Math

Digital SAT

Geometry and Trigonometry

SAT Math Geometry and Trigonometry

1. The table gives the perimeters of similar triangles ABC and DEF, where AB corresponds to DE. The length of AB is 5. What is the length of DE?

Triangle	Perimeter
Triangle ABC	30
Triangle DEF	90

A. 12

B. 15

C. 18

D. 20

2. Circle C has a radius of $5x$ and circle D has a radius of $25x$. The area of circle D is how many times the area of circle C?

A. 10

B. 15

C. 20

D. 25

3. A wooden cube used in a public health education demonstration has an edge length of 3 centimeters. If the cube weighs 5.61 grams, what is the density of the cube in grams per cubic centimeter?

A. 0.2068

B. 0.2070

C. 0.2082

D. 0.2078

4. In ΔABC, $\angle B$ is a right angle and the length of BC is 180 millimeters. If $cos(A) = \frac{4}{5}$, what is the length, in millimeters, of AB?

A. 200

B. 220

C. 240

D. 260

5. For two acute angles, $\angle A$ and $\angle B$, sin(A) = cos(B). The measures, in degrees, of $\angle A$ and $\angle B$ are 2x + 30 and 5x - 10, respectively. What is the value of x?

A. 8

B. 9

C. 10

D. 11

6. Circle C has a radius of 2x and circle D has a radius of 50x. The area of circle D is how many times the area of circle C?

7. A wooden cube is carved from a log, and its edges measure 4 centimeters. If the cube is then sanded down, causing each edge to decrease in length by 0.5 centimeters, what will be the volume of the newly shaped cube, in cubic centimeters?

8. For two acute angles, $\angle A$ and $\angle B$, $sin(A) = cos(B)$. The measures, in degrees, of $\angle A$ and $\angle B$ are $2x + 30$ and $5x - 10$, respectively. What is the value of x?

A. 8

B. 9

C. 10

D. 11

9. A solid sphere has a radius of 15 feet. If a certain VR simulation requires the volume of an object to be equal to the volume of the sphere, what is the volume of the sphere in cubic feet?

A. 3000π

B. 4500π

C. 5000π

D. 6000π

10. The table gives the perimeters of similar triangles DEF and GHI, where DE corresponds to GH. If the length of DE is 16, what is the length of GH?

Triangle	Perimeter
Triangle DEF	80
Triangle GHI	240

A. 24

B. 32

C. 48

D. 64

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SAT Math Geometry and Trigonometry Solutions

1. The table gives the perimeters of similar triangles ABC and DEF, where AB corresponds to DE. The length of AB is 5. What is the length of DE?

Triangle	Perimeter
Triangle ABC	30
Triangle DEF	90

A. 12

B. 15

C. 18

D. 20

Answer

B

Solution

This problem tests the student's understanding of similar triangles and the concept of proportionality. The student should be able to use the known perimeters to find the length of a corresponding side in similar triangles. To solve this problem, the student should start by recognizing that if two triangles are similar, the ratios of their corresponding side lengths are equal to the ratio of their perimeters. First, calculate the ratio of the given perimeters from the table. Then, use this ratio to find the length of DE by setting up a proportion with the known length of AB, which corresponds to DE. Always remember that the key to solving problems with similar triangles is setting up correct proportions. Make sure to match the corresponding sides correctly and use the information given in the problem, such as the length of AB and the perimeters provided, to find the unknown length. Be careful not to mix up the corresponding sides. Double-check which sides correspond to each other based on the problem's description. Also, ensure you correctly calculate the ratio of the perimeters before applying it to the corresponding sides. This type of problem is a classic example of applying the properties of similar triangles and proportional relationships, which is a fundamental skill in geometry. Being able to correctly identify and apply these properties is crucial on the SAT, as it shows a student's ability to understand and manipulate geometric concepts effectively.

Let the length of DE be x., The ratio of the perimeters is 30: 90, which simplifies to 1: 3., Since AB corresponds to DE, the ratio of AB to DE is also 1: 3., Thus, we have $5 : x = 1 : 3$., Cross-multiplying gives $5\times3 = x\times1$., Therefore, $x = 15$.

2. Circle C has a radius of $5x$ and circle D has a radius of $25x$. The area of circle D is how many times the area of circle C?

A. 10

B. 15

C. 20

D. 25

Answer

D

Solution

This problem aims to assess the student's understanding of the relationship between the radius and the area of a circle. Specifically, it examines the ability to apply the formula for the area of a circle and to work with ratios. To solve this problem, the student should start by recalling the formula for the area of a circle, which is $A = \pi r^2$. Calculate the area of both circles using their respective radii, then compare the two areas by forming a ratio. Remember that the area of a circle increases with the square of its radius. When comparing areas of circles, you can often simplify your work by setting up a ratio instead of calculating exact areas. In this problem, simplify the ratio of the radii first to see the effect on the area. Be careful not to confuse the ratio of the radii with the ratio of the areas. The radius is linear, while the area is quadratic. Also, ensure that you square the radii correctly and apply the π factor consistently. This type of problem is common in SAT geometry questions and tests the ability to understand and manipulate geometric formulas, specifically circles. It also assesses the student's skill in working with proportions and recognizing how changes in one dimension (radius) affect another dimension (area). Mastery of these concepts is crucial not only for geometry but also for more advanced topics in mathematics.

Calculate the area of circle C: $A_C = \pi(5x)^2 = 25\pi x^2$, Calculate the area of circle D:

$A_D = \pi(25x)^2 = 625\pi x^2$, The ratio of the area of circle D to circle C is:
$\frac{A_D}{A_C} = \frac{625\pi x^2}{25\pi x^2} = 25$., Thus, the area of circle D is 25 times the area of circle C.

3. A wooden cube used in a public health education demonstration has an edge length of 3 centimeters. If the cube weighs 5.61 grams, what is the density of the cube in grams per cubic centimeter?

A. 0.2068

B. 0.2070

C. 0.2082

D. 0.2078

Answer

D

Solution

This problem aims to test the student's understanding of geometric properties of a cube, specifically how to calculate the volume, and then apply the formula for density. The student needs to be familiar with basic volume formulas and the concept of density as mass per unit volume. 1. Calculate the volume of the cube using the formula for the volume of a cube ($V = a^3$ where 'a' is the edge length). 2. Use the given mass and the volume to calculate the density using the formula ($Density = \frac{Mass}{Volume}$). Remember that the volume of a cube is found by cubing the edge length. Write down all given information and use the density formula directly after calculating the volume. This helps in organizing thoughts and reducing careless errors. Be careful with units and ensure consistency throughout the calculation. Miscalculating the volume by forgetting to cube the edge length is a common mistake. Verify that the density units are in grams per cubic centimeter as required by the problem. This problem tests fundamental skills in geometry and unit analysis, which are crucial for many SAT math problems. Understanding the relationships between edge length, volume, and density is key. Efficiently solving such problems requires a clear grasp of basic formulas and careful unit management, which are essential skills for SAT success.

The formula for calculating the volume of a cube is $Volume = edge\ length^3$, For this cube, the volume is $3^3 = 27$ cubic centimeters., Density is given by

$Density\ = \frac{Mass}{Volume}$, Substituting the known values: $Density\ = \frac{5.61}{27}$ grams per cubic centimeter., Performing the division: $\frac{5.61}{27}\ =\ 0.207777...$, Rounding to the fourth digit, we get $Density \cong 0.2078$ grams per cubic centimeter.

4. In ΔABC, ∠B is a right angle and the length of BC is 180 millimeters. If $cos(A)\ = \frac{4}{5}$, what is the length, in millimeters, of AB?

A. 200

B. 220

C. 240

D. 260

Answer

C

Solution

This problem is designed to test the student's understanding of right-angle trigonometry, specifically the ability to use the cosine function to find the length of a side in a right triangle.
The student should recognize that in right triangle ΔABC, with ∠B as the right angle, the cosine of angle A is defined as the ratio of the adjacent side (AB) to the hypotenuse (AC). Given that $cos(A)\ = \frac{4}{5}$, the student needs to set up the equation $\frac{AB}{AC}\ = \frac{4}{5}$. Since BC is given as 180 millimeters, and BC is the side opposite angle A, the student can use the Pythagorean theorem to find AC first before finding AB. Remember that the Pythagorean theorem can be used to find the hypotenuse when you have one side and the cosine ratio. Set up a ratio equation using $cos(A)\ = \frac{adjacent}{hypotenuse}$, and solve for the unknown side. Double-check your calculations by ensuring the triangle's side lengths satisfy the Pythagorean theorem. Be careful not to confuse the sides of the triangle. Ensure you correctly identify which side is opposite and which is adjacent to angle A. Also, ensure that your calculations are exact, and consider simplifying fractions or square roots accurately. This problem assesses the student's proficiency in applying trigonometric ratios to solve for missing side lengths in right triangles. Mastery of this concept is essential for solving more complex trigonometry problems in the SAT. The ability to correctly interpret and apply the cosine function is a crucial skill in the geometry section of the test.

Since $cos(A) = \frac{4}{5}$, we have $\frac{AB}{AC} = \frac{4}{5}$., We need to find the length of AB., Since BC is 180 millimeters, BC is opposite to angle A., In a right triangle, we use the Pythagorean identity: $(sin)^2(A) + (cos)^2(A) = 1$., Given $cos(A) = \frac{4}{5}$, find $(sin)^2(A)$: $\left(\frac{4}{5}\right)^2 + (sin)^2(A) = 1$., $\frac{16}{25} + (sin)^2(A) = 1$., $(sin)^2(A) = \frac{9}{25}$, therefore $sin(A) = \frac{3}{5}$., Using $sin(A)$, we have $sin(A) = \frac{BC}{AC} = \frac{3}{5}$., $AC = \frac{BC}{sin(A)} = \frac{180}{\frac{3}{5}} = 180\times\frac{5}{3} = 300\, millimeters$., Now, using $cos(A) = \frac{4}{5}$, solve for AB: $AB = cos(A)\times AC = \frac{4}{5}\times300 = 240\, millimeters$.

5. For two acute angles, ∠A and ∠B, sin(A) = cos(B). The measures, in degrees, of ∠A and ∠B are 2x + 30 and 5x - 10, respectively. What is the value of x?

A. 8

B. 9

C. 10

D. 11

Answer

C

Solution

This problem tests the student's understanding of the complementary angle relationship between sine and cosine, where sin(A) = cos(B) implies A + B = 90 degrees. It also examines their algebraic manipulation skills to solve for the variable x.
To solve this problem, first apply the trigonometric identity that sin(A) = cos(B) means A + B = 90 degrees. Set up the equation (2x + 30) + (5x - 10) = 90. Simplify and solve this linear equation for x.
Remember the relationship between the sine and cosine of complementary angles: sin(θ) = cos(90° - θ). This is crucial for setting up the correct equation. Also, carefully combine like terms when solving the equation.
Be cautious with the angle measures' expressions. Ensure you correctly combine and simplify the terms in the equation. Double-check your arithmetic when solving for x to avoid simple calculation errors.
This problem is a classic example of testing fundamental trigonometric identities and algebraic skills in one. It assesses your ability to connect geometric angle relationships with algebraic equations, which is essential for solving

trigonometry-related problems on the SAT. Mastery of these concepts and careful calculation will help you excel in this section.

Combine the expressions for A and B: (2x + 30) + (5x - 10) = 90, Simplify: 2x + 30 + 5x - 10 = 90, Combine like terms: 7x + 20 = 90, Subtract 20 from both sides: 7x = 70, Divide both sides by 7: x = 10

6. Circle C has a radius of 2x and circle D has a radius of 50x. The area of circle D is how many times the area of circle C?

Answer

625

Solution

This problem tests the student's understanding of the formula for the area of a circle and their ability to use ratios to compare the areas of two circles based on their radii.
To solve this problem, students should first recall the formula for the area of a circle, $A = \pi r^2$, where r is the radius. Next, they calculate the area of both circles using their given radii: Circle C with a radius of 2x and Circle D with a radius of 50x. After finding the areas, students should set up a ratio of the area of Circle D to the area of Circle C and simplify the ratio.
Remember that when comparing areas of circles, the ratio of the areas is the square of the ratio of their radii. This can simplify the calculations significantly.
Be careful with squaring the radii correctly. A common mistake is not squaring the entire expression, which can lead to an incorrect ratio. Also, ensure that you simplify the ratio completely.
This type of problem is common in SAT geometry sections, as it assesses both the understanding of geometric formulas and the ability to manipulate algebraic expressions. Mastery of such problems requires familiarity with basic geometric formulas and an ability to apply algebraic principles, such as simplifying ratios. Practicing these skills will help improve accuracy and speed on test day.

Calculate the area of circle C:, $Area\ of\ circle\ C = \pi(2x)^2 = 4\pi x^2$, Calculate the area of circle D:, $Area\ of\ circle\ D = \pi(50x)^2 = 2500\pi x^2$, Determine how many times the area of circle D is compared to circle C:,
$Number\ of\ times\ = \frac{2500\pi x^2}{4\pi x^2} = \frac{2500}{4} = 625$

7. A wooden cube is carved from a log, and its edges measure 4 centimeters. If the cube is then sanded down, causing each edge to decrease in length by 0.5 centimeters, what will be the volume of the newly shaped cube, in cubic centimeters?

Answer

42.875 cubic centimeters

Solution

This problem tests the student's understanding of volume calculations for geometric shapes, specifically cubes, and requires the ability to apply volume formulas after modifying dimensions.
To solve this problem, first, calculate the original volume of the cube using the formula for the volume of a cube ($V = (side)^3$). Then, adjust the edge length by subtracting 0.5 cm to account for the sanding down process. Finally, calculate the new volume using the adjusted edge length.
Remember that when the dimensions of a cube change, even slightly, it can significantly impact the volume due to the cubic relationship. Always perform the calculations step by step to ensure accuracy.
Be careful not to confuse the reduction in edge length with a reduction in volume. Ensure that you subtract the 0.5 cm from each edge before recalculating the volume. Also, double-check your arithmetic to ensure that cube calculations are correct.
This type of problem is a classic example of testing geometric reasoning and arithmetic skills. It requires students to accurately apply a formula and understand how dimensional changes affect the volume. Being able to handle such problems efficiently is crucial for the SAT, as it demonstrates a solid grasp of basic geometry and measurement principles.

Determine the new edge length by subtracting 0.5 cm from the original length of each edge., New edge length = 4 cm - 0.5 cm = 3.5 cm., Calculate the volume of the new cube using the formula for the volume of a cube, $V = a^3$, where 'a' is the edge length., Substitute the new edge length into the formula: $V = (3.5cm)^3$., Calculate the cube of the new edge length: $V = 3.5cm \times 3.5cm \times 3.5cm$., V = 42.875 cubic centimeters.

8. For two acute angles, $\angle A$ and $\angle B$, $sin(A) = cos(B)$. The measures, in degrees, of $\angle A$ and $\angle B$ are $2x + 30$ and $5x - 10$, respectively. What is the value of x?

A. 8

B. 9

C. 10

D. 11

Answer

C

Solution

This problem aims to assess the student's understanding of trigonometric identities, specifically the relationship between sine and cosine for complementary angles, and their ability to solve for unknown variables in angle measures. The key to solving this problem is knowing the trigonometric identity $sin(A) = cos(90° - A)$. Since $sin(A) = cos(B)$, we can set up the equation $A = 90° - B$. Substitute the given expressions for $\angle A$ and $\angle B$ and solve for x. Remember that for any angle θ, $sin(\theta) = cos(90° - \theta)$. Use this identity to set up your equation. Carefully substitute the given expressions for $\angle A$ and $\angle B$ into the equation $A = 90° - B$ and solve for x step-by-step. Be careful with your algebra when solving for x. Ensure you correctly distribute and combine like terms. Also, double-check your trigonometric identity and make sure you are substituting correctly. This problem is a classic example of using trigonometric identities to find unknown values. It tests the student's knowledge of the complementary angle relationship between sine and cosine, as well as their algebraic manipulation skills. Being familiar with these identities and solving equations accurately is essential for success in the SAT math section.

Start by setting up the equation from the condition $A + B = 90$ degrees., Substitute the expressions for A and B:, $2x + 30 + 5x - 10 = 90$, Combine like terms:, $7x + 20 = 90$, Subtract 20 from both sides to isolate the term with x:, $7x = 70$, Divide both sides by 7 to solve for x:, $x = 10$

9. A solid sphere has a radius of 15 feet. If a certain VR simulation requires the volume of an object to be equal to the volume of the sphere, what is the volume of the sphere in cubic feet?

A. 3000π

B. 4500π

C. 5000π

D. 6000π

Answer

B

Solution

This question aims to assess the student's ability to calculate the volume of a sphere using the correct formula. It also tests their understanding of geometric properties and their ability to apply these in a real-world context. To solve this problem, students need to recall the formula for the volume of a sphere, which is $V = \frac{4}{3}\pi r^3$. They should then substitute the given radius (15 feet) into the formula and compute the volume. Make sure to remember the formula for the volume of a sphere: $V = \frac{4}{3}\pi r^3$. Write it down first to help guide your calculations. Also, it might be helpful to use a calculator to ensure accuracy, especially when dealing with π. Be careful with the units and ensure that all measurements are in feet. Additionally, make sure to correctly cube the radius (15 feet) and multiply by π. Misplacing a decimal point or making a minor error in calculation can lead to an incorrect answer. This type of problem is common in the SAT to test geometric understanding and the ability to apply formulas in practical situations. It is essential to be comfortable with key geometric formulas and practice substituting values accurately. Remember to double-check your work to avoid small mistakes that could lead to incorrect answers. Mastery of these skills will be beneficial not only in the SAT but also in future mathematical applications.

Using the formula for the volume of a sphere: $V = \frac{4}{3}\pi r^3$., Substitute $r = 15$ into the formula., Calculate $r^3 = (15)^3 = 3375$., Substitute $r^3 = 3375$ into the volume formula: $V = \frac{4}{3}\pi \times 3375$., Simplify the expression: $V = \frac{4 \times 3375}{3}\pi$., Calculate the multiplication: $4 \times 3375 = 13500$., Divide by 3: $\frac{13500}{3} = 4500$., Substitute back: $V = 4500\pi$ cubic feet.

10. The table gives the perimeters of similar triangles DEF and GHI, where DE corresponds to GH. If the length of DE is 16, what is the length of GH?

Triangle	Perimeter
Triangle DEF	80
Triangle GHI	240

A. 24

B. 32

C. 48

D. 64

Answer

C

Solution

This problem tests the student's understanding of the concept of similarity in geometry, particularly focusing on how the perimeters and corresponding side lengths of similar triangles are related.
To solve this problem, the student should recognize that the perimeters of similar triangles are proportional to the corresponding side lengths. Given the perimeter of both triangles, the student can set up a proportion to find the missing length of GH.
Remember that the ratio of any pair of corresponding side lengths in similar triangles is equal to the ratio of their perimeters. Use this ratio to set up a proportion between the length of DE and GH.
Be careful to ensure that the sides being compared are indeed corresponding sides. Also, make sure to solve the proportion correctly to avoid calculation errors.
This type of problem is common in SAT geometry questions as it assesses the ability to apply the properties of similar figures, which is a fundamental concept in geometry. Mastery of setting up and solving proportions is crucial for these problems. Practicing similar problems can improve speed and accuracy in solving them during the test.

For similar triangles, the ratio of the lengths of corresponding sides is equal to the ratio of their perimeters., Given: DE corresponds to GH, Perimeter of DEF = 80, Perimeter of GHI = 240., The ratio of the perimeters is 80:240, which simplifies to 1:3., Therefore, the ratio of DE to GH is also 1:3., If DE = 16, then GH must be 16 multiplied by this ratio, 3., Hence, $GH = 16 \times 3 = 48$.

