# Math

## Digital SAT

Algebra

Advanced

Geometry and Trigonometry

Problem Solving and Data Analysis

# Table of Contents

PART: MATH

## MATH for the SAT

# Math

## Digital SAT

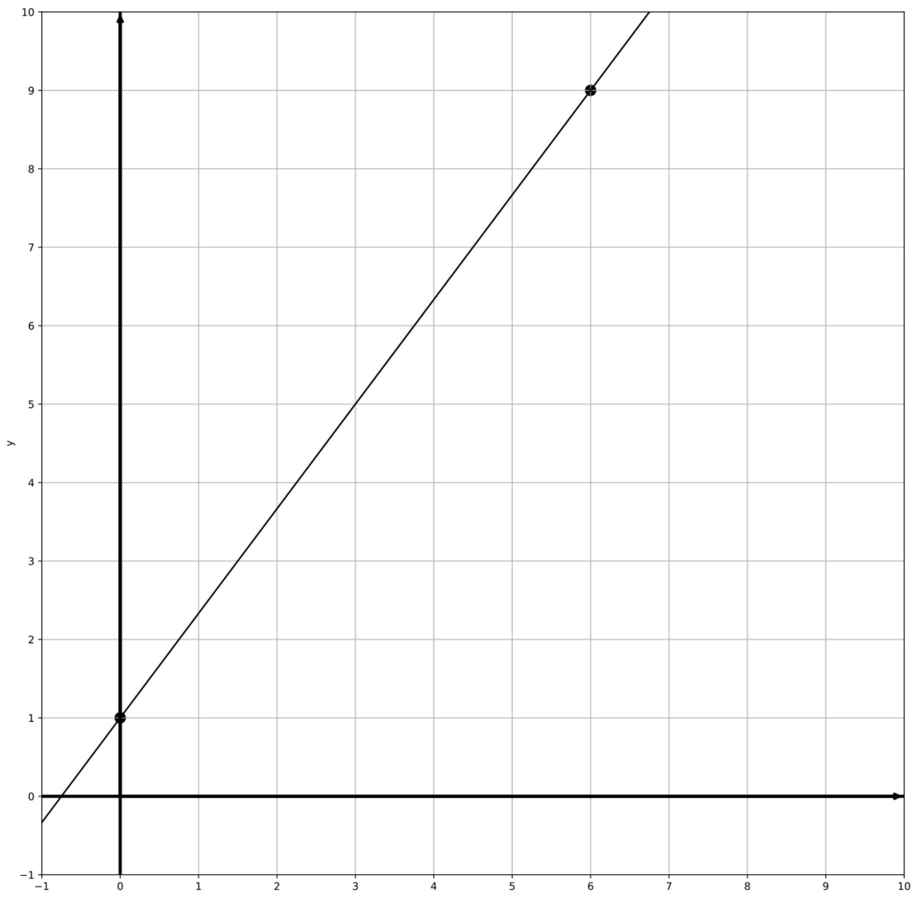
# 1

Algebra

Studyola

# SAT Math Algebra

1. The graph of line g is shown in the xy-plane. Line k is defined by the equation $40x + py = w$. If line k is graphed in this xy-plane, resulting in the graph of a system of two linear equations, the system of two linear equations will have infinitely many solutions. Given the points (0, 1) and (6, 9), what is the value of $p + w$ where those points lie on the same line as line k?




2. The function f is defined by $f(x) = \frac{1}{m}x - 5$, where m is an integer constant and $59 \leq m \leq 61$. For the graph of $y = f(x) + 8$ in the xy-plane, what is the x-coordinate of a possible x-intercept?

3. For the linear function p, the slope of the graph of $y = p(x)$ is 8, and it is known that $p(c) = 189$ and $p(33) = 165$. For the linear function t, it is given that $t(c) = 10$ and $t(3) =- 89$. What is the slope of the graph of $y = t(x)$ in the xy-plane?

A. 1

B. 2

C. 3

D. 4

4. A company produces eco-friendly products. The total revenue of the company in the year 2023 is represented by the equation $831 = 71 + 76(x - 8)$, where x represents the number of years since 2015. If the revenue is expected to reach 831 dollars in 2023, how many years since 2015 has the company been operating?

A. 10 years

B. 15 years

C. 18 years

D. 20 years

5. If $86(x + 9) = 36(x + 9) + 150$, what is the value of $x + 9$?

A. -9

B. -6

C. 3

D. 6

6. The function f is defined by $f(x) = \frac{4}{85}x + 23$. What is the value of $f(340)$?

7. The table shows two values of x and their corresponding values of y. The graph of the linear equation representing this relationship passes through the point $(\frac{1}{2}, a)$. What is the value of a?

| x | y |
|---|---|
| -3 | 2 |
| 95 | 198 |

8. What value of x is the solution to the equation $89x + 395 = 168$?

A. $-\frac{227}{89}$

B. $-\frac{17}{5}$

C. $\frac{5}{4}$

D. 35

9. For the linear function f, the graph of $y = f(x)$ in the xy-plane has a slope of 5 and passes through the point $(0, -45)$. Which equation defines f?

A. $f(x) = 5x - 45$

B. $f(x) = 5x + 45$

C. $f(x) = 45x - 5$

D. $f(x) = -5x - 45$

10. A researcher is studying a population of bacteria that can grow at specific temperature ranges. The temperature must be at least 81 degrees Fahrenheit and no more than 109 degrees Fahrenheit for optimal growth. Which inequality represents this temperature range, where x is the temperature in degrees Fahrenheit?

A. $x \leq 190$

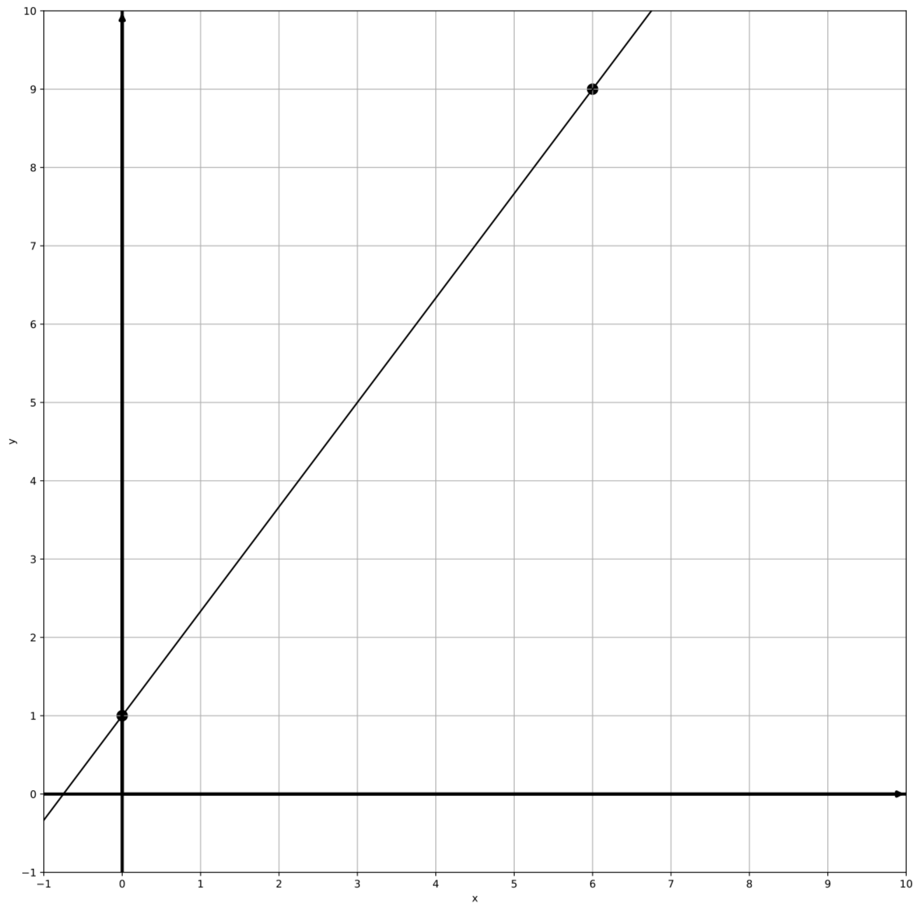
B. $81 \leq x \leq 109$

C. $x \geq 81$

D. $x \leq 109$

# SAT Math Algebra Solutions

1. The graph of line g is shown in the xy-plane. Line k is defined by the equation $40x + py = w$. If line k is graphed in this xy-plane, resulting in the graph of a system of two linear equations, the system of two linear equations will have infinitely many solutions. Given the points (0, 1) and (6, 9), what is the value of $p + w$ where those points lie on the same line as line k?



## Answer

-60

## Solution

Concept Check : The question requires students to understand linear equations, specifically how to derive the equation of a line from given points and analyze the relationship between two lines in a graph. It also expects knowledge of how to find solutions for systems of equations and recognize when they have infinitely many solutions, which occurs when the lines are coincident.

Solution Strategy : To approach this problem, students should first rewrite the equation of line k in slope-intercept form ($y = mx + b$) to identify its slope and y-intercept. Next, they need to check if the provided points (0, 1) and (6, 9) can be used to derive a linear equation. This involves calculating the slope between the two points and then finding the equation of the line that passes through both points. Finally, students should compare the derived equation with the equation of line k to determine if they are the same line, which would confirm that the points lie on the same line.

Quick Wins : Start by converting the given equation of line k into slope-intercept form to make it easier to compare with the equation you derive from the two points. Remember to use the slope formula ($m = \frac{y_2 - y_1}{x_2 - x_1}$) to find the slope between the two points. Don't forget to check that the derived line equation matches line k, which will confirm that the points lie on the same line.

Mistake Alert : Be cautious when calculating the slope to avoid arithmetic errors. Make sure to double-check the conversion of line k into slope-intercept form. When deriving the equation from the points, ensure that you correctly apply the point-slope form of a linear equation. Lastly, remember that for the system to have infinitely many solutions, the two lines must be identical, not just parallel.

SAT Know-How : This problem falls under the category of Algebra, specifically focusing on the graphs of linear equations and functions. It assesses skills in deriving linear equations from points, understanding the concept of systems of equations, and recognizing conditions for overlapping lines. Mastering these concepts is essential for success in SAT math, as they are commonly tested in various forms.

Step 1: Find the equation of line g.
Calculate the slope of line g: $slope = \frac{9-1}{6-0} = \frac{8}{6} = \frac{4}{3}$.
Using point (0, 1) and the slope $\frac{4}{3}$, the equation of line g is: $y = \frac{4}{3}x + 1$.
Rewriting in standard form: $4x - 3y = -3$.
Step 2: Equate line g with line k.
Given equation for line k: $40x + py = w$.
Since lines are the same, multiply line g's equation by a factor to match line k's coefficients.
Multiply $4x - 3y = -3$ by 10 to obtain: $40x + py = w$.
Step 3: Find p and w.
Line k's form: $40x - 30y = -30$ => $p = -30$, $w = -30$.
Therefore, $p + w = -30 - 30 = -60$.

2. The function f is defined by $f(x) = \frac{1}{m}x - 5$, where m is an integer constant and $59 \leq m \leq 61$. For the graph of $y = f(x) + 8$ in the xy-plane, what is the x-coordinate of a possible x-intercept?

## Answer

-177, -180, -183

## Solution

Concept Check : The student is expected to understand how to manipulate linear functions and determine x-intercepts. Specifically, they need to know how to adjust the function to find the x-intercept after a transformation (in this case, adding 8 to the function). This requires knowledge of how to set the function equal to zero to find the x-intercept.

Solution Strategy : To find the x-coordinate of the x-intercept of the function $y = f(x) + 8$, the student should first rewrite the function as $f(x) + 8 = 0$. This will lead to setting up the equation to solve for x. The student should also remember that the x-intercept occurs when $y = 0$. After simplifying the equation, the next step involves substituting the values of m (from the given range) to find the corresponding x-intercepts.

Quick Wins : To solve this problem efficiently, first, correctly rewrite the function with the transformation applied. Make sure to carefully simplify and isolate x. Since m is an integer that can take on specific values, consider calculating the x-intercept for each integer value of m within the range 59 to 65. This will give you multiple possible x-intercepts to consider, enhancing your understanding of how changes in m affect the function.

Mistake Alert : Be cautious not to misinterpret the transformation of the function. Adding 8 to the function shifts the graph vertically, so ensure that you correctly calculate the new equation before proceeding to find the x-intercept. Additionally, double-check your arithmetic when substituting values for m, as small mistakes could lead to incorrect x-intercept values.

SAT Know-How : This problem falls under the category of algebra, specifically focusing on the graphs of linear equations and functions. It assesses the student's ability to manipulate linear equations and understand transformations involving vertical shifts. Mastering this type of problem will enhance problem-solving skills related to functions, a crucial aspect of the SAT math section.

First, substitute the given function into the transformed function:
$y = f(x) + 8 = (\frac{1}{m}x - 5) + 8$
Simplify the equation:
$y = \frac{1}{m}x - 5 + 8 \rightarrow y = \frac{1}{m}x + 3$
To find the x-intercept, set $y = 0$:
$0 = \frac{1}{m}x + 3 \rightarrow -3 = \frac{1}{m}x \rightarrow x = -3m$
Substitute possible values of m within the given range:

When $m = 59, x = -3 \times 59 = -177$
When $m = 60, x = -3 \times 60 = -180$
When $m = 61, x = -3 \times 61 = -183$
Thus, possible x-intercepts are: -177, -180, -183

3. For the linear function p, the slope of the graph of $y = p(x)$ is 8, and it is known that $p(c) = 189$ and $p(33) = 165$. For the linear function t, it is given that $t(c) = 10$ and $t(3) =- 89$. What is the slope of the graph of $y = t(x)$ in the xy-plane?

A. 1

B. 2

C. 3

D. 4

## Answer

C

## Solution

Concept Check : The question is designed to assess the student's understanding of linear functions and how to calculate slopes. Students should know that the slope of a linear function can be determined using the formula $\frac{change\ in\ y}{change\ in\ x}$ and apply this knowledge to the given values.

Solution Strategy : To find the slope of the linear function t, students need to recognize that the slope can be calculated by using the two points provided: $t(c) = 10$ and $t(3) =- 89$. They should set up the slope formula, substituting the values for y and x from these two points. The thought process involves identifying the coordinates and applying the slope formula correctly.

Quick Wins : When calculating the slope, remember to use the correct order for the points in the slope formula: $\frac{(y_2 - y_1)}{(x_2 - x_1)}$. It's also helpful to note that you can label the points as (c, 10) and (3, -89) to keep track of them. Additionally, if you're unsure about the order, visualize the points on a graph to determine which corresponds to which value.

Mistake Alert : Be careful not to mix up the values of c and 3 when substituting into the slope formula. Additionally, ensure that you are subtracting the y-values correctly and that you are using the x-values that correspond to those y-values. A common mistake is to forget to consider the signs of the changes in y and x.

SAT Know-How : This type of problem falls under the category of Algebra, specifically focusing on the graphs of linear equations and functions. It assesses the student's ability to compute slopes from given points, a fundamental skill in understanding linear relationships. Mastering such problems is crucial for SAT success, as it enhances problem-solving skills and numerical reasoning.

First, use the conditions for the function p to determine the value of c:
We know that the slope of p is 8, so we can write: $8 = \frac{165-189}{33-c}$.

Simplify: $8 = \frac{-24}{33-c}$.
Multiply both sides by (33 - c): $8 \times (33 - c) = -24$.
Expand the left side: $264 - 8c = -24$.
Add 24 to both sides: $264 = 8c - 24$.
Add 24 to both sides: $288 = 8c$.
Divide both sides by 8: $c = 36$.
Now, find the slope of the function t using the slope formula with the known points (c, 10) and (3, -89):
$t(36) = 10$ and $t(3) = -89$, so the slope $m = \frac{10-(-89)}{36-3}$.
Simplify the numerator: $10 - (-89) = 99$.
Subtract in the denominator: $36 - 3 = 33$.
So, $m = \frac{99}{33} = 3$.

4. A company produces eco-friendly products. The total revenue of the company in the year 2023 is represented by the equation $831 = 71 + 76(x - 8)$, where x represents the number of years since 2015. If the revenue is expected to reach 831 dollars in 2023, how many years since 2015 has the company been operating?

A. 10 years

B. 15 years

C. 18 years

D. 20 years

## Answer

C

## Solution

Concept Check : The intent of the question is to assess the student's ability to interpret a linear equation in the context of a real-life scenario. Students are expected to understand how to manipulate and solve linear equations, and they should be familiar with the concepts of revenue, time variables, and how to relate them in the context of the problem.

Solution Strategy : To approach this problem, students should first identify what the variable 'x' represents in the context of the problem, which is the number of years since 2015. Then, they should recognize that the equation given can be rearranged to isolate 'x'. This will require applying algebraic principles, such as distributing terms, combining like terms, and solving for the variable. It's essential to keep track of the relationships between the years and the revenue stated in the equation.

Quick Wins : A helpful tip is to break down the equation step-by-step. Start by simplifying the right side and isolating 'x' on one side of the equation. It can also be beneficial to convert the equation into a more familiar linear form. Additionally, double-check your calculations at each step to ensure accuracy. Finally, remember to interpret the final value of 'x' in the context of the problem to ensure it makes sense.

Mistake Alert : Students should be careful not to make common mistakes such as misreading the equation or mistakenly adding or subtracting terms incorrectly. Additionally, pay attention to the meaning of the variable 'x' to avoid misinterpreting what the solution represents. It's also important to check that the context of the problem aligns with the solution you arrive at, ensuring that it is a reasonable answer in terms of the years since the company began operating.

SAT Know-How : This problem falls under the category of Algebra, specifically focusing on linear equation word problems. It assesses the student's skills in interpreting, manipulating, and solving linear equations in real-world contexts. Mastery of these skills is essential for success on the SAT, as it demonstrates the ability to connect mathematical concepts to practical situations.

1. Begin by simplifying the equation: $831 = 71 + 76(x - 8)$.
2. Distribute 76 in the term $76(x - 8)$: $76x - 608$.
3. The equation becomes: $831 = 71 + 76x - 608$.
4. Combine like terms on the right side: $71 - 608 =- 537$.
5. The equation now is: $831 = 76x - 537$.
6. Add 537 to both sides to isolate the term with x: $831 + 537 = 76x$.
7. This simplifies to: $1368 = 76x$.
8. Divide both sides by 76 to solve for x: $x = \frac{1368}{76}$.
9. Calculate the division: $x = 18$.
10. Therefore, the company has been operating for 18 years since 2015.

5. If $86(x+9)=36(x+9)+150$, what is the value of $x+9$?

A. -9

B. -6

C. 3

D. 6

## Answer

C

## Solution

Concept Check : The intent of this question is to assess the student's ability to solve a linear equation using the substitution method. Students are expected to understand how to isolate variables and perform algebraic operations to find the value of 'x + 9.'

Solution Strategy : To approach this problem, the student should start by recognizing that both sides of the equation contain the term '(x + 9).' The first step is to distribute the coefficients (86 and 36) to the term '(x + 9)' on both sides. After simplifying both sides, the student should then rearrange the equation to isolate 'x' or directly solve for 'x + 9' by manipulating the equation appropriately.

Quick Wins : A helpful tip is to combine like terms after distributing the coefficients. This will make it easier to isolate the variable. Also, consider rewriting the equation in terms of 'x + 9' directly, which could simplify your calculations. Always double-check your calculations after each step to ensure accuracy.

Mistake Alert : Students should be cautious about making errors in distribution and combining like terms. It's easy to miscalculate coefficients or to accidentally drop a term when rearranging the equation. Double-check your work to avoid these common mistakes, especially when dealing with negative numbers or when moving terms from one side of the equation to the other.

SAT Know-How : This problem falls under the category of Algebra, specifically focusing on solving linear equations and inequalities through substitution. It assesses the student's skills in distributing terms, combining like terms, and isolating variables. Mastering these techniques is essential for success in SAT math, as it reinforces the foundational skills needed for more complex algebraic concepts.

Step 1: Simplify both sides by subtracting $36(x+9)$ from both sides of the equation.
$86(x+9)-36(x+9)=150$

Step 2: Combine like terms on the left side.
$50(x+9)=150$

Step 3: Isolate $(x + 9)$ by dividing both sides by 50.
$x + 9 = \frac{150}{50}$
Step 4: Simplify the fraction.
$x + 9 = 3$

6. The function f is defined by $f(x) = \frac{4}{85}x + 23$. What is the value of $f(340)$?

## Answer

39

## Solution

Concept Check : The intent of the question is to assess the student's understanding of linear functions and their ability to evaluate a function for a specific input. Students should know how to substitute a value for x in the given function and perform basic arithmetic operations.

Solution Strategy : To approach this problem, the student should first identify what f(340) means, which involves substituting 340 in place of x in the function f(x). The next step is to carefully perform the arithmetic operations as defined by the function, ensuring to follow the order of operations correctly.

Quick Wins : When evaluating a function, always start by clearly substituting the value into the function. It may be helpful to break the problem down into smaller steps: first calculate the multiplication, and then add the constant. Keeping track of each step can help prevent mistakes.

Mistake Alert : Be careful with arithmetic operations, especially when dealing with fractions and large numbers. Double-check each step to ensure that you haven't made any calculation errors, particularly with addition and multiplication. It's also essential to ensure that you're substituting the correct value for x.

SAT Know-How : This problem type falls under the category of Algebra, specifically focusing on evaluating linear functions. It assesses the student's ability to substitute values and perform calculations accurately. Developing a systematic approach to function evaluation will enhance problem-solving efficiency in the SAT exam.

Step 1: Substitute $x = 340$ into the function.
$f(340) = \frac{4}{85} \times 340 + 23$
Step 2: Calculate the multiplication.
$\frac{4}{85} \times 340 = 4 \times \frac{340}{85}$
Since $\frac{340}{85} = 4$, the expression simplifies to $4 \times 4 = 16$.
Step 3: Add the constant term 23 to the result.
$f(340) = 16 + 23 = 39$

7. The table shows two values of x and their corresponding values of y. The graph of the linear equation representing this relationship passes through the point $(\frac{1}{2}, a)$. What is the value of a?

| x | y |
|---|---|
| -3 | 2 |
| 95 | 198 |

## Answer

$-\frac{369}{190}$

## Solution

Concept Check : The intent of this question is to assess the student's understanding of linear equations, specifically the ability to interpret a data table that relates x and y values. Students are expected to know how to use the slope-intercept form of a linear equation or point-slope form to find the value of y (denoted as 'a') when a specific x value is provided.

Solution Strategy : To approach this problem, the student should first identify the linear relationship indicated by the two given points in the table. The student should calculate the slope of the line using the formula for slope (change in y over change in x). Once the slope is known, they can use the point-slope form or slope-intercept form to derive the equation of the line. By substituting $x = \frac{1}{2}$ into the equation, the student can solve for the corresponding y value, which is 'a'.

Quick Wins : Make sure to carefully read the values from the table and double-check the calculations for slope. When deriving the equation of the line, remember that the point-slope form can be very helpful: $y - y_1 = m(x - x_1)$, where m is the slope and $(x_1, y_1)$ is one of the points from the table. If needed, ensure that the resulting equation is simplified before substituting $x = \frac{1}{2}$.

Mistake Alert : A common mistake is miscalculating the slope, which can lead to an incorrect equation. Also, be careful when substituting $x = \frac{1}{2}$ into the equation; it's easy to make arithmetic errors if not careful. Lastly, ensure that the units are consistent when interpreting the values from the table.

SAT Know-How : This problem is a type of algebra question focusing on the graphs of linear equations. It assesses the student's ability to interpret data, calculate the slope, and derive the equation of a line. Mastering these skills is crucial for solving similar SAT problems effectively, as it requires not just computational skills but also an understanding of the relationship between variables in linear functions.

First, calculate the slope (m) of the line using the formula for the slope of a line between two points $(x_1, y_1)$ and $(x_2, y_2)$:

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{198-(-3)}{95-0} = \frac{201}{95}$

Simplify the slope: $m = \frac{201}{95}$

Next, use the point-slope form of a line to find the equation of the line. We can use the point (0, -3):

$y - y_1 = m(x - x_1)$

$y - (-3) = (\frac{201}{95})(x - 0)$

$y + 3 = \frac{201}{95}x$

Subtract 3 from both sides to solve for y:

$y = \frac{201}{95}x - 3$

Now, substitute $x = \frac{1}{2}$ to find the value of a:

$a = (\frac{201}{95})(\frac{1}{2}) - 3$

Calculate a:

$a = \frac{201}{190} - 3$

Convert 3 to a fraction with a denominator of 190:

$3 = \frac{570}{190}$

$a = \frac{201-570}{190} = -\frac{369}{190}$

8. What value of x is the solution to the equation $89x + 395 = 168$?

A. $-\frac{227}{89}$

B. $-\frac{17}{5}$

C. $\frac{5}{4}$

D. 35

## Answer

A

## Solution

Concept Check : The intent of this question is to assess the student's understanding of solving linear equations. Specifically, it tests the ability to isolate the variable x using algebraic manipulation, which typically involves operations such as addition, subtraction, multiplication, and division.

Solution Strategy : To approach this problem, the student should begin by isolating the term containing x. This usually involves subtracting 395 from both sides of the equation to eliminate the constant term on the left side. Following that, the student would divide both sides by 89 to solve for x. Keeping track of the order of operations is crucial throughout this process.

Quick Wins : A helpful tip is to write down each step of your calculation clearly. This will not only help you avoid mistakes but also make it easier to double-check your work. Remember to balance the equation by performing the same operation on both sides. Finally, always recheck your final answer by substituting it back into the original equation to ensure it holds true.

Mistake Alert : Students should be cautious not to make common mistakes such as forgetting to perform the same operation on both sides of the equation or miscalculating arithmetic operations. It's also important to be careful with negative signs and to ensure that the final value of x is simplified correctly.

SAT Know-How : This problem falls under the category of algebra, specifically focusing on solving linear equations. It assesses the student's ability to manipulate equations and understand the properties of equality. Mastering these skills is essential for success in algebra and is a common type of question encountered in the SAT exam, emphasizing the importance of clear, logical reasoning and attention to detail.

Step 1: Start with the given equation: $89x + 395 = 168$.
Step 2: Subtract 395 from both sides to begin isolating x: $89x = 168 - 395$.
Step 3: Simplify the right-hand side: $168 - 395 =- 227$.
Now we have: $89x =- 227$.

Step 4: Divide both sides by 89 to solve for x: $x =-\frac{227}{89}$.
Step 5: Ensure the fraction is in its simplest form. Since -227 and 89 have no common factors other than 1, the fraction $-\frac{227}{89}$ is already simplified.

9. For the linear function f, the graph of $y = f(x)$ in the xy-plane has a slope of 5 and passes through the point $(0, -45)$. Which equation defines f?

A. $f(x) = 5x - 45$

B. $f(x) = 5x + 45$

C. $f(x) = 45x - 5$

D. $f(x) =- 5x - 45$

## Answer

A

## Solution

Concept Check : The question intends for the student to understand the concept of linear functions, specifically how to use the slope-intercept form of a linear equation ($y = mx + b$ ) to find the equation of a line. The student is expected to know how to identify the slope and y-intercept of a linear function.

Solution Strategy : To solve this problem, the student should recognize that the slope-intercept form of a line is given by $y = mx + b$, where m represents the slope and b represents the y-intercept. Given the slope of 5 and the point (0, -45), the student should substitute these values into the equation to find the linear function f.

Quick Wins : Remember that the slope-intercept form is very helpful in these types of problems. The slope (m) is the coefficient of x, and the y-intercept (b) is the constant term. If a point is given where x equals 0, it directly gives you the y-intercept. Be sure to correctly substitute the values and simplify the equation properly.

Mistake Alert : Be careful not to confuse the point (0, -45) with the slope. It is easy to mistakenly use the y-coordinate as the slope or vice versa. Additionally, double-check your arithmetic when substituting values into the equation to avoid simple calculation errors.

SAT Know-How : This problem falls under the category of Algebra, specifically focused on linear equations and functions. It assesses the student's ability to apply the slope-intercept form of a linear equation to find a function. Mastering this skill is crucial for solving various types of algebraic problems on the SAT, showcasing the importance of understanding the relationship between slope, intercepts, and linear functions.

1. Understand the General Form of a Linear Function: The equation of a line in slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept.
2. Identify the Given Slope: The problem states that the slope m is 5.
3. Determine the Y-Intercept: Since the line passes through the point (0, -45), the y-intercept b is -45.
4. Express the function $f(x)$: Substitute the values of m and b into the slope-intercept form: $f(x) = 5x - 45$.

10. A researcher is studying a population of bacteria that can grow at specific temperature ranges. The temperature must be at least 81 degrees Fahrenheit and no more than 109 degrees Fahrenheit for optimal growth. Which inequality represents this temperature range, where x is the temperature in degrees Fahrenheit?

A. $x \le 190$

B. $81 \le x \le 109$

C. $x \ge 81$

D. $x \le 109$

## Answer

B

## Solution

Concept Check : The question requires students to understand how to express a real-world situation using a linear inequality. Specifically, students need to know how to translate the constraints of a temperature range into mathematical notation, recognizing that 'at least' corresponds to a greater than or equal to sign (≥) and 'no more than' corresponds to a less than or equal to sign (≤).

Solution Strategy : Begin by identifying the key phrases in the problem that indicate the boundaries of the temperature range. Note that 'at least 81 degrees' suggests that the temperature x is greater than or equal to 81, while 'no more than 109 degrees' indicates that the temperature x is less than or equal to 109. This leads to a compound inequality that can be formulated to represent these constraints.

Quick Wins : When you encounter word problems, highlight or underline the important phrases that indicate constraints or limits. Creating a visual representation, such as a number line, can also help to clarify the range of values. Remember to use appropriate inequality symbols: ≥ for 'at least' and ≤ for 'no more than'.

Mistake Alert : Be careful not to confuse the terms 'at least' and 'no more than' with their opposite meanings. Misinterpreting these phrases can lead to incorrect inequalities. Also, ensure that you are using the variable (x in this case) consistently to represent the temperature throughout your work.

SAT Know-How : This problem falls under the category of algebra, specifically focusing on linear inequalities derived from real-world contexts. It assesses the student's ability to interpret and translate conditions into mathematical expressions. Mastering this skill is crucial for success in the SAT, as it demonstrates not only comprehension of inequalities but also the ability to apply mathematical reasoning to everyday situations.

1. We need to represent the minimum and maximum temperatures in an inequality.
2. The temperature must be at least 81 degrees Fahrenheit, which means x is greater than or equal to 81.

3. The temperature must be no more than 109 degrees Fahrenheit, which means x is less than or equal to 109.
4. Combining these two conditions, we get the inequality $81 \leq x \leq 109$, which represents the temperature range for optimal growth.

# Math

Digital SAT

2

Advanced

Studyola

# SAT Math Advanced

1. For the given polynomial function $f(x) = 2x^5 - 9x^3 + 8x^2 + 18$, what is the value of $b$ if the graph of $y = f(x)$ passes through the point (0, b)?

2. An exponential function f is defined by the equation $f(x) = a(b)^x$, where a is the initial value and b is the ratio. If $f(15) = 6561f(7)$, what is the value of b?

3. The function f is a quadratic function. In the xy-plane, the graph of $y = f(x)$ has a vertex at (6, -4) and passes through the points (-3, 239) and (-5, 359). What is the value of $f(7) - f(4)$?

A. -15

B. -12

C. -9

D. -5

4. Which expression is equivalent to $- 6x^5y^7(7x^7 + 4x^4 + 84)$?

A. $- 42x^{12}y^6 - 24x^9y^7 - 504x^5y^7$

B. $- 42x^{12}y^7 - 24x^6y^7 - 504x^5y^6$

C. $- 42x^{12}y^7 - 24x^9y^6 - 504x^5y^7$

D. $- 42x^{12}y^7 - 24x^9y^7 - 504x^5y^7$

5. The function f is defined by $f(x) = 7x^2 + 6x - 38$. What is the value of $f(2)$?

6. What is an x-coordinate of an x-intercept of the graph $y = 3(x - 4)(x - 7)(x + 5)$ in the xy-plane?

A. -5

B. -4

C. 0

D. 10

7. A financial analyst is modeling the projected economic activity in a city using the equation for active jobs: $y = -0.3x^2 - 3x + 86$, where $y$ represents the number of active jobs and $x$ is the number of months since January 2020. What is the best interpretation of the y-intercept of the graph of this equation in the xy-plane?

A. At the end of January 2020, the projected number of active jobs was 0.

B. At the end of January 2020, the projected number of active jobs was 86, suggesting a lack of job activity or data errors.

C. At the end of July 2020, the projected number of active jobs was 86.

D. At the end of July 2020, the projected number of active jobs was 0.

8. The equation $4|x - 97| + 7 = k^2 + 110k + 3032$ has exactly one solution for the variable $x$. Which of the following could be the value of $k$?

A. -55 only

B. 0 only

C. -55 and 0

D. -55 and 55

9. A physicist is studying a certain radioactive substance whose quantity decreases over time. The equation representing the quantity of this substance in grams after x years is given by: $f(x) = 1100(0.41)^x$. Which of the following is the best interpretation of 1100 in this context?

A. The estimated amount of substance remaining after 1 year.

B. The estimated amount of substance remaining after 2 years.

C. The estimated initial quantity of the substance at time 0.

D. The estimated quantity of substance after 6 years.

10. A solution to the given system of equations is (x, y). What is a possible value of x?

$x - 76 = y \ (x - 76)^2 = y$

A. 75

B. 76

C. 78

D. 80

# SAT Math Advanced Solutions

1. For the given polynomial function $f(x) = 2x^5 - 9x^3 + 8x^2 + 18$, what is the value of $b$ if the graph of $y = f(x)$ passes through the point (0, b)?

## Answer

18

## Solution

Concept Check : The question is asking students to find the y-intercept of a polynomial function. Students should know that the y-intercept occurs where x equals zero, and they should be able to evaluate the function at that point. This requires understanding of polynomial evaluation.

Solution Strategy : To find the y-intercept, substitute x = 0 into the polynomial function. This will involve evaluating each term of the polynomial at x = 0, which will simplify the calculation significantly since any term with x in it will equal zero.

Quick Wins : Remember that the y-intercept is always found by setting x to zero in the equation. When evaluating polynomials, any term that includes x will drop out (become zero), making your calculations easier. Focus primarily on the constant term for the y-intercept.

Mistake Alert : Be careful to correctly substitute x = 0 into each term of the polynomial. Double-check that you are not mistakenly performing operations on terms that should evaluate to zero. Also, ensure you properly account for the constant term, as it directly gives you the y-intercept value.

SAT Know-How : This problem falls under the category of Advanced Math, specifically focusing on polynomial functions and their properties. It assesses a student's ability to evaluate polynomials to find specific points, such as the y-intercept. Mastering this skill is crucial for solving similar SAT problems efficiently.

Substitute x = 0 into the polynomial function f(x):
$f(0) = 2(0)^5 - 9(0)^3 + 8(0)^2 + 18$
Simplify each term after substitution:
f(0) = 0 - 0 + 0 + 18
Thus, f(0) = 18.
Therefore, the value of b is 18.

2. An exponential function f is defined by the equation $f(x) = a(b)^x$, where a is the initial value and b is the ratio. If $f(15) = 6561f(7)$, what is the value of b?

## Answer

3

## Solution

Concept Check : The intent of this question is to assess the student's understanding of exponential functions, specifically the properties of exponential growth and how to manipulate the function's properties using given values. The student is expected to know how to apply the definition of exponential functions and possibly logarithmic properties to solve for the variable b.

Solution Strategy : To approach this problem, the student should first substitute the values of x into the function $f(x)$ to express $f(15)$ and $f(7)$ in terms of a and b. This will lead to an equation that can be simplified. The next step is to set up the equation based on the relationship given in the problem, which states that $f(15)$ is 6561 times $f(7)$. This should lead to an equation that can be solved for b. The student should carefully track the bases and the exponents involved in the exponential function.

Quick Wins : When dealing with exponential equations, remember that if two exponential expressions are equal, their bases and exponents can be related. It can be helpful to express both $f(15)$ and $f(7)$ in the same form, possibly factoring out common elements. If you encounter a number like 6561, it might be useful to express it as a power of a smaller number (for example, recognize that $6561 = 3^8$) to simplify comparisons. This will help you leverage the properties of exponents effectively.

Mistake Alert : Be cautious with the arithmetic when applying the exponential properties. Ensure that you accurately apply the exponent rules, especially when manipulating the equation to isolate b. Double-check your calculations when substituting values into the exponential function to avoid common mistakes, such as miscalculating powers or neglecting the coefficients.

SAT Know-How : This problem is categorized under Advanced Math and focuses on nonlinear functions, specifically exponential functions. It assesses the student's ability to analyze exponential relationships and manipulate equations effectively. Understanding the properties of exponents and being able to express relationships clearly is crucial for solving such problems in the SAT context.

Use the definition of the exponential function to express $f(15)$ and $f(7)$:
$f(15) = a(b)^{15}, f(7) = a(b)^7$
Substitute these expressions into the given condition $f(15) = 6561f(7)$:
$a(b)^{15} = 6561a(b)^7$
Divide both sides by $a(b)^7$ to isolate $b^8$:

$\frac{b^{15}}{b^{7}} = 6561 \rightarrow b^{(15-7)} = 6561 \rightarrow b^{8} = 6561$
To find b, take the eighth root of both sides:
$b = (6561)^{\frac{1}{8}}$
Since 6561 $= 3^{8}$, we find:
$b = 3$
Therefore, the value of b is 3.

3. The function f is a quadratic function. In the xy-plane, the graph of $y = f(x)$ has a vertex at (6, -4) and passes through the points (-3, 239) and (-5, 359). What is the value of $f(7) - f(4)$?

A. -15

B. -12

C. -9

D. -5

## Answer

C

## Solution

Concept Check : The intent of this question is to assess the student's understanding of quadratic functions, particularly in determining the function's equation when given its vertex and specific points on the graph. Students are expected to know how to use the vertex form of a quadratic function and how to manipulate it based on known points.

Solution Strategy : To solve the problem, the student should start by recalling the vertex form of a quadratic function, which is given by the equation: $f(x) = a(x - h)^2 + k$, where (h, k) is the vertex of the parabola. Here, the vertex is given as (6, -4), so h = 6 and k = -4. The next step is to substitute the coordinates of the points (-3, 239) and (-5, 359) into the equation to create two equations that can be solved to find the value of 'a'. Once 'a' is determined, the full function can be established, and then f(7) and f(4) can be calculated to find their difference.

Quick Wins : 1. Remember the vertex form of a quadratic function is essential in problems like this. 2. Substitute points carefully and ensure you maintain correct signs. 3. When substituting the points into the equation, simplify carefully to avoid arithmetic errors. 4. After finding the function, evaluate it at the required points to find the difference.

Mistake Alert : 1. Be cautious when substituting the points; double-check that you plug the correct x and y values into the equation. 2. Pay special attention to the negative signs when working with the vertex's coordinates and the points; it's easy to make minor mistakes here. 3. Ensure that you correctly calculate the values of f(7) and f(4) before subtracting them.

SAT Know-How : This problem belongs to the Advanced Math category and focuses on quadratic functions and their graphs. It assesses skills such as understanding the vertex form of a quadratic equation, substituting values correctly, and performing arithmetic operations accurately. Mastering these skills is crucial for solving quadratic-related problems on the SAT, as it enhances your problem-solving efficiency and reduces the chances of mistakes.

Start by writing the quadratic function in vertex form: $f(x) = a(x - 6)^2 - 4$.
Substitute the point (-3, 239) into the equation to solve for a: $239 = a(-3 - 6)^2 - 4$.

Simplify: 239 = a(81) - 4.
Add 4 to both sides: 243 = 81a.
Solve for a: $a = \frac{243}{81} = 3$.
Verify a with the point (-5, 359):
$359 = 3(-5-6)^2 - 4$.
Simplify: 359 = 3(121) - 4.
Calculate: 359 = 363 - 4.
This confirms a = 3 is correct.
Now, calculate f(7) and f(4):
$f(x) = 3(x-6)^2 - 4$.
$f(7) = 3(7-6)^2 - 4 = 3(1)^2 - 4 = 3 - 4 =- 1$.
$f(4) = 3(4-6)^2 - 4 = 3(-2)^2 - 4 = 3(4) - 4 = 12 - 4 = 8$.
Calculate f(7) - f(4): -1 - 8 = -9.

4. Which expression is equivalent to $-6x^5y^7(7x^7+4x^4+84)$?

A. $-42x^{12}y^6-24x^9y^7-504x^5y^7$

B. $-42x^{12}y^7-24x^6y^7-504x^5y^6$

C. $-42x^{12}y^7-24x^9y^6-504x^5y^7$

D. $-42x^{12}y^7-24x^9y^7-504x^5y^7$

## Answer

D

## Solution

Concept Check : The question tests the student's understanding of polynomial operations, specifically multiplication. Students should know how to distribute the term $-6x^5y^7$ across each term in the polynomial $(7x^7+4x^4+84)$ and apply the rules of exponents when multiplying terms with the same base.

Solution Strategy : To solve this problem, the student should first recognize that they need to distribute $-6x^5y^7$ to each term inside the parentheses. This means multiplying $-6x^5y^7$ *by* $7x^7$, then by $4x^4$, and finally by 84. The student should keep in mind how to handle the coefficients and the variables while applying the laws of exponents, specifically that when multiplying like bases, you add the exponents.

Quick Wins : Start by rewriting the expression clearly; it helps to see each part. Remember to multiply the coefficients (the numbers in front) together and then handle the variable parts separately. When multiplying the variable parts, add the exponents of like bases. It can be helpful to write down intermediate steps to avoid confusion, ensuring that each term is calculated correctly before moving on to the next.

Mistake Alert : Be careful with the signs—multiplying by -6 means that the sign of each resulting term will flip. Also, double-check the exponents; it's easy to make a mistake when adding exponents, especially with higher-degree polynomials. Pay attention to the arrangement of your final expression to ensure all terms are included and properly simplified.

SAT Know-How : This problem is an example of operations with higher-degree polynomials, specifically focusing on polynomial multiplication. It assesses skills such as distribution, combining like terms, and the laws of exponents. Mastering these concepts is crucial for success in the SAT's math section, particularly in advanced math questions.

Step 1: Distribute $-6x^5y^7$ across each term in the parenthesis: $-6x^5y^7\times 7x^7$, $-6x^5y^7\times 4x^4$, and $-6x^5y^7\times 84$.

Step 2: Calculate $-6 \times 7 = -42$, $x^5 \times x^7 = x^{(5+7)} = x^{12}$, $y^7$ remains unchanged. So the first term becomes $-42x^{12}y^7$.
Step 3: Calculate $-6 \times 4 = -24$, $x^5 \times x^4 = x^9$, $y^7$ remains unchanged. So the second term becomes $-24x^9y^7$.
Step 4: Calculate $-6 \times 84 = -504$, $x^5$ remains unchanged, $y^7$ remains unchanged. So the third term becomes $-504x^5y^7$.
Step 5: Combine all terms to form the expression: $-42x^{12}y^7 - 24x^9y^7 - 504x^5y^7$.

5. The function f is defined by $f(x) = 7x^2 + 6x - 38$. What is the value of $f(2)$?

## Answer

2

## Solution

Concept Check : The intent of this question is to assess the student's understanding of evaluating a quadratic function. Students are expected to know how to substitute a given value into a function and compute the result, demonstrating their proficiency with basic algebraic operations.

Solution Strategy : To solve this problem, the student should start by substituting the given value, which is 2, into the function $f(x)$. This involves replacing x with 2 in the expression $7x^2 + 6x - 38$. After substitution, they will need to follow the order of operations (PEMDAS/BODMAS) to simplify the expression step by step.

Quick Wins : When substituting, it's helpful to write out the function clearly and confirm each step as you go. Break down the calculation into manageable parts, calculating $7(2^2)$, $6(2)$, and then combining these results along with the constant term -38. This systematic approach can minimize errors.

Mistake Alert : Be careful with arithmetic, especially when squaring numbers and performing addition or subtraction. It's easy to make mistakes with signs, so double-check your calculations to ensure accuracy. Also, remember to follow the order of operations closely.

SAT Know-How : This problem falls under the category of Advanced Math, focusing on nonlinear functions, particularly quadratics. It assesses the student's skill in evaluating functions and performing algebraic operations. Mastering such problems enhances problem-solving capabilities and prepares students for the types of questions they may encounter on the SAT.

Start with the function: $f(x) = 7x^2 + 6x - 38$.
Substitute $x = 2$ into the function: $f(2) = 7(2)^2 + 6(2) - 38$.
Calculate $7(2)^2$: $2^2 = 4$, so $7(4) = 28$.
Calculate $6(2)$: $6 \times 2 = 12$.
Combine the terms: $28 + 12 - 38$.
Perform the arithmetic: $28 + 12 = 40$.
Subtract 38 from 40: $40 - 38 = 2$.
Thus, the value of $f(2)$ is 2.

6. What is an x-coordinate of an x-intercept of the graph $y = 3(x - 4)(x - 7)(x + 5)$ in the xy-plane?

A. -5

B. -4

C. 0

D. 10

## Answer

A

## Solution

Concept Check : The question asks students to find the x-coordinate of an x-intercept of a polynomial function. Students should know that the x-intercepts occur where the value of y is zero, which means solving the equation set to zero. This requires understanding polynomial functions and their roots.

Solution Strategy : To find the x-intercept, the student should set the polynomial expression equal to zero: 3(x - 4)(x - 7)(x + 5) = 0. This will lead to a product of factors equal to zero, which means setting each factor equal to zero to find the corresponding x-values. The student should be prepared to solve for x by isolating each factor.

Quick Wins : First, remember that if a product of factors equals zero, at least one of the factors must be zero. This means you will need to consider each factor individually. In this case, the factors are (x - 4), (x - 7), and (x + 5). Set each factor to zero and solve for x to find the x-intercepts. It may help to write down each equation separately for clarity.

Mistake Alert : Be careful not to overlook any factors when setting them to zero. It's easy to accidentally skip one if you're not thorough. Also, remember that the polynomial has real roots, so ensure that you are not making assumptions about the nature of the roots without checking the factors. Lastly, double-check your arithmetic when solving for x to avoid simple calculation errors.

SAT Know-How : This problem falls under the category of Advanced Math, specifically dealing with polynomials and their roots. It tests the student's ability to manipulate polynomial equations and solve for specific values (x-intercepts). Mastery of this skill is crucial for handling more complex polynomial functions in the SAT, as it demonstrates understanding of foundational algebraic concepts.

Set y = 0 in the equation: 0 = 3(x - 4)(x - 7)(x + 5).
Since the product is zero, at least one of the factors must be zero.
Solve each factor for x:
1. x - 4 = 0 → x = 4
2. x - 7 = 0 →x = 7

3. $x + 5 = 0 \rightarrow x = -5$
Therefore, the x-coordinates of the x-intercepts are 4, 7, and -5.

7. A financial analyst is modeling the projected economic activity in a city using the equation for active jobs: $y = -0.3x^2 - 3x + 86$, where $y$ represents the number of active jobs and $x$ is the number of months since January 2020. What is the best interpretation of the y-intercept of the graph of this equation in the xy-plane?

A. At the end of January 2020, the projected number of active jobs was 0.

B. At the end of January 2020, the projected number of active jobs was 86, suggesting a lack of job activity or data errors.

C. At the end of July 2020, the projected number of active jobs was 86.

D. At the end of July 2020, the projected number of active jobs was 0.

## Answer

B

## Solution

Concept Check : The intent of this question is to assess the student's understanding of quadratic functions, specifically the interpretation of the y-intercept in the context of a real-world application. Students should recognize that the y-intercept represents the value of y when x is zero, which corresponds to a specific point in time (January 2020 in this case).

Solution Strategy : To approach this problem, students should first identify what the y-intercept represents in the given equation. This involves substituting $x = 0$ into the equation to find the value of y. Then, they should interpret this value in the context of the problem, considering what it means in terms of active jobs at the starting point of the timeframe given (January 2020).

Quick Wins : When interpreting the y-intercept, remember that it gives you the starting value of the dependent variable when the independent variable is zero. In this case, think about what 'months since January 2020' means—$x = 0$ will indicate January 2020, so the y-intercept will tell you the number of active jobs at that time. Always read the problem carefully to ensure you understand the real-world connection.

Mistake Alert : Be careful not to confuse the y-intercept with other points on the graph. The y-intercept is specifically the point where the graph crosses the y-axis (x = 0). Additionally, ensure that you correctly substitute x = 0 into the equation; miscalculating the equation may lead to incorrect interpretations. Remember that negative values can have significant implications in real-world contexts.

SAT Know-How : This problem falls under the category of Advanced Math, focusing on quadratic and exponential word problems. It tests the student's ability to interpret mathematical models in real-world contexts, specifically through the lens of the y-intercept. Mastering this concept is crucial for SAT problem-solving, as it reinforces the connection between algebraic equations and their applications in various scenarios.

1. Identify the y-intercept in the given equation $y =- 0.3x^2 - 3x + 86$.
2. The y-intercept occurs when $x = 0$.
3. Substitute $x = 0$ into the equation: $y =- 0.3(0)^2 - 3(0)6 + 86$.
4. Simplify: $y = 86$.
5. The y-intercept of 86 means that at the start of January 2020, the projected number of active jobs is 86.

8. The equation $4|x - 97| + 7 = k^2 + 110k + 3032$ has exactly one solution for the variable $x$. Which of the following could be the value of $k$?

A. -55 only

B. 0 only

C. -55 and 0

D. -55 and 55

## Answer

A

## Solution

Concept Check : The intent of this question is to assess the student's understanding of absolute value equations and how the number of solutions can be determined based on the properties of the equations involved. Students should be familiar with the concept of absolute values and the conditions under which an equation has one solution, particularly in relation to quadratic functions.

Solution Strategy : To solve this problem, students should start by isolating the absolute value expression on one side of the equation. They will need to recognize that for the equation to have exactly one solution, the expression inside the absolute value must equal zero at that solution, and the quadratic expression on the right side must also take a specific form. This typically involves finding the vertex of the quadratic function and ensuring it touches the x-axis at exactly one point. Students should think about how to manipulate the equation to find values of k that meet these conditions.

Quick Wins : Consider rewriting the equation in a form that allows you to analyze the absolute value term. Remember that an absolute value equation has one solution when the expression inside the absolute value is zero, or when the graph of the quadratic touches the line represented by the absolute value. You may want to identify the vertex of the quadratic equation on the right side and find values of k that make it equal to the expression represented by the absolute value. Calculating the discriminant of the quadratic can also provide insight into the number of solutions.

Mistake Alert : Be careful not to overlook the conditions for the absolute value to yield exactly one solution. It's crucial to ensure that the quadratic's vertex aligns correctly for it to touch the axis at just one point. Additionally, pay attention to any algebraic manipulations and signs when isolating the absolute value term, as mistakes in these steps can lead to incorrect conclusions about the number of solutions.

SAT Know-How : This problem falls under the category of Advanced Math, specifically focusing on radical, rational, and absolute value equations. It assesses skills related to manipulating and analyzing equations to determine the number of solutions. The key takeaway is to understand how absolute values interact with quadratic functions and to use properties of both to solve for possible values of k. Mastering these concepts is essential for effective problem-solving in SAT math.

Step 1: Start with the absolute value equation: $4|x - 97| + 7 = k^2 + 110k + 3032$.
Step 2: Isolate the absolute value term: $4|x - 97| = k^2 + 110k + 3025$.
Step 3: Divide both sides by 4: $|x - 97| = \frac{k^2+110k+3025}{4}$.
Step 4: For |x - 97| to have exactly one solution, the expression $\frac{k^2+110k+3025}{4}$ must be 0.
Step 5: Solve the equation: $k^2 + 110k + 3025 = 0$.
Step 6: Factor the quadratic equation: $k^2 + 110k + 3025 = (k + 55)^2$.
Step 7: Set the factored expression to 0: $(k + 55)^2 = 0$.
Step 8: Solve for k: k + 55 = 0, so k = -55.

9. A physicist is studying a certain radioactive substance whose quantity decreases over time. The equation representing the quantity of this substance in grams after x years is given by: $f(x) = 1100(0.41)^x$. Which of the following is the best interpretation of 1100 in this context?

A. The estimated amount of substance remaining after 1 year.

B. The estimated amount of substance remaining after 2 years.

C. The estimated initial quantity of the substance at time 0.

D. The estimated quantity of substance after 6 years.

## Answer

C

## Solution

Concept Check : The question is designed to test the student's understanding of exponential decay, particularly in the context of real-world applications such as radioactive decay. The student is expected to know how to interpret the parameters of an exponential function, specifically the initial quantity and the decay factor.

Solution Strategy : To approach this problem, the student should identify the components of the given exponential function. They should recognize that in the equation of the form $f(x) = a(b)^x$, 'a' represents the initial value of the quantity when $x = 0$. The student should think through the meaning of the parameters in relation to the problem context to determine what 1100 signifies.

Quick Wins : A good strategy is to substitute $x = 0$ into the function to see what value you get. This will help clarify what the initial quantity is. Also, it can be helpful to visualize the process of radioactive decay and how it relates to the parameters in the equation. Remember that the base of the exponent (0.41 in this case) indicates the rate of decay, whereas the coefficient provides the starting amount.

Mistake Alert : Be careful not to confuse the initial quantity with the decay factor. The number 1100 is significant at the start of the observation (when $x = 0$), while the base of the exponent, 0.41, tells you how much of the substance remains after each year. Misinterpreting these can lead to incorrect conclusions.

SAT Know-How : This problem falls under the category of advanced math, specifically focusing on exponential word problems related to real-life scenarios like radioactive decay. It assesses the student's ability to interpret mathematical models and understand the significance of parameters. Mastering this type of problem enhances your skills in applying mathematical concepts to analyze and interpret real-world data effectively.

Step 1: Understand the general form of an exponential decay function. It is typically given by $f(x) = a(b)^x$, where 'a' is the initial quantity, 'b' is the decay factor, and 'x' is the time elapsed.

Step 2: Compare the given function $f(x) = 1100(0.41)^x$ with the general form. Here, $'a' = 1100$ and $'b' = 0.41$.

Step 3: Interpret the meaning of $'a' = 1100$. This value represents the initial quantity of the substance before any decay has occurred, i.e., at time $x = 0$.

Step 4: Verify by setting $x = 0$ in $f(x) = 1100(0.41)^x$. The equation becomes $f(0) = 1100(0.41)^0 = 1100 \times 1 = 1100$.

Step 5: Therefore, 1100 represents the initial amount of the substance, confirming the interpretation that it is the quantity present at time $x = 0$.

10. A solution to the given system of equations is (x, y). What is a possible value of x?
$x - 76 = y\ (x - 76)^2 = y$

A. 75

B. 76

C. 78

D. 80

## Answer

B

## Solution

Concept Check : The intent of the question is to assess the student's understanding of systems of equations, specifically a combination of linear and quadratic equations. Students should know how to manipulate both types of equations and recognize how to find possible solutions for x based on the given equations.

Solution Strategy : To solve the problem, the student should first substitute the expression for y from the linear equation into the quadratic equation. This will create a single equation in terms of x, which can then be solved for possible values of x. The student should be prepared to explore the nature of the quadratic equation resulting from this substitution, which may yield one or two solutions for x.

Quick Wins : 1. Always start by identifying one equation that is easier to manipulate—here, the linear equation is simpler. 2. Substitute expressions carefully to avoid mistakes. 3. When working with quadratic equations, remember to check for both possible solutions, as some quadratics can yield two values for x. 4. Use the quadratic formula or factoring techniques if applicable.

Mistake Alert : Be cautious about sign errors when rearranging the equations. Pay special attention to the quadratic form; sometimes students overlook negative solutions or fail to correctly simplify expressions. Ensure you check both possible solutions after solving the quadratic equation, as it can have two valid x-values.

SAT Know-How : This problem falls under the category of Advanced Math and specifically tests skills in solving systems of linear and quadratic equations. It requires the ability to manipulate equations and recognize the relationship between linear and quadratic functions. Developing these problem-solving skills is essential for success on the SAT, especially in recognizing how different types of equations can interact.

1. Substitute equation 1 into equation 2, since both are equal to y.
2. Set up the equation: $(x - 76)^2 = x - 76$.
3. Rearrange the equation: $(x - 76)^2 - (x - 76) = 0$.
4. Factor the equation: (x - 76)(x - 76 - 1) = (x - 76)(x - 77) = 0.

5. Solve the factors individually:
- If (x - 76) = 0, then x = 76.
- If (x - 77) = 0, then x = 77.
6. Thus, possible values for x are 76 and 77.

# Math

## Digital SAT

# 3

## Geometry and Trigonometry

Studyola

# SAT Math Geometry and Trigonometry

1. Circle C has a radius of 2x and circle D has a radius of 50x. The area of circle D is how many times the area of circle C?

2. Circle C has a radius of $5x$ and circle D has a radius of $25x$. The area of circle D is how many times the area of circle C?

A. 10

B. 15

C. 20

D. 25

3. The table gives the perimeters of similar triangles DEF and PQR, where DE corresponds to PQ. The length of DE is 16. What is the length of PQ?

| Triangle | Perimeter |
|---|---|
| Triangle DEF | 80 |
| Triangle PQR | 240 |

4. A circle in the xy-plane has its center at (4, -2) and has a radius of 5. An equation of this circle is given by $x^2 + y^2 + ax + by + c = 0$, where a, b, and c are constants. What is the value of c?

5. What is the center of the circle in the $xy$-plane defined by the equation $(x + 5)^2 + (y - 3)^2 = 16$?

6. A wooden cube is carved from a log, and its edges measure 4 centimeters. If the cube is then sanded down, causing each edge to decrease in length by 0.5 centimeters, what will be the volume of the newly shaped cube, in cubic centimeters?

7. In $\Delta ABC$, $\angle B$ is a right angle and the length of BC is 180 millimeters. If $cos(A) = \frac{4}{5}$, what is the length, in millimeters, of AB?

A. 200

B. 220

C. 240

D. 260

8. In triangle DEF, the measure of angle D is 32°, the measure of angle E is 90°, and the measure of angle F is $\frac{m}{3}°$. What is the value of m?

A. 162

B. 168

C. 174

D. 180

9. A wooden cube used in a public health education demonstration has an edge length of 3 centimeters. If the cube weighs 5.61 grams, what is the density of the cube in grams per cubic centimeter?

A. 0.2068

B. 0.2070

C. 0.2082

D. 0.2078

10. In the $xy$-plane, circle N is represented by the equation $(x + 1)^2 + (y - 2)^2 = 9$. Circle Q has the same center as circle N but has a radius that is 1.5 times the radius of circle N. What is the equation for circle Q?

# SAT Math Geometry and Trigonometry Solutions

1. Circle C has a radius of 2x and circle D has a radius of 50x. The area of circle D is how many times the area of circle C?

## Answer

625

## Solution

This problem tests the student's understanding of the formula for the area of a circle and their ability to use ratios to compare the areas of two circles based on their radii.
To solve this problem, students should first recall the formula for the area of a circle, $A = \pi r^2$, where r is the radius. Next, they calculate the area of both circles using their given radii: Circle C with a radius of 2x and Circle D with a radius of 50x. After finding the areas, students should set up a ratio of the area of Circle D to the area of Circle C and simplify the ratio.
Remember that when comparing areas of circles, the ratio of the areas is the square of the ratio of their radii. This can simplify the calculations significantly.
Be careful with squaring the radii correctly. A common mistake is not squaring the entire expression, which can lead to an incorrect ratio. Also, ensure that you simplify the ratio completely.
This type of problem is common in SAT geometry sections, as it assesses both the understanding of geometric formulas and the ability to manipulate algebraic expressions. Mastery of such problems requires familiarity with basic geometric formulas and an ability to apply algebraic principles, such as simplifying ratios. Practicing these skills will help improve accuracy and speed on test day.

Calculate the area of circle C:, $Area\ of\ circle\ C = \pi(2x)^2 = 4\pi x^2$, Calculate the area of circle D:, $Area\ of\ circle\ D = \pi(50x)^2 = 2500\pi x^2$, Determine how many times the area of circle D is compared to circle C:, $Number\ of\ times\ = \frac{2500\pi x^2}{4\pi x^2} = \frac{2500}{4} = 625$

2. Circle C has a radius of $5x$ and circle D has a radius of $25x$. The area of circle D is how many times the area of circle C?

A. 10

B. 15

C. 20

D. 25

## Answer

D

## Solution

This problem aims to assess the student's understanding of the relationship between the radius and the area of a circle. Specifically, it examines the ability to apply the formula for the area of a circle and to work with ratios.
To solve this problem, the student should start by recalling the formula for the area of a circle, which is $A = \pi r^2$. Calculate the area of both circles using their respective radii, then compare the two areas by forming a ratio.
Remember that the area of a circle increases with the square of its radius. When comparing areas of circles, you can often simplify your work by setting up a ratio instead of calculating exact areas. In this problem, simplify the ratio of the radii first to see the effect on the area.
Be careful not to confuse the ratio of the radii with the ratio of the areas. The radius is linear, while the area is quadratic. Also, ensure that you square the radii correctly and apply the $\pi$ factor consistently.
This type of problem is common in SAT geometry questions and tests the ability to understand and manipulate geometric formulas, specifically circles. It also assesses the student's skill in working with proportions and recognizing how changes in one dimension (radius) affect another dimension (area). Mastery of these concepts is crucial not only for geometry but also for more advanced topics in mathematics.

Calculate the area of circle C: $A_C = \pi(5x)^2 = 25\pi x^2$

Calculate the area of circle D: $A_D = \pi(25x)^2 = 625\pi x^2$

The ratio of the area of circle D to circle C is: $\frac{A_D}{A_C} = \frac{625\pi x^2}{25\pi x^2} = 25$.

Thus, the area of circle D is 25 times the area of circle C.

3. The table gives the perimeters of similar triangles DEF and PQR, where DE corresponds to PQ. The length of DE is 16. What is the length of PQ?

| Triangle | Perimeter |
|---|---|
| Triangle DEF | 80 |
| Triangle PQR | 240 |

## Answer

48

## Solution

This problem tests the student's understanding of similar triangles, particularly their ability to use the relationship between the perimeters of similar triangles to find the corresponding side lengths.
To solve this problem, students need to recognize that the perimeters of similar triangles are in the same ratio as their corresponding side lengths. They should set up a proportion using the given perimeter values and the known side length DE to find the unknown side length PQ.
Remember that in similar triangles, the ratio of the perimeters is equal to the ratio of any pair of corresponding side lengths. Set up a proportion using the given perimeter and side length information to find the unknown side length.
Be careful to correctly identify the corresponding sides in the triangles and ensure that the ratios are set up correctly. Misidentifying corresponding sides or mixing up the ratios can lead to incorrect answers.
This problem is a classic example of testing knowledge on similar triangles and their properties. It evaluates the student's ability to apply proportional reasoning in geometry. Understanding and correctly applying the concept of similarity is crucial in solving such problems efficiently on the SAT.

Since the triangles are similar, the ratio of any two corresponding sides of similar triangles is equal to the ratio of their perimeters.
Let the length of PQ be x.
The ratio of the perimeters of the triangles is $\frac{80}{240} = \frac{1}{3}$.
The ratio of the corresponding sides DE and PQ is $\frac{16}{x}$.
So, we set up the equation: $\frac{16}{x} = \frac{1}{3}$.
Solving for x, we cross-multiply: $16 \times 3 = x \times 1$.
This simplifies to $48 = x$.
Thus, the length of PQ is 48.

4. A circle in the xy-plane has its center at (4, -2) and has a radius of 5. An equation of this circle is given by $x^2 + y^2 + ax + by + c = 0$, where a, b, and c are constants. What is the value of c?

## Answer

-5

## Solution

This problem tests the student's ability to work with the standard form of a circle equation and their understanding of how to manipulate and expand it into the general quadratic form. It assesses the student's skills in algebraic manipulation and geometric understanding of a circle's properties.

Start by recalling the standard equation of a circle, which is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius. Substitute the given center (4, -2) and radius 5 into this equation to get $(x - 4)^2 + (y + 2)^2 = 25$. Then, expand this equation to form $x^2 + y^2 - 8x + 4y + 20 = 0$. Compare this with $x^2 + y^2 + ax + by + c = 0$ to find the values of a, b, and c.

When expanding the squared terms, carefully distribute and simplify each step to avoid errors. Focus on correctly aligning the terms with the general quadratic form. Remember that the coefficient of $x^2$ *and* $y^2$ should remain 1, matching the provided equation format. Be careful with the signs when substituting the center coordinates into $(x - h)^2 + (y - k)^2$. It's easy to make mistakes with signs, especially when handling $(y + 2)^2$. Additionally, ensure that all terms are correctly expanded and combined to match the general form.

This problem is a classic example of testing the ability to manipulate geometric equations, specifically those of circles. It evaluates algebraic manipulation skills and understanding of circle properties in coordinate geometry. Mastery of these concepts is crucial for success in SAT geometry questions, and this exercise helps develop proficiency in transforming and comparing different equation forms.

Substitute the center (h, k) = (4, -2) and radius r = 5 into the equation $(x - h)^2 + (y - k)^2 = r^2$., This gives $(x - 4)^2 + (y + 2)^2 = 25$., Expand $(x - 4)^2$ to get $x^2 - 8x + 16$., Expand $(y + 2)^2$ to get $y^2 + 4y + 4$., Combine these to form: $x^2 - 8x + 16 + y^2 + 4y + 4 = 25$., Rearrange to the form $x^2 + y^2 + ax + by + c = 0$., Combine terms: $x^2 + y^2 - 8x + 4y + 20 = 25$., Subtract 25 from both sides to isolate c: $x^2 + y^2 - 8x + 4y + 20 - 25 = 0$., Simplify: $x^2 + y^2 - 8x + 4y - 5 = 0$., From this, c is found to be -5.

5. What is the center of the circle in the $xy$-plane defined by the equation $(x + 5)^2 + (y - 3)^2 = 16$?

## Answer

(-5, 3)

## Solution

This problem tests the student's understanding of the standard form of a circle's equation and their ability to identify the center and radius from this form. Students should recognize that the equation $(x - h)^2 + (y - k)^2 = r^2$ represents a circle centered at (h, k) with radius r.
To solve this problem, students need to identify the form of the given equation $(x + 5)^2 + (y - 3)^2 = 16$ and compare it to the standard form of a circle's equation $(x - h)^2 + (y - k)^2 = r^2$. Recognize that the equation can be rewritten as $(x - (-5))^2 + (y - 3)^2 = 16$, indicating that the center of the circle (h, k) is (-5, 3).
When dealing with circle equations, always rewrite the equation in the form $(x - h)^2 + (y - k)^2 = r^2$ to easily identify the center (h, k) and the radius r. Remember that the signs in the equation are opposite to those in the center coordinates.
Be careful with the signs when determining the center of the circle. In the equation $(x - h)^2 + (y - k)^2 = r^2$, the center is at (h, k), so you must pay attention to the minus signs in the equation to correctly identify the positive or negative values of h and k. Additionally, make sure not to confuse the squared term 16 with the radius; the radius is the square root of 16, which is 4.
This type of problem is common in SAT math sections and assesses a student's ability to work with the standard equation of a circle. Recognizing the structure of the equation and understanding how to manipulate it to extract the center and radius is crucial. Mastery of these concepts is essential for success in geometry and trigonometry problems on the SAT. Practice with a variety of circle equations to become comfortable with quickly identifying the center and radius.

Identify the standard form from the given equation: $(x + 5)^2 + (y - 3)^2 = 16$ compared to $(x - h)^2 + (y - k)^2 = r^2$.
Rewrite $(x + 5)^2$ as $(x - (-5))^2$ to match the standard form.
Similarly, rewrite $(y - 3)^2$., By comparing, we see that $h = -5$ and $k = 3$, and $r^2 = 16$.
Thus, the center of the circle is (-5, 3).

6. A wooden cube is carved from a log, and its edges measure 4 centimeters. If the cube is then sanded down, causing each edge to decrease in length by 0.5 centimeters, what will be the volume of the newly shaped cube, in cubic centimeters?

## Answer

42.875 cubic centimeters

## Solution

This problem tests the student's understanding of volume calculations for geometric shapes, specifically cubes, and requires the ability to apply volume formulas after modifying dimensions.
To solve this problem, first, calculate the original volume of the cube using the formula for the volume of a cube ($V = (side)^3$). Then, adjust the edge length by subtracting 0.5 cm to account for the sanding down process. Finally, calculate the new volume using the adjusted edge length.
Remember that when the dimensions of a cube change, even slightly, it can significantly impact the volume due to the cubic relationship. Always perform the calculations step by step to ensure accuracy.
Be careful not to confuse the reduction in edge length with a reduction in volume. Ensure that you subtract the 0.5 cm from each edge before recalculating the volume. Also, double-check your arithmetic to ensure that cube calculations are correct.
This type of problem is a classic example of testing geometric reasoning and arithmetic skills. It requires students to accurately apply a formula and understand how dimensional changes affect the volume. Being able to handle such problems efficiently is crucial for the SAT, as it demonstrates a solid grasp of basic geometry and measurement principles.

Determine the new edge length by subtracting 0.5 cm from the original length of each edge., New edge length = 4 cm - 0.5 cm = 3.5 cm., Calculate the volume of the new cube using the formula for the volume of a cube, $V = a^3$, where 'a' is the edge length., Substitute the new edge length into the formula: $V = (3.5cm)^3$., Calculate the cube of the new edge length: $V = 3.5cm \times 3.5cm \times 3.5cm$., V = 42.875 cubic centimeters.

7. In ΔABC, ∠B is a right angle and the length of BC is 180 millimeters. If $cos(A) = \frac{4}{5}$, what is the length, in millimeters, of AB?

A. 200

B. 220

C. 240

D. 260

## Answer

C

## Solution

This problem is designed to test the student's understanding of right-angle trigonometry, specifically the ability to use the cosine function to find the length of a side in a right triangle.
The student should recognize that in right triangle ΔABC, with ∠B as the right angle, the cosine of angle A is defined as the ratio of the adjacent side (AB) to the hypotenuse (AC). Given that $cos(A) = \frac{4}{5}$, the student needs to set up the equation $\frac{AB}{AC} = \frac{4}{5}$. Since BC is given as 180 millimeters, and BC is the side opposite angle A, the student can use the Pythagorean theorem to find AC first before finding AB.
Remember that the Pythagorean theorem can be used to find the hypotenuse when you have one side and the cosine ratio. Set up a ratio equation using $cos(A) = \frac{adjacent}{hypotenuse}$, and solve for the unknown side. Double-check your calculations by ensuring the triangle's side lengths satisfy the Pythagorean theorem.
Be careful not to confuse the sides of the triangle. Ensure you correctly identify which side is opposite and which is adjacent to angle A. Also, ensure that your calculations are exact, and consider simplifying fractions or square roots accurately.
This problem assesses the student's proficiency in applying trigonometric ratios to solve for missing side lengths in right triangles. Mastery of this concept is essential for solving more complex trigonometry problems in the SAT. The ability to correctly interpret and apply the cosine function is a crucial skill in the geometry section of the test.

Since $cos(A) = \frac{4}{5}$, we have $\frac{AB}{AC} = \frac{4}{5}$., We need to find the length of AB., Since BC is 180 millimeters, BC is opposite to angle A., In a right triangle, we use the Pythagorean identity: $(sin)^2(A) + (cos)^2(A) = 1$., Given $cos(A) = \frac{4}{5}$, find $(sin)^2(A)$: $\left(\frac{4}{5}\right)^2 + (sin)^2(A) = 1$., $\frac{16}{25} + (sin)^2(A) = 1$., $(sin)^2(A) = \frac{9}{25}$, therefore $sin(A) = \frac{3}{5}$., Using $sin(A)$, we have $sin(A) = \frac{BC}{AC} = \frac{3}{5}$., $AC = \frac{BC}{sin(A)} = \frac{180}{\frac{3}{5}} = 180 \times \frac{5}{3} = 300\ millimeters$., Now, using $cos(A) = \frac{4}{5}$, solve for AB: $AB = cos(A) \times AC = \frac{4}{5} \times 300 = 240\ millimeters$.

8. In triangle DEF, the measure of angle D is 32°, the measure of angle E is 90°, and the measure of angle F is $\frac{m}{3}$°. What is the value of m?

A. 162

B. 168

C. 174

D. 180

## Answer

C

## Solution

This problem tests the student's understanding of the properties of angles in a triangle, particularly the fact that the sum of the interior angles in a triangle is always 180 degrees. It also requires the student to solve for a variable within a given expression.
To solve this problem, recognize that the sum of the angles in any triangle is 180 degrees. Given that angle E is 90 degrees, angle D is 32 degrees, and angle F is expressed as $\frac{m}{3}$ degrees, set up an equation: $32 + 90 + \frac{m}{3} = 180$. Solve this equation for m by first combining the known angles and then isolating the variable.
Remember that for any triangle, the sum of the interior angles is always 180 degrees. Also, pay attention to how the angle F is expressed in terms of m. Rearranging and solving linear equations accurately will help you find the correct value of m.
Be careful with arithmetic operations, especially when working with fractions. Ensure that you properly isolate the variable m after combining like terms. Common mistakes include arithmetic errors or miscalculating the value of expressions.
This problem is a classic example of testing the understanding of basic geometric principles such as the sum of interior angles in a triangle. It requires algebraic manipulation skills to isolate and solve for a variable. Such questions are designed to evaluate both geometric understanding and algebraic problem-solving abilities. Mastery of these concepts is crucial for success in the SAT math section.

The sum of the angles in triangle DEF is: 32° + 90° + F = 180°, Substituting the given measures: 32° + 90° + $\frac{m}{3}$° = 180°, Combine the known angles: 122° + $\frac{m}{3}$° = 180°, Subtract 122° from both sides: $\frac{m}{3}$° = 58°, Solve for m by multiplying both sides by 3: m = 58° $\times$ 3, Calculate m: m = 174°

9. A wooden cube used in a public health education demonstration has an edge length of 3 centimeters. If the cube weighs 5.61 grams, what is the density of the cube in grams per cubic centimeter?

A. 0.2068

B. 0.2070

C. 0.2082

D. 0.2078

## Answer

D

## Solution

This problem aims to test the student's understanding of geometric properties of a cube, specifically how to calculate the volume, and then apply the formula for density. The student needs to be familiar with basic volume formulas and the concept of density as mass per unit volume.
1. Calculate the volume of the cube using the formula for the volume of a cube ($V = a^3$ where 'a' is the edge length).
2. Use the given mass and the volume to calculate the density using the formula ($Density = \frac{Mass}{Volume}$).
Remember that the volume of a cube is found by cubing the edge length. Write down all given information and use the density formula directly after calculating the volume. This helps in organizing thoughts and reducing careless errors.
Be careful with units and ensure consistency throughout the calculation. Miscalculating the volume by forgetting to cube the edge length is a common mistake. Verify that the density units are in grams per cubic centimeter as required by the problem.
This problem tests fundamental skills in geometry and unit analysis, which are crucial for many SAT math problems. Understanding the relationships between edge length, volume, and density is key. Efficiently solving such problems requires a clear grasp of basic formulas and careful unit management, which are essential skills for SAT success.

The formula for calculating the volume of a cube is $Volume = edge\ length^3$
For this cube, the volume is $3^3 = 27$ cubic centimeters.
Density is given by $Density = \frac{Mass}{Volume}$, Substituting the known values: $Density = \frac{5.61}{27}$ grams per cubic centimeter.
Performing the division: $\frac{5.61}{27} = 0.207777...$
Rounding to the fourth digit, we get $Density \cong 0.2078$ grams per cubic centimeter.

10. In the $xy$-plane, circle N is represented by the equation $(x + 1)^2 + (y - 2)^2 = 9$. Circle Q has the same center as circle N but has a radius that is 1.5 times the radius of circle N. What is the equation for circle Q?

## Answer

$(x + 1)^2 + (y - 2)^2 = 20.25$

## Solution

This question tests the student's ability to understand and manipulate the equation of a circle in the standard form. It assesses their knowledge of how changes in the radius affect the equation and their ability to calculate and apply these changes accurately.
First, identify the center and radius of circle N from its equation. The equation given is in the form $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius. For circle N, the center is (-1, 2) and the radius is $\sqrt{9} = 3$. Since circle Q has the same center and a radius 1.5 times that of circle N, calculate the new radius: $1.5 \times 3 = 4.5$. Then, write the equation for circle Q using the new radius.
Keep in mind the standard equation of a circle, $(x - h)^2 + (y - k)^2 = r^2$. When the problem states that the radius is 1.5 times larger, you only need to multiply the original radius by 1.5 and square it to get the new $r^2$ value for the equation of circle Q.
Make sure not to confuse the radius with the squared radius when writing the equation.
After calculating the new radius, remember to square it to find the correct $r^2$ value for the equation.
This type of question is common in SAT geometry sections, focusing on the student's ability to manipulate and understand geometric equations, specifically those of circles. Mastery of this concept is crucial as it often appears in various forms across different questions. Practice with similar problems will help in quickly identifying the center and radius from circle equations, as well as applying transformations like scaling the radius.

1. Determine the center and radius of Circle N from the given equation $(x + 1)^2 + (y - 2)^2 = 9$.
- Center of Circle N: (-1, 2)
- Radius of Circle N: $\sqrt{9} = 3$
2. Calculate the radius of Circle Q by multiplying Circle N's radius by 1.5.
- Radius of Circle Q: $3 \times 1.5 = 4.5$
3. Write the equation for Circle Q using its center and newly calculated radius.
- The equation of Circle Q: $(x + 1)^2 + (y - 2)^2 = (4.5)^2$
- Simplify the equation: $(x + 1)^2 + (y - 2)^2 = 20.25$

# Math

## Digital SAT

# 4

Provlem Solving and Data Analysis

# SAT Math Problem Solving and Data Analysis

1. In a survey of 70 individuals regarding their support for international policies, the results indicated that 12 supported Country A, 25 supported Country B, and 33 supported Country C. If one individual is selected at random, what is the probability that the individual supports Country B?

| Type | Frequency |
|---|---|
| Country A | 12 |
| Country B | 25 |
| Country C | 33 |

A. $\frac{1}{2}$

B. $\frac{3}{5}$

C. $\frac{5}{14}$

D. $\frac{33}{70}$

2. What is the median of the data set shown? data set = [4, 50, 8, 23, 15, 42, 16]

3. How many liters are equivalent to 3.5 gallons? (1 gallon = 3.78541 liters)

4. A car travels at a constant acceleration of 4.5 meters per second squared. What is this acceleration, in feet per minute squared, rounded to the nearest tenth? (Use 1 foot = 0.3048 meters)

A. 53149.0

B. 53149.2

C. 53150.0

D. 53150.2

5. Each side of rectangle C has a length of 10 feet and a width of 4 feet. If both dimensions of rectangle C are multiplied by a scale factor of 2 to create rectangle D, what is the length, in feet, of each side of rectangle D?

A. 20 feet

B. 8 feet

C. 16 feet

D. 12 feet

6. The table shows the distribution of different big data technologies adopted by two technology companies. If a technology represented in the table is selected at random, what is the probability of selecting a technology related to Company A, given that the technology is related to Data Storage? (Express your answer as a decimal or fraction, not as a percent.)

| Technologies | Company A | Company B | Total |
|---|---|---|---|
| Data Storage | 40 | 30 | 70 |
| Data Mining | 25 | 35 | 60 |
| Data Analytics | 20 | 20 | 40 |
| Data Visualization | 15 | 15 | 30 |
| Total | 100 | 100 | 200 |

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. $\frac{3}{5}$

D. $\frac{4}{7}$

7. What is the median of the data set shown? data set = [3, 7, 9, 1, 5, 8, 2]

8. In a survey of 70 adults regarding their political ideologies, the results were classified as either Conservative, Liberal, or Moderate, as shown in the frequency table. If one adult is selected at random, what is the probability that the selected adult identifies as Liberal?

| Political Ideology | Frequency |
|---|---|
| Conservative | 25 |
| Liberal | 30 |
| Moderate | 15 |

9. The table shows the energy output in gigawatt-hours (GWh) for two countries with respect to their renewable and traditional energy resources. If an energy output is selected at random, what is the probability that the output is from Country A, given that it is from a renewable resource? (Express your answer as a decimal or fraction, not as a percent.)

| Countries | Renewable Resources (G | Traditional Energy (GW | Total Output (GWh) |
|---|---|---|---|
| Country A | 120 | 50 | 170 |
| Country B | 80 | 20 | 100 |
| Total | 200 | 70 | 270 |

10. A car travels at a speed of 5.2 meters per second. What is this speed in kilometers per hour, rounded to the nearest tenth? (Use 1 kilometer = 1,000 meters.)

A. 18.5 km/h

B. 18.6 km/h

C. 18.7 km/h

D. 18.8 km/h

# SAT Math Problem Solving and Data Analysis Solutions

1. In a survey of 70 individuals regarding their support for international policies, the results indicated that 12 supported Country A, 25 supported Country B, and 33 supported Country C. If one individual is selected at random, what is the probability that the individual supports Country B?

| Type | Frequency |
|---|---|
| Country A | 12 |
| Country B | 25 |
| Country C | 33 |

A. $\frac{1}{2}$

B. $\frac{3}{5}$

C. $\frac{5}{14}$

D. $\frac{33}{70}$

### Answer

C

### Solution

This problem tests the student's understanding of basic probability concepts and their ability to interpret and use frequency data to calculate probabilities.
To approach this problem, students need to identify the total number of individuals surveyed, which is 70, and the number of individuals who support Country B, which is 25. The probability that a randomly selected individual supports Country B is the ratio of the number of supporters of Country B to the total number of individuals surveyed.
Remember that probability is calculated as the number of favorable outcomes divided by the total number of possible outcomes. Always double-check the numbers given in the problem to ensure accuracy.
A common mistake is to incorrectly sum the frequencies or misinterpret the question, such as finding the probability for the wrong country. Pay close attention to the details provided in the problem.
This type of problem is straightforward and requires a solid understanding of basic probability concepts. It evaluates the student's ability to interpret data and apply probability formulas accurately. In SAT exams, being adept at these kinds of problems can

lead to quick wins, freeing up time for more complex questions.

To find the probability that an individual supports Country B, we use the formula:, $Probability\ = \frac{Number\ of\ individuals\ supporting\ Country\ B}{Total\ number\ of\ individuals\ surveyed}$, Substitute the given values into the formula:, $Probability\ = \frac{25}{70}$, Simplify the fraction:, $Probability\ = \frac{5}{14}$

2. What is the median of the data set shown? data set = [4, 50, 8, 23, 15, 42, 16]

## Answer

16

## Solution

The problem is designed to test the student's understanding of how to find the median in a data set. It assesses the student's ability to organize data and identify the middle value.
To solve the problem, the student needs to first ensure the data set is in numerical order, which it already is. Then, since there are seven numbers, the median is the one in the fourth position when the numbers are listed in order.
Remember that the median is the middle number in a list of numbers. If the list has an odd number of entries, the median is the exact middle number. If it has an even number of entries, you would take the average of the two middle numbers.
Be careful not to confuse the median with the mean or mode. They are different measures of central tendency. Ensure the data is sorted before attempting to find the median.
This type of problem is fundamental in understanding statistical data analysis. Finding the median is a basic skill that helps in understanding the distribution of data. On the SAT, this type of problem tests your ability to work with and interpret data accurately, which is essential for data analysis.

Count the numbers in the data set: 7 numbers., Identify the middle position: (7 + 1) / 2 = 4, The median is the fourth number in the list., Looking at the data set: [4, 50, 8, 23, 15, 42, 16], sorting the data set: [4, 8, 15, 16, 23, 42, 50], The fourth number is 16.

3. How many liters are equivalent to 3.5 gallons? (1 gallon = 3.78541 liters)

## Answer

13.2489

## Solution

This problem aims to test the student's understanding of unit conversion, specifically converting from gallons to liters using a given conversion factor.
To solve this problem, students need to multiply the number of gallons by the conversion factor to find the equivalent amount in liters. Thus, they should calculate $3.5 gallons \times 3.78541 liters/gallon$.
Always write down the units and ensure they cancel out correctly in the conversion process. This will help you keep track of the conversion you are performing and avoid errors.
Make sure to use the exact conversion factor provided in the problem. Also, be careful with multiplication and ensure you are not missing any decimal points which could lead to incorrect answers.
This type of problem assesses your ability to perform basic unit conversions, a vital skill in the Problem Solving and Data Analysis section. It requires careful attention to detail and precision in calculations. Practicing unit conversions with various units can help improve speed and accuracy on such questions in the SAT.

1. Use the conversion factor: $1 gallon = 3.78541 liters$.
2. Multiply the number of gallons by the conversion factor to find the equivalent number of liters.
3. Calculation: $3.5 gallons \times 3.78541 liters/gallon = 13.248935 liters$.
4. According to the guidelines, round the decimal to the fourth digit: 13.2489.
5. Therefore, 3.5 gallons is approximately 13.2489 liters.

4. A car travels at a constant acceleration of 4.5 meters per second squared. What is this acceleration, in feet per minute squared, rounded to the nearest tenth? (Use 1 foot = 0.3048 meters)

A. 53149.0

B. 53149.2

C. 53150.0

D. 53150.2

## Answer

B

## Solution

This problem tests the student's ability to perform unit conversions, specifically converting from meters per second squared to feet per minute squared. It also evaluates the student's understanding of unit relationships and their ability to handle multi-step conversions.
To solve this problem, the student should follow these steps:
1) Convert meters to feet by using the conversion factor 1 foot = 0.3048 meters.
2) Convert seconds squared to minutes squared by recognizing that there are 60 seconds in a minute and squaring that conversion factor.
3) Combine these conversions to find the acceleration in feet per minute squared.
First, remember to deal with one unit conversion at a time. It might help to write down each step to keep track of your conversions. Additionally, always double-check your conversion factors and ensure that units cancel out correctly.
Be careful with squaring the time conversion factor. Remember that you need to square the entire conversion factor (60 seconds per minute) to convert seconds squared to minutes squared. Also, ensure you do not round off too early in your calculations, as this can lead to inaccuracies.
This unit conversion problem is a common type in SAT math, reflecting real-world scenarios where multiple unit conversions are necessary. It tests both basic arithmetic skills and understanding of unit relationships. Mastering these types of problems is crucial for the Problem Solving and Data Analysis section of the SAT.

Given acceleration: 4.5 meters per second squared.
First, convert meters to feet: $\frac{4.5}{0.3048}$ feet per meter.
Result: $\frac{4.5}{0.3048} = 14.7637795276$ feet per second squared.
Now convert seconds squared to minutes squared: (14.7637795276 feet / $(second)^2$) × ($(60)^2$) seconds squared per minute squared.
Calculation: $14.7637795276 \times 3600 = 53149.2062996$ feet per minute squared.
Round the result to the nearest tenth: 53149.2

5. Each side of rectangle C has a length of 10 feet and a width of 4 feet. If both dimensions of rectangle C are multiplied by a scale factor of 2 to create rectangle D, what is the length, in feet, of each side of rectangle D?

A. 20 feet

B. 8 feet

C. 16 feet

D. 12 feet

## Answer

A

## Solution

This problem intends to assess the student's understanding of how scale factors affect the dimensions of geometric figures, specifically rectangles, and requires knowledge of basic multiplication and properties of rectangles.
To solve this problem, students should recognize that multiplying each dimension of rectangle C by a given scale factor will yield the dimensions of rectangle D. The length and width of rectangle C are 10 feet and 4 feet, respectively. By multiplying these dimensions by the scale factor of 2, students can find the new dimensions of rectangle D.
Remember that when you multiply both the length and width by a scale factor, you are essentially enlarging the rectangle proportionally. Make sure to apply the scale factor to both dimensions separately.
Be careful not to confuse the scale factor with addition. It's important to multiply each dimension by the scale factor, not add it. Additionally, ensure that you apply the scale factor to both the length and the width.
This problem is a straightforward application of scaling, a fundamental concept in geometry and proportional reasoning. It tests the student's ability to apply multiplication to geometric figures and understand the properties of similar shapes. Mastery of this type of problem is essential for success in the SAT's Problem Solving and Data Analysis category, as it reflects a student's ability to handle real-world mathematical situations.

Calculate the new length: $10\ feet\ \times 2 = 20\ feet$, Calculate the new width: $4\ feet\ \times 2 = 8\ feet$, Both sides of rectangle D are calculated.

6. The table shows the distribution of different big data technologies adopted by two technology companies. If a technology represented in the table is selected at random, what is the probability of selecting a technology related to Company A, given that the technology is related to Data Storage? (Express your answer as a decimal or fraction, not as a percent.)

| Technologies | Company A | Company B | Total |
|---|---|---|---|
| Data Storage | 40 | 30 | 70 |
| Data Mining | 25 | 35 | 60 |
| Data Analytics | 20 | 20 | 40 |
| Data Visualization | 15 | 15 | 30 |
| Total | 100 | 100 | 200 |

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. $\frac{3}{5}$

D. $\frac{4}{7}$

## Answer

D

## Solution

This problem tests the student's understanding of conditional probability, particularly how to calculate the probability of an event given a specific condition using a table of data. It checks if the student can interpret data in a tabular format and apply probability formulas correctly.

To solve this problem, students need to identify the relevant data in the table concerning technologies related to Data Storage and then focus only on those entries. They must then calculate the probability that, given a technology is related to Data Storage, it is related to Company A. This involves using the conditional probability formula: $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$.

First, isolate the rows or columns that pertain to Data Storage. Focus on these entries, ignoring all other technologies. Then count the total number of Data Storage entries, and specifically those associated with Company A. Use these counts to set up your fraction for conditional probability.

Ensure that you are only considering the technologies related to Data Storage when calculating probabilities. A common mistake is to include unrelated categories, which can lead to incorrect answers. Also, remember to express your final answer as a decimal or fraction as instructed.

This type of SAT problem is designed to assess the student's capability in handling conditional probabilities in a real-world context using tables. Mastery of this problem involves attentiveness to detail and the ability to filter relevant data from a larger dataset. It highlights the importance of methodical data handling and precise calculation, critical skills in both academic and professional data analysis contexts.

First, find the total number of technologies related to Data Storage: 70.
Next, find the number of Data Storage technologies related to Company A: 40.
Calculate the probability $P(A|B)$ as the ratio of Data Storage technologies related to Company A to the total Data Storage technologies:
$$P(A|B) = \frac{Number\ of\ Data\ Storage\ technologies\ related\ to\ Company\ A}{Total\ number\ of\ Data\ Storage\ technologies} = \frac{40}{70} = \frac{4}{7}$$

7. What is the median of the data set shown? data set = [3, 7, 9, 1, 5, 8, 2]

## Answer

5

## Solution

The problem aims to assess the student's understanding of finding the median from a data set. It checks if the student can correctly identify the middle value of an ordered data set, which is a fundamental concept in statistics.
To solve this problem, the student should first arrange the data set in ascending order. Once the data is ordered, the student should identify the middle value. Since the data set has an odd number of elements, the median is the middle number of the ordered list.
Remember, the median is the middle value of a data set arranged in order. For an odd number of data points, it's simply the middle one. For an even number, it's the average of the two middle values. Always ensure your data is ordered before searching for the median.
A common mistake is forgetting to order the data before finding the median. Always double-check that the data is sorted correctly. Also, ensure you correctly count to find the middle position, especially if the data set is long.
This problem is a classic example of testing basic statistical skills related to identifying central tendencies. Being able to find the median is crucial as it provides insight into the distribution of data. In the SAT, this type of problem tests accuracy and attention to detail, ensuring students understand and apply statistical concepts accurately.

Step 1: Arrange the data set in ascending order.
The given data set is [3, 7, 9, 1, 5, 8, 2].
Arranging it in ascending order gives [1, 2, 3, 5, 7, 8, 9].
Step 2: Determine the number of values in the data set.
The data set has 7 values, which is an odd number.
Step 3: Find the middle value.
Since there are 7 values, the median is the 4th value in the ordered data set.
The 4th value is 5.
Therefore, the median of the data set is 5.

8. In a survey of 70 adults regarding their political ideologies, the results were classified as either Conservative, Liberal, or Moderate, as shown in the frequency table. If one adult is selected at random, what is the probability that the selected adult identifies as Liberal?

| Political Ideology | Frequency |
| --- | --- |
| Conservative | 25 |
| Liberal | 30 |
| Moderate | 15 |

## Answer

$\frac{3}{7}$

## Solution

This problem is designed to assess the student's understanding of simple probability concepts, specifically how to calculate probability using a frequency table. The goal is to check if students can interpret data presented in a tabular form and apply basic probability principles.
To solve this problem, students should first identify the total number of adults surveyed, which is given as 70. Then, they need to find the frequency of adults who identify as Liberal from the table. The probability is then calculated by dividing the number of Liberals by the total number of surveyed adults.
Focus on extracting the correct data from the table. Ensure you correctly identify the number of adults who identify as Liberal. Remember that probability is the ratio of the favorable outcome to the total number of possible outcomes.
Be cautious about misreading the frequency table. Ensure you are dividing the correct numbers – the number of Liberals should be the numerator, and the total number of adults should be the denominator. Double-check your calculations to avoid simple arithmetic errors.
This type of problem is fundamental in understanding probability and data interpretation. It tests the ability to read data from tables and apply mathematical concepts to solve real-world problems. Mastering these skills is crucial for success in the SAT math section, as it demonstrates proficiency in problem-solving and analytical reasoning.

1. To find the probability, divide the number of favorable outcomes by the total number of outcomes.
2. In this context, a favorable outcome is an adult identifying as Liberal.
3. Therefore, the probability P that the selected adult identifies as Liberal is given by:
$P(Liberal) = \frac{Number of Liberals}{Total Number of Adults}$
4. Substitute the values into the equation:
$P(Liberal) = \frac{30}{70}$
5. Simplify the fraction:

$P(Liberal) = \frac{3}{7}$

6. The simplified probability that a randomly selected adult identifies as Liberal is $\frac{3}{7}$.

9. The table shows the energy output in gigawatt-hours (GWh) for two countries with respect to their renewable and traditional energy resources. If an energy output is selected at random, what is the probability that the output is from Country A, given that it is from a renewable resource? (Express your answer as a decimal or fraction, not as a percent.)

| Countries | Renewable Resources (G | Traditional Energy (GW | Total Output (GWh) |
|---|---|---|---|
| Country A | 120 | 50 | 170 |
| Country B | 80 | 20 | 100 |
| Total | 200 | 70 | 270 |

## Answer

$\frac{3}{5}$

## Solution

This problem assesses the student's understanding of conditional probability, particularly the ability to use a contingency table to find the probability of one event given another event has occurred.
To solve this problem, you should first focus on the rows or columns that represent renewable resources. Then, identify all the energy outputs from renewable resources. Next, focus on the subset of these outputs that are from Country A. Use the formula for conditional probability: $P(A|B) = \frac{P(A \cap B)}{P(B)}$.
Carefully read the table to accurately identify the row or column representing renewable resources. Ensure you correctly sum the values associated with Country A and the total for renewable resources. Double-check your calculations when applying the conditional probability formula.
A common mistake is to misinterpret the table or to mistakenly include values not related to renewable resources. Ensure that you are only considering outputs from renewable resources when calculating the probability.
This type of question is typical in the SAT's Problem Solving and Data Analysis section. It tests your understanding of conditional probability and your ability to accurately interpret data from tables. Mastery of these skills is crucial for success in this area, as they are fundamental to data analysis and interpreting real-world data scenarios.

To find the probability that an energy output is from Country A given that it is from a renewable resource, we'll use the formula for conditional probability: $P(A|B) = \frac{P(A \cap B)}{P(B)}$.
$P(A \cap B)$ is the probability that the output is from Country A and a renewable resource, which is the amount of renewable energy from Country A divided by the total energy output: $P(A \cap B) = \frac{120}{270}$.
P(B) is the probability that an output is from a renewable resource, which is the total renewable energy output divided by the total energy output: $P(B) = \frac{200}{270}$.

Substitute these into the formula: $P(A|B) = \frac{\frac{120}{270}}{\frac{200}{270}}$.
Simplify the expression: $P(A|B) = \frac{120}{200}$.
Further simplify the fraction: $\frac{120}{200} = \frac{3}{5}$.
Thus, the probability that an energy output is from Country A given that it is from a renewable resource is $\frac{3}{5}$.

10. A car travels at a speed of 5.2 meters per second. What is this speed in kilometers per hour, rounded to the nearest tenth? (Use 1 kilometer = 1,000 meters.)

A. 18.5 km/h

B. 18.6 km/h

C. 18.7 km/h

D. 18.8 km/h

## Answer

C

## Solution

This problem tests the student's ability to convert units of speed from meters per second to kilometers per hour. It assesses understanding of unit conversion principles and multiplication skills.
To solve this problem, students need to first understand the conversion factor between meters and kilometers, and seconds and hours. The speed is given in meters per second, and it needs to be converted to kilometers per hour. This requires multiplying the speed by the conversion factors: 1,000 meters per kilometer and 3,600 seconds per hour.
Remember that 1 kilometer is 1,000 meters and there are 3,600 seconds in an hour. To convert from meters per second to kilometers per hour, multiply the speed by 3.6 (since $\frac{3{,}600}{1{,}000} = 3.6$).
A common mistake is to forget to convert both the meters to kilometers and the seconds to hours. Ensure to multiply by 3.6, not 3,600 or 1,000, as this already accounts for both conversions.
This type of problem is common in SAT exams as it evaluates the student's ability to perform unit conversions, which is essential in solving real-world problems. By practicing such questions, students can improve their accuracy and speed in handling unit conversion tasks, which are key skills in the Problem Solving and Data Analysis section.

Start with the speed in meters per second: 5.2 m/s., Convert meters to kilometers by multiplying by 0.001: 5.2 m/s × 0.001 km/m = 0.0052 km/s., Convert seconds to hours by multiplying by 3,600: 0.0052 km/s × 3600 s/h = 18.72 km/h., The speed in kilometers per hour is 18.72 km/h, which is rounded to the nearest tenth as 18.7 km/h.

Studyola