Digital

SAT Math

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SAT Math Problems

1. If x and y are numbers greater than 1 and $\sqrt[4]{x^3}$ is equivalent to $\sqrt[6]{y^4}$, for what value of b is x^{3b-2} equal to y?

A. $\frac{5}{24}$

B. 1

C. $\frac{25}{24}$

D. $\frac{5}{8}$

2. A circle in the xy-plane has its center at (2, -5) and has a radius of 7. An equation of this circle is represented by the form $x^2 + y^2 + ax + by + c = 0$, where a, b, and c are constants. What is the value of c?

3. A solution to the given system of equations is (x, y). What is a possible value of x?
$y = \frac{1}{2}(x - 4)^2 + 7, y = 2x + 5$

A. 4

B. 6

C. 8

D. 10

4. A wooden cube is carved from a log, and its edges measure 4 centimeters. If the cube is then sanded down, causing each edge to decrease in length by 0.5 centimeters, what will be the volume of the newly shaped cube, in cubic centimeters?

5. An exponential function f is defined by $f(x) = a(b)^x$, where a is a constant greater than 0 and b is a constant greater than 1. If $f(3) = 16f(1)$, what is the value of b?

A. 2

B. 3

C. 4

D. 5

6. For two acute angles, $\angle$A and $\angle$B, sin(A) = cos(B). The measures, in degrees, of $\angle$A and $\angle$B are 2x + 30 and 5x - 10, respectively. What is the value of x?

A. 8

B. 9

C. 10

D. 11

7. For the polynomial function defined as $f(x) = 3x^4 - 5x^3 + 2x^2 + 7$, what is the value of the y-intercept?

8. If m and p are numbers greater than 1 and $\sqrt[4]{m^3}$ is equivalent to $\sqrt[6]{p^4}$, for what value of b is m^{3b-2} equal to p?

9. One solution to the given equation can be expressed as $x = \frac{-9+\sqrt{m}}{2}$, where m is a constant. What is the value of m? $x^2 + 9x + 6 = 0$

10. In the xy-plane, line m has a slope of -3 and a y-intercept of (0, 12). What is the x-coordinate of the x-intercept of line m?

SAT Math Solutions

1. If x and y are numbers greater than 1 and $\sqrt[4]{x^3}$ is equivalent to $\sqrt[6]{y^4}$, for what value of b is x^{3b-2} equal to y?

A. $\frac{5}{24}$

B. 1

C. $\frac{25}{24}$

D. $\frac{5}{8}$

Answer

C

Solution

This problem aims to test students' understanding of equations involving radical and rational exponents and their ability to manipulate and simplify such expressions.
First, rewrite the given expressions with rational exponents:
$\sqrt[4]{x^3} = x^{\frac{3}{4}}$ and $\sqrt[6]{y^4} = y^{\frac{4}{6}} = y^{\frac{2}{3}}$. Set these equal to each other: $x^{\frac{3}{4}} = y^{\frac{2}{3}}$. Next, express y in terms of x: $y = \left(x^{\frac{3}{4}}\right)^{\frac{3}{2}} = x^{\frac{9}{8}}$.

Now, set the given equation $x^{(3b-2)} = y$: $x^{(3b-2)} = x^{\frac{9}{8}}$.
Equate the exponents: $3b - 2 = \frac{9}{8}$.
Solve for b: $3b = \frac{9}{8} + 2 = \frac{9}{8} + \frac{16}{8} = \frac{25}{8}$, $b = \frac{25}{24}$.
When dealing with radical and rational exponents, it is often helpful to convert all expressions to rational exponents. This simplifies the process of comparing and manipulating the equations.
Be careful with the arithmetic operations, especially when adding fractions. Ensure that you correctly find a common denominator before performing the addition. Also, double-check your exponent rules to avoid mistakes.
This problem effectively gauges a student's proficiency in handling radical and rational exponents. It requires a solid understanding of exponent laws and careful algebraic manipulation. Mastery of these concepts is crucial for solving advanced mathematics problems efficiently on the SAT.

Start with the equation: $\sqrt[4]{x^3} = \sqrt[6]{y^4}$.

Rewrite each side using fractional exponents: $\left(x^3\right)^{\frac{1}{4}} = \left(y^4\right)^{\frac{1}{6}}$., Simplify: $x^{\frac{3}{4}} = y^{\frac{2}{3}}$.

Raise both sides to the power of 12 to eliminate the fractions: $\left(x^{\frac{3}{4}}\right)^{12} = \left(y^{\frac{2}{3}}\right)^{12}$.

This becomes $x^9 = y^8$.

We now know $y = x^{\frac{9}{8}}$., Set $x^{3b-2} = y = x^{\frac{9}{8}}$.

Since the bases are the same, equate the exponents: $3b - 2 = \frac{9}{8}$.

Solve for b: $3b = \frac{9}{8} + 2$.

Convert 2 to a fraction: $2 = \frac{16}{8}$., So, $3b = \frac{9}{8} + \frac{16}{8} = \frac{25}{8}$.

Solve for b by dividing both sides by 3: $b = \frac{25}{8} \times \frac{1}{3} = \frac{25}{24}$.

Therefore, the value of b is $\frac{25}{24}$.

2. A circle in the xy-plane has its center at (2, -5) and has a radius of 7. An equation of this circle is represented by the form $x^2 + y^2 + ax + by + c = 0$, where a, b, and c are constants. What is the value of c?

Answer

-20

Solution

This problem is designed to test the student's ability to understand and manipulate the standard equation of a circle. It assesses the student's skills in expanding the equation and identifying constants when given the center and radius of a circle.

First, recall the standard equation of a circle in the form $(x - h)^2 + (y - k)^2 = r^2$, where h, k is the center and r is the radius. Substitute the given center (2, -5) and radius 7 into this equation to obtain $(x - 2)^2 + (y + 5)^2 = 49$. Expand this equation to match the given form $x^2 + y^2 + ax + by + c = 0$.

When expanding $(x - 2)^2 + (y + 5)^2 = 49$, remember to carefully expand each square: $(x - 2)^2 = x^2 - 4x + 4$ and $(y + 5)^2 = y^2 + 10y + 25$. Combine these and compare with the given form to find a, b, and c.

Avoid common mistakes such as incorrectly expanding the binomials or losing negative signs. Double-check your algebra as missing a sign or number can lead to incorrect constants.

This problem is a classic application of circle equations and requires algebraic manipulation skills. Understanding how to transition from the standard form to the general form is crucial. Such problems often appear on the SAT to evaluate both conceptual understanding and procedural fluency in algebra and geometry. Mastery of these skills will aid in solving similar problems efficiently.

The standard form of the equation of a circle with center (h, k) and radius r is:
$(x - h)^2 + (y - k)^2 = r^2$.
Substitute the given center and radius into the standard form equation:
$(x - 2)^2 + (y + 5)^2 = 49$.
Expand the terms: $(x - 2)^2 = x^2 - 4x + 4$ and $(y + 5)^2 = y^2 + 10y + 25$.
Now, combine these expanded terms:
$x^2 - 4x + 4 + y^2 + 10y + 25 = 49$.
Rearrange the equation to match the given form $x^2 + y^2 + ax + by + c = 0$:
$x^2 + y^2 - 4x + 10y + 4 + 25 - 49 = 0$.
Simplify the constant terms: $4 + 25 - 49 =- 20$.
The equation of the circle in the desired form is: $x^2 + y^2 - 4x + 10y - 20 = 0$.
Thus, the value of c is -20.

3. A solution to the given system of equations is (x, y). What is a possible value of x?
$y = \frac{1}{2}(x - 4)^2 + 7$, $y = 2x + 5$

A. 4

B. 6

C. 8

D. 10

Answer

D

Solution

This problem is designed to test the student's ability to solve a system of equations involving a quadratic and a linear equation. It checks the understanding of graph intersections and algebraic solutions.
To solve this problem, students should set the equations equal to each other since both are equal to y, and then solve for x. This involves expanding the quadratic equation, setting up a quadratic equation in standard form, and then using methods such as factoring, completing the square, or the quadratic formula to find the

possible values of x.
When dealing with quadratic and linear systems, remember that solutions correspond to the intersection points of a parabola and a line. It might be helpful to sketch the graphs to visualize potential solutions before solving algebraically. Also, check if the quadratic equation can be easily factored after expansion to simplify calculations.
Be careful when expanding the quadratic expression and ensure that all terms are correctly simplified. Additionally, check all potential solutions in the original equations to verify they are valid, as extraneous solutions can sometimes arise.
This type of problem is common in SAT's advanced math section, as it assesses not only algebraic manipulation skills but also conceptual understanding of graph intersections and their physical interpretations. Mastery of these concepts can be beneficial, as it combines different branches of mathematics into a single problem, which is a frequent characteristic of SAT questions.

Set $\frac{1}{2}(x - 4)^2 + 7$ equal to $2x + 5$: $\frac{1}{2}(x - 4)^2 + 7 = 2x + 5$

Subtract 7 from both sides: $\frac{1}{2}(x - 4)^2 = 2x - 2$

Multiply every term by 2 to eliminate the fraction: $(x - 4)^2 = 4x - 4$

Expand $(x - 4)^2$: $x^2 - 8x + 16 = 4x - 4$

Rearrange all terms to one side: $x^2 - 8x + 16 - 4x + 4 = 0$

Combine like terms: $x^2 - 12x + 20 = 0$

Use the quadratic formula to solve for x, where $a = 1, b = -12,\ and\ c = 20$:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \cdot 1 \cdot 20}}{2 \cdot 1}, x = \frac{12 \pm \sqrt{144 - 80}}{2}, x = \frac{12 \pm \sqrt{64}}{2}, x = \frac{12 \pm 8}{2}$

Calculate the possible values: $x = \frac{12+8}{2} = 10, x = \frac{12-8}{2} = 2$

Possible values of x are 10 and 2.

4. A wooden cube is carved from a log, and its edges measure 4 centimeters. If the cube is then sanded down, causing each edge to decrease in length by 0.5 centimeters, what will be the volume of the newly shaped cube, in cubic centimeters?

Answer

42.875 cubic centimeters

Solution

This problem tests the student's understanding of volume calculations for geometric shapes, specifically cubes, and requires the ability to apply volume formulas after modifying dimensions.
To solve this problem, first, calculate the original volume of the cube using the formula for the volume of a cube ($V = (side)^3$). Then, adjust the edge length by subtracting 0.5 cm to account for the sanding down process. Finally, calculate the new volume using the adjusted edge length.
Remember that when the dimensions of a cube change, even slightly, it can significantly impact the volume due to the cubic relationship. Always perform the calculations step by step to ensure accuracy.
Be careful not to confuse the reduction in edge length with a reduction in volume. Ensure that you subtract the 0.5 cm from each edge before recalculating the volume. Also, double-check your arithmetic to ensure that cube calculations are correct.
This type of problem is a classic example of testing geometric reasoning and arithmetic skills. It requires students to accurately apply a formula and understand how dimensional changes affect the volume. Being able to handle such problems efficiently is crucial for the SAT, as it demonstrates a solid grasp of basic geometry and measurement principles.

Determine the new edge length by subtracting 0.5 cm from the original length of each edge., New edge length = 4 cm - 0.5 cm = 3.5 cm., Calculate the volume of the new cube using the formula for the volume of a cube, $V = a^3$, where 'a' is the edge length., Substitute the new edge length into the formula: $V = (3.5cm)^3$., Calculate the cube of the new edge length: $V = 3.5cm \times 3.5cm \times 3.5cm$., V = 42.875 cubic centimeters.

5. An exponential function f is defined by $f(x) = a(b)^x$, where a is a constant greater than 0 and b is a constant greater than 1. If $f(3) = 16f(1)$, what is the value of b?

A. 2

B. 3

C. 4

D. 5

Answer

C

Solution

This problem tests the student's understanding of exponential functions, specifically manipulating and solving equations involving exponential growth. The student must recognize how to apply properties of exponents and solve for the base of the exponential function.
To solve this problem, start by expressing the given information in terms of the function. Given the function $f(x) = a(b)^x$ and the condition $f(3) = 16f(1)$, substitute $x = 3$ and $x = 1$ into the function to create two equations. Then, set up a ratio or an equation to solve for the unknown variable b.
Remember that you can express the function at different points in terms of the same base and exponent. Since $f(3) = ab^3$ and $f(1) = ab^1$, you can set up the equation $a(b)^3 = 16a(b)^1$. Dividing both sides by $a(b)^1$ simplifies the equation and helps isolate b.
Be careful with algebraic manipulations, especially when dividing both sides of the equation. Ensure you maintain the properties of exponents correctly and do not lose any terms in the process. Also, remember that since a is a constant greater than 0, it will cancel out easily.
This problem is a good test of understanding and manipulating exponential functions, which is a critical skill in advanced mathematics. It requires students to apply properties of exponents and solve equations systematically. Ensuring accuracy in algebraic steps is crucial to avoid simple mistakes. Mastery of these concepts is essential for success in the SAT Math section, particularly in more challenging problems involving nonlinear functions.

Starting with the equation $b^3 = 16b$.

Divide both sides by b (since $b > 1$, b is not zero): $b^2 = 16$.

Taking the square root of both sides gives $b = \sqrt{16}$.
Therefore, $b = 4$.

6. For two acute angles, ∠A and ∠B, sin(A) = cos(B). The measures, in degrees, of ∠A and ∠B are 2x + 30 and 5x - 10, respectively. What is the value of x?

A. 8

B. 9

C. 10

D. 11

Answer

C

Solution

This problem tests the student's understanding of the complementary angle relationship between sine and cosine, where sin(A) = cos(B) implies A + B = 90 degrees. It also examines their algebraic manipulation skills to solve for the variable x.
To solve this problem, first apply the trigonometric identity that sin(A) = cos(B) means A + B = 90 degrees. Set up the equation (2x + 30) + (5x - 10) = 90. Simplify and solve this linear equation for x.
Remember the relationship between the sine and cosine of complementary angles: $\sin(\theta) = \cos(90° - \theta)$. This is crucial for setting up the correct equation. Also, carefully combine like terms when solving the equation.
Be cautious with the angle measures' expressions. Ensure you correctly combine and simplify the terms in the equation. Double-check your arithmetic when solving for x to avoid simple calculation errors.
This problem is a classic example of testing fundamental trigonometric identities and algebraic skills in one. It assesses your ability to connect geometric angle relationships with algebraic equations, which is essential for solving trigonometry-related problems on the SAT. Mastery of these concepts and careful calculation will help you excel in this section.

Combine the expressions for A and B: (2x + 30) + (5x - 10) = 90
Simplify: 2x + 30 + 5x - 10 = 90
Combine like terms: 7x + 20 = 90
Subtract 20 from both sides: 7x = 70
Divide both sides by 7: x = 10

7. For the polynomial function defined as $f(x) = 3x^4 - 5x^3 + 2x^2 + 7$, what is the value of the y-intercept?

Answer

7

Solution

The question is designed to test the student's understanding of polynomial functions, specifically how to find the y-intercept of a polynomial graph. It assesses the student's ability to substitute values into a polynomial function and evaluate the expression correctly.
To find the y-intercept of the polynomial function, substitute $x = 0$ into the function $f(x)$ since the y-intercept occurs where the graph crosses the y-axis. Evaluate $f(0)$ to find the constant term, which will be the y-intercept.
Remember that the y-intercept of any function is found by setting $x = 0$. This simplifies the polynomial since all terms involving x will become zero, leaving only the constant term. This term is the y-intercept.
Be careful when substituting $x = 0$ into the polynomial. Ensure that each term involving x is correctly evaluated to zero, and only the constant term is considered. Avoid making arithmetic errors when evaluating the polynomial function.
This type of question is a straightforward test of a student's ability to work with polynomial functions. It focuses on evaluating a function at a particular point, which is a fundamental skill in algebra. Understanding how to find the y-intercept is crucial, as it provides insight into the behavior of a graph. In the context of the SAT, being able to quickly identify and compute the y-intercept can save valuable time during the test.

To find the y-intercept, evaluate $f(0)$.

Substitute $x = 0$ into the function: $f(0) = 3(0)^4 - 5(0)^3 + 2(0)^2 + 7$.
Simplify each term: $3(0) = 0, - 5(0) = 0, 2(0) = 0$.
Thus, $f(0) = 0 + 0 + 0 + 7 = 7$.
The y-intercept is the value of f(x) when $x = 0$, which is 7.

8. If m and p are numbers greater than 1 and $\sqrt[4]{m^3}$ is equivalent to $\sqrt[6]{p^4}$, for what value of b is m^{3b-2} equal to p?

Answer

$\frac{25}{24}$

Solution

This problem tests the student's understanding of radical and rational exponents and their ability to manipulate and equate expressions involving these concepts.
To solve this problem, students need to express both sides of the equation in terms of rational exponents. Recognize that $\frac{3}{4}$ is the exponent for m and $\frac{4}{6} = \frac{2}{3}$ is the exponent for p. Equate these two expressions to find a relationship between m and p. Once the relationship is established, use it to solve for the value of b in $m^{3b-2} = p$.
Convert all radical expressions to expressions with rational exponents. Then, set the exponents equal to each other. Remember to simplify fractions when possible to make calculations easier.
Be careful with the manipulation of exponents and ensure that all expressions are simplified correctly. It's easy to make mistakes when converting radicals to rational exponents, especially when dealing with fractional exponents.
This problem is a classic example of testing the manipulation of radical and rational exponents, which is a common skill in advanced mathematics. It assesses the student's ability to simplify and equate expressions involving these exponents.
Mastery of converting between different forms of exponents and solving equations is crucial for success in similar SAT math problems.

Start by expressing the radical expressions in exponential form:
$m^{\frac{3}{4}} = p^{\frac{4}{6}}$, Simplify the exponent on the right:
$p^{\frac{4}{6}} = p^{\frac{2}{3}}$
Thus, $m^{\frac{3}{4}} = p^{\frac{2}{3}}$.
Isolate the p:
$m^{\frac{3}{4}} = p^{\frac{2}{3}} \to p^{\frac{2}{3}\times\frac{3}{2}} = m^{\frac{3}{4}\times\frac{3}{2}} \to p = m^{\frac{9}{8}}$
Since $p = m^{3b-2}$, $m^{\frac{9}{8}} = m^{3b-2} \to \frac{9}{8} = 3b - 2$
Calculate the equation:
$3b - 2 = \frac{9}{8} \to 3b = \frac{9}{8} + 2 \to 3b = \frac{9}{8} + \frac{16}{8} = \frac{25}{8} \to b = \frac{25}{24}$
Thus, $b = \frac{25}{24}$

9. One solution to the given equation can be expressed as $x = \frac{-9+\sqrt{m}}{2}$, where m is a constant. What is the value of m? $x^2 + 9x + 6 = 0$

Answer

57

Solution

The problem tests the student's ability to solve quadratic equations using the quadratic formula and understand how the discriminant affects the solutions of the equation.
First, recognize that the equation is a quadratic in the standard form $ax^2 + bx + c = 0$. Identify a = 1, b = 7, and c = 6. Use the quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ to find the roots. Compare the given expression $x = \frac{-7+\sqrt{m}}{2}$ with the formula's roots to find m.

Focus on the discriminant part of the quadratic formula, $b^2 - 4ac$, since it is the part under the square root and affects the value of m. Calculate it carefully to determine m.
Be careful with the signs when comparing the roots from the quadratic formula to the given expression. Ensure that you match the correct part of the formula to the given form, as a sign error could lead to an incorrect value of m.
This problem evaluates your understanding of the quadratic formula and your ability to manipulate and compare algebraic expressions. It emphasizes the importance of the discriminant in determining the nature of the solutions. Mastery of this type of problem is crucial for success in advanced math sections of standardized tests like the SAT.

Step 1: Identify the parameters in the quadratic equation. Here, a = 1, b = 7, c = 6.

Step 2: Use the quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

Step 3: Substitute a, b, and c into the quadratic formula: $x = \frac{-(9) \pm \sqrt{9^2-4(1)(6)}}{2(1)}$.

Step 4: Calculate the discriminant: $b^2 - 4ac = 81 - 24 = 57$

Step 5: Substitute the discriminant back into the formula: $x = \frac{-9 \pm \sqrt{57}}{2}$.

Step 6: Compare with the given form $x = \frac{-9+\sqrt{m}}{2}$

Therefore, $m = 57$

10. In the xy-plane, line m has a slope of -3 and a y-intercept of (0, 12). What is the x-coordinate of the x-intercept of line m?

Answer

4

Solution

This problem tests the student's understanding of the equation of a line in slope-intercept form $y = mx + b$ and their ability to determine the x-intercept from this equation.
To find the x-coordinate of the x-intercept, the student must set y = 0 in the equation of the line and solve for x. The equation of the line can be written as $y =- 3x + 12$. Setting y to 0 gives the equation $0 =- 3x + 12$. Solving for x will yield the x-intercept.
Remember that the x-intercept is where the line crosses the x-axis, which means y is zero at this point. You can always use the slope-intercept form of the equation to find intercepts quickly.
Be careful with the signs when solving the equation. It's easy to make a mistake with negative slopes or intercepts, so double-check your calculations.
This problem is a classic test of understanding linear equations and their intercepts. It assesses the ability to manipulate algebraic equations and understand the geometric interpretation of a line's slope and intercepts. Mastery of these concepts is essential for success on the SAT, as they are fundamental to algebra and graph interpretation.

Start with the equation: $0 =- 3x + 12$.
Subtract 12 from both sides: $- 12 =- 3x$.
Divide both sides by -3: $x = \frac{-12}{-3}$.
Simplify the fraction: $x = 4$.