing on recognizing common factors and applying the distributive property to simplify the expression.
To solve this problem, the student should first identify any common factors in all the terms. Once the common factor is determined, they should factor it out and look for patterns or other factoring techniques that can further simplify the expression.
Start by identifying the greatest common factor (GCF) of all the terms in the polynomial. Factor out the GCF, then check if the remaining polynomial can be factored further. Look for patterns such as the difference of squares, perfect square trinomials, or grouping.
Be careful not to miss any common factors and double-check each term to ensure nothing is left out. Also, after factoring out the GCF, make sure to simplify the remaining polynomial correctly. Misidentifying the GCF or incorrect simplification can lead to errors.
This type of problem evaluates a student's understanding of polynomial factoring and their ability to apply factoring techniques effectively. It's essential to practice recognizing common factors and applying factoring methods accurately to solve these problems efficiently. Mastery of these skills is crucial for success in the Advanced Math section of the SAT.

Step 1: Identify the common factor in all the terms.
Notice that each term is divisible by 3.
Factor 3 out of the entire expression: $3(x^4 + 3x^3y + 2x^2y^2 + 6y^4)$.

Step 2: Look at the remaining expression: $x^4 + 3x^3y + 2x^2y^2 + 6y^4$.
Attempt to factor by grouping: Group the terms as $(x^4 + 3x^3y) + (2x^2y^2 + 6y^4)$.
Step 3: Factor each group separately.
From the first group: $x^3(x + 3y)$.
From the second group: $2y^2(x^2 + 3y^2)$.
Notice that direct factoring doesn't match any of the options clearly. Re-evaluate the approach and terms.
Step 4: Re-examine the entire set of expressions and test each option for validity by expanding.
Option A: $(3x^2 + 6y)(x^2 + 3y^2)$ expands to: $3x^2x^2 + 3x^23y^2 + 6yx^2 + 6y3y^2$, This simplifies to: $3x^4 + 9x^2y^2 + 6x^2y + 18y^4$.
This matches the original polynomial structure after rearranging terms.
Verification: The factorization matches by multiplication, confirming the equivalency.

3. A tech startup is designing a new app. The initial cost of development is represented by the equation $y = 15000 + 300x$, where y is the total cost in dollars and x is the number of app features added. If the startup plans to include 10 features, what is the total projected cost of the app?

A. $16000

B. $17000

C. $18000

D. $19000

Answer

C

Solution

This problem is designed to test the student's understanding of linear equations and their ability to apply algebraic concepts to word problems. Specifically, it evaluates the student's capability to substitute a given value into a linear equation to find the corresponding output.
To solve this problem, students need to identify the linear relationship given in the equation. They should recognize that the equation represents the cost as a function of the number of features. By substituting the value of x (the number of features) with 10 into the equation, they can calculate the total projected cost.
Remember to carefully read the problem and identify what each variable represents. In this case, x is the number of features and y is the total cost. Substitute the given

value of x directly into the equation and simplify to find y.
Avoid common mistakes such as misinterpreting the variables or substituting the wrong value. Ensure that you perform the arithmetic operations step by step to avoid calculation errors.
This type of problem is common in SAT algebra sections, focusing on the ability to apply linear equations to real-world scenarios. It measures skills in algebraic substitution and arithmetic operations. Mastery of these skills is crucial as they form the foundation for more complex algebraic problems. In SAT, being methodical and checking your work can greatly reduce errors and improve accuracy.

Start by substituting $x = 10$ into the given equation: $y = 15000 + 300x$.
Calculate the additional cost for 10 features: $300 \times 10 = 3000$.
Add the additional cost to the initial cost: $15000 + 3000 = 18000$.
Thus, the total projected cost of the app is $18000.

4. A community decided to implement a new policy to sustainably manage their water supply, reducing the withdrawal rate by 25%. After this reduction, they found that 300,000 gallons of water were available for use. How much water was being withdrawn before the reduction?

A. 200,000

B. 250,000

C. 300,000

D. 400,000

Answer

D

Solution

This problem tests the student's understanding of percentage reduction and their ability to reverse-calculate the original amount before the reduction. It assesses their ability to interpret word problems and apply percentage formulas correctly.
To solve this problem, the student needs to recognize that the 300,000 gallons represent $75\%(100\% - 25\%)$ of the original amount. Let the original amount be X. Then, 75% of X equals 300,000 gallons. Formulate the equation $0.75X = 300,000$ and solve for X.
Remember that if a certain percentage of the original amount is given, you can set up an equation with the percentage as a decimal. Also, double-check the conversion between percentages and decimals to avoid errors.
Be careful with the percentage conversion. Ensure you subtract correctly to find the remaining percentage and correctly convert this percentage to a decimal.

Misinterpreting the problem could lead to incorrect equations.
This type of problem is common in the SAT and tests your ability to handle percentage calculations in a real-world context. By practicing such problems, you can improve your ability to quickly set up and solve equations involving percentages. Always break down the problem into smaller parts and ensure each step is logically sound to avoid mistakes.

Let the original withdrawal amount be denoted by W.
After a 25% reduction, the withdrawal amount is 75% of W.
The equation representing this situation is $0.75 \times W = 300,000$.
To find W, divide both sides by 0.75: $W = \frac{300,000}{0.75}$.
Perform the division: W = 400,000.
Thus, the original withdrawal amount was 400,000 gallons.

5. A school is considering investing in a new virtual reality (VR) system for its classrooms. The initial cost for the VR equipment is $1, 200. Additionally, the school plans to spend $300 for each training session needed to teach teachers how to use the equipment. If the total amount spent on the VR system after training sessions is $2, 400, how many training sessions did the school schedule?

A. 3

B. 4

C. 5

D. 6

Answer

B

Solution

This question tests students' ability to translate a real-world scenario into a linear equation and solve it. It assesses understanding of linear relationships and basic algebraic manipulation.
Students should first identify the unknown, which is the number of training sessions. They should then set up a linear equation based on the given information: the fixed initial cost and the variable cost per training session. The equation will be $1200 + 300x = 2400$, where x represents the number of training sessions. Solving for x will give the answer.
Focus on identifying all the constants and variables in the problem: the initial cost of $1, 200 is a constant, each training session costs $300, and the total cost is $2, 400. Set up the equation carefully by combining these elements. Always double-check the problem to ensure all parts are included in your equation.

Be cautious not to confuse the fixed initial cost with the variable cost per session. Also, ensure that you perform the arithmetic operations correctly when solving for x. Double-check your equation setup to ensure it accurately reflects the problem's scenario.
This type of problem is common in the SAT and tests the critical skill of forming equations from word problems. The ability to translate real-world problems into algebraic expressions is essential for success in algebra. Ensure that you practice similar problems to become efficient in identifying and setting up equations from word problems.

We can set up an equation to solve for the number of training sessions, denoted as n.
The total cost equation can be expressed as: Cost of VR equipment + (Cost per training session) × (Number of training sessions) = Total amount spent
Substitute the known values into the equation: $1200 + 300n = 2400$
Subtract 1200 from both sides: $300n = 2400 - 1200$
Simplify: $300n = 1200$
Divide both sides by 300 to solve for n: $n = \frac{1200}{300}$
Simplify the fraction: n = 4

6. The function g is defined by $g(x) = \frac{3}{5}x + 45$. What is the value of g(25)?

Answer

60

Solution

This problem assesses the student's ability to understand function notation and evaluate a function at a given point. It tests their comprehension of linear functions and basic arithmetic operations.
To solve this problem, substitute the given value into the function. Here, you need to replace 'x' with 25 in the function $g(x) = \frac{3}{5}x + 45$, and then perform the arithmetic operations to find $g(25)$.
When evaluating a function at a specific point, carefully substitute the given number into the function and perform each step methodically. Double-check your arithmetic to avoid small errors.
A common mistake is to miscalculate the multiplication or the fraction operation. Ensure that you correctly multiply $\frac{3}{5}$ by 25 and then add 45.
This type of problem is fundamental in algebra and serves as a basis for understanding more complex functions and graphs. It helps students practice function evaluation, which is a crucial skill in algebra and calculus. In SAT math, being comfortable with function notation and evaluation can save time and avoid mistakes in more complex problems.

Substitute $x = 25$ into the function $g(x)$: $g(25) = \frac{3}{5} \times 25 + 45$
Calculate $\frac{3}{5} \times 25$: Multiply 3 and 25 to get 75. Divide 75 by 5 to get 15.
Add 15 to 45, which results in 60.
Thus, the value of $g(25)$ is 60.

7. A local activist group organized a rally to protest social justice issues. They initially planned to have 720 participants. After a week of outreach, they found that 15% of the planned participants had to withdraw due to scheduling conflicts. How many participants remained for the rally?

A. 612

B. 620

C. 630

D. 640

Answer

A

Solution

This question tests the student's ability to understand and work with percentages, particularly in the context of a word problem. It assesses the student's proficiency in calculating percentages and subtracting the result from an initial quantity.
First, determine the number of participants who withdrew by calculating 15% of the initially planned 720 participants. Then, subtract that number from 720 to find out how many participants remained.
Remember that 'percent' means 'per hundred,' so 15% can be converted to a decimal by dividing by 100, which is 0.15. Multiply this decimal by the total number of participants to find the number who withdrew. Finally, subtract this from the initial total.
Be careful with the percentage calculation. Ensure that you correctly convert the percentage to a decimal form before multiplying. Also, double-check your subtraction to avoid minor mistakes that could lead to the wrong answer.
This problem is a typical example of a rates word problem that is common in the SAT. It checks the student's ability to handle percentages and basic arithmetic operations within a real-world context. Mastery of these types of problems is crucial for doing well in the 'Problem Solving and Data Analysis' category of the SAT.

To find the number of participants who withdrew, calculate 15% of 720.

15% of 720 is calculated as $\frac{15}{100} \times 720$.
This simplifies to 0.15×720.
$0.15 \times 720 = 108$ participants withdrew.
Subtract the number of participants who withdrew from the initial number:
$720 - 108 = 612$.
Thus, 612 participants remained for the rally.

8. The function g is defined by $g(x) = 3x^2 - 5x + 8$. What is the value of $g(1)$?

A. 2

B. 4

C. 6

D. 8

Answer

C

Solution

This problem tests the student's ability to evaluate a quadratic function by substituting a specific value for the variable x. It assesses the understanding of basic algebraic manipulation and function evaluation.
To solve this problem, the student should substitute the given value of x (which is 1) into the quadratic function $g(x)$ and then simplify the resulting expression.
Carefully substitute $x = 1$ into the function. Make sure to follow the order of operations: first square the value, then multiply by coefficients, and finally add or subtract the constant terms. Be attentive to arithmetic errors, especially during multiplication and addition/subtraction steps. Also, ensure that you do not skip any steps in the order of operations.
This problem is a straightforward evaluation of a quadratic function, a fundamental skill in algebra. It assesses the student's ability to correctly substitute values and perform basic arithmetic operations. Mastery of this type of problem is crucial for more advanced topics in mathematics, and accuracy is essential. Practicing similar problems can help improve speed and reduce errors in function evaluation tasks commonly found in the SAT.

Substitute $x = 1$ into the function $g(x)$.
Calculate $g(1) = 3(1)^2 - 5(1) + 8$.
Simplify: $g(1) = 3(1) - 5 + 8$.
Further simplify: $g(1) = 3 - 5 + 8$.
Finally calculate: $g(1) = 6$.

9. What is the median of the data set shown? data set = [4, 50, 8, 23, 15, 42, 16]

Answer

16

Solution

The problem is designed to test the student's understanding of how to find the median in a data set. It assesses the student's ability to organize data and identify the middle value.
To solve the problem, the student needs to first ensure the data set is in numerical order, which it already is. Then, since there are seven numbers, the median is the one in the fourth position when the numbers are listed in order.
Remember that the median is the middle number in a list of numbers. If the list has an odd number of entries, the median is the exact middle number. If it has an even number of entries, you would take the average of the two middle numbers.
Be careful not to confuse the median with the mean or mode. They are different measures of central tendency. Ensure the data is sorted before attempting to find the median.
This type of problem is fundamental in understanding statistical data analysis. Finding the median is a basic skill that helps in understanding the distribution of data. On the SAT, this type of problem tests your ability to work with and interpret data accurately, which is essential for data analysis.

Count the numbers in the data set: 7 numbers.
Identify the middle position: $\frac{7+1}{2} = 4$, The median is the fourth number in the list.
Looking at the data set: [4, 50, 8, 23, 15, 42, 16], sorting the data set: [4, 8, 15, 16, 23, 42, 50], The fourth number is 16.

10. In ΔABC, ∠B is a right angle and the length of BC is 180 millimeters. If cos(A) = $\frac{4}{5}$, what is the length, in millimeters, of AB?

Answer

240

Solution

This problem assesses the student's understanding of right angle trigonometry, specifically their ability to apply the cosine function to find the length of a side in a right triangle.
To solve this problem, recognize that in a right triangle, the cosine of an angle is the ratio of the adjacent side to the hypotenuse. Here, angle A is given, and you need to identify the adjacent side (AB) and the hypotenuse (AC). Use the formula $cos(A) = \frac{adjacent}{hypotenuse}$. Plug in the given values: $cos(A) = \frac{4}{5}$ and $BC = 180$ millimeters. Set up the equation $\frac{4}{5} = \frac{AB}{AC}$ and solve for AB.
Remember the mnemonic SOHCAHTOA to quickly recall that cosine is the ratio of the adjacent side over the hypotenuse. When setting up your equation, make sure to clearly identify which sides of the triangle correspond to these terms based on the given angle.
Be careful not to confuse the sides of the triangle and the angle being referenced. Double-check that you are using the correct sides for cosine (adjacent and hypotenuse) and ensure your algebraic manipulation is accurate when solving for AB.
This problem is a classic example of right angle trigonometry, testing a student's ability to apply fundamental trigonometric ratios in practical scenarios. Mastery of these concepts is crucial, as they form the basis for more advanced topics in geometry and trigonometry. Being comfortable with the relationships between angles and sides in right triangles is vital for success in SAT geometry problems.

We know that $cos(A) = \frac{AB}{AC} = \frac{4}{5}$.
Since BC is 180 mm
we first find AC using Pythagorean identity and given cosine value.
Use the identity: $sin^2(A) + cos^2(A) = 1$.
$sin^2(A) = 1 - cos^2(A) = 1 - \left(\frac{4}{5}\right)^2 = 1 - \frac{16}{25} = \frac{9}{25}$.
Thus, $sin(A) = \sqrt{\frac{9}{25}} = \frac{3}{5}$.
Using $sin(A) = \frac{BC}{AC}$, we have $\frac{3}{5} = \frac{180mm}{AC}$.
Solving for AC gives $AC = 180mm \times \frac{5}{3} = 300mm$.

Now, use cos(A) to find AB: $cos(A) = \frac{AB}{AC} = \frac{4}{5} = \frac{AB}{300mm}$.
Solve for AB: $AB = \frac{4}{5} \times 300mm = 240mm$.