SAT Math Advanced

1. For a polynomial function, the graph of $y = f(x)$ in the xy-plane contains the points (7, 0), (0, 0), (6, 0), (5, 0), (3, 0), and (4, 0). Which of the following must be a factor of $f(x)$?

A. $x^2 + 9x - 18$

B. $x^3 - 18x^2 + 107x - 210$

C. $x^2 + 7x$

D. $x^3 - 15x^2 - 74x - 120$

2. The function f is defined by $f(x) = 29 + 44\sqrt{x}$. What is the value of $f(25)$?

A. 29

B. 125

C. 249

D. 300

3. A physicist is studying a certain radioactive substance whose quantity decreases over time. The equation representing the quantity of this substance in grams after x years is given by: $f(x) = 1100(0.41)^x$. Which of the following is the best interpretation of 1100 in this context?

A. The estimated amount of substance remaining after 1 year.

B. The estimated amount of substance remaining after 2 years.

C. The estimated initial quantity of the substance at time 0.

D. The estimated quantity of substance after 6 years.

4. The function f is a quadratic function. In the xy-plane, the graph of $y = f(x)$ has a vertex at (6, -4) and passes through the points (-3, 239) and (-5, 359). What is the value of $f(7) - f(4)$?

A. -15

B. -12

C. -9

D. -5

5. A diplomatic cable containing sensitive information is sent to a foreign country. The function given by $h(t) = 925 + (975 - 925)(8.52)^{-0.317t}$ describes the rate of decay of information secrecy, in some units, t hours after the cable is sent. What was the initial rate of information secrecy when the cable was sent?

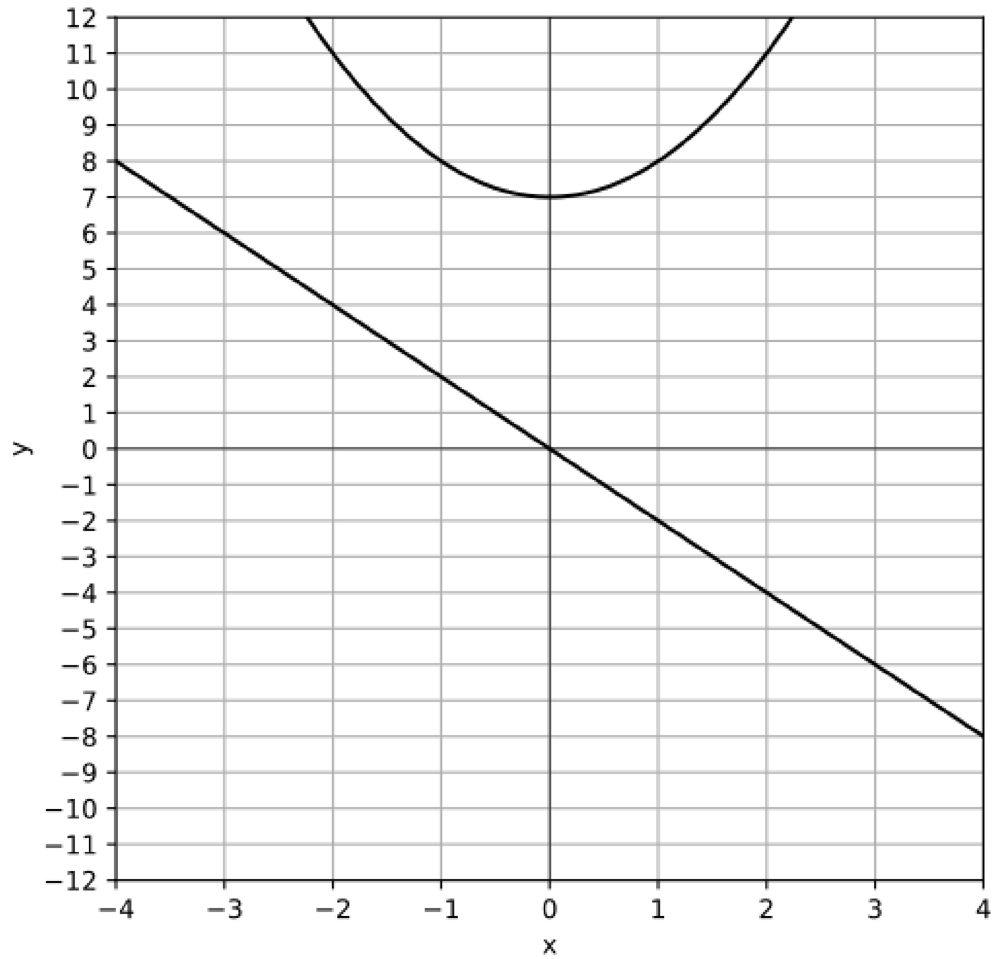
A. 825

B. 925

C. 975

D. 1025

6. The graph of a system of a linear and a quadratic equation is shown. Which system of equations is represented by the graph?



A. $y = x^2 + 7$ and y = -2x

B. $y = x^2 - 7$ and y = -2x

C. $y = x^2 - 7$ and y = 2x

D. $y = x^2 + 7$ and y = 2x

7. A financial analyst is modeling the projected economic activity in a city using the equation for active jobs: $y = -0.3x^2 - 3x + 86$, where y represents the number of active jobs and x is the number of months since January 2020. What is the best interpretation of the y-intercept of the graph of this equation in the xy-plane?

A. At the end of January 2020, the projected number of active jobs was 0.

B. At the end of January 2020, the projected number of active jobs was 86, suggesting a lack of job activity or data errors.

C. At the end of July 2020, the projected number of active jobs was 86.

D. At the end of July 2020, the projected number of active jobs was 0.

8. What is the y-intercept of the function defined by $f(x) = 2^x + 15$ in the xy-plane?

A. (0, 15)

B. (0, 16)

C. (0, 17)

D. (0, 18)

9. Which expression is equivalent to $-6x^5y^7(7x^7 + 4x^4 + 84)$?

A. $-42x^{12}y^6 - 24x^9y^7 - 504x^5y^7$

B. $-42x^{12}y^7 - 24x^6y^7 - 504x^5y^6$

C. $-42x^{12}y^7 - 24x^9y^6 - 504x^5y^7$

D. $-42x^{12}y^7 - 24x^9y^7 - 504x^5y^7$

10. The functions f and g are defined by the equations shown, where a is an integer constant, and a <0. If y = f(x) and y = g(x) are graphed in the xy-plane, which of the following equations displays, as a constant or coefficient, the y-coordinate of the y-intercept of the graph of the corresponding function? $f(x) = a(-4.2)^{x+29}$
$g(x) = a(-4.2)^{x} + 29$

A. f(x) only

B. g(x) only

C. f(x) and g(x)

D. Neither f(x) nor g(x)

SAT Math Advanced Solutions

1. For a polynomial function, the graph of $y = f(x)$ in the xy-plane contains the points (7, 0), (0, 0), (6, 0), (5, 0), (3, 0), and (4, 0). Which of the following must be a factor of $f(x)$?

A. $x^2 + 9x - 18$

B. $x^3 - 18x^2 + 107x - 210$

C. $x^2 + 7x$

D. $x^3 - 15x^2 - 74x - 120$

Answer

B

Solution

Concept Check : The intent of this question is to assess the student's understanding of polynomial functions and their roots. The student is expected to know that if a polynomial has a root at a certain x-value, then (x - that value) is a factor of the polynomial. The roots provided in the problem indicate where the polynomial crosses the x-axis, which directly relates to identifying factors.

Solution Strategy : To approach this problem, the student should first recognize that the given points where the polynomial crosses the x-axis (the x-values of the points) are the roots of the polynomial function. For each root, a corresponding factor can be formed. The student should consider each x-coordinate of the roots provided and develop the factors based on the form (x - root). The task is to identify which of these factors must be part of the polynomial function.

Quick Wins : A helpful tip is to list out all the roots provided in the problem and write down their corresponding factors. For example, if one of the roots is x = 0, then the factor is (x - 0) or simply x. Additionally, recognizing that any factor can be multiplied by a constant does not affect the roots, so focus on the linear factors derived from the roots directly. This can help in quickly identifying the correct factors without getting distracted by the polynomial's leading coefficient.

Mistake Alert : Be careful not to confuse the roots with the factors. Each root (x-value) corresponds to a factor, but it's important to ensure you are setting them up correctly as (x - root). Also, pay attention to whether a root is repeated, as this might suggest a squared factor, but for this question, you only need to identify which factors must exist, not their multiplicities.

SAT Know-How : This problem falls under the category of Advanced Math, specifically focusing on polynomials and their factors related to their roots. The skills assessed include the ability to connect roots of polynomial functions to their corresponding factors and understand the fundamental relationship between these concepts in polynomial graphing and function analysis. Mastering this problem type is crucial for success in the SAT, as it tests understanding of polynomial behavior and algebraic manipulation.

The polynomial function f(x) can be expressed in terms of its factors: f(x) = x(x - 3)(x - 4)(x - 5)(x - 6)(x - 7).
Examine each option to determine if it contains these factors:
Option A: $x^2 + 9x - 18$. Attempt to factor:
- $x^2 + 9x - 18$ = (x - 3)(x + 6), which doesn't match any pair of zeros.
Option B: $x^3 - 18x^2 + 107x - 210$. Attempt to factor by identifying roots or using synthetic division:
- Using synthetic division, try x = 3, x = 4, x = 5, x = 6, and x = 7. Upon testing, x = 5 is a root.
- Factoring out (x - 5) from $x^3 - 18x^2 + 107x - 210$ leaves a quadratic factor: ($x^2 - 13x + 42$).
- Further factor ($x^2 - 13x + 42$) = (x - 6)(x - 7), confirming x = 6 and x = 7 are roots.
- Thus, $x^3 - 18x^2 + 107x - 210$ = (x - 5)(x - 6)(x - 7), which includes factors for zeros.
Option C: $x^2 + 7x$. Attempt to factor:
- $x^2 + 7x$ = x(x + 7), which doesn't match the zeros.
Option D: $x^3 - 15x^2 - 74x - 120$. Attempt to factor by identifying roots or using synthetic division:
- Using synthetic division, try x = 3, x = 4, x = 5, x = 6, and x = 7. None are roots.
- This polynomial does not factor to match the zeros.

2. The function f is defined by $f(x) = 29 + 44\sqrt{x}$. What is the value of $f(25)$?

A. 29

B. 125

C. 249

D. 300

Answer

C

Solution

Concept Check : The question is aimed at assessing the student's understanding of evaluating functions, particularly those involving irrational numbers such as square roots. The student is expected to know how to substitute a value into the function and perform the calculation accurately.

Solution Strategy : To solve the problem, the student should follow these steps: First, identify the given function, which is $f(x) = 29 + 44\sqrt{x}$. Then, substitute x with the value 25. This means calculating 44 times the square root of 25, and then adding 29 to the result. It's important to clearly follow the order of operations during this process.

Quick Wins : Remember to calculate the square root first before multiplying by 44. The square root of 25 is a common value (5), so it may be easier to recall. After finding 44 times the square root, don't forget to add 29 at the end to find the final value of $f(25)$. Keeping track of each step will help prevent errors.

Mistake Alert : Be careful with the square root calculation; it's easy to miscalculate if you're unsure of the value. Also, ensure you remember to perform the addition after the multiplication. Double-check your substitution to make sure the correct number is used in place of x.

SAT Know-How : This problem falls under the category of Advanced Math, focusing on nonlinear functions that include irrational components. It assesses the student's ability to evaluate functions accurately and apply basic operations. Mastering this type of problem is key for SAT success, as it reinforces the importance of function evaluation and precision in calculations.

Substitute $x = 25$ into the function: $f(x) = 29 + 44\sqrt{x}$.
Calculate $\sqrt{25}$, which is 5.

Substitute $\sqrt{25} = 5$ into the function: $f(25) = 29 + 44 \times 5$.
Perform the multiplication: $44 \times 5 = 220$.
Add the results: $29 + 220 = 249$.

3. A physicist is studying a certain radioactive substance whose quantity decreases over time. The equation representing the quantity of this substance in grams after x years is given by: $f(x) = 1100(0.41)^x$. Which of the following is the best interpretation of 1100 in this context?

A. The estimated amount of substance remaining after 1 year.

B. The estimated amount of substance remaining after 2 years.

C. The estimated initial quantity of the substance at time 0.

D. The estimated quantity of substance after 6 years.

Answer

C

Solution

Concept Check : The question is designed to test the student's understanding of exponential decay, particularly in the context of real-world applications such as radioactive decay. The student is expected to know how to interpret the parameters of an exponential function, specifically the initial quantity and the decay factor.

Solution Strategy : To approach this problem, the student should identify the components of the given exponential function. They should recognize that in the equation of the form $f(x) = a(b)^x$, 'a' represents the initial value of the quantity when $x = 0$. The student should think through the meaning of the parameters in relation to the problem context to determine what 1100 signifies.

Quick Wins : A good strategy is to substitute $x = 0$ into the function to see what value you get. This will help clarify what the initial quantity is. Also, it can be helpful to visualize the process of radioactive decay and how it relates to the parameters in the equation. Remember that the base of the exponent (0.41 in this case) indicates the rate of decay, whereas the coefficient provides the starting amount.

Mistake Alert : Be careful not to confuse the initial quantity with the decay factor. The number 1100 is significant at the start of the observation (when $x = 0$), while the base of the exponent, 0.41, tells you how much of the substance remains after each year. Misinterpreting these can lead to incorrect conclusions.

SAT Know-How : This problem falls under the category of advanced math, specifically focusing on exponential word problems related to real-life scenarios like radioactive decay. It assesses the student's ability to interpret mathematical models and understand the significance of parameters. Mastering this type of problem enhances your skills in applying mathematical concepts to analyze and interpret real-world data effectively.

Step 1: Understand the general form of an exponential decay function. It is typically given by $f(x) = a(b)^x$, where 'a' is the initial quantity, 'b' is the decay factor, and 'x' is the time elapsed.
Step 2: Compare the given function $f(x) = 1100(0.41)^x$ with the general form. Here, $'a' = 1100$ and $'b' = 0.41$.
Step 3: Interpret the meaning of $'a' = 1100$. This value represents the initial quantity of the substance before any decay has occurred, i.e., at time $x = 0$.
Step 4: Verify by setting $x = 0$ in $f(x) = 1100(0.41)^x$. The equation becomes $f(0) = 1100(0.41)^0 = 1100 \times 1 = 1100$.
Step 5: Therefore, 1100 represents the initial amount of the substance, confirming the interpretation that it is the quantity present at time $x = 0$.

4. The function f is a quadratic function. In the xy-plane, the graph of $y = f(x)$ has a vertex at (6, -4) and passes through the points (-3, 239) and (-5, 359). What is the value of $f(7) - f(4)$?

A. -15

B. -12

C. -9

D. -5

Answer

C

Solution

Concept Check : The intent of this question is to assess the student's understanding of quadratic functions, particularly in determining the function's equation when given its vertex and specific points on the graph. Students are expected to know how to use the vertex form of a quadratic function and how to manipulate it based on known points.

Solution Strategy : To solve the problem, the student should start by recalling the vertex form of a quadratic function, which is given by the equation: $f(x) = a(x - h)^2 + k$, where (h, k) is the vertex of the parabola. Here, the vertex is given as (6, -4), so h = 6 and k = -4. The next step is to substitute the coordinates of the points (-3, 239) and (-5, 359) into the equation to create two equations that can be solved to find the value of 'a'. Once 'a' is determined, the full function can be established, and then f(7) and f(4) can be calculated to find their difference.

Quick Wins : 1. Remember the vertex form of a quadratic function is essential in problems like this. 2. Substitute points carefully and ensure you maintain correct signs. 3. When substituting the points into the equation, simplify carefully to avoid arithmetic errors. 4. After finding the function, evaluate it at the required points to find the difference.

Mistake Alert : 1. Be cautious when substituting the points; double-check that you plug the correct x and y values into the equation. 2. Pay special attention to the negative signs when working with the vertex's coordinates and the points; it's easy to make minor mistakes here. 3. Ensure that you correctly calculate the values of f(7) and f(4) before subtracting them.

SAT Know-How : This problem belongs to the Advanced Math category and focuses on quadratic functions and their graphs. It assesses skills such as understanding the vertex form of a quadratic equation, substituting values correctly, and performing arithmetic operations accurately. Mastering these skills is crucial for solving quadratic-related problems on the SAT, as it enhances your problem-solving efficiency and reduces the chances of mistakes.

Start by writing the quadratic function in vertex form: $f(x) = a(x - 6)^2 - 4$.
Substitute the point (-3, 239) into the equation to solve for a:
$239 = a(-3 - 6)^2 - 4$.
Simplify: 239 = a(81) - 4.
Add 4 to both sides: 243 = 81a.
Solve for a: $a = \frac{243}{81} = 3$.
Verify a with the point (-5, 359):
$359 = 3(-5 - 6)^2 - 4$.
Simplify: 359 = 3(121) - 4.
Calculate: 359 = 363 - 4.
This confirms a = 3 is correct.
Now, calculate f(7) and f(4):
$f(x) = 3(x - 6)^2 - 4$.
$f(7) = 3(7 - 6)^2 - 4 = 3(1)^2 - 4 = 3 - 4 = -1$.
$f(4) = 3(4 - 6)^2 - 4 = 3(-2)^2 - 4 = 3(4) - 4 = 12 - 4 = 8$.
Calculate f(7) - f(4): -1 - 8 = -9.

5. A diplomatic cable containing sensitive information is sent to a foreign country. The function given by $h(t) = 925 + (975 - 925)(8.52)^{-0.317t}$ describes the rate of decay of information secrecy, in some units, t hours after the cable is sent. What was the initial rate of information secrecy when the cable was sent?

A. 825

B. 925

C. 975

D. 1025

Answer

C

Solution

Concept Check : The intent of the question is to assess the student's understanding of exponential decay functions, specifically how to evaluate such functions at a given point in time. Students should know how to interpret the parameters in the function and calculate the initial value when time is zero.

Solution Strategy : To solve the problem, the student should recognize that the initial rate of information secrecy corresponds to the value of the function when time, t, equals 0. This requires substituting 0 for t in the given function h(t) and simplifying the expression to find the initial rate.

Quick Wins : A good approach to similar problems is to always identify what value of t corresponds to the initial condition. In this case, $t = 0$ gives the initial rate. Remember to carefully substitute the value into the function and simplify correctly. It can be helpful to break down the function into parts to see how each term contributes to the overall value.

Mistake Alert : Be cautious to ensure you substitute the value of t correctly and watch for any negative signs or operations that might affect the outcome. Pay close attention to the order of operations, especially when dealing with exponentials and constants in the function.

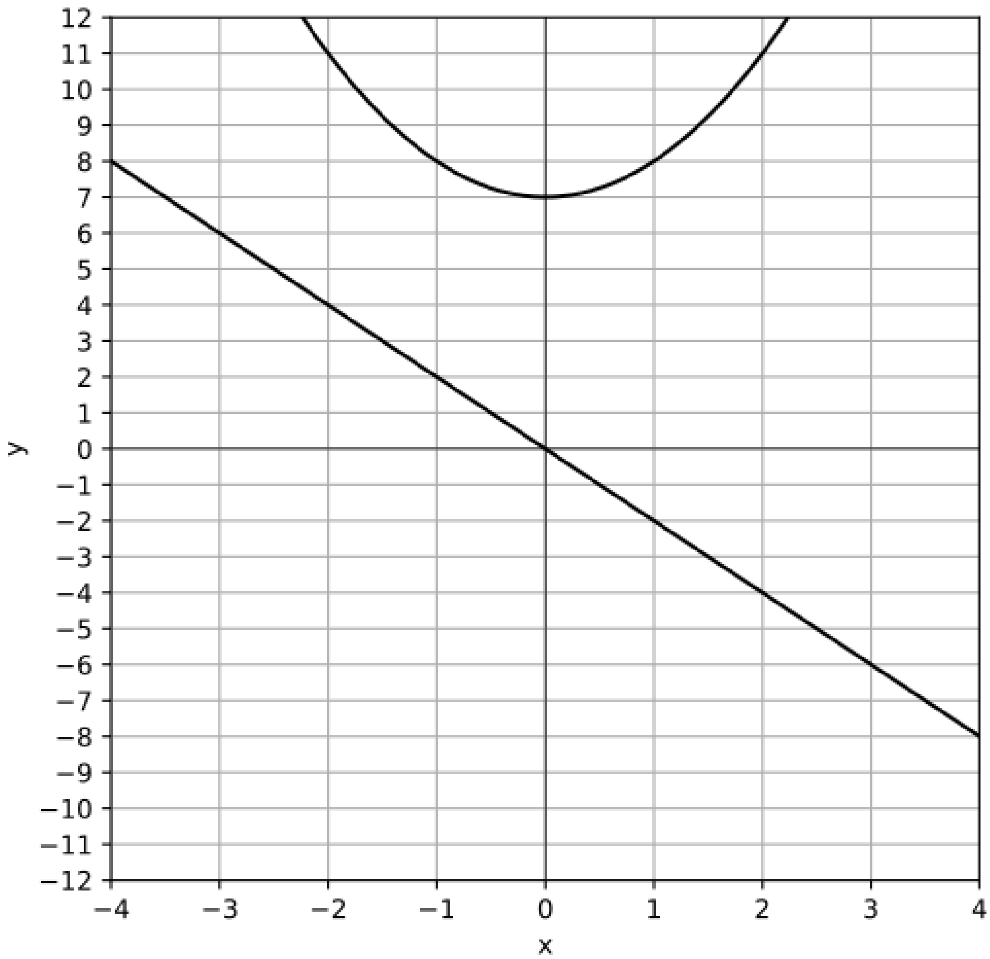
SAT Know-How : This problem falls under 'Advanced Math' and specifically focuses on exponential word problems. It assesses the student's ability to evaluate exponential functions and understand the implications of the parameters within the context of decay. This type of problem helps students develop the necessary skills for reasoning through mathematical functions and applying them in practical scenarios, essential for success in the SAT.

To find the initial rate of secrecy, we evaluate h(0) using the given function.

Substitute $t = 0$ into the function: $h(0) = 925 + (975 - 925)(8.52)^0$.
Therefore, $h(0) = 925 + (975 - 925) \times 1$.

Since any non-zero number raised to the power of 0 is 1, $8.(52)^0 = 1$.
Now calculate: $h(0) = 925 + (975 - 925) = 975$.
Thus, The initial rate of information secrecy when the cable was sent is 975.

6. The graph of a system of a linear and a quadratic equation is shown. Which system of equations is represented by the graph?



A. $y = x^2 + 7$ and y = -2x

B. $y = x^2 - 7$ and y = -2x

C. $y = x^2 - 7$ and y = 2x

D. $y = x^2 + 7$ and y = 2x

Answer

C

Solution

Concept Check : The intent of this question is to assess the student's understanding of the relationship between linear and quadratic equations as represented graphically. Students are expected to identify the equations that correspond to the features of the graph, such as intersections, vertex, and the general shape of the quadratic function.

Solution Strategy : To approach this problem, students should carefully analyze the graph provided. They should look for key features such as the vertex of the quadratic, the slope of the linear equation, and points of intersection. By identifying the characteristics of the lines and curves, they can set up equations that represent these features. Additionally, they might consider the standard forms of both equations: y = mx + b for the linear equation and $y = ax^2 + bx + c$ for the quadratic equation.

Quick Wins : One effective strategy is to focus on the intersection points of the graph, as these will help you identify solutions to the system of equations. Additionally, remember that the vertex of the quadratic will give you information about the maximum or minimum point of the parabola. If the graph is labeled with coordinates, use those points to substitute into your equations to find the correct coefficients.

Mistake Alert : Be careful not to confuse the orientation of the quadratic (whether it opens upwards or downwards) and the slope of the linear function. Pay attention to the curves and lines; minor visual misinterpretations can lead to incorrect equations. Also, ensure that you check all possible answer choices against the graph rather than relying solely on one characteristic.

SAT Know-How : This problem falls under the category of Advanced Math, specifically focusing on quadratic graphs and their systems with linear equations. It assesses skills in analyzing graphical representations, deriving equations, and understanding the interaction between different types of functions. Mastering these concepts is crucial for effective problem-solving on the SAT.

1. The quadratic equations provided are of the form $y = x^2 \pm c$. The graph of the parabola should open upwards (standard $y = x^2$) and be shifted vertically by c units.
2. The linear equations provided are of the form y = mx. A positive slope (m >0) indicates a line that increases as x increases, while a negative slope (m <0) indicates a line that decreases as x increases.
3. Examine each pair of equations in the options:
- Option A: $y = x^2 + 7$ and y = -2x

The parabola opens upwards and is shifted up by 7 units, and the line has a negative slope so it decreases.

- Option B: $y = x^2 - 7$ and y = -2x

The parabola opens upwards and is shifted down by 7 units, and the line has a negative slope so it decreases.

- Option C: $y = x^2 - 7$ and y = 2x

The parabola opens upwards and is shifted down by 7 units, and the line has a positive slope so it increases.

- Option D: $y = x^2 + 7$ and y = 2x

The parabola opens upwards and is shifted up by 7 units, and the line has a positive slope so it increases.

4. Based solely on the descriptions given, the intersection of the line and parabola in the graph may help identify the correct pair. The parabola at $y = x^2 + 7$ intersects differently depending on whether the line increases or decreases.
5. The correct option will show two points of intersection between the parabola and the line. The intersection pattern can help determine the system of equations.

7. A financial analyst is modeling the projected economic activity in a city using the equation for active jobs: $y = -0.3x^2 - 3x + 86$, where y represents the number of active jobs and x is the number of months since January 2020. What is the best interpretation of the y-intercept of the graph of this equation in the xy-plane?

A. At the end of January 2020, the projected number of active jobs was 0.

B. At the end of January 2020, the projected number of active jobs was 86, suggesting a lack of job activity or data errors.

C. At the end of July 2020, the projected number of active jobs was 86.

D. At the end of July 2020, the projected number of active jobs was 0.

Answer

B

Solution

Concept Check : The intent of this question is to assess the student's understanding of quadratic functions, specifically the interpretation of the y-intercept in the context of a real-world application. Students should recognize that the y-intercept represents the value of y when x is zero, which corresponds to a specific point in time (January 2020 in this case).

Solution Strategy : To approach this problem, students should first identify what the y-intercept represents in the given equation. This involves substituting $x = 0$ into the equation to find the value of y. Then, they should interpret this value in the context of the problem, considering what it means in terms of active jobs at the starting point of the timeframe given (January 2020).

Quick Wins : When interpreting the y-intercept, remember that it gives you the starting value of the dependent variable when the independent variable is zero. In this case, think about what 'months since January 2020' means—$x = 0$ will indicate January 2020, so the y-intercept will tell you the number of active jobs at that time. Always read the problem carefully to ensure you understand the real-world connection.

Mistake Alert : Be careful not to confuse the y-intercept with other points on the graph. The y-intercept is specifically the point where the graph crosses the y-axis (x = 0). Additionally, ensure that you correctly substitute x = 0 into the equation; miscalculating the equation may lead to incorrect interpretations. Remember that negative values can have significant implications in real-world contexts.

SAT Know-How : This problem falls under the category of Advanced Math, focusing on quadratic and exponential word problems. It tests the student's ability to interpret mathematical models in real-world contexts, specifically through the lens of the y-intercept. Mastering this concept is crucial for SAT problem-solving, as it reinforces the connection between algebraic equations and their applications in various scenarios.

1. Identify the y-intercept in the given equation $y =- 0.3x^2 - 3x + 86$.
2. The y-intercept occurs when $x = 0$.
3. Substitute $x = 0$ into the equation: $y =- 0.3(0)^2 - 3(0)6 + 86$.
4. Simplify: $y = 86$.
5. The y-intercept of 86 means that at the start of January 2020, the projected number of active jobs is 86.

8. What is the y-intercept of the function defined by $f(x) = 2^x + 15$ in the xy-plane?

A. (0, 15)

B. (0, 16)

C. (0, 17)

D. (0, 18)

Answer

B

Solution

Concept Check : The question aims to assess the student's understanding of exponential functions and their properties, particularly how to find the y-intercept of a function. The student should know that the y-intercept occurs where the value of x is zero.

Solution Strategy : To find the y-intercept of the function $f(x) = 2^x + 15$, the student should substitute x with 0 in the function. This will involve evaluating the expression at that specific point. The thought process should involve recognizing that for any function, the y-intercept is found by determining the function's value when x equals zero.

Quick Wins : Remember that the y-intercept is simply the value of the function when x = 0. For exponential functions like $f(x) = 2^x$, it's important to know that 2^0 equals 1. Therefore, when you're substituting, you can quickly compute the value without needing to graph the function. It's also helpful to remember the general form of an exponential function and how it behaves as x changes.

Mistake Alert : Be careful not to confuse the y-intercept with the x-intercept. The y-intercept is found by setting x to 0, while the x-intercept involves setting $f(x)$ to 0 and solving for x. Also, ensure you do not overlook the constant term in the function, as it significantly affects the final result when calculating the y-intercept.

SAT Know-How : This problem falls under the 'Advanced Math' category focusing on nonlinear functions, specifically exponential functions. It tests the student's ability to evaluate a function at a specific point, which is a crucial skill in understanding function behavior. Mastering this concept is essential for success on the SAT, as it lays the groundwork for more complex mathematical reasoning.

Step 1: Understand that the y-intercept is found by setting $x = 0$.
Step 2: Substitute 0 for x in the function: $f(x) = 2^{x} + 15$ becomes $f(0) = 2^{0} + 15$.
Step 3: Calculate 2^{0}. Any number raised to the power of 0 is 1, so $2^{0} = 1$.
Step 4: Calculate $f(0) = 1 + 15$.
Step 5: Simplify the expression: $f(0) = 16$.
Step 6: The y-intercept of the function is the point where $x = 0$and $y = 16$, so the y-intercept is (0, 16).

9. Which expression is equivalent to $-6x^5y^7(7x^7+4x^4+84)$?

A. $-42x^{12}y^6-24x^9y^7-504x^5y^7$

B. $-42x^{12}y^7-24x^6y^7-504x^5y^6$

C. $-42x^{12}y^7-24x^9y^6-504x^5y^7$

D. $-42x^{12}y^7-24x^9y^7-504x^5y^7$

Answer

D

Solution

Concept Check : The question tests the student's understanding of polynomial operations, specifically multiplication. Students should know how to distribute the term $-6x^5y^7$ across each term in the polynomial $(7x^7+4x^4+84)$ and apply the rules of exponents when multiplying terms with the same base.

Solution Strategy : To solve this problem, the student should first recognize that they need to distribute $-6x^5y^7$ to each term inside the parentheses. This means multiplying $-6x^5y^7\ by\ 7x^7$, then by $4x^4$, and finally by 84. The student should keep in mind how to handle the coefficients and the variables while applying the laws of exponents, specifically that when multiplying like bases, you add the exponents.

Quick Wins : Start by rewriting the expression clearly; it helps to see each part. Remember to multiply the coefficients (the numbers in front) together and then handle the variable parts separately. When multiplying the variable parts, add the exponents of like bases. It can be helpful to write down intermediate steps to avoid confusion, ensuring that each term is calculated correctly before moving on to the next.

Mistake Alert : Be careful with the signs—multiplying by -6 means that the sign of each resulting term will flip. Also, double-check the exponents; it's easy to make a mistake when adding exponents, especially with higher-degree polynomials. Pay attention to the arrangement of your final expression to ensure all terms are included and properly simplified.

SAT Know-How : This problem is an example of operations with higher-degree polynomials, specifically focusing on polynomial multiplication. It assesses skills such as distribution, combining like terms, and the laws of exponents. Mastering these concepts is crucial for success in the SAT's math section, particularly in advanced math questions.

Step 1: Distribute $-\ 6x^5y^7$ across each term in the parenthesis: $-\ 6x^5y^7 \times 7x^7$, $-\ 6x^5y^7 \times 4x^4$, and $-\ 6x^5y^7 \times 84$.
Step 2: Calculate $-\ 6 \times 7 =-\ 42$, $x^5 \times x^7 = x^{(5+7)} = x^{12}$, y^7 remains unchanged. So the first term becomes $-\ 42x^{12}y^7$.
Step 3: Calculate $-\ 6 \times 4 =-\ 24$, $x^5 \times x^4 = x^9$, y^7 remains unchanged. So the second term becomes $-\ 24x^9y^7$.
Step 4: Calculate $-\ 6 \times 84 =-\ 504$, x^5 remains unchanged, y^7 remains unchanged. So the third term becomes $-\ 504x^5y^7$.
Step 5: Combine all terms to form the expression: $-\ 42x^{12}y^7 - 24x^9y^7 - 504x^5y^7$.

10. The functions f and g are defined by the equations shown, where a is an integer constant, and a <0. If y = f(x) and y = g(x) are graphed in the xy-plane, which of the following equations displays, as a constant or coefficient, the y-coordinate of the y-intercept of the graph of the corresponding function? $f(x) = a(-4.2)^{x+29}$
$g(x) = a(-4.2)^{x} + 29$

A. f(x) only

B. g(x) only

C. f(x) and g(x)

D. Neither f(x) nor g(x)

Answer

B

Solution

Concept Check : The intent of the question is to assess the student's understanding of exponential functions, specifically how to determine the y-intercept of these functions based on their equations. Students are expected to apply their knowledge of function properties and intercepts in the context of exponential graphs.

Solution Strategy : To approach this problem, the student should first recall the definition of the y-intercept. The y-intercept is found by evaluating the function at x = 0. Therefore, the student needs to substitute x = 0 into both equations for f(x) and g(x) and simplify to find the y-intercept of each function. This will involve understanding how the constant 'a' affects the output of the functions and considering what happens when x is replaced by 0.

Quick Wins : Remember that for any function f(x), the y-intercept is found at f(0). Substitute x = 0 into both f(x) and g(x) to find their respective y-values at the intercept. Pay attention to the role of the constant 'a' and how it interacts with the base of the exponential function. Additionally, make sure to evaluate any constants or coefficients accurately after substitution.

Mistake Alert : Be cautious when dealing with negative bases in exponential functions, as this can lead to complex behavior depending on the exponent. In this case, ensure that you carefully follow the algebra when substituting x = 0. Also, double-check that you are correctly interpreting the outputs, especially since 'a' is negative, which can affect the sign of the y-intercept. Lastly, ensure you are not confusing the y-intercept with other characteristics of the graph.

SAT Know-How : This problem falls under the category of Advanced Math, specifically focusing on exponential graphs and their properties. It assesses the student's ability to analyze and interpret the y-intercept of exponential functions. Mastering this type of problem requires a solid grasp of function evaluation and the behavior of exponential functions, which are critical skills in the SAT math section.

To find the y-intercept of f(x), substitute x = 0: $f(0) = a(-4.2)^{0+29} = a(-4.2)^{29}$.
To find the y-intercept of g(x), substitute x = 0:
$g(0) = a(-4.2)^{0} + 29 = a(1) + 29 = a + 29$.
The y-coordinate of the y-intercept of f(x) is $a(-4.2)^{29}$, which is not explicitly displayed as a constant or coefficient in its equation.
The y-coordinate of the y-intercept of g(x) is a + 29. The value 29 is explicitly shown as a constant in the equation $g(x) = a(-4.2)^{x} + 29$.
Thus, the y-coordinate of the y-intercept is displayed as a constant in the function g(x).

