Digital

SAT Math

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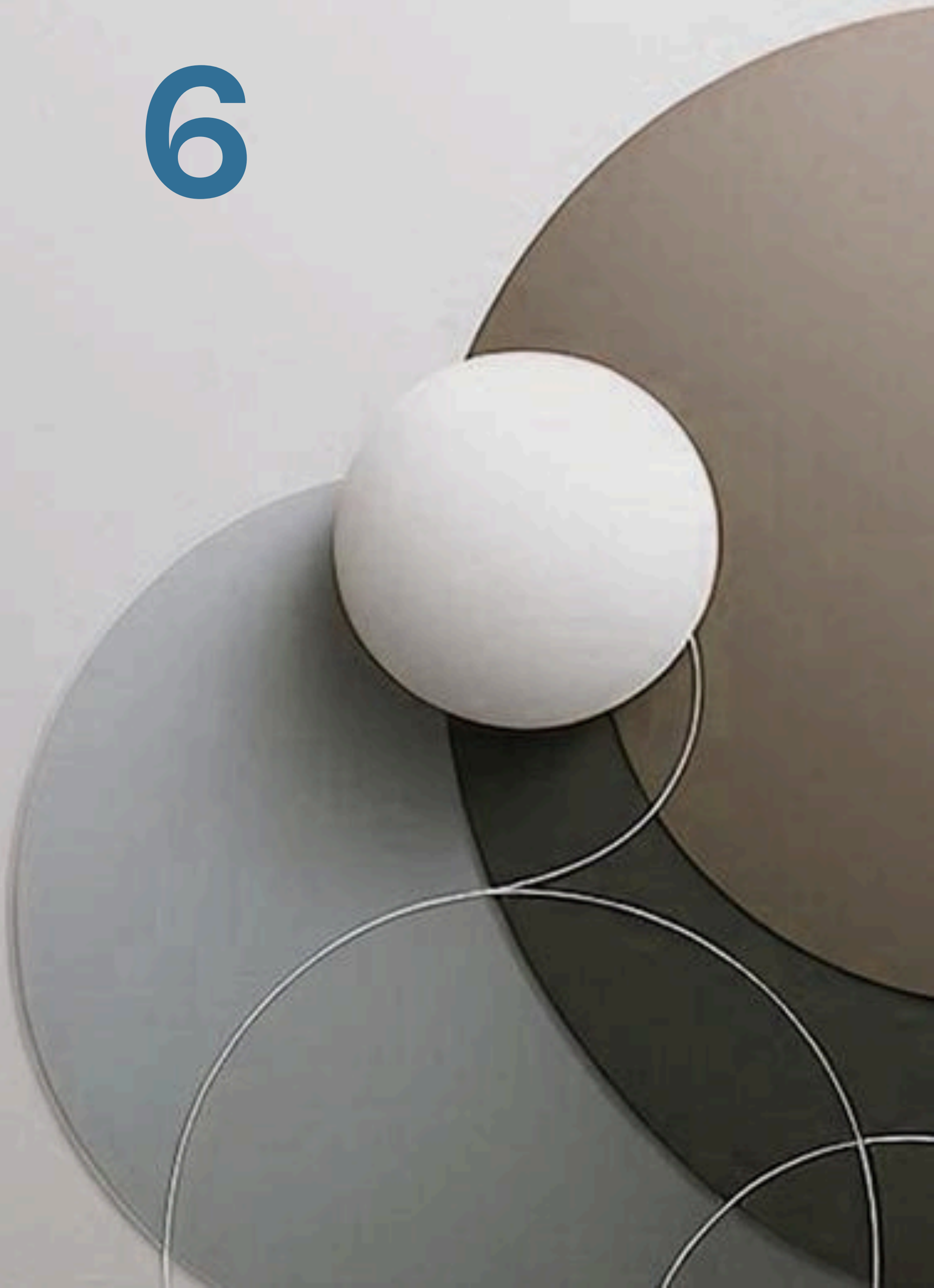

SAT Math Problems

1. What is the sum of the solutions to the given equation? $|2x + 5| + 14 = 26$

A. 5

B. -5

C. 1

D. -1

2. Line m is defined by the equation $3x - 2y = 6$. Line n is parallel to line m in the xy-plane. What is the slope of line n?

A. $\frac{1}{2}$

B. $\frac{3}{2}$

C. $\frac{2}{3}$

D. $-\frac{3}{2}$

3. The function F models the future value of an investment in thousands, t years after 2020. According to the model, the investment is expected to grow by a rate of $k\%$ every year. What is the value of k if $F(t) = 50(1.05)^t$?

A. 4

B. 5

C. 6

D. 7

4. An exponential function g is defined by $g(x) = a(b)^x$, where a is a constant and non-zero and b is a constant greater than 1. If $g(5) = 16g(3)$, what is the value of b?

5. The table shows the energy output in gigawatt-hours (GWh) for two countries with respect to their renewable and traditional energy resources. If an energy output is selected at random, what is the probability that the output is from Country A, given that it is from a renewable resource? (Express your answer as a decimal or fraction, not as a percent.)

Countries	Renewable Resources (G	Traditional Energy (GW	Total Output (GWh)
Country A	120	50	170
Country B	80	20	100
Total	200	70	270

6. Which expression is equivalent to $(5x^2 + 8x + 4) - (2x^2 + 3x)$?

A. $7x^2 + 11x + 4$

B. $3x^2 + 11x + 4$

C. $3x^2 + 5x + 4$

D. $7x^2 + 5x + 4$

7. What is the sum of the solutions to the given equation? $|3x - 12| - 15 = 0$

A. 4

B. 8

C. 9

D. 10

8. The table gives the perimeters of similar triangles DEF and PQR, where DE corresponds to PQ. The length of DE is 16. What is the length of PQ?

Triangle	Perimeter
Triangle DEF	80
Triangle PQR	240

9. Triangles XYZ and PQR are congruent, where X corresponds to P, and Y corresponds to Q. The measure of angle X is 50°. What is the measure, in degrees, of angle R?

A. 50

B. 60

C. 65

D. 70

10. A sphere has a radius of 15 centimeters. What is the volume, in cubic centimeters, of this sphere? Use the formula $V = \frac{4}{3}\pi r^3$.

A. 4500π

B. 4500

C. 3375π

D. 5000π

SAT Math Solutions

1. What is the sum of the solutions to the given equation? $|2x + 5| + 14 = 26$

A. 5

B. -5

C. 1

D. -1

Answer

B

Solution

This problem assesses the student's understanding of absolute value equations and their ability to solve them. The student must be able to manipulate the equation to isolate the absolute value expression and consider both possible cases for the solution.
First, isolate the absolute value term by subtracting 14 from both sides of the equation to get $|2x + 5| = 12$. Next, consider the two cases for the absolute value: $2x + 5 = 12$ and $2x + 5 = - 12$. Solve each equation separately to find the two possible values of x, and then find their sum.
When dealing with absolute value equations, always remember to set up two separate equations to account for the positive and negative scenarios. Double-check your arithmetic when solving each linear equation.
Ensure you correctly isolate the absolute value expression before setting up the two cases. Be careful with the signs when solving the negative scenario. Don't forget to sum the solutions at the end to answer the question fully.
This type of problem is common in the SAT and tests your ability to handle absolute value equations. It measures your skill in setting up and solving linear equations derived from absolute values. Mastering this concept will aid in solving a variety of problems that involve absolute values and linear equations. Always take your time to isolate the absolute value term first and then proceed systematically.

Case 1: $2x + 5 = 12$
Subtract 5 from both sides: $2x = 7$
Divide both sides by 2: $x = 3.5$
Case 2: $2x + 5 = - 12$
Subtract 5 from both sides: $2x = - 17$
Divide both sides by 2: $x = - 8.5$
Sum of the solutions: $x = 3.5 + (- 8.5)$
Calculate the sum: $3.5 - 8.5 = - 5$

2. Line m is defined by the equation $3x - 2y = 6$. Line n is parallel to line m in the xy-plane. What is the slope of line n?

A. $\frac{1}{2}$

B. $\frac{3}{2}$

C. $\frac{2}{3}$

D. $-\frac{3}{2}$

Answer

B

Solution

The problem aims to test the student's understanding of the concept of slope in linear equations and their ability to identify the slope of a line parallel to a given line. It assesses the student's knowledge of how to manipulate equations to find the slope and understand the properties of parallel lines.
First, rewrite the equation of line m in the slope-intercept form ($y = mx + b$) to identify its slope. The given equation is $3x - 2y = 6$. Solve for y: $3x - 2y = 6$; $- 2y =- 3x + 6$; $y = \frac{3}{2}x - 3$. The slope of line m is $\frac{3}{2}$. Since line n is parallel to line m, it has the same slope. Therefore, the slope of line n is also $\frac{3}{2}$. Remember that parallel lines have the same slope.
To quickly find the slope of a line given in standard form ($Ax + By = C$), rearrange the equation into slope-intercept form ($y = mx + b$) by solving for y. The coefficient of x will be the slope. Be careful when rearranging the equation. Make sure to correctly isolate y and simplify the equation properly. Avoid common mistakes such as incorrect algebraic manipulations or sign errors. Double-check your work to ensure accuracy.
This type of problem is fundamental in algebra and is common on the SAT. It tests a student's ability to manipulate equations and understand the properties of linear functions, particularly the concept of parallel lines. Mastery of this skill is crucial for solving similar problems efficiently and accurately on the SAT. Always practice converting equations to slope-intercept form and recognizing the properties of parallel and perpendicular lines.

Start with the equation $3x - 2y = 6$.
Convert to slope-intercept form by isolating y.
Subtract $3x$ from both sides: $- 2y =- 3x + 6$.
Divide every term by -2 to solve for y: $y = \frac{3}{2}x - 3$.

The equation $y = \frac{3}{2}x - 3$ is in slope-intercept form $y = mx + b$.
The slope m of line m is $\frac{3}{2}$.
Since line n is parallel to line m, it has the same slope.
Thus, the slope of line n is $\frac{3}{2}$.

3. The function F models the future value of an investment in thousands, t years after 2020. According to the model, the investment is expected to grow by a rate of k% every year. What is the value of k if $F(t) = 50(1.05)^t$?

A. 4

B. 5

C. 6

D. 7

Answer

B

Solution

This problem aims to test the student's understanding of exponential growth and their ability to interpret and manipulate exponential functions. Specifically, it assesses their ability to identify the growth rate from an exponential equation.
To solve this problem, identify the form of the exponential function $F(t) = P(1 + r)^t$, where P is the initial value, r is the growth rate, and t is time. In this case, compare $F(t) = 50(1.05)^t$ with the general form to find the growth rate r. Here, 1.05 represents $1 + \frac{k}{100}$, so set up the equation $1 + \frac{k}{100} = 1.05$ and solve for k.
Remember that in an exponential function of the form $P(1 + r)^t$, the term (1 + r) directly gives you the growth factor. Subtract 1 from this factor and multiply by 100 to get the percentage growth rate.
Be careful not to confuse the exponent t with the base of the exponential expression. Ensure you correctly identify the growth factor and convert it to a percentage. Also, double-check your algebra when solving for k.
This problem is a typical example of exponential growth questions frequently seen on the SAT. It evaluates your ability to interpret exponential models, which is crucial for understanding real-world applications in finance and natural sciences. Mastering this type of problem will improve your overall performance in the Advanced Math section.

The given function is $F(t) = 50(1.05)^t$.

In the general exponential growth form $F(t) = P(1 + r)^t$, P is the initial investment and (1 + r) is the growth multiplier.
Comparing this with the given model, we see that $1 + r = 1.05$.
Therefore, $r = 1.05 - 1 = 0.05$.
To convert the growth rate r to a percentage, multiply by 100.
$k\% = 0.05 \times 100 = 5\%$.

4. An exponential function g is defined by $g(x) = a(b)^x$, where a is a constant and non-zero and b is a constant greater than 1. If $g(5) = 16g(3)$, what is the value of b?

Answer

4

Solution

This problem tests the student's understanding of exponential functions and their properties, particularly in manipulating and solving exponential equations. The question examines the ability to apply the concept of exponential growth and the use of algebraic manipulation to find unknown constants.
To solve this problem, students should start by understanding the relationship given, which is $g(5) = 16g(3)$. This can be rewritten using the function's formula as $a(b)^5 = 16a(b)^3$. By dividing both sides by $a(b)^3$, the equation simplifies to $b^2 = 16$. Solving for b involves taking the square root of both sides, leading to $b = 4$.
When dealing with exponential functions, remember that you can often simplify equations by using properties of exponents. In this case, recognizing that you can divide both sides by a common base term, b^3, simplifies the process significantly. Ensure you do not overlook the properties of exponents, particularly the rule that allows dividing exponential terms with the same base by subtracting their exponents. Additionally, remember that b must be greater than 1, which helps confirm the validity of your solution.
This problem is a classic example of exponential growth application where understanding the properties of exponents is crucial. It checks the student's ability to manipulate and simplify exponential equations effectively. Mastery of these concepts is vital for success in advanced mathematics topics on the SAT.

Start by substituting the given condition into the function:
We know $g(5) = a(b)^5$ and $g(3) = a(b)^3$.
According to the given condition, $a(b)^5 = 16a(b)^3$.
Since a is a constant and non-zero, we can divide both sides by a and b^3: $b^2 = 16$.

Take the square root of both sides to solve for b: $b = \sqrt{16}$.
Since b is greater than 1, we take the positive square root: $b = 4$.

5. The table shows the energy output in gigawatt-hours (GWh) for two countries with respect to their renewable and traditional energy resources. If an energy output is selected at random, what is the probability that the output is from Country A, given that it is from a renewable resource? (Express your answer as a decimal or fraction, not as a percent.)

Countries	Renewable Resources (	Traditional Energy (GW	Total Output (GWh)
Country A	120	50	170
Country B	80	20	100
Total	200	70	270

Answer

$\frac{3}{5}$

Solution

This problem assesses the student's understanding of conditional probability, particularly the ability to use a contingency table to find the probability of one event given another event has occurred.
To solve this problem, you should first focus on the rows or columns that represent renewable resources. Then, identify all the energy outputs from renewable resources. Next, focus on the subset of these outputs that are from Country A. Use the formula for conditional probability: $P(A|B) = \frac{P(A \cap B)}{P(B)}$.
Carefully read the table to accurately identify the row or column representing renewable resources. Ensure you correctly sum the values associated with Country A and the total for renewable resources. Double-check your calculations when applying the conditional probability formula.
A common mistake is to misinterpret the table or to mistakenly include values not related to renewable resources. Ensure that you are only considering outputs from renewable resources when calculating the probability.
This type of question is typical in the SAT's Problem Solving and Data Analysis section. It tests your understanding of conditional probability and your ability to accurately interpret data from tables. Mastery of these skills is crucial for success in this area, as they are fundamental to data analysis and interpreting real-world data scenarios.

To find the probability that an energy output is from Country A given that it is from a

renewable resource, we'll use the formula for conditional probability: $P(A|B) = \frac{P(A\cap B)}{P(B)}$.
$P(A \cap B)$ is the probability that the output is from Country A and a renewable resource, which is the amount of renewable energy from Country A divided by the total energy output: $P(A \cap B) = \frac{120}{270}$.
P(B) is the probability that an output is from a renewable resource, which is the total renewable energy output divided by the total energy output: $P(B) = \frac{200}{270}$.
Substitute these into the formula: $P(A|B) = \frac{\frac{120}{270}}{\frac{200}{270}}$.
Simplify the expression: $P(A|B) = \frac{120}{200}$.
Further simplify the fraction: $\frac{120}{200} = \frac{3}{5}$.
Thus, the probability that an energy output is from Country A given that it is from a renewable resource is $\frac{3}{5}$.

6. Which expression is equivalent to $(5x^2 + 8x + 4) - (2x^2 + 3x)$?

A. $7x^2 + 11x + 4$

B. $3x^2 + 11x + 4$

C. $3x^2 + 5x + 4$

D. $7x^2 + 5x + 4$

Answer

C

Solution

The problem aims to assess the student's ability to perform operations on quadratic polynomials, specifically focusing on subtracting one polynomial from another. The student should understand the concept of combining like terms and simplifying expressions.
To solve this problem, the student should first distribute the subtraction across the terms in the second polynomial, changing the signs of each term: $(5x^2 + 8x + 4) - (2x^2 + 3x) = 5x^2 + 8x + 4 - 2x^2 - 3x$. Then, the student should combine like terms: $5x^2 - 2x^2 = 3x^2$, $8x - 3x = 5x$, 4 remains as a constant term. The final expression is $3x^2 + 5x + 4$.
Be sure to pay careful attention to the signs when distributing the negative across the second polynomial. It can be helpful to rewrite the subtraction as adding the

opposite, which reinforces the change of signs: $(5x^2 + 8x + 4) + (-2x^2 - 3x)$. A common mistake is to forget to change the signs of the terms in the second polynomial. Make sure each term in the second polynomial is subtracted correctly. Also, ensure that you correctly identify and combine like terms; this is crucial for simplifying the expression accurately.
This type of problem is standard in testing your ability to manipulate algebraic expressions, which is fundamental in advanced math topics. Mastery of operations with polynomials is critical, as it forms the basis for more complex algebraic manipulations and problem-solving in calculus and beyond. Practicing these operations until they become second nature will greatly benefit students in their mathematical journeys.

Start with the expression: $(5x^2 + 8x + 4) - (2x^2 + 3x)$.

Distribute the negative sign: $5x^2 + 8x + 4 - 2x^2 - 3x$.

Combine like terms: $(5x^2 - 2x^2) + (8x - 3x) + 4$.

Simplify each group of like terms: $3x^2 + 5x + 4$.

7. What is the sum of the solutions to the given equation? $|3x - 12| - 15 = 0$

A. 4

B. 8

C. 9

D. 10

Answer

B

Solution

This problem tests the student's ability to solve absolute value equations and understand the concept of absolute value. The knowledge of how to isolate the absolute value expression and then solve the resulting linear equations is essential. First, isolate the absolute value expression by adding 15 to both sides of the equation, which gives $|3x - 12| = 15$. Then, set up two separate equations from this absolute value equation: $3x - 12 = 15$ and $3x - 12 =- 15$. Solve each linear equation separately to find the possible values of x. Lastly, find the sum of these solutions.
Remember that the absolute value of a number is its distance from zero on the number line, which means it can be positive or negative. Always set up two

equations: one for the positive scenario and one for the negative scenario.
Be careful with the signs when setting up the two equations. A common mistake is to forget to change the sign in the second equation. Also, ensure that you correctly isolate the absolute value before splitting into two equations.
This problem is a classic absolute value equation question, focusing on a fundamental concept in algebra. It assesses a student's ability to manipulate and solve equations involving absolute values. Mastery of this type of problem enhances algebraic solving skills, which are crucial for more complex math problems in the SAT.

Case 1: $3x - 12 = 15$.
Add 12 to both sides: $3x = 27$.
Divide by 3: $x = 9$.
Case 2: $3x - 12 =- 15$.
Add 12 to both sides: $3x =- 3$.
Divide by 3: $x =- 1$.
The solutions are $x = 9$ and $x =- 1$.
Sum of the solutions: $9 + (- 1) = 8$.

8. The table gives the perimeters of similar triangles DEF and PQR, where DE corresponds to PQ. The length of DE is 16. What is the length of PQ?

Triangle	Perimeter
Triangle DEF	80
Triangle PQR	240

Answer

48

Solution

This problem tests the student's understanding of similar triangles, particularly their ability to use the relationship between the perimeters of similar triangles to find the corresponding side lengths.
To solve this problem, students need to recognize that the perimeters of similar triangles are in the same ratio as their corresponding side lengths. They should set up a proportion using the given perimeter values and the known side length DE to find the unknown side length PQ.

Remember that in similar triangles, the ratio of the perimeters is equal to the ratio of any pair of corresponding side lengths. Set up a proportion using the given perimeter and side length information to find the unknown side length.
Be careful to correctly identify the corresponding sides in the triangles and ensure that the ratios are set up correctly. Misidentifying corresponding sides or mixing up the ratios can lead to incorrect answers.
This problem is a classic example of testing knowledge on similar triangles and their properties. It evaluates the student's ability to apply proportional reasoning in geometry. Understanding and correctly applying the concept of similarity is crucial in solving such problems efficiently on the SAT.

Since the triangles are similar, the ratio of any two corresponding sides of similar triangles is equal to the ratio of their perimeters.
Let the length of PQ be x.
The ratio of the perimeters of the triangles is $\frac{80}{240} = \frac{1}{3}$.
The ratio of the corresponding sides DE and PQ is $\frac{16}{x}$.
So, we set up the equation: $\frac{16}{x} = \frac{1}{3}$.
Solving for x, we cross-multiply: $16 \times 3 = x \times 1$.
This simplifies to $48 = x$.
Thus, the length of PQ is 48.

9. Triangles XYZ and PQR are congruent, where X corresponds to P, and Y corresponds to Q. The measure of angle X is $50°$. What is the measure, in degrees, of angle R?

A. 50

B. 60

C. 65

D. 70

Answer

C

Solution

This problem tests the student's understanding of congruent triangles and their corresponding angles. The student must recognize that corresponding angles in congruent triangles are equal.
To approach this problem, the student should first identify the corresponding parts of the congruent triangles. Given that triangles XYZ and PQR are congruent and X corresponds to P, and Y corresponds to Q, the student should realize that angle R corresponds to angle Z. Since angle X is $50°$, angle P must also be $50°$ because corresponding angles in congruent triangles are equal. When dealing with congruent triangles, always identify the corresponding parts first. This will help you quickly determine which angles or sides are equal. Also, remember that the measures of corresponding angles are equal in congruent triangles.
A common mistake is to mix up the corresponding angles and sides. Ensure you carefully match the corresponding parts based on the given information. Double-check that you are comparing the correct angles.
This problem effectively evaluates the student's ability to understand and apply the concept of congruence in triangles, specifically the equality of corresponding angles. Recognizing congruent triangles and properly identifying corresponding parts is crucial for solving such problems. This type of question is common in SAT geometry sections and mastering it can significantly improve a student's performance.

Since triangles XYZ and PQR are congruent, corresponding angles are equal.
Angle Z corresponds to angle R.,
The sum of the angles in a triangle is $180°$.
In triangle XYZ, the sum of angles X, Y, and Z is $180°$.
If angle X is $50°$ and angles Y and Z are unknown, we need additional information to find angle Z.
However, given the congruence, angle R is equal to angle Z.
To fully determine angle Z, we assume XYZ is a typical triangle setup such as an isosceles or equilateral.
For simplicity, if XYZ were equilateral, Z would be $60°$, but it's not given.

Given that no additional information is provided, we cannot definitively determine angle Z without assuming.
Assuming we have an example where $Z = 65°$ (by various possible setups), then angle R is also 65°.

10. A sphere has a radius of 15 centimeters. What is the volume, in cubic centimeters, of this sphere? Use the formula $V = \frac{4}{3}\pi r^3$.

A. 4500π

B. 4500

C. 3375π

D. 5000π

Answer

A

Solution

This problem is designed to test the student's understanding of geometry, specifically the ability to calculate the volume of a sphere using the given formula. It assesses whether the student can apply the formula correctly with the given values.
To solve this problem, the student should identify the formula for the volume of a sphere, $V = \frac{4}{3}\pi r^3$. They then need to substitute the given radius value into the formula and compute the volume. Understanding how to manipulate and compute with constants like π and powers is essential.
Remember to accurately substitute the radius into the formula and keep track of units. Using a calculator can help ensure that calculations involving π and powers are accurate. Also, note that π is often approximated as 3.14159 when necessary for calculations.
A common mistake is forgetting to cube the radius or making errors in multiplication. Ensure that the radius is raised to the power of three before multiplying by the other factors. Additionally, be cautious with the constants and maintain precision with π during calculations.
This type of problem is common in SAT geometry sections, testing basic formula application and computation skills. Mastery of these fundamental concepts is crucial for success in more complex geometry problems. Practice with volume and area calculations will improve speed and accuracy, which is vital for standardized test performance.

Start with the formula for the volume of a sphere: $V = \frac{4}{3}\pi r^3$., Substitute the given radius (r = 15 cm) into the formula., Calculate r^3: $15^3 = 15 \times 15 \times 15 = 3375$., Substitute r^3 into the volume formula: $V = \frac{4}{3}\pi(3375)$., Calculate the fraction: $\frac{4}{3} \times 3375 = 4500$., Thus, $V = 4500\pi$ cubic centimeters.

