

Digital SAT Math 10



SAT Math Problems

1. In triangle DEF , the measure of angle D is 60° , the measure of angle E is 80° , and the measure of angle F is $\left(\frac{m}{3}\right)^\circ$. What is the value of m ?

- A. 100
- B. 110
- C. 120
- D. 130

2. For two acute angles, $\angle A$ and $\angle B$, $\sin(A) = \cos(B)$. The measures, in degrees, of $\angle A$ and $\angle B$ are $2x + 30$ and $5x - 10$, respectively. What is the value of x ?

- A. 8
- B. 9
- C. 10
- D. 11

3. In a factory that has recently implemented automated machines, there are currently 200 workers. Following automation, the company found that the probability of a worker being reassigned to a new role in the automation process is 0.25, while the probability of a worker remaining in their current role is 0.55. What is the probability that a randomly selected worker will either be reassigned or remain in their current role?

- A. 0.75
- B. 0.80
- C. 1.00
- D. 0.85

4. If x and y are numbers greater than 1 and $\sqrt[4]{x^5}$ is equivalent to $\sqrt[6]{y^3}$, for what value of b is x^{3b+2} equal to y ?

A. $\frac{1}{6}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

5. A city implemented a new public policy aiming to reduce air pollution. The estimated reduction in air pollution levels, measured in tons, in the first five years after the policy is modeled by the function $f(x) = 500(0.90)^x$, where x is the number of years since the policy was enacted. What does the value 500 represent in this context?

A. The amount of air pollution measured in tons after 5 years

B. The estimated air pollution level in tons during the baseline year before the policy was enacted

C. The percentage decrease in air pollution level each year

D. The total reduction in air pollution expected after 5 years

6. For the linear function g , the graph of $y = g(x)$ in the xy -plane has a slope of 12 and passes through the point $(0, 5)$. Which equation defines g ?

7. For a polynomial function, the graph of $y = f(x)$ in the xy -plane contains the points $(-4, 0)$, $(0, 0)$, $(3, 0)$, and $(5, 0)$. Which of the following must be a factor of $f(x)$?

- A. $x^2 + 2x - 8$
- B. $x^2 - 8x + 15$
- C. $x^2 - 3x - 10$
- D. $x^2 - 5x$

8. What is the center of the circle in the xy -plane defined by the equation $(x + 3)^2 + (y - 4)^2 = 16$?

- A. $(-3, 4)$
- B. $(3, -4)$
- C. $(4, 3)$
- D. $(-4, 3)$

9. In triangle DEF, the measure of angle D is 55° , the measure of angle E is 90° , and the measure of angle F is $\frac{m}{3}^\circ$. What is the value of m ?

10. One solution to the given equation can be written as $x = \frac{-7 + \sqrt{k}}{2}$, where k is a constant. What is the value of k ? $x^2 + 7x + 10 = 0$

- A. 9
- B. 16
- C. 25
- D. 36

SAT Math Solutions

1. In triangle DEF , the measure of angle D is 60° , the measure of angle E is 80° , and the measure of angle F is $\left(\frac{m}{3}\right)^\circ$. What is the value of m ?

- A. 100
- B. 110
- C. 120
- D. 130

Answer

C

Solution

This problem tests the student's understanding of the angle sum property of triangles and their ability to use algebra to solve for an unknown variable. To solve this problem, the student needs to apply the angle sum property of a triangle, which states that the sum of the interior angles of a triangle is 180° . Set up an equation where the sum of the given angles equals 180° , and solve for the unknown variable m .

Remember that the sum of the angles in any triangle is always 180° . Use this property to set up your equation. Also, be careful with your algebraic manipulation when solving for m .

Ensure that you correctly interpret the given angle measures and carefully solve the equation step by step. It's easy to make mistakes with simple arithmetic or algebraic manipulation, so double-check your work.

This type of problem is common in SAT geometry questions. It assesses the student's ability to apply fundamental properties of triangles and perform basic algebra.

Mastering these types of problems will help students improve their problem-solving speed and accuracy, which is crucial for the SAT.

Step 1:

Substitute the given angle measures into the equation: $60^\circ + 80^\circ + \left(\frac{m}{3}\right)^\circ = 180^\circ$.

Step 2: Combine the known angle measures: $140^\circ + \left(\frac{m}{3}\right)^\circ = 180^\circ$.

Step 3: Subtract 140° from both sides to isolate the term with m : $\left(\frac{m}{3}\right)^\circ = 40^\circ$.

Step 4: Multiply both sides by 3 to solve for m : $m = 40 \times 3$.

Step 5: Simplify: $m = 120$.

2. For two acute angles, $\angle A$ and $\angle B$, $\sin(A) = \cos(B)$. The measures, in degrees, of $\angle A$ and $\angle B$ are $2x + 30$ and $5x - 10$, respectively. What is the value of x ?

- A. 8
- B. 9
- C. 10
- D. 11

Answer

C

Solution

This problem aims to assess the student's understanding of trigonometric identities, specifically the relationship between sine and cosine for complementary angles, and their ability to solve for unknown variables in angle measures.

The key to solving this problem is knowing the trigonometric identity $\sin(A) = \cos(90^\circ - A)$. Since $\sin(A) = \cos(B)$, we can set up the equation $A = 90^\circ - B$.

Substitute the given expressions for $\angle A$ and $\angle B$ and solve for x .

Remember that for any angle θ , $\sin(\theta) = \cos(90^\circ - \theta)$. Use this identity to set up your equation. Carefully substitute the given expressions for $\angle A$ and $\angle B$ into the equation $A = 90^\circ - B$ and solve for x step-by-step.

Be careful with your algebra when solving for x . Ensure you correctly distribute and combine like terms. Also, double-check your trigonometric identity and make sure you are substituting correctly.

This problem is a classic example of using trigonometric identities to find unknown values. It tests the student's knowledge of the complementary angle relationship between sine and cosine, as well as their algebraic manipulation skills. Being familiar with these identities and solving equations accurately is essential for success in the SAT math section.

Start by setting up the equation from the condition $A + B = 90$ degrees.

Substitute the expressions for A and B; $2x + 30 + 5x - 10 = 90$

Combine like terms: $7x + 20 = 90$

Subtract 20 from both sides to isolate the term with x : $7x = 70$

Divide both sides by 7 to solve for x : $x = 10$

3. In a factory that has recently implemented automated machines, there are currently 200 workers. Following automation, the company found that the probability of a worker being reassigned to a new role in the automation process is 0.25, while the probability of a worker remaining in their current role is 0.55. What is the probability that a randomly selected worker will either be reassigned or remain in their current role?

- A. 0.75
- B. 0.80
- C. 1.00
- D. 0.85

Answer

B

Solution

This problem tests the student's understanding of basic probability concepts, particularly the ability to calculate the probability of combined events using addition rules. It assesses their ability to interpret word problems and convert them into mathematical expressions.

To solve this problem, students need to recognize that the probability of a worker being reassigned or remaining in their current role can be found by adding the individual probabilities of these events. Specifically, they need to add the probability of being reassigned (0.25) to the probability of remaining in the current role (0.55). Remember that probabilities are always between 0 and 1, and the sum of probabilities for all possible outcomes must equal 1. Since the problem states the probabilities directly, your task is simply to add them together. Be careful with the arithmetic to avoid simple mistakes.

Be sure to only add the probabilities of mutually exclusive events. In this problem, being reassigned and remaining in the current role are mutually exclusive, so their probabilities can be added directly. Also, ensure that you read the problem carefully to understand which probabilities are given and what is being asked.

This problem is a straightforward application of basic probability rules, specifically the addition rule for mutually exclusive events. It evaluates the student's ability to interpret and solve word problems involving probability. Mastery of this type of problem is essential for success in the SAT math section, as it requires both conceptual understanding and attention to detail.

Since the events 'being reassigned' and 'remaining in the current role' are mutually exclusive, their combined probability can be calculated by adding their individual probabilities.

$$P(\text{reassigned or remaining}) = P(\text{reassigned}) + P(\text{remaining}), P(\text{reassigned or remaining}) = 0.80$$

4. If x and y are numbers greater than 1 and $\sqrt[4]{x^5}$ is equivalent to $\sqrt[6]{y^3}$, for what value of b is x^{3b+2} equal to y ?

A. $\frac{1}{6}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

Answer

A

Solution

This problem intends to assess the student's understanding of how to manipulate and solve equations involving radical and rational exponents. It evaluates the ability to equate expressions with different bases and exponents by finding a common ground and applying algebraic principles.

To approach this problem, first express each radical in terms of rational exponents: $\sqrt[4]{x^5} = x^{\frac{5}{4}}$, $\sqrt[6]{y^3} = y^{\frac{1}{2}}$. Since these are equal, set $x^{\frac{5}{4}} = y^{\frac{1}{2}}$. Then express y in terms of x : $y = \left(x^{\frac{5}{4}}\right)^2 = x^{\frac{5}{2}}$. Next, equate x^{3b+2} to y : $x^{3b+2} = x^{\frac{5}{2}}$. Solve for b by setting the exponents equal: $3b + 2 = \frac{5}{2}$.

When working with equations involving radicals and rational exponents, always convert the radicals to rational exponents first. This simplifies the comparison and manipulation of the expressions. Ensure all expressions are in terms of the same base or can be manipulated into a common base.

Be cautious with the exponent rules, especially when equating two expressions. A common mistake is to miscalculate or overlook the conversion between radicals and rational exponents. Double-check the algebraic manipulation to ensure the exponents are correctly equated.

This type of problem is common in advanced math sections of standardized tests like the SAT. It assesses your ability to handle radicals and rational exponents systematically. Mastery of these concepts is crucial as they form the foundation for more complex algebraic problems. Practice converting between radicals and exponents and solving equations to improve speed and accuracy in this topic.

Start by expressing the roots as exponents. $\sqrt[4]{x^5} = x^{\frac{5}{4}}$ and $\sqrt[6]{y^3} = y^{\frac{1}{2}}$.

We have the equation: $x^{\frac{5}{4}} = y^{\frac{1}{2}}$.

Raise both sides to the power of 4 to eliminate the fourth root on the left:

$$\left(x^{\frac{5}{4}}\right)^4 = \left(y^{\frac{1}{2}}\right)^4.$$

This simplifies to $x^5 = y^2$.

We need to find b such that $x^{3b+2} = y$.

Substituting $y = x^{\frac{5}{2}}$ into $x^{3b+2} = x^{\frac{5}{2}}$, we equate the exponents: $3b + 2 = \frac{5}{2}$.

Solve for b : $3b + 2 = \frac{5}{2}$.

Subtract 2 from both sides: $3b = \frac{5}{2} - 2$.

Substitute to get: $3b = \frac{1}{2}$.

Divide by 3: $b = \frac{1}{2} \div 3 = \frac{1}{6}$.

The value of b is $\frac{1}{6}$.

5. A city implemented a new public policy aiming to reduce air pollution. The estimated reduction in air pollution levels, measured in tons, in the first five years after the policy is modeled by the function $f(x) = 500(0.90)^x$, where x is the number of years since the policy was enacted. What does the value 500 represent in this context?

- A. The amount of air pollution measured in tons after 5 years
- B. The estimated air pollution level in tons during the baseline year before the policy was enacted
- C. The percentage decrease in air pollution level each year
- D. The total reduction in air pollution expected after 5 years

Answer

B

Solution

This problem aims to test students' understanding of exponential functions and their ability to interpret parameters in real-world contexts. Specifically, it evaluates whether students can identify what the initial value in an exponential decay function represents.

To solve this problem, you should focus on understanding the components of the exponential function given. Recognize that in the context of the function, the term '500' represents the initial value or starting amount at year zero (when the policy was first enacted).

Remember that in an exponential function of the form $y = a(b)^x$, the 'a' term represents the initial value before any changes occur as a result of the exponential process. In this problem, identify what the situation was at the start (year 0). Be careful not to confuse the initial value with the rate of change or the decay factor. The initial value is the amount present at the beginning ($x = 0$), while the decay factor in this problem is 0.90. This problem is a good example of how SAT math questions often require both an understanding of mathematical concepts and the ability to apply these concepts to real-world scenarios. Recognizing the meaning of parameters in an exponential function is crucial. To excel in such problems, practice identifying and interpreting each component of the function accurately.

The initial value of the function is 500. In exponential decay functions, the initial value represents the starting amount before any decay has occurred. In this context, 500 represents the air pollution level measured in tons during the baseline year before the policy was enacted. As the function models a reduction, and 500 is the amount without decay applied, it is the amount before any reduction.

6. For the linear function g , the graph of $y = g(x)$ in the xy -plane has a slope of 12 and passes through the point $(0, 5)$. Which equation defines g ?

Answer

1

Solution

This problem tests the student's ability to understand and apply the concept of a linear equation in slope-intercept form. The student needs to identify the slope and y-intercept to form the equation of the line.

The student should recognize that the equation of a line in slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept. Given the slope ($m = 12$) and the y-intercept ($b = 5$, as the line passes through the point $(0, 5)$), the student should substitute these values into the slope-intercept form to find the equation $g(x) = 12x + 5$.

Remember the slope-intercept form $y = mx + b$ as this is critical for quickly forming the equation of a line when given the slope and y-intercept. Recognize points like $(0, b)$ as the y-intercept when given directly.

Be careful not to confuse the slope with the y-intercept. Also, ensure you substitute the correct values into the equation. Double-check that the point given is indeed the y-intercept ($x=0$), which simplifies identifying the equation.

This problem is a classic example of testing foundational algebra skills, focusing on the application of the slope-intercept form. Mastery of this concept is essential as it

frequently appears in SAT math sections. Quickly identifying and using the slope and y-intercept to formulate linear equations is a valuable skill that will aid in efficiently solving similar problems.

Since the line passes through the point $(0, 5)$, this point is the y-intercept of the line. Therefore, $b = 5$. With a slope of $m = 12$, and the y-intercept $b = 5$, the equation of the line can be written in slope-intercept form as: $y = 12x + 5$. This equation defines the linear function g , as it incorporates both the given slope and the point through which the line passes.

7. For a polynomial function, the graph of $y = f(x)$ in the xy -plane contains the points $(-4, 0)$, $(0, 0)$, $(3, 0)$, and $(5, 0)$. Which of the following must be a factor of $f(x)$?

- A. $x^2 + 2x - 8$
- B. $x^2 - 8x + 15$
- C. $x^2 - 3x - 10$
- D. $x^2 - 5x$

Answer

B

Solution

This problem is designed to test students' understanding of polynomial functions and their roots. Specifically, it checks if students can identify the factors of a polynomial given its zeros.

To solve this problem, students need to recognize that the x-coordinates of the points where the function intersects the x-axis correspond to the zeros of the polynomial function. Therefore, for each zero, there is a corresponding factor of the polynomial.

Remember that if a polynomial function has zeros at $x = a$, $x = b$, and $x = c$, then $(x - a)$, $(x - b)$, and $(x - c)$ are factors of the polynomial. In this case, you should identify the zeros from the given points and then write the corresponding factors. Be careful not to confuse the x-coordinates of the points with the y-coordinates. The zeros of the polynomial are the x-values where $y = 0$, which are the x-intercepts.

Also, ensure you consider all given points to avoid missing any factors.

This type of problem is common in the SAT as it evaluates a student's ability to analyze polynomial graphs and understand the relationship between the roots and factors of polynomial functions. By mastering this skill, students can efficiently handle similar problems on the test. Always double-check the zeros and

corresponding factors to avoid simple mistakes.

Factors of the polynomial based on the roots are $(x + 4)$, x , $(x - 3)$, and $(x - 5)$. We check each option to see if they match any of these factors or a product of them:

Option A: $x^2 + 2x - 8$.

Factoring gives $(x + 4)(x - 2)$, which has roots $x = -4$ and $x = 2$.

Hence, not all roots match.

Option B: $x^2 - 8x + 15$.

Factoring gives $(x - 3)(x - 5)$, which has roots $x = 3$ and $x = 5$.

These match the roots of the polynomial.

Option C: $x^2 - 3x - 10$.

Factoring gives $(x - 5)(x + 2)$, which has roots $x = 5$ and $x = -2$.

Not all roots match.

Option D: $x^2 - 5x$. Factoring gives $x(x - 5)$, which has roots $x = 0$ and $x = 5$.

Not all roots match.

The correct factor that corresponds to the given roots is found in Option B, which provides the factors $(x - 3)$ and $(x - 5)$, matching two of the roots.

8. What is the center of the circle in the xy -plane defined by the equation

$$(x + 3)^2 + (y - 4)^2 = 16?$$

A. $(-3, 4)$

B. $(3, -4)$

C. $(4, 3)$

D. $(-4, 3)$

Answer

A

Solution

This problem tests the student's understanding of the standard form of a circle's equation in the xy -plane and their ability to identify the center and radius from this form.

The standard form of a circle's equation is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius of the circle. In the given equation

$(x + 3)^2 + (y - 4)^2 = 16$, compare it to the standard form to identify the center and radius. Here, $h = -3$ and $k = 4$, so the center of the circle is $(-3, 4)$.

Remember that in the equation $(x - h)^2 + (y - k)^2 = r^2$, the values of h and k are

taken directly but with opposite signs.

This is because the equation is written as $(x - h)$ and $(y - k)$.

Be careful with the signs when identifying h and k . In this problem, since the equation has $(x + 3)$, it means $h = -3$. Similarly, $(y - 4)$ means $k = 4$. Students often make mistakes with these signs.

This problem is a fundamental exercise in understanding and working with the standard form of a circle's equation. Mastery of this concept is crucial, as it frequently appears in various forms on the SAT. Being able to quickly and accurately identify the center and radius will save valuable time and reduce errors on the exam.

The equation is $(x + 3)^2 + (y - 4)^2 = 16$.

This can be rewritten as $(x - (-3))^2 + (y - 4)^2 = 16$.

From the standard form $(x - h)^2 + (y - k)^2 = r^2$, $h = -3$ and $k = 4$.

Thus, the center of the circle is at the point $(-3, 4)$.

9. In triangle DEF, the measure of angle D is 55° , the measure of angle E is 90° , and the measure of angle F is $\frac{m}{3}^\circ$. What is the value of m ?

Answer

105

Solution

This problem aims to test the student's understanding of the properties of angles in a triangle, specifically the fact that the sum of the angles in a triangle is always 180° . To solve this problem, a student needs to recall that the sum of the angles in a triangle is 180° . Given the measures of angles D and E, the student should set up an equation: $55^\circ + 90^\circ + \frac{m}{3}^\circ = 180^\circ$. Solving this equation will yield the value of m .

Remember to first add the known angles together. Once you have their sum, subtract it from 180° to find the measure of angle F. Then, solve for m by multiplying both sides of the equation by 3.

Be careful with arithmetic operations, especially when dealing with fractions.

Ensure that you correctly perform the multiplication step to solve for m .

This type of problem reinforces the fundamental concept of the sum of angles in a triangle being 180° . It also assesses basic algebraic manipulation skills. On the SAT, such problems test both your geometric knowledge and your ability to apply algebraic techniques efficiently. Practice problems like these to enhance your speed and accuracy.

Calculate the sum of angles D and E: $55^\circ + 90^\circ = 145^\circ$.

Set up the equation for the sum of angles in the triangle: $145^\circ + \frac{m}{3}^\circ = 180^\circ$.

Subtract 145° from both sides to isolate $\frac{m}{3}$: $\frac{m}{3} = 180^\circ - 145^\circ$.

This simplifies to $\frac{m}{3} = 35^\circ$.

Multiply both sides by 3 to solve for m: $m = 35 \times 3$.

This results in $m = 105$.

10. One solution to the given equation can be written as $x = \frac{-7+\sqrt{k}}{2}$, where k is a constant. What is the value of k? $x^2 + 7x + 10 = 0$

- A. 9
- B. 16
- C. 25
- D. 36

Answer

A

Solution

The problem aims to test the student's understanding of solving quadratic equations using the quadratic formula. It specifically evaluates the student's ability to identify and manipulate the components of the formula, and to recognize the relationship between the given solution and the quadratic equation.

To solve the problem, follow these steps:

1. Recognize that the given equation is in the standard quadratic form,

$$ax^2 + bx + c = 0.$$

2. Use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve for x.

3. Compare the given solution form with the quadratic formula to identify the discriminant \sqrt{k} and solve for the value of k.

Remember that the quadratic formula is derived from completing the square of a quadratic equation. The discriminant $b^2 - 4ac$ under the square root sign determines the nature of the roots. In this case, equate the discriminant to k and solve for it.

Be careful with signs when working with the quadratic formula. It's easy to make mistakes with negative signs, especially when dealing with the term $-b$ and the discriminant. Also, ensure you correctly identify the values of a, b, and c from the quadratic equation.

This type of problem is common on the SAT and tests a student's ability to apply the quadratic formula accurately. The key skills evaluated include recognizing the

standard form of a quadratic equation, correctly applying the quadratic formula, and manipulating algebraic expressions. Mastery of these skills is essential for success in advanced mathematics topics on the SAT.

The standard form of the quadratic equation is $ax^2 + bx + c = 0$ where $a=1$, $b=7$, $c=10$.

Using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Substitute values: $x = \frac{-7 \pm \sqrt{7^2 - 4 \times 1 \times 10}}{2 \times 1}$.

Calculate the discriminant: $b^2 - 4ac = 7^2 - 4 \times 1 \times 10 = 49 - 40 = 9$.

Complete the formula: $x = \frac{-7 \pm \sqrt{9}}{2}$.

Thus, $k = 9$.

