Math

Digital SAT

3

Geometry and Trigonometry

Studyola

SAT Math Geometry and Trigonometry

1. Circle C has a radius of 2x and circle D has a radius of 50x. The area of circle D is how many times the area of circle C?

2. Circle C has a radius of $5x$ and circle D has a radius of $25x$. The area of circle D is how many times the area of circle C?

A. 10

B. 15

C. 20

D. 25

3. The table gives the perimeters of similar triangles DEF and PQR, where DE corresponds to PQ. The length of DE is 16. What is the length of PQ?

Triangle	Perimeter
Triangle DEF	80
Triangle PQR	240

4. A circle in the xy-plane has its center at (4, -2) and has a radius of 5. An equation of this circle is given by $x^2 + y^2 + ax + by + c = 0$, where a, b, and c are constants. What is the value of c?

5. What is the center of the circle in the xy-plane defined by the equation $(x + 5)^2 + (y - 3)^2 = 16$?

6. A wooden cube is carved from a log, and its edges measure 4 centimeters. If the cube is then sanded down, causing each edge to decrease in length by 0.5 centimeters, what will be the volume of the newly shaped cube, in cubic centimeters?

7. In ΔABC, $\angle B$ is a right angle and the length of BC is 180 millimeters. If $cos(A) = \frac{4}{5}$, what is the length, in millimeters, of AB?

A. 200

B. 220

C. 240

D. 260

8. In triangle DEF, the measure of angle D is 32°, the measure of angle E is 90°, and the measure of angle F is $\frac{m}{3}°$. What is the value of m?

A. 162

B. 168

C. 174

D. 180

9. A wooden cube used in a public health education demonstration has an edge length of 3 centimeters. If the cube weighs 5.61 grams, what is the density of the cube in grams per cubic centimeter?

A. 0.2068

B. 0.2070

C. 0.2082

D. 0.2078

10. In the xy-plane, circle N is represented by the equation $(x + 1)^2 + (y - 2)^2 = 9$. Circle Q has the same center as circle N but has a radius that is 1.5 times the radius of circle N. What is the equation for circle Q?

SAT Math Geometry and Trigonometry Solutions

1. Circle C has a radius of 2x and circle D has a radius of 50x. The area of circle D is how many times the area of circle C?

Answer

625

Solution

This problem tests the student's understanding of the formula for the area of a circle and their ability to use ratios to compare the areas of two circles based on their radii.
To solve this problem, students should first recall the formula for the area of a circle, $A = \pi r^2$, where r is the radius. Next, they calculate the area of both circles using their given radii: Circle C with a radius of 2x and Circle D with a radius of 50x. After finding the areas, students should set up a ratio of the area of Circle D to the area of Circle C and simplify the ratio.
Remember that when comparing areas of circles, the ratio of the areas is the square of the ratio of their radii. This can simplify the calculations significantly.
Be careful with squaring the radii correctly. A common mistake is not squaring the entire expression, which can lead to an incorrect ratio. Also, ensure that you simplify the ratio completely.
This type of problem is common in SAT geometry sections, as it assesses both the understanding of geometric formulas and the ability to manipulate algebraic expressions. Mastery of such problems requires familiarity with basic geometric formulas and an ability to apply algebraic principles, such as simplifying ratios. Practicing these skills will help improve accuracy and speed on test day.

Calculate the area of circle C:, $Area\ of\ circle\ C = \pi(2x)^2 = 4\pi x^2$, Calculate the area of circle D:, $Area\ of\ circle\ D = \pi(50x)^2 = 2500\pi x^2$, Determine how many times the area of circle D is compared to circle C:, $Number\ of\ times = \frac{2500\pi x^2}{4\pi x^2} = \frac{2500}{4} = 625$

2. Circle C has a radius of $5x$ and circle D has a radius of $25x$. The area of circle D is how many times the area of circle C?

A. 10

B. 15

C. 20

D. 25

Answer

D

Solution

This problem aims to assess the student's understanding of the relationship between the radius and the area of a circle. Specifically, it examines the ability to apply the formula for the area of a circle and to work with ratios.
To solve this problem, the student should start by recalling the formula for the area of a circle, which is $A = \pi r^2$. Calculate the area of both circles using their respective radii, then compare the two areas by forming a ratio.
Remember that the area of a circle increases with the square of its radius. When comparing areas of circles, you can often simplify your work by setting up a ratio instead of calculating exact areas. In this problem, simplify the ratio of the radii first to see the effect on the area.
Be careful not to confuse the ratio of the radii with the ratio of the areas. The radius is linear, while the area is quadratic. Also, ensure that you square the radii correctly and apply the π factor consistently.
This type of problem is common in SAT geometry questions and tests the ability to understand and manipulate geometric formulas, specifically circles. It also assesses the student's skill in working with proportions and recognizing how changes in one dimension (radius) affect another dimension (area). Mastery of these concepts is crucial not only for geometry but also for more advanced topics in mathematics.

Calculate the area of circle C: $A_C = \pi(5x)^2 = 25\pi x^2$

Calculate the area of circle D: $A_D = \pi(25x)^2 = 625\pi x^2$

The ratio of the area of circle D to circle C is: $\frac{A_D}{A_C} = \frac{625\pi x^2}{25\pi x^2} = 25$.

Thus, the area of circle D is 25 times the area of circle C.

3. The table gives the perimeters of similar triangles DEF and PQR, where DE corresponds to PQ. The length of DE is 16. What is the length of PQ?

Triangle	Perimeter
Triangle DEF	80
Triangle PQR	240

Answer

48

Solution

This problem tests the student's understanding of similar triangles, particularly their ability to use the relationship between the perimeters of similar triangles to find the corresponding side lengths.
To solve this problem, students need to recognize that the perimeters of similar triangles are in the same ratio as their corresponding side lengths. They should set up a proportion using the given perimeter values and the known side length DE to find the unknown side length PQ.
Remember that in similar triangles, the ratio of the perimeters is equal to the ratio of any pair of corresponding side lengths. Set up a proportion using the given perimeter and side length information to find the unknown side length.
Be careful to correctly identify the corresponding sides in the triangles and ensure that the ratios are set up correctly. Misidentifying corresponding sides or mixing up the ratios can lead to incorrect answers.
This problem is a classic example of testing knowledge on similar triangles and their properties. It evaluates the student's ability to apply proportional reasoning in geometry. Understanding and correctly applying the concept of similarity is crucial in solving such problems efficiently on the SAT.

Since the triangles are similar, the ratio of any two corresponding sides of similar triangles is equal to the ratio of their perimeters.
Let the length of PQ be x.
The ratio of the perimeters of the triangles is $\frac{80}{240} = \frac{1}{3}$.
The ratio of the corresponding sides DE and PQ is $\frac{16}{x}$.
So, we set up the equation: $\frac{16}{x} = \frac{1}{3}$.
Solving for x, we cross-multiply: $16 \times 3 = x \times 1$.
This simplifies to $48 = x$.
Thus, the length of PQ is 48.

4. A circle in the xy-plane has its center at (4, -2) and has a radius of 5. An equation of this circle is given by $x^2 + y^2 + ax + by + c = 0$, where a, b, and c are constants. What is the value of c?

Answer

-5

Solution

This problem tests the student's ability to work with the standard form of a circle equation and their understanding of how to manipulate and expand it into the general quadratic form. It assesses the student's skills in algebraic manipulation and geometric understanding of a circle's properties.

Start by recalling the standard equation of a circle, which is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius. Substitute the given center (4, -2) and radius 5 into this equation to get $(x - 4)^2 + (y + 2)^2 = 25$. Then, expand this equation to form $x^2 + y^2 - 8x + 4y + 20 = 0$. Compare this with $x^2 + y^2 + ax + by + c = 0$ to find the values of a, b, and c.

When expanding the squared terms, carefully distribute and simplify each step to avoid errors. Focus on correctly aligning the terms with the general quadratic form. Remember that the coefficient of x^2 *and* y^2 should remain 1, matching the provided equation format. Be careful with the signs when substituting the center coordinates into $(x - h)^2 + (y - k)^2$. It's easy to make mistakes with signs, especially when handling $(y + 2)^2$. Additionally, ensure that all terms are correctly expanded and combined to match the general form.

This problem is a classic example of testing the ability to manipulate geometric equations, specifically those of circles. It evaluates algebraic manipulation skills and understanding of circle properties in coordinate geometry. Mastery of these concepts is crucial for success in SAT geometry questions, and this exercise helps develop proficiency in transforming and comparing different equation forms.

Substitute the center (h, k) = (4, -2) and radius r = 5 into the equation $(x - h)^2 + (y - k)^2 = r^2$., This gives $(x - 4)^2 + (y + 2)^2 = 25$., Expand $(x - 4)^2$ to get $x^2 - 8x + 16$., Expand $(y + 2)^2$ to get $y^2 + 4y + 4$., Combine these to form: $x^2 - 8x + 16 + y^2 + 4y + 4 = 25$., Rearrange to the form $x^2 + y^2 + ax + by + c = 0$., Combine terms: $x^2 + y^2 - 8x + 4y + 20 = 25$., Subtract 25 from both sides to isolate c: $x^2 + y^2 - 8x + 4y + 20 - 25 = 0$., Simplify: $x^2 + y^2 - 8x + 4y - 5 = 0$., From this, c is found to be -5.

5. What is the center of the circle in the xy-plane defined by the equation $(x + 5)^2 + (y - 3)^2 = 16$?

Answer

(-5, 3)

Solution

This problem tests the student's understanding of the standard form of a circle's equation and their ability to identify the center and radius from this form. Students should recognize that the equation $(x - h)^2 + (y - k)^2 = r^2$ represents a circle centered at (h, k) with radius r.
To solve this problem, students need to identify the form of the given equation $(x + 5)^2 + (y - 3)^2 = 16$ and compare it to the standard form of a circle's equation $(x - h)^2 + (y - k)^2 = r^2$. Recognize that the equation can be rewritten as $(x - (- 5))^2 + (y - 3)^2 = 16$, indicating that the center of the circle (h, k) is (-5, 3).
When dealing with circle equations, always rewrite the equation in the form $(x - h)^2 + (y - k)^2 = r^2$ to easily identify the center (h, k) and the radius r. Remember that the signs in the equation are opposite to those in the center coordinates.
Be careful with the signs when determining the center of the circle. In the equation $(x - h)^2 + (y - k)^2 = r^2$, the center is at (h, k), so you must pay attention to the minus signs in the equation to correctly identify the positive or negative values of h and k. Additionally, make sure not to confuse the squared term 16 with the radius; the radius is the square root of 16, which is 4.
This type of problem is common in SAT math sections and assesses a student's ability to work with the standard equation of a circle. Recognizing the structure of the equation and understanding how to manipulate it to extract the center and radius is crucial. Mastery of these concepts is essential for success in geometry and trigonometry problems on the SAT. Practice with a variety of circle equations to become comfortable with quickly identifying the center and radius.

Identify the standard form from the given equation: $(x + 5)^2 + (y - 3)^2 = 16$ compared to $(x - h)^2 + (y - k)^2 = r^2$.
Rewrite $(x + 5)^2$ as $(x - (- 5))^2$ to match the standard form.
Similarly, rewrite $(y - 3)^2$., By comparing, we see that $h =- 5$ and $k = 3$, and $r^2 = 16$.
Thus, the center of the circle is (-5, 3).

6. A wooden cube is carved from a log, and its edges measure 4 centimeters. If the cube is then sanded down, causing each edge to decrease in length by 0.5 centimeters, what will be the volume of the newly shaped cube, in cubic centimeters?

Answer

42.875 cubic centimeters

Solution

This problem tests the student's understanding of volume calculations for geometric shapes, specifically cubes, and requires the ability to apply volume formulas after modifying dimensions.
To solve this problem, first, calculate the original volume of the cube using the formula for the volume of a cube ($V = (side)^3$). Then, adjust the edge length by subtracting 0.5 cm to account for the sanding down process. Finally, calculate the new volume using the adjusted edge length.
Remember that when the dimensions of a cube change, even slightly, it can significantly impact the volume due to the cubic relationship. Always perform the calculations step by step to ensure accuracy.
Be careful not to confuse the reduction in edge length with a reduction in volume. Ensure that you subtract the 0.5 cm from each edge before recalculating the volume. Also, double-check your arithmetic to ensure that cube calculations are correct.
This type of problem is a classic example of testing geometric reasoning and arithmetic skills. It requires students to accurately apply a formula and understand how dimensional changes affect the volume. Being able to handle such problems efficiently is crucial for the SAT, as it demonstrates a solid grasp of basic geometry and measurement principles.

Determine the new edge length by subtracting 0.5 cm from the original length of each edge., New edge length = 4 cm - 0.5 cm = 3.5 cm., Calculate the volume of the new cube using the formula for the volume of a cube, $V = a^3$, where 'a' is the edge length., Substitute the new edge length into the formula: $V = (3.5cm)^3$., Calculate the cube of the new edge length: $V = 3.5cm \times 3.5cm \times 3.5cm$., V = 42.875 cubic centimeters.

7. In ΔABC, ∠B is a right angle and the length of BC is 180 millimeters. If $cos(A) = \frac{4}{5}$, what is the length, in millimeters, of AB?

A. 200

B. 220

C. 240

D. 260

Answer

C

Solution

This problem is designed to test the student's understanding of right-angle trigonometry, specifically the ability to use the cosine function to find the length of a side in a right triangle.
The student should recognize that in right triangle ΔABC, with ∠B as the right angle, the cosine of angle A is defined as the ratio of the adjacent side (AB) to the hypotenuse (AC). Given that $cos(A) = \frac{4}{5}$, the student needs to set up the equation $\frac{AB}{AC} = \frac{4}{5}$. Since BC is given as 180 millimeters, and BC is the side opposite angle A, the student can use the Pythagorean theorem to find AC first before finding AB.
Remember that the Pythagorean theorem can be used to find the hypotenuse when you have one side and the cosine ratio. Set up a ratio equation using $cos(A) = \frac{adjacent}{hypotenuse}$, and solve for the unknown side. Double-check your calculations by ensuring the triangle's side lengths satisfy the Pythagorean theorem.
Be careful not to confuse the sides of the triangle. Ensure you correctly identify which side is opposite and which is adjacent to angle A. Also, ensure that your calculations are exact, and consider simplifying fractions or square roots accurately.
This problem assesses the student's proficiency in applying trigonometric ratios to solve for missing side lengths in right triangles. Mastery of this concept is essential for solving more complex trigonometry problems in the SAT. The ability to correctly interpret and apply the cosine function is a crucial skill in the geometry section of the test.

Since $cos(A) = \frac{4}{5}$, we have $\frac{AB}{AC} = \frac{4}{5}$., We need to find the length of AB., Since BC is 180 millimeters, BC is opposite to angle A., In a right triangle, we use the Pythagorean identity: $(sin)^2(A) + (cos)^2(A) = 1$., Given $cos(A) = \frac{4}{5}$, find $(sin)^2(A)$: $\left(\frac{4}{5}\right)^2 + (sin)^2(A) = 1$., $\frac{16}{25} + (sin)^2(A) = 1$., $(sin)^2(A) = \frac{9}{25}$, therefore $sin(A) = \frac{3}{5}$., Using $sin(A)$, we have $sin(A) = \frac{BC}{AC} = \frac{3}{5}$., $AC = \frac{BC}{sin(A)} = \frac{180}{\frac{3}{5}} = 180 \times \frac{5}{3} = 300\, millimeters$., Now, using $cos(A) = \frac{4}{5}$, solve for AB: $AB = cos(A) \times AC = \frac{4}{5} \times 300 = 240\, millimeters$.

8. In triangle DEF, the measure of angle D is 32°, the measure of angle E is 90°, and the measure of angle F is $\frac{m}{3}°$. What is the value of m?

A. 162

B. 168

C. 174

D. 180

Answer

C

Solution

This problem tests the student's understanding of the properties of angles in a triangle, particularly the fact that the sum of the interior angles in a triangle is always 180 degrees. It also requires the student to solve for a variable within a given expression.
To solve this problem, recognize that the sum of the angles in any triangle is 180 degrees. Given that angle E is 90 degrees, angle D is 32 degrees, and angle F is expressed as $\frac{m}{3}$ degrees, set up an equation: $32 + 90 + \frac{m}{3} = 180$. Solve this equation for m by first combining the known angles and then isolating the variable.
Remember that for any triangle, the sum of the interior angles is always 180 degrees. Also, pay attention to how the angle F is expressed in terms of m. Rearranging and solving linear equations accurately will help you find the correct value of m.
Be careful with arithmetic operations, especially when working with fractions. Ensure that you properly isolate the variable m after combining like terms. Common mistakes include arithmetic errors or miscalculating the value of expressions.
This problem is a classic example of testing the understanding of basic geometric principles such as the sum of interior angles in a triangle. It requires algebraic manipulation skills to isolate and solve for a variable. Such questions are designed to evaluate both geometric understanding and algebraic problem-solving abilities. Mastery of these concepts is crucial for success in the SAT math section.

The sum of the angles in triangle DEF is: 32° + 90° + F = 180°, Substituting the given measures: 32° + 90° + $\frac{m}{3}°$ = 180°, Combine the known angles: 122° + $\frac{m}{3}°$ = 180°, Subtract 122° from both sides: $\frac{m}{3}°$ = 58°, Solve for m by multiplying both sides by 3: m = 58° $\times$ 3, Calculate m: m = 174°

9. A wooden cube used in a public health education demonstration has an edge length of 3 centimeters. If the cube weighs 5.61 grams, what is the density of the cube in grams per cubic centimeter?

A. 0.2068

B. 0.2070

C. 0.2082

D. 0.2078

Answer

D

Solution

This problem aims to test the student's understanding of geometric properties of a cube, specifically how to calculate the volume, and then apply the formula for density. The student needs to be familiar with basic volume formulas and the concept of density as mass per unit volume.
1. Calculate the volume of the cube using the formula for the volume of a cube ($V = a^3$ where 'a' is the edge length).
2. Use the given mass and the volume to calculate the density using the formula ($Density = \frac{Mass}{Volume}$).
Remember that the volume of a cube is found by cubing the edge length. Write down all given information and use the density formula directly after calculating the volume. This helps in organizing thoughts and reducing careless errors.
Be careful with units and ensure consistency throughout the calculation. Miscalculating the volume by forgetting to cube the edge length is a common mistake. Verify that the density units are in grams per cubic centimeter as required by the problem.
This problem tests fundamental skills in geometry and unit analysis, which are crucial for many SAT math problems. Understanding the relationships between edge length, volume, and density is key. Efficiently solving such problems requires a clear grasp of basic formulas and careful unit management, which are essential skills for SAT success.

The formula for calculating the volume of a cube is $Volume = edge\ length^3$
For this cube, the volume is $3^3 = 27$ cubic centimeters.
Density is given by $Density = \frac{Mass}{Volume}$, Substituting the known values: $Density = \frac{5.61}{27}$ grams per cubic centimeter.
Performing the division: $\frac{5.61}{27} = 0.207777...$
Rounding to the fourth digit, we get $Density \cong 0.2078$ grams per cubic centimeter.

10. In the xy-plane, circle N is represented by the equation $(x + 1)^2 + (y - 2)^2 = 9$. Circle Q has the same center as circle N but has a radius that is 1.5 times the radius of circle N. What is the equation for circle Q?

Answer

$$(x + 1)^2 + (y - 2)^2 = 20.25$$

Solution

This question tests the student's ability to understand and manipulate the equation of a circle in the standard form. It assesses their knowledge of how changes in the radius affect the equation and their ability to calculate and apply these changes accurately.
First, identify the center and radius of circle N from its equation. The equation given is in the form $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius. For circle N, the center is (-1, 2) and the radius is $\sqrt{9} = 3$. Since circle Q has the same center and a radius 1.5 times that of circle N, calculate the new radius: $1.5 \times 3 = 4.5$. Then, write the equation for circle Q using the new radius.
Keep in mind the standard equation of a circle, $(x - h)^2 + (y - k)^2 = r^2$. When the problem states that the radius is 1.5 times larger, you only need to multiply the original radius by 1.5 and square it to get the new r^2 value for the equation of circle Q.
Make sure not to confuse the radius with the squared radius when writing the equation.
After calculating the new radius, remember to square it to find the correct r^2 value for the equation.
This type of question is common in SAT geometry sections, focusing on the student's ability to manipulate and understand geometric equations, specifically those of circles. Mastery of this concept is crucial as it often appears in various forms across different questions.
Practice with similar problems will help in quickly identifying the center and radius from circle equations, as well as applying transformations like scaling the radius.

1. Determine the center and radius of Circle N from the given equation $(x + 1)^2 + (y - 2)^2 = 9$.
- Center of Circle N: (-1, 2)
- Radius of Circle N: $\sqrt{9} = 3$
2. Calculate the radius of Circle Q by multiplying Circle N's radius by 1.5.
- Radius of Circle Q: $3 \times 1.5 = 4.5$
3. Write the equation for Circle Q using its center and newly calculated radius.
- The equation of Circle Q: $(x + 1)^2 + (y - 2)^2 = (4.5)^2$
- Simplify the equation: $(x + 1)^2 + (y - 2)^2 = 20.25$

