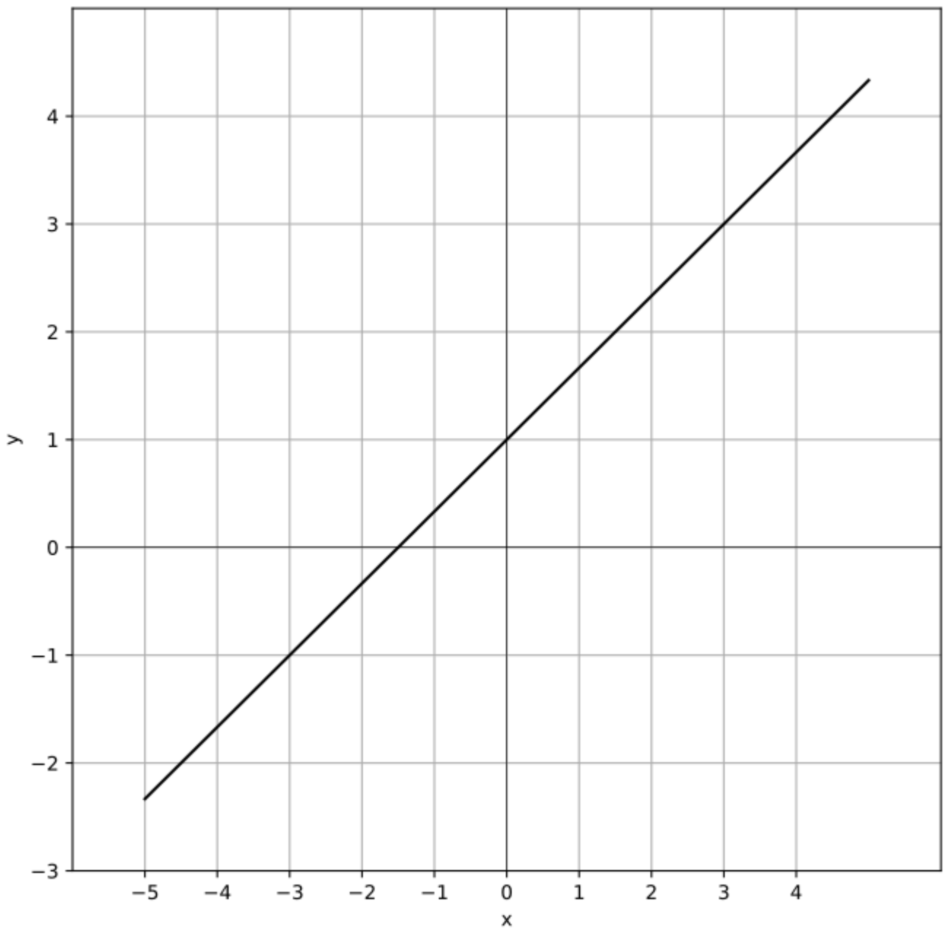
Digital

SAT Math

13

SAT Math Problems

1. What equation defines the linear function shown in the graph?



2. The function f is defined by $f(x) = \frac{2}{n}x - 10$, where n is an integer constant and $5 \leq n \leq 8$. For the graph of $y = f(x) + 15$ in the xy-plane, what is the x-coordinate of a possible x-intercept?

A. -13

B. -15

C. -17

D. -19

3. What is the median of the data set shown? data set = [3, 7, 9, 1, 5, 8, 2]

4. Rob holds a fundraising event for a political movement advocating for climate policy reform. If he raised $800 and decided to donate 15% of the funds to support a local environmental group, how much money will he donate?

A. $100

B. $110

C. $120

D. $130

5. A car travels at a constant acceleration of 4.5 meters per second squared. What is this acceleration, in feet per minute squared, rounded to the nearest tenth? (Use 1 foot = 0.3048 meters)

A. 53149.0

B. 53149.2

C. 53150.0

D. 53150.2

6. For a polynomial function, the graph of $y = f(x)$ in the xy-plane contains the points (2, 0), (3, 0), (-1, 0), and (5, 0). Which of the following must be a factor of $f(x)$?

A. $x^2 - 5x + 6$

B. $x^2 - 4x + 3$

C. $x^2 - 2x - 15$

D. $x^2 - 8x + 12$

7. A wooden cube used in a public health education demonstration has an edge length of 3 centimeters. If the cube weighs 5.61 grams, what is the density of the cube in grams per cubic centimeter?

A. 0.2068

B. 0.2070

C. 0.2082

D. 0.2078

8. Which expression is equivalent to $5x^4(3x^3 + 2x^2 - 7)$?

A. $15x^7 + 10x^6 - 35x^5$

B. $15x^7 + 10x^6 + 35x^4$

C. $15x^{12} + 10x^8 - 35x^4$

D. $15x^7 + 10x^6 - 35x^4$

9. Liam is collecting data on educational access for girls in a community. If the number of girls with access to education is 75% of the total number of girls, and the total number of girls is 540, how many girls have access to education?

10. What is the y-coordinate of the y-intercept of the graph of $y = f(x) - 3$ in the xy-plane? $f(x) = 2(5x - 4)$

A. -11

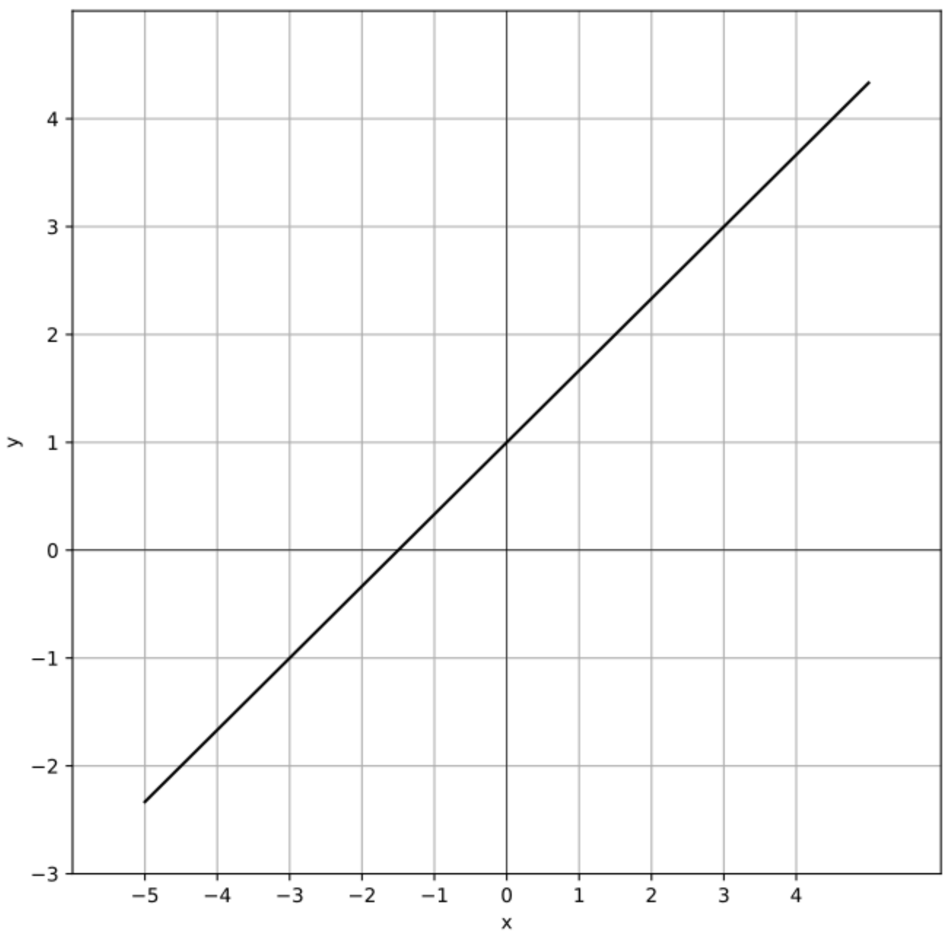
B. -8

C. -5

D. -3

SAT Math Solutions

1. What equation defines the linear function shown in the graph?



Answer

$f(x) = \frac{2}{3}x + 1$

Solution

This problem tests the student's ability to understand and apply the concept of linear equations, specifically the slope-intercept form. The student must be able to identify the slope and y-intercept from a graph and use these to formulate the equation of the line.

The student should recall the slope-intercept form of a linear equation, which is $y = mx + b$, where m is the slope and b is the y-intercept. Here, the slope m is given as $\frac{2}{3}$, and the y-intercept b is given as 1. Substituting these values into the equation will give the equation of the line.
Remember that the y-intercept is the point where the line crosses the y-axis, so it is always in the form of (0, b). Also, when given a slope, ensure you understand that it represents the rate of change or 'rise over run', which in this case is 2/3.
Be careful not to confuse the slope with the y-intercept. Also, ensure that you substitute the values correctly into the slope-intercept form. Watch out for common mistakes like reversing the numerator and denominator of the slope.
This type of problem is fundamental in understanding how linear equations represent straight lines in a graph. It reinforces the concept of the slope-intercept form and the importance of accurately identifying and applying the slope and y-intercept. Mastering this concept is crucial for solving more complex algebraic equations and understanding linear relationships in various contexts. In the SAT, such problems test the student's ability to quickly translate graphical information into algebraic expressions, a vital skill for success.

Find slope and y-intercept in graph: slope is $\frac{2}{3}$ and y-intercept is (0,1)

Substitute slope and y-intercept: $f(x) = mx + b$, Where the slope is $\frac{2}{3}$ and the y-intercept is 1
So, $f(x) = \frac{2}{3}x + 1$.

2. The function f is defined by $f(x) = \frac{2}{n}x - 10$, where n is an integer constant and $5 \leq n \leq 8$. For the graph of $y = f(x) + 15$ in the xy-plane, what is the x-coordinate of a possible x-intercept?

A. -13

B. -15

C. -17

D. -19

Answer

B

Solution

This problem tests the student's understanding of linear equations and their graphs, specifically focusing on finding the x-intercept of a transformed function. It also checks the student's ability to manipulate algebraic expressions and understand the

effect of constants on the graph of the function.
To solve this problem, recognize that the x-intercept is the value of x when y equals zero. The function provided is transformed to $y = f(x) + 15$. Set y to zero and solve for x: $0 = \frac{2}{n}x - 10 + 15$. Simplify the equation to find x in terms of n, then substitute possible integer values for n between 5 and 8 as given in the problem to find the possible x-intercept.
Remember that the x-intercept occurs when $y = 0$. Carefully handle the algebraic manipulation and ensure you substitute each possible value of n to find potential solutions. Check your work by substituting back to see if the value makes the entire expression zero.
A common mistake is to forget adding or subtracting constants when manipulating the function. Make sure not to overlook the +15 when setting up the equation for finding the x-intercept. Additionally, ensure all integer values of n within the specified range are checked.
This problem is a good test of basic algebraic manipulation and understanding of linear functions. It requires careful attention to detail, especially in handling transformations of functions. SAT problems like this assess whether students can apply algebraic concepts in slightly more complex contexts, which is a critical skill for success in this section.

Substitute $y = f(x) + 15$ into the equation: $y = (\frac{2}{n}x - 10) + 15$
Simplify: $y = \frac{2}{n}x + 5$
To find the x-intercept, set y = 0: $0 = \frac{2}{n}x + 5$
Subtract 5 from both sides: $-\frac{2}{n}x = 5$
Multiply both sides by $-\frac{n}{2}$: $x =- \frac{5n}{2}$
Considering the integer values of n:
For n = 5: $x =- \frac{5(5)}{2} =- \frac{25}{2} =- 12.5$
For n = 6: $x =- \frac{5(6)}{2} =- \frac{30}{2} =- 15$
For n = 7: $x =- \frac{5(7)}{2} =- \frac{35}{2} =- 17.5$
For $n = 8$: $x =- \frac{5(8)}{2} =- \frac{40}{2} =- 20$
Possible x-coordinates are -12.5, -15, -17.5, and -20.

3. What is the median of the data set shown? data set = [3, 7, 9, 1, 5, 8, 2]

Answer

5

Solution

The problem aims to assess the student's understanding of finding the median from a data set. It checks if the student can correctly identify the middle value of an ordered data set, which is a fundamental concept in statistics.
To solve this problem, the student should first arrange the data set in ascending order. Once the data is ordered, the student should identify the middle value. Since the data set has an odd number of elements, the median is the middle number of the ordered list.
Remember, the median is the middle value of a data set arranged in order. For an odd number of data points, it's simply the middle one. For an even number, it's the average of the two middle values. Always ensure your data is ordered before searching for the median.
A common mistake is forgetting to order the data before finding the median. Always double-check that the data is sorted correctly. Also, ensure you correctly count to find the middle position, especially if the data set is long.
This problem is a classic example of testing basic statistical skills related to identifying central tendencies. Being able to find the median is crucial as it provides insight into the distribution of data. In the SAT, this type of problem tests accuracy and attention to detail, ensuring students understand and apply statistical concepts accurately.

Step 1: Arrange the data set in ascending order.
The given data set is [3, 7, 9, 1, 5, 8, 2].
Arranging it in ascending order gives [1, 2, 3, 5, 7, 8, 9].
Step 2: Determine the number of values in the data set.
The data set has 7 values, which is an odd number.
Step 3: Find the middle value.
Since there are 7 values, the median is the 4th value in the ordered data set.
The 4th value is 5.
Therefore, the median of the data set is 5.

4. Rob holds a fundraising event for a political movement advocating for climate policy reform. If he raised \$800 and decided to donate 15% of the funds to support a local environmental group, how much money will he donate?

A. \$100

B. \$110

C. \$120

D. \$130

Answer

C

Solution

This problem is designed to test the student's ability to calculate percentages and apply this understanding to a real-world context. It examines whether students can interpret percentage problems and execute the basic arithmetic needed to find a percentage of a given amount.
To solve this problem, students need to identify that they are required to calculate 15% of \$800. This involves multiplying \$800 by 0.15 (since 15% is the same as $\frac{15}{100}$ or 0.15). The result of this calculation will give the amount that Rob will donate to the local environmental group.
When calculating percentages, it can be helpful to convert the percentage into a decimal by dividing by 100. This makes it straightforward to multiply by the given amount. In this case, converting 15% to 0.15 and then multiplying by \$800 simplifies the calculation process.
One common mistake is to forget to convert the percentage into a decimal form before multiplying. Additionally, ensure that the multiplication is performed correctly and double-check calculations to avoid simple arithmetic errors.
Remember, the amount given is initially in dollars, so the final answer should also reflect a monetary value.
This problem is a straightforward percentage calculation that is typical of the 'Percentages' unit in the 'Problem Solving and Data Analysis' category. It assesses a fundamental skill that is applicable in many real-life scenarios. Being adept at such calculations is crucial for effectively handling financial decisions and data analysis tasks. Mastery of this type of problem is important for succeeding in the SAT math section, as it builds a foundation for more complex percentage problems encountered later.

To find 15% of \$800, we convert 15% to a decimal: 15% = 0.15.
Multiply the total amount by the decimal: \$800 $\times$ 0.15.
Calculation: $800 \times 0.15 = 120$.
Thus, Rob will donate \$120 to the local environmental group.

5. A car travels at a constant acceleration of 4.5 meters per second squared. What is this acceleration, in feet per minute squared, rounded to the nearest tenth? (Use 1 foot = 0.3048 meters)

A. 53149.0

B. 53149.2

C. 53150.0

D. 53150.2

Answer

B

Solution

This problem tests the student's ability to perform unit conversions, specifically converting from meters per second squared to feet per minute squared. It also evaluates the student's understanding of unit relationships and their ability to handle multi-step conversions.
To solve this problem, the student should follow these steps:
1) Convert meters to feet by using the conversion factor 1 foot = 0.3048 meters.
2) Convert seconds squared to minutes squared by recognizing that there are 60 seconds in a minute and squaring that conversion factor.
3) Combine these conversions to find the acceleration in feet per minute squared.
First, remember to deal with one unit conversion at a time. It might help to write down each step to keep track of your conversions. Additionally, always double-check your conversion factors and ensure that units cancel out correctly.
Be careful with squaring the time conversion factor. Remember that you need to square the entire conversion factor (60 seconds per minute) to convert seconds squared to minutes squared. Also, ensure you do not round off too early in your calculations, as this can lead to inaccuracies.
This unit conversion problem is a common type in SAT math, reflecting real-world scenarios where multiple unit conversions are necessary. It tests both basic arithmetic skills and understanding of unit relationships. Mastering these types of problems is crucial for the Problem Solving and Data Analysis section of the SAT.

Given acceleration: 4.5 meters per second squared.
First, convert meters to feet: $\frac{4.5}{0.3048}$ feet per meter.
Result: $\frac{4.5}{0.3048} = 14.7637795276$ feet per second squared.
Now convert seconds squared to minutes squared: (14.7637795276 feet / $(second)^2$) $\times$ $((60)^2)$ seconds squared per minute squared.
Calculation: $14.7637795276 \times 3600 = 53149.2062996$ feet per minute squared.
Round the result to the nearest tenth: 53149.2

6. For a polynomial function, the graph of $y = f(x)$ in the xy-plane contains the points (2, 0), (3, 0), (-1, 0), and (5, 0). Which of the following must be a factor of $f(x)$?

A. $x^2 - 5x + 6$

B. $x^2 - 4x + 3$

C. $x^2 - 2x - 15$

D. $x^2 - 8x + 12$

Answer

A

Solution

This problem tests your understanding of polynomial functions and their factors, specifically how the x-intercepts of a polynomial relate to its factors.
To solve this problem, recognize that each x-intercept of the polynomial function corresponds to a factor of the function. The x-intercepts given are (2, 0), (3, 0), (-1, 0), and (5, 0). Therefore, the factors of the polynomial are (x - 2), (x - 3), (x + 1), and (x - 5).
Remember that if a polynomial has a root at x = a, then (x - a) is a factor of the polynomial. Listing out the given x-intercepts can help you quickly determine the factors.
Be cautious not to confuse the x-intercepts with y-intercepts, as they indicate different things. Also, ensure that you do not overlook any negative signs when determining factors from the intercepts.
This type of problem is common on the SAT as it assesses your ability to connect graphical information to algebraic expressions. Mastery of this concept is crucial because it highlights your understanding of the relationship between a polynomial's roots and its factors, a fundamental concept in advanced algebra.

Given roots are x = 2, x = 3, x = -1, and x = 5. Therefore, f(x) must include factors (x - 2), (x - 3), (x + 1), and (x - 5).
The task is to determine which option is necessarily a factor of f(x).
Option A: $x^2 - 5x + 6$ can be factored as $(x - 2)(x - 3)$. This matches two of the roots, indicating it is a factor.
Option B: $x^2 - 4x + 3$ can be factored as $(x - 1)(x - 3)$. This does not match with the needed roots.
Option C: $x^2 - 2x - 15$ can be factored as $(x + 3)(x - 5)$. This does not match with the needed roots

Option D: $x^2 - 8x + 12$ can be factored as $(x - 2)(x - 6)$. This does not match with the needed roots.

7. A wooden cube used in a public health education demonstration has an edge length of 3 centimeters. If the cube weighs 5.61 grams, what is the density of the cube in grams per cubic centimeter?

A. 0.2068

B. 0.2070

C. 0.2082

D. 0.2078

Answer

D

Solution

This problem aims to test the student's understanding of geometric properties of a cube, specifically how to calculate the volume, and then apply the formula for density. The student needs to be familiar with basic volume formulas and the concept of density as mass per unit volume.
1. Calculate the volume of the cube using the formula for the volume of a cube ($V = a^3$ where 'a' is the edge length).
2. Use the given mass and the volume to calculate the density using the formula ($Density = \frac{Mass}{Volume}$).
Remember that the volume of a cube is found by cubing the edge length. Write down all given information and use the density formula directly after calculating the volume. This helps in organizing thoughts and reducing careless errors.
Be careful with units and ensure consistency throughout the calculation.
Miscalculating the volume by forgetting to cube the edge length is a common mistake. Verify that the density units are in grams per cubic centimeter as required by the problem.
This problem tests fundamental skills in geometry and unit analysis, which are crucial for many SAT math problems. Understanding the relationships between edge length, volume, and density is key. Efficiently solving such problems requires a clear grasp of basic formulas and careful unit management, which are essential skills for SAT success.

The formula for calculating the volume of a cube is $Volume = edge\ length^3$
For this cube, the volume is $3^3 = 27$ cubic centimeters.

Density is given by $Density = \frac{Mass}{Volume}$, Substituting the known values:
$Density = \frac{5.61}{27}$ grams per cubic centimeter.
Performing the division: $\frac{5.61}{27} = 0.207777...$
Rounding to the fourth digit, we get $Density \cong 0.2078$ grams per cubic centimeter.

8. Which expression is equivalent to $5x^4(3x^3 + 2x^2 - 7)$?

A. $15x^7 + 10x^6 - 35x^5$

B. $15x^7 + 10x^6 + 35x^4$

C. $15x^{12} + 10x^8 - 35x^4$

D. $15x^7 + 10x^6 - 35x^4$

Answer

D

Solution

This problem assesses a student's understanding of polynomial operations, specifically focusing on the multiplication of polynomials with a degree greater than two. The student must demonstrate their ability to distribute a monomial across a polynomial expression and simplify the result.

To solve this problem, the student needs to distribute the monomial, which is $5x^4$, over each term in the polynomial inside the parenthesis. This involves multiplying $5x^4$ by each term in the polynomial $(3x^3, 2x^2, and\ -7)$ and combining the results. When distributing the monomial, remember to add the exponents of the x terms. For instance, when multiplying x^4 by x^3, you add the exponents to get x^7. Also, keep track of the coefficients and make sure to multiply them correctly.
Be careful with the signs when multiplying. It's easy to make mistakes with negative numbers, so when multiplying the -7 term, ensure you apply the negative sign correctly. Additionally, ensure all terms are combined at the end to form a correct polynomial expression.
This type of problem is common in the Advanced Math section of the SAT and tests a fundamental algebra skill: operations with polynomials. Mastery of these skills is crucial as they form the basis for more complex algebraic manipulations. Efficiently distributing terms and simplifying expressions is an essential skill in algebra that will be used in various contexts throughout the test.

Distribute $5x^4$ to each term in the parentheses: $5x^4 \times 3x^3 = 15x^{(4+3)} = 15x^7$,

$5x^4 \times 2x^2 = 10x^{(4+2)} = 10x^6, 5x^4 \times (-7) = -35x^4$
Combine the results: $15x^7 + 10x^6 - 35x^4$.

9. Liam is collecting data on educational access for girls in a community. If the number of girls with access to education is 75% of the total number of girls, and the total number of girls is 540, how many girls have access to education?

Answer

405

Solution

This question assesses the student's ability to understand and apply percentage concepts in a real-world context. It tests the student's capacity to calculate percentages and apply them to word problems to find a solution.
To solve this problem, the number of girls is given as 540, and 75% of them have access to education. Calculate 75% of 540 to find the number of girls with access.
When dealing with percentages, remember that you can find a percentage of a number by converting the percentage to a decimal and then multiplying. For instance, 75% becomes 0.75, and you multiply by 540 to get the result.
Students often make mistakes by confusing the percentages or applying the percentage to the wrong number. Ensure that the percentage is applied to the correct total number of girls, not some other number mentioned in the problem.
This type of problem is common in the SAT as it evaluates the student's ability to apply mathematical concepts to real-world scenarios. It is essential to read the problem carefully, identify the correct data points, and apply the percentage calculations accurately. Mastery of this type of problem demonstrates a solid understanding of percentages and their applications, which is a valuable skill in the SAT math section.

1. Calculate the number of girls with access to education.
The number of girls with access to education is 75% of the total number of girls.
$Number\ of\ girls\ with\ access$ = 75% of 540
$Number\ of\ girls\ with\ access = 0.75 \times 540$
$Number\ of\ girls\ with\ access = 405$

10. What is the y-coordinate of the y-intercept of the graph of $y = f(x) - 3$ in the xy-plane? $f(x) = 2(5x - 4)$

A. -11

B. -8

C. -5

D. -3

Answer

A

Solution

This problem assesses a student's ability to understand and manipulate linear functions, particularly focusing on finding the y-intercept of a modified function. It tests knowledge of how transformations affect the graph of a function and basic algebraic operations.
To solve this problem, first understand that the y-intercept of a function $y = f(x)$ occurs when $x = 0$. Start by finding $f(0)$ for the given function$f(x) = 2(5x - 4)$. Then, adjust this y-intercept by subtracting 3, as indicated by the modified function $y = f(x) - 3$.
Always remember that the y-intercept is found by setting x to 0. For composite functions involving transformations like $y = f(x) - 3$, first find the y-intercept of the original function and then apply the transformation to this intercept.
Be careful with the order of operations when evaluating $f(0)$. Ensure that you correctly substitute $x = 0$ and perform all arithmetic operations accurately. Also, do not forget to apply the transformation (subtracting 3) to the original y-intercept.
This type of problem is common in SAT algebra sections and tests the student's proficiency in handling linear functions and their transformations. Understanding function transformations and correctly applying them to find the y-intercept is a crucial skill. By practicing similar problems, students can improve their ability to quickly and accurately solve these types of questions.

First, solve for $f(x)$ when $x = 0$.
Substitute$x = 0$ into $f(x)$:, $f(x) = 2(5(0) - 4) = 2(-4) =- 8$
Now substitute $f(x)$ into $y = f(x) - 3$:, $y =- 8 - 3 =- 11$
Therefore, the y-coordinate of the y-intercept is -11.

