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Digital SAT

Math

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Algebra
Advanced
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2024.10

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Algebra

2024.10

SAT Math Algebra

1. A research study shows that the average number of hours students spend on community service per year, x, can be estimated by the function $f(x) = 5x + 89$. Which statement best interprets the value 89 in this context?

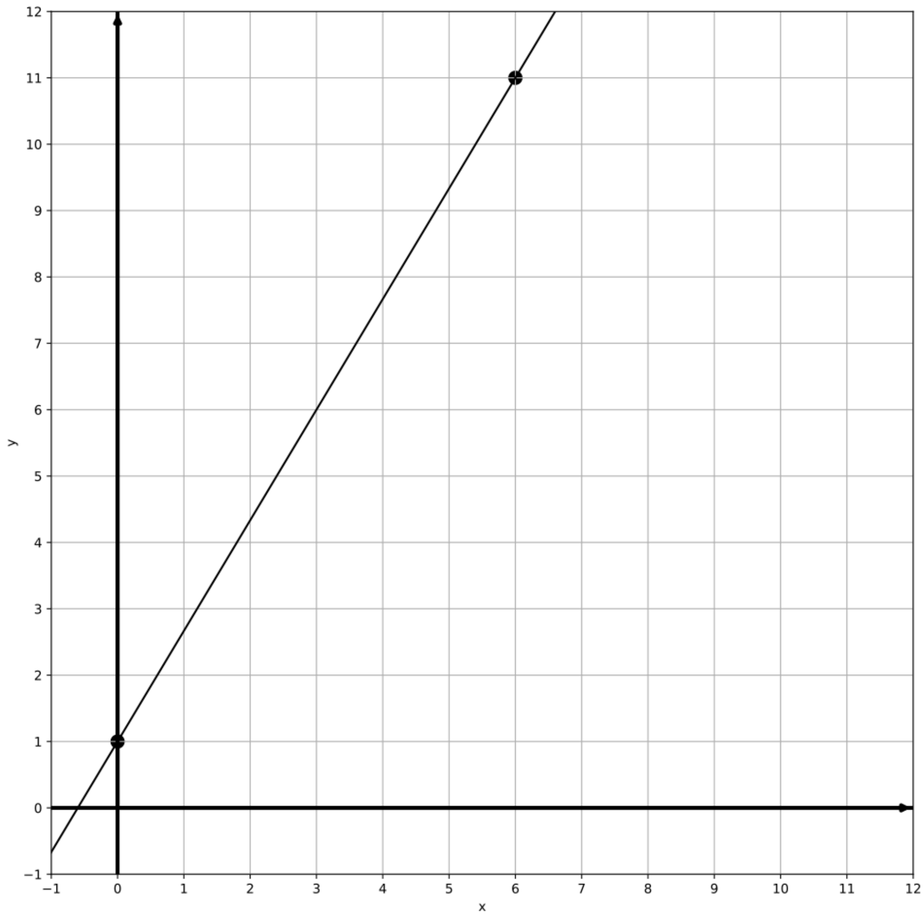
A. Students will spend a total of 89 hours on community service after 5 years.

B. The average number of community service hours is expected to grow by 5 hours each year.

C. The average hours of community service increased from 89 hours after each year.

D. The estimated number of community service hours was 89 hours when no additional hours are counted.

2. The graph of line g is shown in the xy-plane. Line k is defined by $70x + py = w$, where p and w are constants. If line k is graphed in this xy-plane, resulting in the graph of a system of two linear equations, the system of two linear equations will have infinitely many solutions. What is the value of $p + w$?



3. If $86(x + 9) = 36(x + 9) + 150$, what is the value of $x + 9$?

A. -9

B. -6

C. 3

D. 6

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4. Based on the table, which equation represents the linear relationship between the number of diplomatic missions, d, and the associated costs, c?

input	output
83	-4870
54	-2985
14	-385

A. c + 65d = 525

B. c - 65d = -525

C. c + 65d = -525

D. c - 65d = 525

5. The graph of a linear function g(x) is given by the equation $g(x) = 3f(x - 5) + 5$. what is the y-intercept of the linear function f in the xy-plane?

x	g(x)
0	-25
5	20
8	47

A. (0, -10)

B. (0, 0)

C. (0, 5)

D. (0, 10)

6. A space research organization is planning a mission that involves launching two types of satellites. The total cost of launching x number of communication satellites and y number of observation satellites is represented by the equation $65.6x + 287.0y = 7052$. How much more does it cost to launch one observation satellite than one communication satellite?

7. A solar panel's energy output is modeled by the function f, which gives the estimated energy output in watts, x years after the panel was first installed. Based on the equation f(x) = 4x + 55, what does 55 represent in this context?

A. The solar panel will be producing 55 watts each year.

B. The solar panel's energy output will reach 55 watts after 2 years.

C. The estimated energy output of the solar panel was 55 watts when it was first installed.

D. The solar panel is expected to stop working after 55 years.

8. A certain school has a total of 46 students enrolled in a club. The number of students in the club is at least 32 more than the number of students who have signed up for a volunteer activity. If we let x represent the number of students signed up for the volunteer activity, which inequality represents this situation?

A. $x + 32 \leq 46$

B. $x + 46 \geq 32$

C. $x + 32 \geq 46$

D. $x + 46 \leq 32$

9. What is the y-coordinate of the y-intercept of the graph of y = g(x) in the xy-plane? g(x) = f(x) - 8, where f(x) = 7(74x - 22)

A. -176

B. -170

C. -162

D. -154

10. A company produces eco-friendly products. The total revenue of the company in the year 2023 is represented by the equation $831 = 71 + 76(x - 8)$, where x represents the number of years since 2015. If the revenue is expected to reach 831 dollars in 2023, how many years since 2015 has the company been operating?

A. 10 years

B. 15 years

C. 18 years

D. 20 years

SAT Math Algebra Solutions

1. A research study shows that the average number of hours students spend on community service per year, x, can be estimated by the function $f(x) = 5x + 89$. Which statement best interprets the value 89 in this context?

A. Students will spend a total of 89 hours on community service after 5 years.

B. The average number of community service hours is expected to grow by 5 hours each year.

C. The average hours of community service increased from 89 hours after each year.

D. The estimated number of community service hours was 89 hours when no additional hours are counted.

Answer

D

Solution

Concept Check : The intent of this question is to assess the student's understanding of function interpretation, specifically the concept of the y-intercept in the context of a linear equation. The student is expected to recognize what the constant term in the linear equation represents regarding the average number of hours spent on community service.

Solution Strategy : To approach this problem, the student should first identify the components of the linear equation provided, $f(x) = 5x + 89$. The student needs to recognize that the term 'x' represents the number of years, and the function f(x) represents the estimated average hours spent on community service. The constant term '89' should then be interpreted in relation to what it signifies in this context.

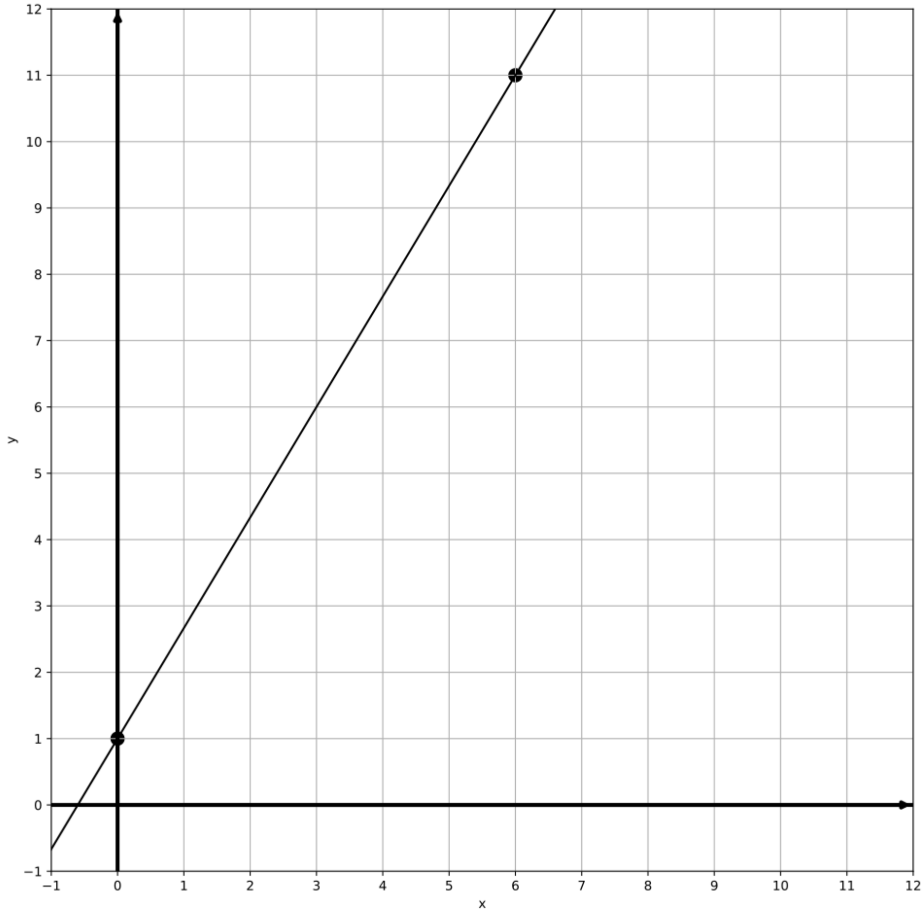
Quick Wins : Consider breaking down the function into its parts: the slope (5) and the y-intercept (89). Remember that the y-intercept indicates the value of the function when x is 0. Think about what it means for students who have not yet spent any time on community service, and how that relates to the average hours in this scenario. This perspective can clarify the meaning of the constant term.

Mistake Alert : Be cautious not to confuse the slope with the y-intercept. The slope (5) indicates how much the average number of hours increases with each additional year, while the y-intercept (89) has a specific meaning that should not be overlooked. Additionally, ensure you are interpreting '89' correctly in the context of the problem—this is not just a number but has relevance to the average hours spent on community service.

SAT Know-How : This problem falls under the category of algebra, specifically focusing on interpreting linear equations and their components. It assesses the student's ability to read and understand the context of mathematical functions. The ability to interpret the y-intercept in a real-world scenario is a valuable skill in SAT problem-solving, highlighting the importance of grasping how mathematical concepts apply beyond mere calculations.

Step 1: Understand the Function $f(x) = 5x + 89$.
The function is in the form $y = mx + b$, where m is the slope and b is the y-intercept.
Step 2: Identify the meaning of the constant term, 89.
In a linear equation, the y-intercept (89 in this case) represents the starting value - the value of the function when $x = 0$.
Step 3: Analyze the context provided by the problem.
In this context, the value 89 represents the estimated initial average hours of community service before any additional time ($x = 0$) is counted.
Step 4: Check each option for alignment with the interpretation of 89.
Option A is incorrect because it confuses the function's application over 5 years.
Option B is incorrect because it describes the slope (5) rather than the y-intercept (89).
Option C is incorrect because it suggests 89 is a change rather than an initial value.
Option D correctly identifies 89 as the initial estimate when no additional hours are factored in.

2. The graph of line g is shown in the xy-plane. Line k is defined by 70x + py = w, where p and w are constants. If line k is graphed in this xy-plane, resulting in the graph of a system of two linear equations, the system of two linear equations will have infinitely many solutions. What is the value of p + w ?



Answer

-84

Solution

Concept Check : The question intends for students to understand the concept of linear equations and their graphical representations. It tests knowledge of how two lines can have infinitely many solutions, which occurs when the lines are identical (i.e., have the same slope and y-intercept). Students should know how to manipulate equations and identify relationships between coefficients.

Solution Strategy : To solve this problem, students should first analyze the graph provided for line g to determine its slope and y-intercept. Then, they need to express line k in slope-intercept form (y = mx + b) to compare it to line g. Since the lines must be identical for there to be infinitely many solutions, students will need to equate the coefficients of x and the constant terms to find the values of p and w.

Quick Wins : 1. Carefully examine the graph of line g to accurately determine its slope and y-intercept. 2. Write the equation of line k in slope-intercept form, which may involve isolating y. 3. Ensure that both lines (g and k) have the same slope and y-intercept, as this is crucial for having infinitely many solutions. 4. Use algebraic techniques to manipulate the equations and solve for p and w effectively.

Mistake Alert : 1. Be cautious with signs when manipulating equations; a small error can lead to incorrect values for p and w. 2. Double-check that both equations represent the same line; ensure that you have correctly identified the slope and y-intercept from the graph. 3. Pay attention to the conditions for infinitely many solutions, as it specifically requires the lines to be identical rather than just parallel.

SAT Know-How : This problem falls under the category of Algebra, specifically focusing on the graphs of linear equations. It assesses the student's ability to analyze linear equations, understand their graphical representations, and manipulate algebraic expressions to find solutions. Mastering this type of problem is essential for SAT success, as it requires a combination of visual interpretation and algebraic reasoning.

1. Find the equation of line g using the points (0, 1) and (6, 11).
The slope of line g is $\frac{11-1}{6-0} = \frac{10}{6} = \frac{5}{3}$.
Using the point (0, 1), the y-intercept b is 1.
Therefore, the equation of line g is $y = \frac{5}{3}x + 1$.
Multiply throughout by 3 to eliminate fractions: 3y = 5x + 3.
Rearranging gives: 5x - 3y = -3.
2. Since line k must be the same as line g for infinitely many solutions:
Line k is given by 70x + py = w.
Comparing with 5x - 3y = -3, we see that line k must be a multiple of line g.
Therefore, $\frac{70}{5} = 14$, so the entire equation must be multiplied by 14.
5x - 3y = -3 becomes 70x - 42y = -42 when multiplied by 14.
Therefore, p = -42 and w = -42.
3. Calculate p + w: p + w = -42 + (-42) = -84.

3. If $86(x + 9) = 36(x + 9) + 150$, what is the value of $x + 9$?

A. -9

B. -6

C. 3

D. 6

Answer

C

Solution

Concept Check : The intent of this question is to assess the student's ability to solve a linear equation using the substitution method. Students are expected to understand how to isolate variables and perform algebraic operations to find the value of 'x + 9.'

Solution Strategy : To approach this problem, the student should start by recognizing that both sides of the equation contain the term '(x + 9).' The first step is to distribute the coefficients (86 and 36) to the term '(x + 9)' on both sides. After simplifying both sides, the student should then rearrange the equation to isolate 'x' or directly solve for 'x + 9' by manipulating the equation appropriately.

Quick Wins : A helpful tip is to combine like terms after distributing the coefficients. This will make it easier to isolate the variable. Also, consider rewriting the equation in terms of 'x + 9' directly, which could simplify your calculations. Always double-check your calculations after each step to ensure accuracy.

Mistake Alert : Students should be cautious about making errors in distribution and combining like terms. It's easy to miscalculate coefficients or to accidentally drop a term when rearranging the equation. Double-check your work to avoid these common mistakes, especially when dealing with negative numbers or when moving terms from one side of the equation to the other.

SAT Know-How : This problem falls under the category of Algebra, specifically focusing on solving linear equations and inequalities through substitution. It assesses the student's skills in distributing terms, combining like terms, and isolating variables. Mastering these techniques is essential for success in SAT math, as it reinforces the foundational skills needed for more complex algebraic concepts.

Step 1: Simplify both sides by subtracting $36(x + 9)$ from both sides of the equation.
$86(x + 9) - 36(x + 9) = 150$

Step 2: Combine like terms on the left side.
$50(x + 9) = 150$
Step 3: Isolate $(x + 9)$ by dividing both sides by 50.
$x + 9 = \frac{150}{50}$
Step 4: Simplify the fraction.
$x + 9 = 3$

4. Based on the table, which equation represents the linear relationship between the number of diplomatic missions, d, and the associated costs, c?

input	output
83	-4870
54	-2985
14	-385

A. c + 65d = 525

B. c - 65d = -525

C. c + 65d = -525

D. c - 65d = 525

Answer

A

Solution

Concept Check : The question is asking the student to identify a linear equation that represents the relationship between two variables: the number of diplomatic missions (d) and the associated costs (c). Students need to understand how to interpret data from a table and the concept of linear relationships, which includes recognizing that they can be expressed in the form of a linear equation ($y = mx + b$).

Solution Strategy : To solve this problem, students should examine the values in the table to determine how changes in the number of diplomatic missions (d) affect the costs (c). They should think about calculating the slope (m) of the line using the formula (change in c)/(change in d) based on two points from the table. Additionally, students should look to identify the y-intercept (b) by establishing the cost when the number of missions is zero, if applicable.

Quick Wins : When examining the table, first identify a pair of (d, c) values to calculate the slope. If possible, choose values that are far apart to minimize calculation errors. After determining the slope, substitute one of the points back into the linear equation format (c = md + b) to solve for b. Make sure to check for consistency across the table to ensure the equation fits all given points.

Mistake Alert : Be careful not to confuse the variables; ensure you are correctly identifying which value corresponds to d and which corresponds to c. Additionally, pay attention to the scale of the table; sometimes the values can be misleading if they are not in a straightforward numerical sequence. Double-check calculations for slope and intercept to avoid simple arithmetic mistakes.

SAT Know-How : This problem is a linear relationship word problem that assesses the student's ability to derive a linear equation from a set of data points. It tests skills in interpreting tables, calculating slopes, and formulating equations. Mastering these concepts is crucial for efficiently solving similar SAT problems and developing a strong foundation in algebra.

Step 1: Calculate the slope (m) using any two points from the table.
Using points (83, -4870) and (54, -2985):
$Slope\ (m) = \frac{c_2 - c_1}{d_2 - d_1} = \frac{-2985+4870}{54-83} = \frac{1885}{-29} = -\ 65.$
Step 2: Use the slope-intercept form $c = md + b$ with one point to find b.
Using point (83, -4870) and $m = -\ 65$:
$-\ 4870 = -\ 65(83) + b \rightarrow -\ 4870 = -\ 5395 + b \rightarrow b = -\ 4870 + 5395 = 525.$
Step 3: Write the equation in the form $c = md + b$.
The equation of the line is $c = -\ 65d + 525$.
Step 4: Rearrange the equation to match the format of the options.
Rearrange c = -65d + 525 to c + 65d = 525.

5. The graph of a linear function g(x) is given by the equation $g(x) = 3f(x - 5) + 5$. what is the y-intercept of the linear function f in the xy-plane?

x	g(x)
0	-25
5	20
8	47

A. (0, -10)

B. (0, 0)

C. (0, 5)

D. (0, 10)

Answer

C

Solution

Concept Check : The question asks students to understand the relationship between a linear function and its transformation. Students are expected to know how to manipulate functions and determine key characteristics, such as the y-intercept, through given values of the transformed function.

Solution Strategy : To solve this problem, students should first analyze the transformation given in the function g. They need to substitute the values of g(0), g(5), and g(8) into the equation to find corresponding values of f. This will involve solving for f(x) based on how g alters the input and output of f. Students should isolate the function f and determine its form to find the y-intercept.

Quick Wins : Start by substituting the known values of g into the transformation equation. This will yield equations in terms of f(x). Once you have those, solve for f(x) in each case. If you can express f(x) in the form of y = mx + b, you can directly identify the y-intercept as b. Also, remember that the y-intercept occurs when x = 0.

Mistake Alert : Be careful with the transformations. The function g involves shifting the input (x - 5) and scaling the output by 3 while also adding 5. Make sure to correctly apply these transformations when substituting values. Additionally, watch out for sign errors when simplifying the equations. Double-check your calculations to avoid misinterpreting the value of f.

SAT Know-How : This problem falls under the category of Algebra, specifically focusing on the graphs of linear equations and functions. It assesses the student's ability to manipulate and understand functions and their transformations, ultimately leading to finding the y-intercept of a linear function. Mastery of function transformations and solving for y-intercepts is crucial for success in SAT math problems.

Step 1: Determine the slope of g(x).
Using g(5) = 20 and g(0) = -25:
$Slope = \frac{20-(-25)}{5-0} = \frac{45}{5} = 9.$
Step 2: Determine the equation of g(x) using one point (x, g(x)) = (5, 20):
We use the point-slope form: $g(x) = 9(x - 5) + 20.$
$g(x) = 9x - 45 + 20 = 9x - 25.$
Step 3: Compare g(x) definition with given transformation $g(x) = 3f(x - 5) + 5.$
$9x - 25 = 3f(x - 5) + 5.$
$9x - 30 = 3f(x - 5).$
$f(x - 5) = \frac{9x-30}{3} = 3x - 10.$
$f(x) = 3(x + 5) - 10 = 3x + 15 - 10 = 3x + 5.$
Step 4: The y-intercept of f(x) is found by setting x = 0.
$f(0) = 3(0) + 5 = 5.$
Thus, the y-intercept of function f is (0, 5).

6. A space research organization is planning a mission that involves launching two types of satellites. The total cost of launching x number of communication satellites and y number of observation satellites is represented by the equation $65.6x + 287.0y = 7052$. How much more does it cost to launch one observation satellite than one communication satellite?

Answer

221.4

Solution

Concept Check : The question aims to assess the student's understanding of linear equations and their ability to analyze cost relationships in a real-world context. Students are expected to recognize how the coefficients in the equation represent costs associated with each type of satellite.

Solution Strategy : To solve the problem, students should focus on the coefficients of x and y in the given equation, which represent the cost per satellite for communication and observation satellites, respectively. The goal is to determine the difference in costs between launching one observation satellite and one communication satellite, which can be found by subtracting the coefficient of x from the coefficient of y.

Quick Wins : Pay close attention to the coefficients in the equation. Since the equation is in the form of total cost based on the number of satellites, the numbers in front of x and y will directly give you the cost per type of satellite. A good strategy is to clearly identify these coefficients and perform the subtraction to find the cost difference.

Mistake Alert : Be careful not to confuse the variables x and y with their coefficients. It's important to remember that x represents the number of communication satellites and y represents the number of observation satellites. Also, ensure that you are subtracting the correct coefficients to find the cost difference.

SAT Know-How : This problem falls under the category of algebra, specifically linear relationships in word problems. It tests the student's ability to interpret a linear equation in terms of real-world applications and to analyze costs associated with different variables. Mastering this type of problem helps develop skills in understanding linear relationships and applying algebraic concepts to practical situations, which is a key component of SAT problem-solving.

Step 1: Identify the cost of launching one observation satellite and one communication satellite from the equation coefficients.
Step 2: The cost of launching one communication satellite is $65. 6.

Step 3: The cost of launching one observation satellite is $287.0.
Step 4: Calculate the difference in cost by subtracting the cost of one communication satellite from the cost of one observation satellite.
Step 5: $Difference = 287.0 - 65.6 = 221.4$.

7. A solar panel's energy output is modeled by the function f, which gives the estimated energy output in watts, x years after the panel was first installed. Based on the equation $f(x) = 4x + 55$, what does 55 represent in this context?

A. The solar panel will be producing 55 watts each year.

B. The solar panel's energy output will reach 55 watts after 2 years.

C. The estimated energy output of the solar panel was 55 watts when it was first installed.

D. The solar panel is expected to stop working after 55 years.

Answer

C

Solution

Concept Check : The intent of the question is to assess the student's understanding of linear functions and their components, specifically identifying the meaning of the y-intercept in the context of a real-world scenario.

Solution Strategy : To approach this problem, the student should recognize that the given equation $f(x) = 4x + 55$ is in slope-intercept form, which is generally represented as $y = mx + b$, where m is the slope and b is the y-intercept. The student should focus on the value of 55, which is the constant term, and interpret its significance in the context of the problem, considering what it represents in relation to the solar panel's energy output.

Quick Wins : A useful tip is to remember that the y-intercept (the constant term) represents the value of the function when x is zero. In this context, think about what happens at the start, when the solar panel has just been installed. Additionally, consider the relationship between the x variable (time in years) and the output; this can help clarify the meaning of the constant.

Mistake Alert : Be careful not to confuse the slope with the y-intercept. The slope (4 in this case) indicates how much the energy output increases each year, while the y-intercept (55) shows the initial output. Misinterpreting these values can lead to incorrect conclusions about the solar panel's performance.

SAT Know-How : This problem belongs to the category of algebra, specifically focusing on linear equations and their interpretation in real-world scenarios. It assesses the student's ability to analyze and understand the components of a linear equation, particularly the significance of the y-intercept in context. Mastering these concepts is essential for solving SAT problems efficiently and accurately.

The function given is $f(x) = 4x + 55$.
The term $4x$ represents the change in energy output per year.
The constant 55 is the y-intercept of the function.
The y-intercept indicates the value of $f(x)$ when $x = 0$ years.
Therefore, $f(0) = 4(0) + 55 = 55$.
This means that when the solar panel was first installed ($x = 0$), the estimated energy output was 55 watts.

8. A certain school has a total of 46 students enrolled in a club. The number of students in the club is at least 32 more than the number of students who have signed up for a volunteer activity. If we let x represent the number of students signed up for the volunteer activity, which inequality represents this situation?

A. $x + 32 \leq 46$

B. $x + 46 \geq 32$

C. $x + 32 \geq 46$

D. $x + 46 \leq 32$

Answer

A

Solution

Concept Check : The intent of the question is to assess the student's understanding of linear inequalities, particularly in the context of a real-world scenario. The student is expected to be familiar with how to translate verbal descriptions into mathematical inequalities and understand the relationship between the total number of students in the club and those signed up for a volunteer activity.

Solution Strategy : To approach this problem, the student should focus on identifying the key components of the situation presented. The total number of students in the club is 46, and there is a relationship described between the number of students in the club and those signed up for the volunteer activity. The student should express this relationship as an inequality, taking into account that the number of students in the club (which is 46) is at least 32 more than the number of students signed up (represented by x). This leads to the formulation of an inequality that captures this relationship.

Quick Wins : When translating word problems to inequalities, it is helpful to break down the problem into smaller parts. Identify what you know (in this case, the total number of students and the relationship described) and define your variable clearly. Use phrases like 'at least' to recognize that you will use greater than or equal to (≥) in your inequality. Remember to carefully consider the direction of the inequality based on the wording of the problem.

Mistake Alert : Be careful with the mathematical symbols and terms used in the problem. Phrases like 'at least' indicate that you should use a greater than or equal to symbol (≥), while 'more than' would use a greater than symbol (>). Additionally, ensure you correctly interpret the relationship between the total number of students and those signed up for the volunteer activity to avoid misformulating the inequality.

SAT Know-How : This problem belongs to the Algebra category, specifically focusing on linear inequality word problems. It assesses the student's ability to interpret a real-world scenario and convert it into a mathematical inequality. Understanding how to extract relevant information and translate it accurately into inequalities is a critical skill for the SAT, and practicing this will aid in developing problem-solving proficiency.

Identify the given relationship: The number of students in the club (46) is at least 32 more than x.
Express this relationship mathematically: 46 is at least $x + 32$.
This can be transformed into an inequality: $46 \geq x + 32$.
Rearrange the terms to find the correct inequality: $x + 32 \leq 46$.
Therefore, the inequality representing the situation is $x + 32 \leq 46$.

9. What is the y-coordinate of the y-intercept of the graph of $y = g(x)$ in the xy-plane? $g(x) = f(x) - 8$, where $f(x) = 7(74x - 22)$

A. -176

B. -170

C. -162

D. -154

Answer

C

Solution

Concept Check : The intent of the question is to assess the student's understanding of composite functions and their ability to find the y-intercept of a linear function. Students are expected to know how to manipulate functions and understand the concept of y-intercepts, which is found by evaluating the function at $x = 0$.

Solution Strategy : To solve this problem, the student should first substitute $x = 0$ into the function $g(x)$ to find the y-coordinate of the y-intercept. Since $g(x)$ is defined in terms of $f(x)$, the student will need to evaluate $f(0)$ first, then apply the transformation defined by $g(x) = f(x) - 8$. This involves calculating $f(0)$, subtracting 8, and arriving at the y-coordinate of the y-intercept for $g(x)$.

Quick Wins : Remember that the y-intercept occurs where $x = 0$. Therefore, always start by substituting $x = 0$ into the function you are analyzing. For the given functions, carefully compute $f(0)$ first, and then apply any transformations to find $g(0)$. Also, keep track of the order of operations when substituting and calculating values.

Mistake Alert : Be cautious while calculating the value of $f(0)$; it is easy to make arithmetic mistakes, especially when dealing with coefficients and constants. Ensure that you are applying the subtraction in $g(x)$ correctly after evaluating $f(0)$ to avoid errors in the final result. Additionally, double-check your calculations to prevent misreading the function definitions.

SAT Know-How : This problem falls under the category of Algebra, specifically focusing on the graphs of linear equations and functions. It tests the student's ability to evaluate composite functions and find y-intercepts, which are essential skills for understanding linear relationships in the SAT. Mastering this type of problem enhances your problem-solving skills and prepares you for similar questions on the exam.

1. Substitute x = 0 into f(x) to find f(0):
f(x) = 7(74x - 22)
f(0) = 7(74(0) - 22) = 7(-22) = -154
2. Use the result from f(0) to find g(0):
g(x) = f(x) - 8
g(0) = f(0) - 8 = -154 - 8 = -162
3. Therefore, the y-coordinate of the y-intercept is -162.

10. A company produces eco-friendly products. The total revenue of the company in the year 2023 is represented by the equation $831 = 71 + 76(x - 8)$, where x represents the number of years since 2015. If the revenue is expected to reach 831 dollars in 2023, how many years since 2015 has the company been operating?

A. 10 years

B. 15 years

C. 18 years

D. 20 years

Answer

C

Solution

Concept Check : The intent of the question is to assess the student's ability to interpret a linear equation in the context of a real-life scenario. Students are expected to understand how to manipulate and solve linear equations, and they should be familiar with the concepts of revenue, time variables, and how to relate them in the context of the problem.

Solution Strategy : To approach this problem, students should first identify what the variable 'x' represents in the context of the problem, which is the number of years since 2015. Then, they should recognize that the equation given can be rearranged to isolate 'x'. This will require applying algebraic principles, such as distributing terms, combining like terms, and solving for the variable. It's essential to keep track of the relationships between the years and the revenue stated in the equation.

Quick Wins : A helpful tip is to break down the equation step-by-step. Start by simplifying the right side and isolating 'x' on one side of the equation. It can also be beneficial to convert the equation into a more familiar linear form. Additionally, double-check your calculations at each step to ensure accuracy. Finally, remember to interpret the final value of 'x' in the context of the problem to ensure it makes sense.

Mistake Alert : Students should be careful not to make common mistakes such as misreading the equation or mistakenly adding or subtracting terms incorrectly. Additionally, pay attention to the meaning of the variable 'x' to avoid misinterpreting what the solution represents. It's also important to check that the context of the problem aligns with the solution you arrive at, ensuring that it is a reasonable answer in terms of the years since the company began operating.

SAT Know-How : This problem falls under the category of Algebra, specifically focusing on linear equation word problems. It assesses the student's skills in interpreting, manipulating, and solving linear equations in real-world contexts. Mastery of these skills is essential for success on the SAT, as it demonstrates the ability to connect mathematical concepts to practical situations.

1. Begin by simplifying the equation: $831 = 71 + 76(x - 8)$.
2. Distribute 76 in the term $76(x - 8)$: $76x - 608$.
3. The equation becomes: $831 = 71 + 76x - 608$.
4. Combine like terms on the right side: $71 - 608 =- 537$.
5. The equation now is: $831 = 76x - 537$.
6. Add 537 to both sides to isolate the term with x: $831 + 537 = 76x$.
7. This simplifies to: $1368 = 76x$.
8. Divide both sides by 76 to solve for x: $x = \frac{1368}{76}$.
9. Calculate the division: $x = 18$.
10. Therefore, the company has been operating for 18 years since 2015.

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Math

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Advanced

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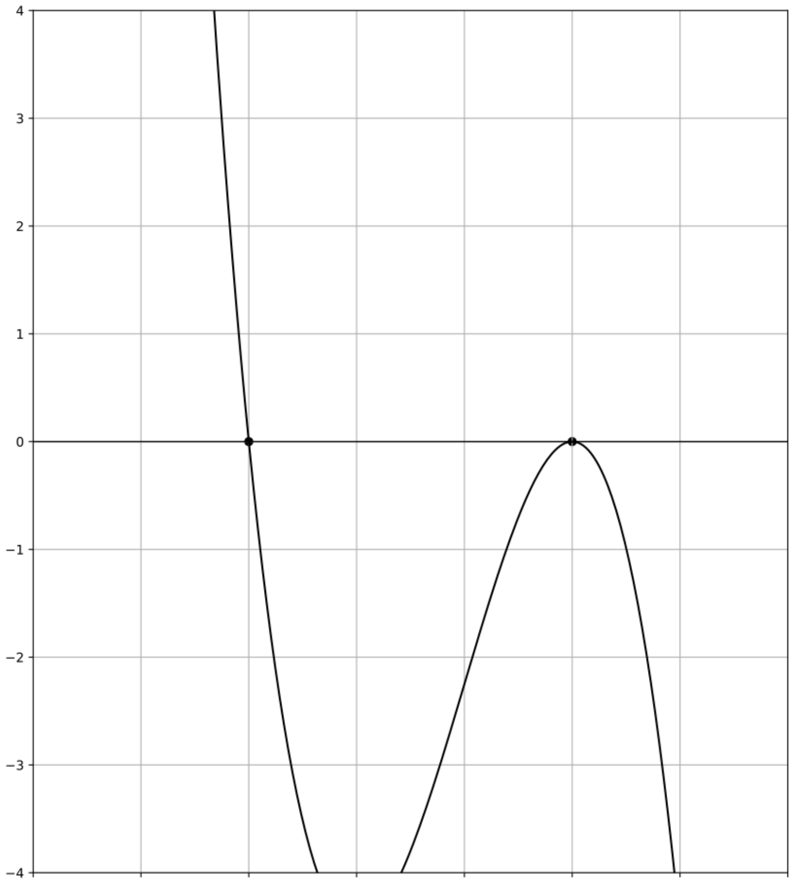
SAT Math Advanced

1. The graph of y = f(x) is shown where $f(x) = ax^3 + bx^2 + cx + d$ and a, b, c and d are constants. For how many distinct values of x does f(x) = 0?



A. One

B. Two

C. Three

D. Four

2. In a certain diplomatic negotiation, the population of a small island nation represented by a particular ethnic group is modeled by the function $f(x) = 8200(0.27)^x$, where x is the number of years since 2010. Which of the following best describes the value 8200 in this context?

A. The estimated population after 10 years f(10) in 2020.

B. The estimated percentage decline in population each year after 2010.

C. The initial population of the ethnic group in the year 2010.

D. The population of the ethnic group at the end of 2010.

3. The function f is defined by $f(x) = 7x^2 + 6x - 38$. What is the value of $f(2)$?

4. The product of a positive number x and the number that is 67 less than x is equal to 1634. What is the value of x?

A. -19

B. 54

C. 67

D. 86

5. For the given function f, the graph of $y = f(x)$ in the xy-plane passes through the point (0, b), where b is a constant. What is the value of b?

$f(x) = 2x^3 - 5x^2 + 9x + 81$

6. The equation $4|x - 97| + 7 = k^2 + 110k + 3032$ has exactly one solution for the variable x. Which of the following could be the value of k?

A. -55 only

B. 0 only

C. -55 and 0

D. -55 and 55

7. The equation $-5|x - 99| - 9 = k^2 - 100k + 2491$ has exactly one solution for the value of k. Which of the following could be the value of k?

A. 50 only

B. -50 and 0

C. -50 only

D. 50 and -50

8. Given that n and k are numbers greater than 1, and $\sqrt[9]{n^{17}} = \sqrt[15]{k^7}$, for what value of a is $n^{3a+2} = k$?

A. 43

B. $\frac{43}{63}$

C. $\frac{85}{63}$

D. $\frac{43}{21}$

9. Which expression is equivalent to $-36x^3 + 54x^2y + 8xy^2 - 12y^3$?

A. $(12x^2 - 4y^2)(-3x + 5y)$

B. $(6x^2 + 3y^2)(-6x + 4y)$

C. $(9x^2 - 2y^2)(-4x + 6y)$

D. $(3x^2 + 2y^2)(-12x + 2y)$

10. What is not an x-coordinate of an x-intercept of the graph $y = 4(x - 7)(x + 9)(x - 2)$ in the xy-plane?

A. -9

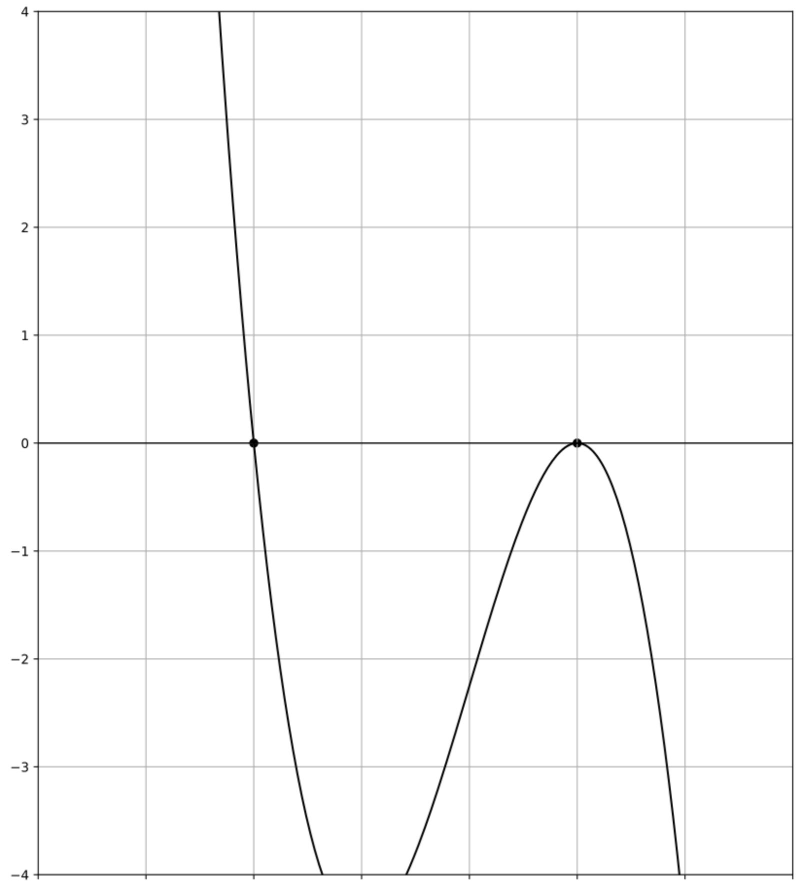
B. 2

C. 7

D. 10

SAT Math Advanced Solutions

1. The graph of y = f(x) is shown where $f(x) = ax^3 + bx^2 + cx + d$ and a, b, c and d are constants. For how many distinct values of x does f(x) = 0?



A. One

B. Two

C. Three

D. Four

Answer

B

Solution

Concept Check : The intent of this question is to assess the student's understanding of polynomial functions, specifically cubic polynomials, and their behavior regarding the number of real roots. Students are expected to know how to analyze the graph of a cubic function to determine the number of distinct x-values at which the function equals zero.

Solution Strategy : To approach this problem, students should examine the graph of the polynomial function provided. They will need to identify the points where the graph intersects the x-axis, as these points correspond to the values of x that satisfy f(x) = 0. Students should also consider the shape of the graph, including the number of turning points and the general behavior of cubic functions, which can give clues about the number of real roots.

Quick Wins : When analyzing the graph, pay attention to the following: 1) Each intersection with the x-axis represents a distinct real root. 2) Count the intersections carefully, as a cubic function can have one, two, or three real roots. 3) If the graph is tangent to the x-axis at a point, this counts as a double root (i.e., one distinct solution). 4) Look for any symmetry or other characteristics of the graph that might simplify your analysis.

Mistake Alert : Be cautious not to overlook any intersections with the x-axis, especially in cases where the curve is very close to the x-axis or where it might appear to just touch the axis without crossing it. Double-check to ensure that points of tangency are counted correctly as distinct roots. Additionally, be aware of any potential misinterpretation of the graph's scale, which can lead to inaccurate counting.

SAT Know-How : This problem falls under the category of Advanced Math and specifically focuses on polynomial functions and their graphs. It assesses the student's ability to identify the number of distinct real solutions to a polynomial equation by interpreting the graphical representation. The skills tested here include understanding polynomial behavior and root finding, which are crucial for success on the SAT.

To find the number of distinct values of x where f(x) = 0, observe the provided roots and the behavior of the graph.
The graph is intersecting the x-axis at two distinct points.
Check for additional roots: The nature of a cubic polynomial implies it can have a third root, which isn't provided explicitly here.
The polynomial form and provided graph data suggest the third root might be a repeated one at either of the given intersections or another unexplored intersection point.

Without more explicit intersections given, the graph might suggest a situation with a triple root or a hidden third root, but given data explicitly shows two distinct real roots.

2. In a certain diplomatic negotiation, the population of a small island nation represented by a particular ethnic group is modeled by the function $f(x) = 8200(0.27)^x$, where x is the number of years since 2010. Which of the following best describes the value 8200 in this context?

A. The estimated population after 10 years f(10) in 2020.

B. The estimated percentage decline in population each year after 2010.

C. The initial population of the ethnic group in the year 2010.

D. The population of the ethnic group at the end of 2010.

Answer

C

Solution

Concept Check : The question is asking students to understand the context of an exponential function, specifically identifying what the coefficient (8200) represents in relation to the modeled population. Students are expected to know how exponential functions function in real-world scenarios, especially regarding growth or decay over time.

Solution Strategy : To solve this problem, students should first recognize the structure of the exponential function provided. The base of the exponent (0.27) indicates a decay factor, suggesting the population is decreasing. The coefficient (8200) typically represents the initial value or starting population at the beginning of the time frame being considered (in this case, the year 2010). Students should analyze the implications of this coefficient in the context of the population's behavior over the years.

Quick Wins : When interpreting coefficients in exponential functions, always consider the context of the problem. Identify what the function models and how the parameters relate to real-world quantities. In exponential decay models, the coefficient often represents the initial amount before any decrease occurs. It can be helpful to sketch a quick graph of the function to visualize how the initial value affects the overall trend.

Mistake Alert : Be careful not to confuse the coefficient with the base of the exponent. The base indicates the rate of change (growth or decay), while the coefficient reflects the starting amount. Additionally, ensure you are clear about what 'x' represents; in this case, it's the number of years since 2010, not the population itself.

SAT Know-How : This problem falls under the category of Advanced Math, specifically focusing on quadratic and exponential word problems. It assesses students' ability to interpret parameters in an exponential model within a real-world context. Understanding the meaning of coefficients and their implications in exponential functions is crucial for solving similar problems on the SAT, showcasing the importance of grasping the relationship between mathematical expressions and their practical applications.

To solve this problem, we need to understand the components of the exponential function $f(x) = 8200(0.27)^x$.

In general, an exponential function of the form $f(x) = a(b)^x$, where a is the initial value or the value when x=0, and b is the base of the exponential function which indicates the growth or decay factor.

In the given function $f(x) = 8200(0.27)^x$, the value 8200 is the coefficient of the exponential term, which represents the initial population size.
When x = 0, which corresponds to the year 2010,
$f(0) = 8200(0.27)^0 = 8200 \times 1 = 8200$.
Thus, 8200 represents the population of the ethnic group at the start of the model, which is the population in the year 2010.

3. The function f is defined by $f(x) = 7x^2 + 6x - 38$. What is the value of $f(2)$?

Answer

2

Solution

Concept Check : The intent of this question is to assess the student's understanding of evaluating a quadratic function. Students are expected to know how to substitute a given value into a function and compute the result, demonstrating their proficiency with basic algebraic operations.

Solution Strategy : To solve this problem, the student should start by substituting the given value, which is 2, into the function $f(x)$. This involves replacing x with 2 in the expression $7x^2 + 6x - 38$. After substitution, they will need to follow the order of operations (PEMDAS/BODMAS) to simplify the expression step by step.

Quick Wins : When substituting, it's helpful to write out the function clearly and confirm each step as you go. Break down the calculation into manageable parts, calculating $7(2^2)$, $6(2)$, and then combining these results along with the constant term -38. This systematic approach can minimize errors.

Mistake Alert : Be careful with arithmetic, especially when squaring numbers and performing addition or subtraction. It's easy to make mistakes with signs, so double-check your calculations to ensure accuracy. Also, remember to follow the order of operations closely.

SAT Know-How : This problem falls under the category of Advanced Math, focusing on nonlinear functions, particularly quadratics. It assesses the student's skill in evaluating functions and performing algebraic operations. Mastering such problems enhances problem-solving capabilities and prepares students for the types of questions they may encounter on the SAT.

Start with the function: $f(x) = 7x^2 + 6x - 38$.
Substitute $x = 2$ into the function: $f(2) = 7(2)^2 + 6(2) - 38$.
Calculate $7(2)^2$: $2^2 = 4$, so $7(4) = 28$.
Calculate $6(2)$: $6 \times 2 = 12$.
Combine the terms: $28 + 12 - 38$.
Perform the arithmetic: $28 + 12 = 40$.
Subtract 38 from 40: $40 - 38 = 2$.
Thus, the value of $f(2)$ is 2.

4. The product of a positive number x and the number that is 67 less than x is equal to 1634. What is the value of x?

A. -19

B. 54

C. 67

D. 86

Answer

D

Solution

Concept Check : The intent of this question is to assess the student's understanding of quadratic equations. The student is expected to recognize how to set up an equation based on a word problem, specifically involving the product of two expressions and a constant. This requires knowledge of forming and solving quadratic equations.

Solution Strategy : To approach the problem, the student should first identify the two expressions mentioned: one is the positive number x, and the other is the number that is 67 less than x, which can be expressed as (x - 67). The next step is to set up the equation by multiplying these two expressions and setting it equal to 1634. This will lead to the formation of a quadratic equation, which can then be rearranged into standard form for solving.

Quick Wins : When forming the equation, be sure to carefully translate the words into mathematical expressions. Remember that 'the product of x and (x - 67)' means you will multiply these two terms. After setting up the equation, try to simplify it as much as possible before applying methods such as factoring, completing the square, or using the quadratic formula to solve for x. Double-check your arithmetic as you go.

Mistake Alert : One common mistake is misinterpreting the relationship between x and (x - 67). Ensure that you correctly express '67 less than x' as (x - 67). Additionally, watch out for errors when expanding the product; it's easy to miscalculate or forget to combine like terms. Finally, remember that since x is a positive number, you should verify that your solution is a valid positive value after solving the quadratic equation.

SAT Know-How : This problem falls under the category of Advanced Math, specifically within the unit of solving quadratic equations. It requires students to apply their skills in translating verbal expressions into mathematical equations and solving quadratic equations effectively. Overall, this problem assesses the student's ability to understand relationships and perform algebraic manipulations, which are critical skills for success on the SAT.

1. Set up the equation based on the problem statement:
$x(x - 67) = 1634$
2. Expand the equation:
$x^2 - 67x = 1634$
3. Rearrange the equation to standard quadratic form ($ax^2 + bx + c = 0$):
$x^2 - 67x - 1634 = 0$
4. Solve the quadratic equation using the quadratic formula:
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
where a = 1, b = -67, c = -1634
5. Calculate the discriminant:
$b^2 - 4ac = (-67)^2 - 4(1)(-1634)$
$b^2 - 4ac = 4489 + 6536$
$b^2 - 4ac = 11025$
6. Since the discriminant is a perfect square, calculate the roots:
$x = \frac{67 \pm \sqrt{11025}}{2}$
$x = \frac{67 \pm 105}{2}$
7. Calculate x for both cases:
$x_1 = \frac{67+105}{2} = \frac{172}{2} = 86$
$x_2 = \frac{67-105}{2} = \frac{-38}{2} = -19$
8. Since x must be positive, x = 86.

5. For the given function f, the graph of $y = f(x)$ in the xy-plane passes through the point (0, b), where b is a constant. What is the value of b?

$f(x) = 2x^3 - 5x^2 + 9x + 81$

Answer

81

Solution

Concept Check : The intent of the question is to assess the student's understanding of polynomial functions and their graphs, specifically the concept of finding the y-intercept of a function. The y-intercept occurs where x = 0, so the student is expected to substitute 0 into the polynomial function to find the corresponding y value, which is represented by the constant b.

Solution Strategy : To approach this problem, the student should recognize that the y-intercept is found by evaluating the function f at x = 0. This involves substituting 0 into the polynomial $f(x) = 2x^3 - 5x^2 + 9x + 81$ and calculating the resulting value. The thought process should focus on plugging in x = 0 and simplifying the expression to directly find the value of b.

Quick Wins : A useful tip is to remember that the y-intercept is simply the value of the function when x equals 0. For higher-degree polynomials, always start by substituting 0 for x and simplify step by step. Additionally, keep in mind the coefficients of the polynomial; when x is 0, all terms containing x will drop out, leaving only the constant term, which simplifies the calculation.

Mistake Alert : Be careful when substituting x = 0; it's easy to miscalculate the terms if you're not careful with the signs or constants. Also, make sure to double-check each term in the polynomial to ensure you haven't skipped or incorrectly computed any of them. It's also important to remember that only the constant term remains when substituting x = 0.

SAT Know-How : This problem falls under the category of Advanced Math, specifically involving polynomial functions and their properties. It tests the student's ability to find the y-intercept of a polynomial function by evaluating it at x = 0. Mastering this concept is crucial for success in the SAT, as it helps in understanding how polynomial graphs behave and provides a foundation for solving more complex equations.

To find b, evaluate the function at x = 0.
Substitute x = 0 into f(x): $f(0) = 2(0)^3 - 5(0)^2 + 9(0) + 81$.
Perform each operation: $2(0)^3 = 0$, $-5(0)^2 = 0$, $9(0) = 0$, and the constant 81 remains.

Therefore, $f(0) = 0 + 0 + 0 + 81 = 81$.
Thus, the value of b is 81.

6. The equation $4|x - 97| + 7 = k^2 + 110k + 3032$ has exactly one solution for the variable x. Which of the following could be the value of k?

A. -55 only

B. 0 only

C. -55 and 0

D. -55 and 55

Answer

A

Solution

Concept Check : The intent of this question is to assess the student's understanding of absolute value equations and how the number of solutions can be determined based on the properties of the equations involved. Students should be familiar with the concept of absolute values and the conditions under which an equation has one solution, particularly in relation to quadratic functions.

Solution Strategy : To solve this problem, students should start by isolating the absolute value expression on one side of the equation. They will need to recognize that for the equation to have exactly one solution, the expression inside the absolute value must equal zero at that solution, and the quadratic expression on the right side must also take a specific form. This typically involves finding the vertex of the quadratic function and ensuring it touches the x-axis at exactly one point. Students should think about how to manipulate the equation to find values of k that meet these conditions.

Quick Wins : Consider rewriting the equation in a form that allows you to analyze the absolute value term. Remember that an absolute value equation has one solution when the expression inside the absolute value is zero, or when the graph of the quadratic touches the line represented by the absolute value. You may want to identify the vertex of the quadratic equation on the right side and find values of k that make it equal to the expression represented by the absolute value. Calculating the discriminant of the quadratic can also provide insight into the number of solutions.

Mistake Alert : Be careful not to overlook the conditions for the absolute value to yield exactly one solution. It's crucial to ensure that the quadratic's vertex aligns correctly for it to touch the axis at just one point. Additionally, pay attention to any algebraic manipulations and signs when isolating the absolute value term, as mistakes in these steps can lead to incorrect conclusions about the number of solutions.

SAT Know-How : This problem falls under the category of Advanced Math, specifically focusing on radical, rational, and absolute value equations. It assesses skills related to manipulating and analyzing equations to determine the number of solutions. The key takeaway is to understand how absolute values interact with quadratic functions and to use properties of both to solve for possible values of k. Mastering these concepts is essential for effective problem-solving in SAT math.

Step 1: Start with the absolute value equation:
$4|x - 97| + 7 = k^2 + 110k + 3032$.
Step 2: Isolate the absolute value term: $4|x - 97| = k^2 + 110k + 3025$.
Step 3: Divide both sides by 4: $|x - 97| = \frac{k^2+110k+3025}{4}$.
Step 4: For |x - 97| to have exactly one solution, the expression $\frac{k^2+110k+3025}{4}$ must be 0.
Step 5: Solve the equation: $k^2 + 110k + 3025$ = 0.
Step 6: Factor the quadratic equation: $k^2 + 110k + 3025 = (k + 55)^2$.
Step 7: Set the factored expression to 0: $(k + 55)^2 = 0$.
Step 8: Solve for k: k + 55 = 0, so k = -55.

7. The equation $-5|x-99|-9=k^2-100k+2491$ has exactly one solution for the value of k. Which of the following could be the value of k?

A. 50 only

B. -50 and 0

C. -50 only

D. 50 and -50

Answer

A

Solution

Concept Check : The question aims to assess the student's understanding of absolute value equations and their properties. Students are expected to know how to manipulate and analyze equations, particularly how to set conditions for the number of solutions an equation can have, especially when dealing with absolute values.

Solution Strategy : To approach this problem, students should recognize that the absolute value equation can be rewritten to isolate the absolute value expression. They need to analyze the right side of the equation, which is a quadratic expression in k, and determine under what conditions this entire equation will yield exactly one solution. This involves understanding the characteristics of quadratic functions and how they relate to the number of solutions of the equation.

Quick Wins : A helpful strategy is to first simplify the equation to isolate the absolute value. Then, consider the implications of having exactly one solution: this typically occurs when the expression inside the absolute value equals zero, or when the quadratic has one real solution (the vertex lies on the x-axis). Checking the discriminant of the quadratic (b^2 - 4ac) can also be useful; for one solution, the discriminant should equal zero.

Mistake Alert : Students should be careful with the manipulation of the quadratic equation, ensuring they do not overlook the conditions under which it provides one solution. Additionally, they should be mindful of the absolute value properties, as mistakes may arise in interpreting the outcomes of the absolute value equation. Be cautious about sign changes when isolating the variable and setting up the equations for both cases of the absolute value.

SAT Know-How : This problem falls under the category of Advanced Math, focusing on radical, rational, and absolute value equations. It evaluates the student's ability to analyze absolute value equations and quadratic expressions, testing their understanding of solution conditions. Mastery of these skills is essential for effective problem-solving on the SAT, particularly in recognizing how to derive conditions for specific numbers of solutions.

Rearrange the given equation: -5|x - 99| - 9 = k^2 - 100k + 2491
Add 9 to both sides: -5|x - 99| = k^2 - 100k + 2500
Divide by -5: $|x - 99| = -\frac{k^2-100k+2500}{5}$
For the equation to have exactly one solution, |x - 99| must be zero.
Set the quadratic expression to zero: $-\frac{k^2-100k+2500}{5} = 0$
Solve for k: k^2 - 100k + 2500 = 0
This is a quadratic equation. Solve using the completing square method or quadratic formula:
Quadratic formula: $k = \frac{100\pm\sqrt{(100)^2-4\times1\times2500}}{2}$
$k = \frac{100\pm\sqrt{10000-10000}}{2}$
k = 50
Thus, the equation has exactly one solution when k = 50.

8. Given that n and k are numbers greater than 1, and $\sqrt[9]{n^{17}} = \sqrt[15]{k^{7}}$, for what value of a is $n^{3a+2} = k$?

A. 43

B. $\frac{43}{63}$

C. $\frac{85}{63}$

D. $\frac{43}{21}$

Answer

B

Solution

Concept Check : The intent of this question is to test the student's understanding of radical and rational exponents, specifically how to manipulate and equate expressions involving roots and powers. Students are expected to know how to express radicals as exponents and how to solve equations involving these expressions.

Solution Strategy : To approach this problem, the student should start by rewriting the radical expressions as fractional exponents. For the equation $\sqrt[9]{n^{17}}$, this becomes $n^{\frac{17}{9}}$, and for $\sqrt[15]{k^{7}}$, it becomes $k^{\frac{7}{15}}$. After rewriting, the student can set the two expressions equal to each other, giving $n^{\frac{17}{9}} = k^{\frac{7}{15}}$. Then, the next step would be to express k in terms of n using the relationship given by $n^{3a+2} = k$, which will lead to an equation involving a single variable, making it easier to solve for 'a'.

Quick Wins : When converting radicals to exponents, remember that the root indicates the denominator of the exponent. Keep track of your fractional exponents and ensure you simplify them correctly. Also, when working with equations that involve powers, it can be helpful to express both sides in terms of a common base if possible. This will often make it easier to isolate the variable you're solving for.

Mistake Alert : Be careful with the laws of exponents when you manipulate the equations. It's easy to mix up multiplication and addition of exponents. Additionally, remember that when you raise an exponent to another power, you multiply the exponents. Double-check your calculations, especially when dealing with fractional values, as these can lead to mistakes if not handled carefully.

SAT Know-How : This problem falls under the category of Advanced Math, specifically focusing on radical and rational exponents. It assesses the student's ability to manipulate and solve equations involving exponents and roots. By practicing problems like this, students can develop a stronger grasp of exponent rules and improve their problem-solving skills, which is vital for success on the SAT.

Step 1: Express the roots as exponents:
$\sqrt[9]{n^{17}} = n^{\frac{17}{9}}$
$\sqrt[15]{k^{7}} = k^{\frac{7}{15}}$
Now equate these two expressions:
$n^{\frac{17}{9}} = k^{\frac{7}{15}}$
Step 2: Solve for k in terms of n:
Raise both sides to the power of $\frac{15}{7}$:
$\left(n^{\frac{17}{9}}\right)^{\frac{15}{7}} = \left(k^{\frac{7}{15}}\right)^{\frac{15}{7}}$
$n^{\frac{17}{9}\cdot\frac{15}{7}} = k^{1} = k$
Calculate $\frac{17}{9} \cdot \frac{15}{7}$:
$\frac{17\times15}{9\times7} = \frac{255}{63} = \frac{85}{21}$
Thus, $k = n^{\frac{85}{21}}$.
Step 3: Equate the exponents of n:
Given $n^{\frac{85}{21}} = n^{3a+2}$, equate the exponents:
$\frac{85}{21} = 3a + 2$
Solve for a:
$3a + 2 = \frac{85}{21}$
Subtract 2 from both sides:
$3a = \frac{85}{21} - \frac{42}{21}$
$3a = \frac{43}{21}$
Divide both sides by 3:
$a = \frac{43}{63}$

9. Which expression is equivalent to $-36x^3 + 54x^2y + 8xy^2 - 12y^3$?

A. $(12x^2 - 4y^2)(-3x + 5y)$

B. $(6x^2 + 3y^2)(-6x + 4y)$

C. $(9x^2 - 2y^2)(-4x + 6y)$

D. $(3x^2 + 2y^2)(-12x + 2y)$

Answer

C

Solution

Concept Check : The intent of the question is to assess the student's ability to factor a polynomial expression. Students should understand the concepts of factoring, including recognizing common factors and applying methods such as grouping or using the distributive property.

Solution Strategy : To solve this problem, students should first look for a common factor among all the terms in the polynomial. After factoring out any common factors, they might consider grouping terms that share a variable or structure. This might involve rearranging the terms or looking for patterns such as differences of cubes or other recognizable forms.

Quick Wins : One useful approach is to first identify the greatest common factor (GCF) of all terms. After factoring out the GCF, examine the remaining expression for further factoring possibilities. If the polynomial has four terms, consider grouping them in pairs to see if a common factor can be factored out from each pair.

Mistake Alert : Be careful not to overlook any common factors, especially if they are negative or involve different variables. Additionally, double-check your factored expression by expanding it back to ensure it matches the original polynomial. It's also easy to misgroup terms, so take your time to ensure that your groupings make sense.

SAT Know-How : This problem falls under the category of Advanced Math, specifically focusing on factoring polynomial expressions. It assesses skills in identifying common factors and applying factoring techniques effectively. Mastering these skills is crucial for SAT problem-solving, as they often appear in various forms on the exam.

Step 1: Group the polynomial terms: $(-36x^3 + 54x^2y)$ + $(8xy^2 - 12y^3)$.
Step 2: Factor out the greatest common factor from each group:
From the first group $(-36x^3 + 54x^2y)$, factor out $-18x^2$:
$-18x^2(2x - 3y)$.
From the second group $(8xy^2 - 12y^3)$, factor out $4y^2$:
$4y^2(2x - 3y)$.
Step 3: The expression can be rewritten using the common factor (2x - 3y):
$(-18x^2 + 4y^2)(2x - 3y)$.
Step 4: Check each option to find a match:
Option A: $(12x^2 - 4y^2)(-3x + 5y)$, does not match.
Option B: $(6x^2 + 3y^2)(-6x + 4y)$, does not match.
Option C: $(9x^2 - 2y^2)(-4x + 6y) = (-18x^2 + 4y^2)(2x - 3y)$, match.
Option D: $(3x^2 + 2y^2)(-12x + 2y)$, does not match. So, answer is Option C

10. What is not an x-coordinate of an x-intercept of the graph $y = 4(x - 7)(x + 9)(x - 2)$ in the xy-plane?

A. -9

B. 2

C. 7

D. 10

Answer

D

Solution

Concept Check : The intent of this question is to assess the student's understanding of polynomial functions and their x-intercepts. Students should know that x-intercepts occur where the output of the function (y) is equal to zero and how to derive the x-coordinate by solving the equation for x.

Solution Strategy : To find the x-intercept, the student should set the polynomial function equal to zero: 4(x - 7)(x + 9)(x - 2) = 0. Then, the student should think about the roots of the polynomial, recognizing that the x-intercepts correspond to the values of x that make any of the factors equal to zero. This means solving the equations (x - 7) = 0, (x + 9) = 0, and (x - 2) = 0.

Quick Wins : Remember that when a product of factors is equal to zero, at least one of the factors must be zero. This principle is known as the Zero Product Property. You can quickly find each x-intercept by simply solving each factor for x, which will give you three potential x-intercepts. Make sure to check each factor carefully.

Mistake Alert : A common mistake is to forget to consider all factors of the polynomial when identifying potential x-intercepts. Double-check that you have accounted for each factor and solved it correctly. Additionally, be careful with your signs when solving equations, especially with negative numbers.

SAT Know-How : This problem is a classic example of finding x-intercepts of a polynomial function, which falls under the category of Advanced Math. It tests the student's ability to apply the Zero Product Property and solve polynomial equations. Mastering these concepts is essential for success on the SAT, as they often appear in various forms throughout the exam.

Set the equation equal to zero: 0 = 4(x - 7)(x + 9)(x - 2).
Divide both sides by 4 (since 4 is not equal to 0, this does not affect the solutions): 0

= (x - 7)(x + 9)(x - 2).
Solve each factor for x:
$x - 7 = 0 \rightarrow x = 7$
$x + 9 = 0 \rightarrow x = -9$
$x - 2 = 0 \rightarrow x = 2$
Therefore, the x-coordinates of the x-intercepts are 7, -9, and 2.

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Math

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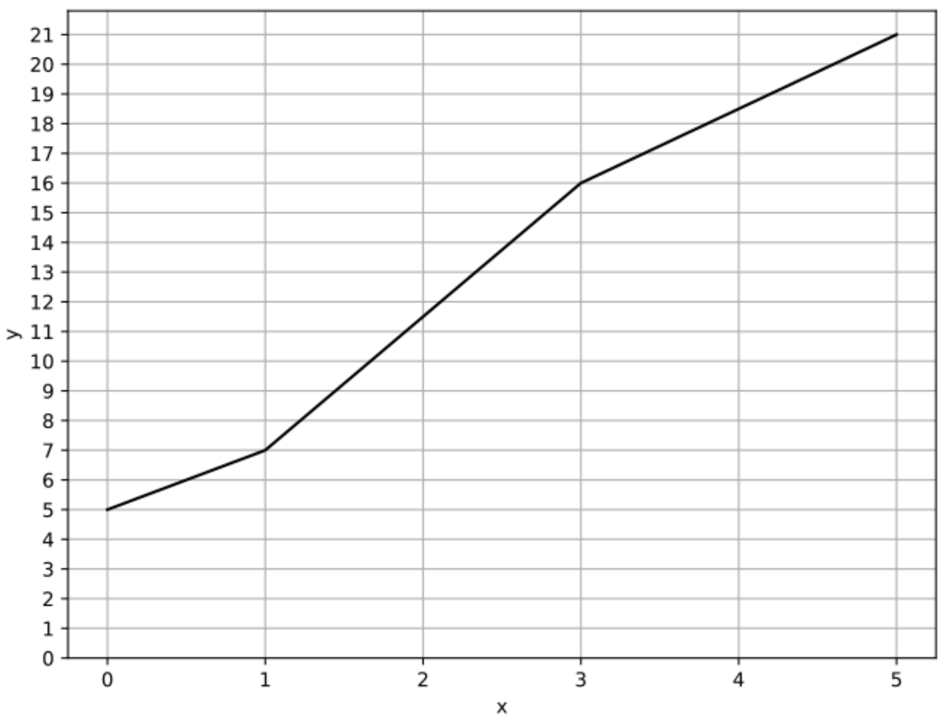
Geometry and Trigonometry

2024.10

SAT Math Geometry and Trigonometry

1. The graph shows the estimated number of users (y, in thousands) of a cryptocurrency platform x years after launch, where x ranges from 0 to 5. which of the following best describes the increase in the estimated number of users from x = 1 to x = 3?



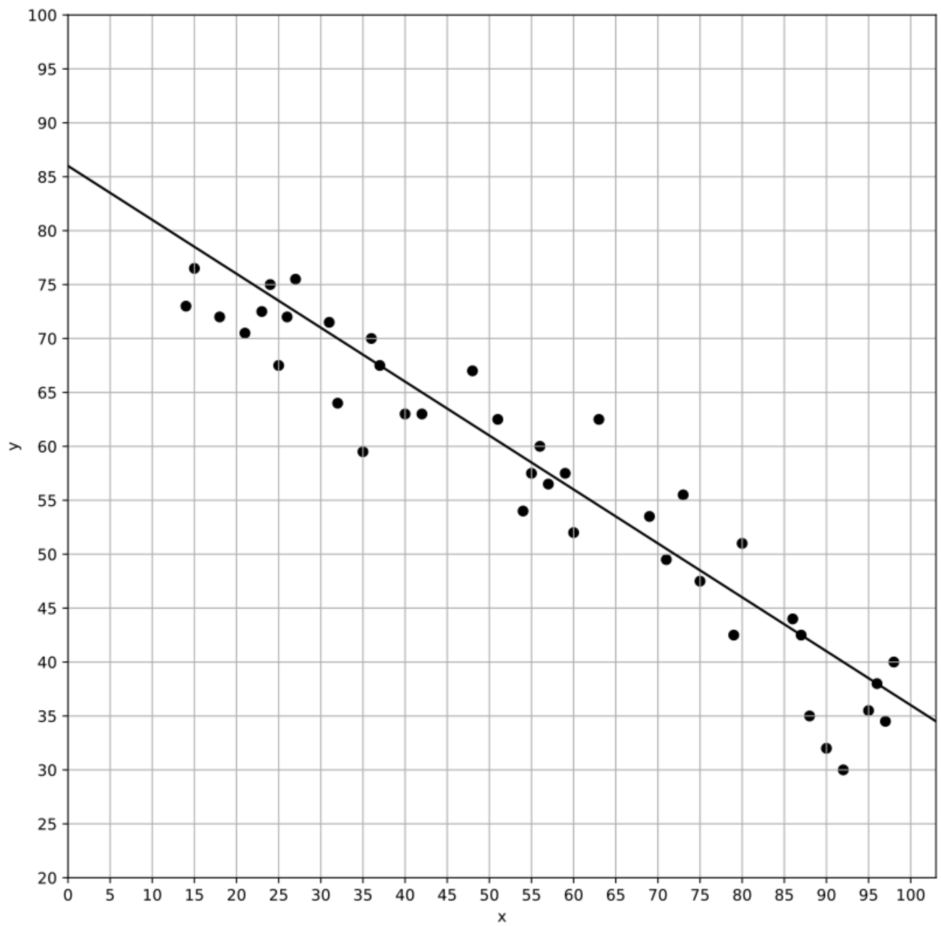
A. Approximately 1.5 times

B. Approximately 2 times

C. Approximately 2.3 times

D. Approximately 3 times

2. The scatterplot shows the relationship between two variables, x and y. If each y-coordinate in the dataset is increased by 11, which of the following is closest to the new y-coordinate of the y-intercept for the line of best fit?



A. 64

B. 75

C. 86

D. 97

3. A car is traveling at a speed of 21 meters per second. Convert this speed to miles per hour, rounding to the nearest tenth. (Use 1 mile = 1,609 meters.)

A. 18.1

B. 32.4

C. 47.0

D. 62.5

4. The positive number a is 2000% of the number c, and c is 10% of the number b. If a - b = wc where w is a constant, what is the value of w?

5. In a city, the number of students enrolled in public schools increased from 2,068,500 to 2,147,166 over a period of 21 years. On average, how many students were added to the enrollment each year?

A. 3,420

B. 3,576

C. 3,746

D. 3,933

6. A certain organization had a membership of 2,000 members in the year 2020. If the membership grows at a steady rate of 5% each year, how many members will the organization have in the year 2025?

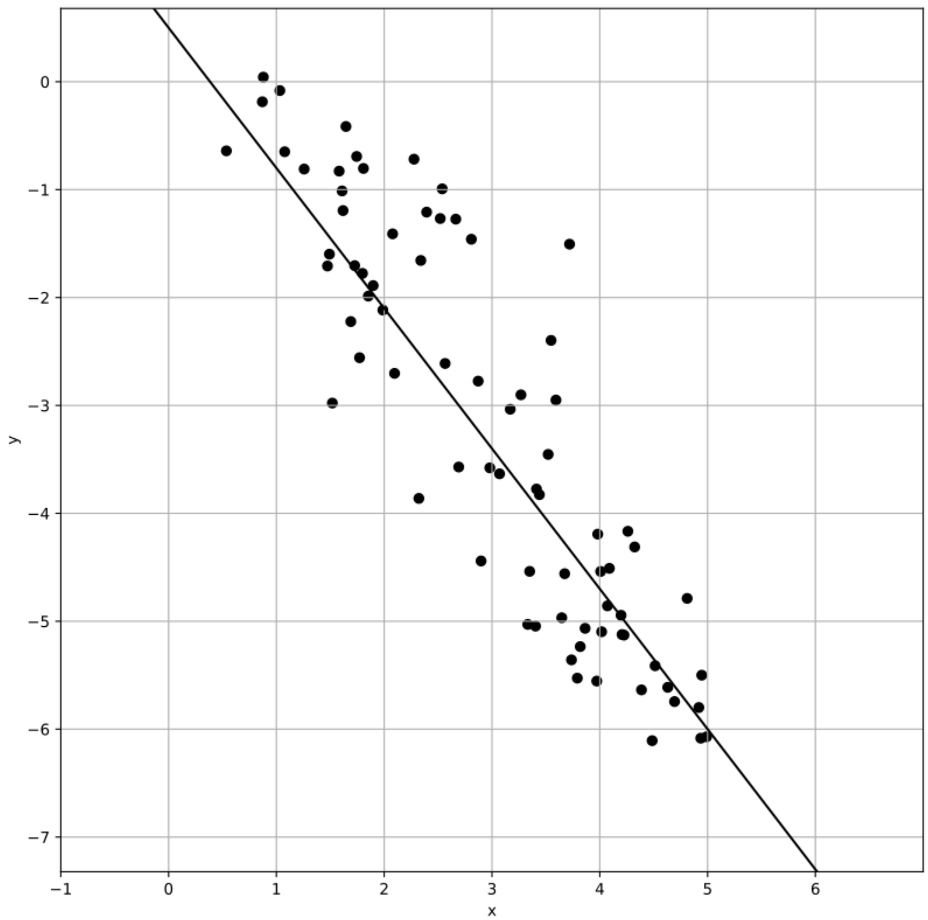
A. 2100

B. 2431

C. 2553

D. 2680

7. Which of the following equations best represents the line of best fit shown for the given scatter plot?



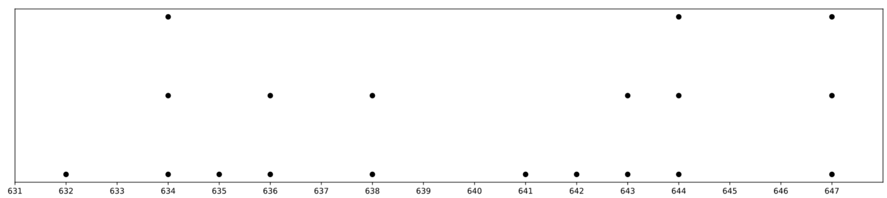
A. y = -1.3x - 0.5

B. y = 1.3x + 0.5

C. y = -1.3x + 0.5

D. y = 1.3x - 0.5

8. A cybersecurity team has recorded the lengths of various incidents over time, represented by the original dataset. After adding a new incident that lasted 303 days, how does the mean and median of the new dataset compare to those of the original dataset?



A. The mean of the new dataset is less than the mean of the original dataset, and the median of the new dataset is less than the median of the original dataset.

B. The mean of the new dataset is greater than the mean of the original dataset, and the median of the new dataset is greater than the median of the original dataset.

C. The mean of the new dataset is equal to the mean of the original dataset, and the median of the new dataset is less than the median of the original dataset.

D. The mean of the new dataset is less than the mean of the original dataset, while the median of the new dataset is greater than the median of the original dataset.

9. What is the median of the data set shown? data set = [56, 44, 90, 95, 41, 22, 69, 19, 4, 43, 28]

10. A company has a total of 750 robotic arms in its production line. If 25% of the robotic arms are being used for assembly, how many robotic arms are currently in use for assembly?

A. 185

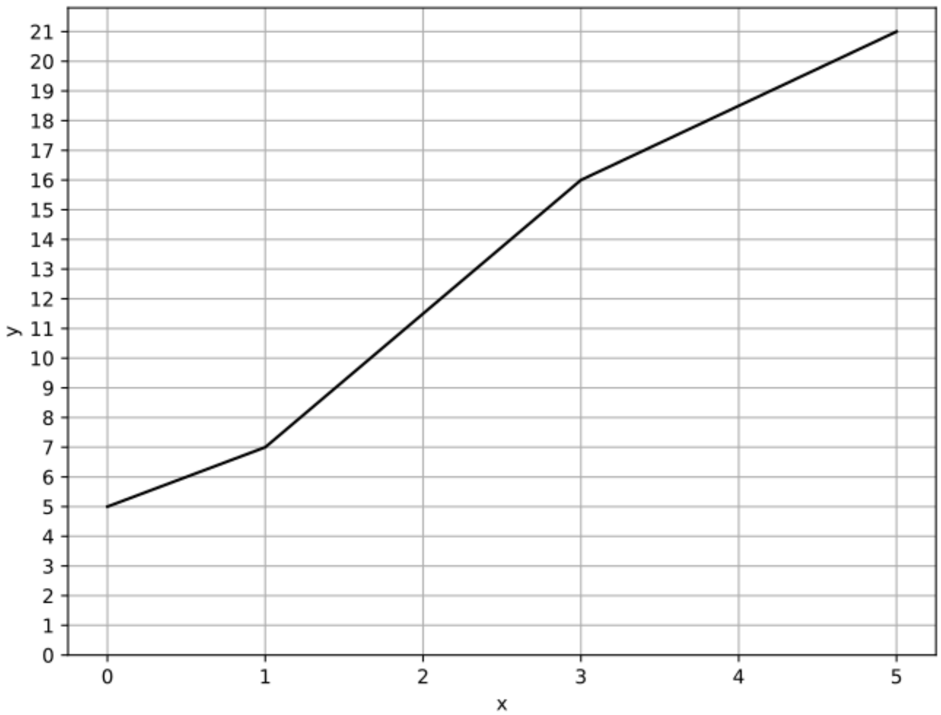
B. 186

C. 187

D. 188

SAT Math Geometry and Trigonometry Solutions

1. The graph shows the estimated number of users (y, in thousands) of a cryptocurrency platform x years after launch, where x ranges from 0 to 5. which of the following best describes the increase in the estimated number of users from x = 1 to x = 3?



A. Approximately 1.5 times

B. Approximately 2 times

C. Approximately 2.3 times

D. Approximately 3 times

Answer

C

Solution

Concept Check : The question expects the student to analyze a line graph and understand linear growth. The focus is on interpreting the data points provided and calculating the increase in users over a specific interval (from x = 1 to x = 3). Students should know how to read graphs and understand the concept of change or increase over time.

Solution Strategy : To solve this problem, students should first identify the values of users at x = 1 and x = 3 from the given data points. Then, they need to calculate the difference in the number of users between these two points. This requires basic subtraction and an understanding of how to interpret the y-axis values in thousands.

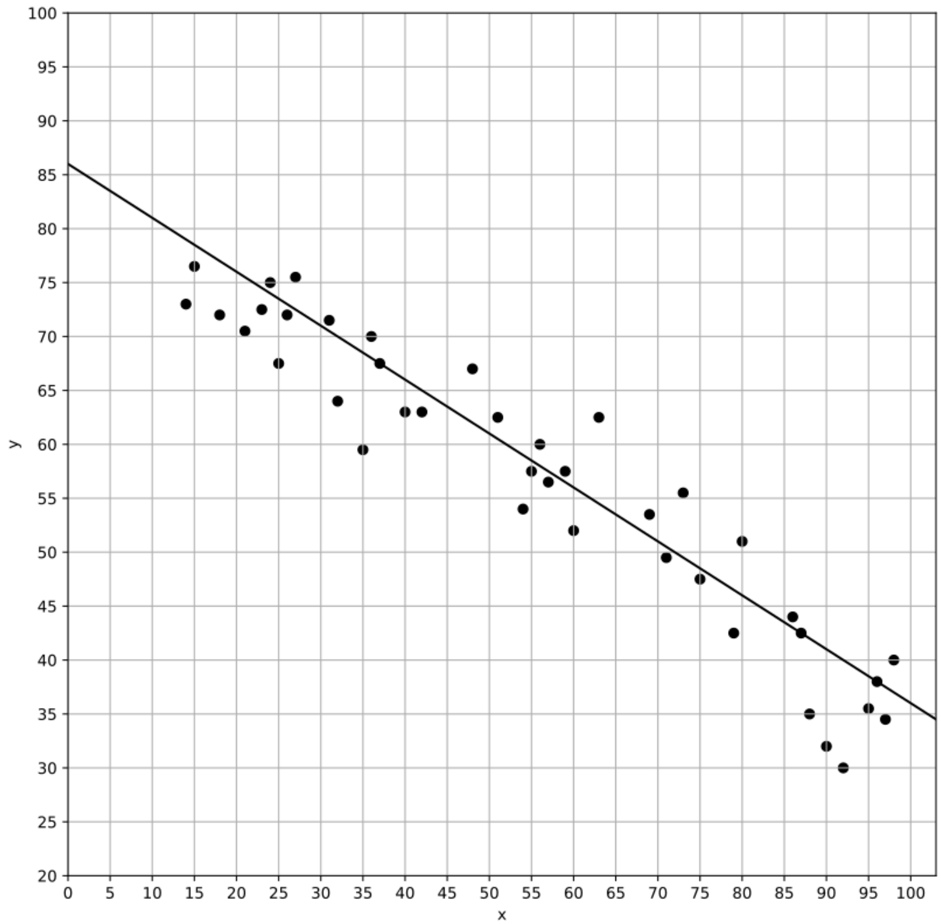
Quick Wins : When analyzing the graph, carefully note the coordinates provided. Always double-check that you are using the correct x-values corresponding to the y-values. It can be helpful to visualize the change by plotting the points on a separate graph if necessary. Also, remember that the y-values are in thousands, so be mindful of the scale when interpreting the numbers.

Mistake Alert : Students may easily make mistakes when reading the graph, especially if they confuse the x and y coordinates. Additionally, be cautious with the units; ensure that you are accounting for the 'thousands' when reporting the final answer. Double-check your arithmetic when calculating the increase to avoid simple calculation errors.

SAT Know-How : This problem falls under the category of Problem Solving and Data Analysis, focusing on linear growth through graph interpretation. It assesses skills in reading and analyzing data points, understanding changes in values, and performing basic arithmetic operations. Mastering these concepts is crucial for success in the SAT, as they frequently appear in various forms in the exam.

First, identify the number of users at x = 1 and x = 3.
At x = 1, the number of users is 7,000 (since y is in thousands).
At x = 3, the number of users is 16,000.
To find how many times the number of users at x = 3 is compared to x = 1, divide the number of users at x = 3 by the number of users at x = 1.
Calculate: 16,000 ÷ 7,000 = 16 ÷ 7
Simplify the fraction $\frac{16}{7}$.
16 ÷ 7 is approximately 2.2857, which suggests to check if it fits one of the option criteria given in the problem.

2. The scatterplot shows the relationship between two variables, x and y. If each y-coordinate in the dataset is increased by 11, which of the following is closest to the new y-coordinate of the y-intercept for the line of best fit?



A. 64

B. 75

C. 86

D. 97

Answer

D

Solution

Concept Check : The intent of the question is to test the student's understanding of the concepts of slope and y-intercept in the context of linear equations, particularly how changes to the data affect the y-intercept of the line of best fit. Students should know how to apply the formula for a linear equation, $y = mx + b$, where m is the slope and b is the y-intercept, and understand the implications of modifying the y-values in a dataset.

Solution Strategy : To approach this problem, the student should start by recalling that the y-intercept (b) of the line of best fit represents the value of y when x is 0. Initially, the y-intercept is given as 86. The problem states that each y-coordinate is increased by 11, which means you need to add 11 to the original y-intercept to find the new y-intercept. This is a straightforward application of the concept that shifting all y-values upward will also shift the y-intercept upward by the same amount.

Quick Wins : Remember that when all y-values are increased by a constant value, the y-intercept will also increase by that same constant. To avoid confusion, write down the original y-intercept, perform the addition clearly, and check your math carefully. Also, be mindful that the slope of the line does not change when only the y-values are adjusted.

Mistake Alert : Be cautious not to confuse the slope with the y-intercept. The slope is not affected by the increase in y-values, so ensure that you are only altering the y-intercept. Double-check your addition to avoid simple arithmetic mistakes, and remember to maintain clarity in distinguishing between the original and new values.

SAT Know-How : This problem falls under the category of Problem Solving and Data Analysis, specifically focusing on scatter plots and linear relationships. It assesses the student's ability to manipulate and understand linear equations and the effects of changing data values on the line of best fit. Mastering such concepts is crucial for solving similar SAT problems efficiently and accurately.

The equation for the original line of best fit is $y = -0.5x + 86$.
Increasing each y-coordinate by 11 affects the entire line, including the y-intercept.
The new y-intercept can be found by adding 11 to the original y-intercept.
New y-intercept = $86 + 11 = 97$.

3. A car is traveling at a speed of 21 meters per second. Convert this speed to miles per hour, rounding to the nearest tenth. (Use 1 mile = 1,609 meters.)

A. 18.1

B. 32.4

C. 47.0

D. 62.5

Answer

C

Solution

Concept Check : The intent of this question is to assess the student's understanding of unit conversion, specifically converting speed from meters per second to miles per hour. The student is expected to know the relationship between meters and miles and the process of converting units using multiplication or division.

Solution Strategy : To solve this problem, the student should first identify the conversion factor needed to convert meters to miles. Then, they should calculate the speed in miles per hour by multiplying the speed in meters per second by the appropriate conversion factor to account for both the distance (meters to miles) and the time (seconds to hours). This involves recognizing that there are 3600 seconds in an hour, which will play a crucial role in the conversion process.

Quick Wins : Start by writing down the conversion factors clearly: 1 mile = 1,609 meters and 1 hour = 3600 seconds. This will help in setting up the conversion correctly. Remember that you need to multiply the speed in meters per second by the number of seconds in an hour (3600) to get the speed in miles per hour. It might be helpful to do the calculations step by step to avoid confusion.

Mistake Alert : Be careful not to confuse the conversion factors. It's easy to misplace the values when multiplying or dividing. Additionally, remember to round the final result to the nearest tenth, as specified in the problem. Double-check your calculations to ensure you haven't made any arithmetic errors during the conversion process.

SAT Know-How : This problem is an example of a unit conversion question within the Problem Solving and Data Analysis category of the SAT. It assesses the student's ability to apply conversion factors accurately and perform calculations involving different units of measurement. Mastering this type of problem enhances your problem-solving skills and helps you manage time effectively during the exam.

To convert from meters per second to miles per hour, use the conversion factors given.

First, convert meters to miles: 21 m/s × (1 mile / 1,609 meters) = $\frac{21}{1{,}609}$ miles/second.

Now, convert seconds to hours: $\frac{21}{1{,}609}$ miles/second × (3,600 seconds / 1 hour) = $\frac{21 \times 3{,}600}{1.609}$ miles/hour.

Calculate the above expression: $\frac{21 \times 3{,}600}{1.609} = \frac{75{,}600}{1{,}609} \approx 46.9857$ miles/hour.

Round 46.9857 to the nearest tenth: 47.0 miles/hour.

4. The positive number a is 2000% of the number c, and c is 10% of the number b. If a - b = wc where w is a constant, what is the value of w?

Answer

10

Solution

Concept Check : The intent of the question is to assess the student's understanding of percentages and their ability to translate percentage relationships into algebraic equations. The student is expected to know how to express one quantity as a percentage of another and manipulate these relationships to find a constant value.

Solution Strategy : To approach the problem, the student should first convert the percentage statements into equations. For instance, since a is 2000% of c, this can be expressed as a = 20c. Next, since c is 10% of b, it can be rewritten as c = 0.1b or b = 10c. After establishing these relationships, the student can substitute these expressions into the equation a - b = wc and solve for w while keeping track of the relationships between a, b, and c.

Quick Wins : A helpful tip is to clearly define each variable and write down each relationship step-by-step. This will help in avoiding confusion. It can also be useful to create a visual representation or chart of the relationships between a, b, and c. Remember to keep track of the units and ensure that when you substitute values, they align correctly with the equations. Lastly, double-check each percentage conversion to ensure accuracy.

Mistake Alert : Be careful with the percentage conversions; it's easy to miscalculate when changing percentages to decimal or whole numbers. Ensure that when you express a as a function of c and b as a function of c, the values are consistent. Additionally, when manipulating the equation a - b = wc, watch for signs (positive/negative) and double-check your substitutions to avoid algebraic errors.

SAT Know-How : This problem is a classic example of problem-solving and data analysis involving percentages. It assesses skills in translating word problems into algebraic equations and manipulating those equations to solve for an unknown constant. Mastering this type of problem enhances one's ability to handle real-world mathematical scenarios, reflecting a critical skill needed for success on the SAT.

Step 1: Replace a with 20c in the equation a - b = wc.
Step 2: Replace b with 10c in the equation.
Step 3: The equation becomes 20c - 10c = wc.
Step 4: Simplify the left side: 10c = wc.
Step 5: Divide both sides by c (assuming $c \neq 0$): 10 = w.

5. In a city, the number of students enrolled in public schools increased from 2,068,500 to 2,147,166 over a period of 21 years. On average, how many students were added to the enrollment each year?

A. 3,420

B. 3,576

C. 3,746

D. 3,933

Answer

C

Solution

Concept Check : The intent of the question is to assess the student's understanding of rates and proportions, specifically how to calculate the average rate of change over a specified period. Students should be familiar with the concept of finding the difference between two values and dividing that by the number of years to find the average increase per year.

Solution Strategy : To approach this problem, students should first find the total increase in the number of students by subtracting the initial enrollment from the final enrollment. Then, they will divide that total increase by the number of years (21 years) to determine the average number of students added each year. This involves basic arithmetic operations and an understanding of averages.

Quick Wins : When calculating the average increase, remember the formula: Average Increase = (Final Value - Initial Value) / Number of Years. Make sure to clearly perform each arithmetic step to avoid confusion. It can also be helpful to write down the numbers involved before performing calculations to keep track of your work.

Mistake Alert : Be cautious with your subtraction; ensure you accurately calculate the total increase. Also, remember to divide by the correct number of years (21) and not to confuse this with the total number of students. Double-check your arithmetic to avoid simple errors.

SAT Know-How : This problem falls under the category of Problem Solving and Data Analysis, focusing on ratios, rates, and proportions. It assesses skills in calculating average rates of change, a fundamental concept in analyzing data over time. Mastering this type of problem helps students build confidence in handling real-world data and prepares them for similar questions on the SAT.

Calculate the total increase in the number of students: Final number - Initial number.
Increase in students = 2,147,166 - 2,068,500 = 78,666.
Now, find the average annual increase by dividing the total increase by the number of years.
Average annual increase = $\frac{78666}{21}$.
Perform the division to find the average: 78,666 ÷ 21 = 3,746.
Therefore, the average number of students added to the enrollment each year is 3,746.

6. A certain organization had a membership of 2,000 members in the year 2020. If the membership grows at a steady rate of 5% each year, how many members will the organization have in the year 2025?

A. 2100

B. 2431

C. 2553

D. 2680

Answer

C

Solution

Concept Check : The question intends to assess the student's understanding of exponential growth, specifically how to apply the formula for exponential growth to determine the future value based on a given growth rate. The student should recognize that a constant percentage growth leads to exponential growth, which involves the concept of compounding over time.

Solution Strategy : To approach this problem, the student should first identify the initial value (the number of members in 2020) and the growth rate. They need to understand that the formula for exponential growth can be expressed as $A = P(1 + r)^n$, where A is the amount after n years, P is the initial amount, r is the growth rate, and n is the number of years. The student should calculate how many years pass from 2020 to 2025 and input the values into the formula to find the total membership in 2025.

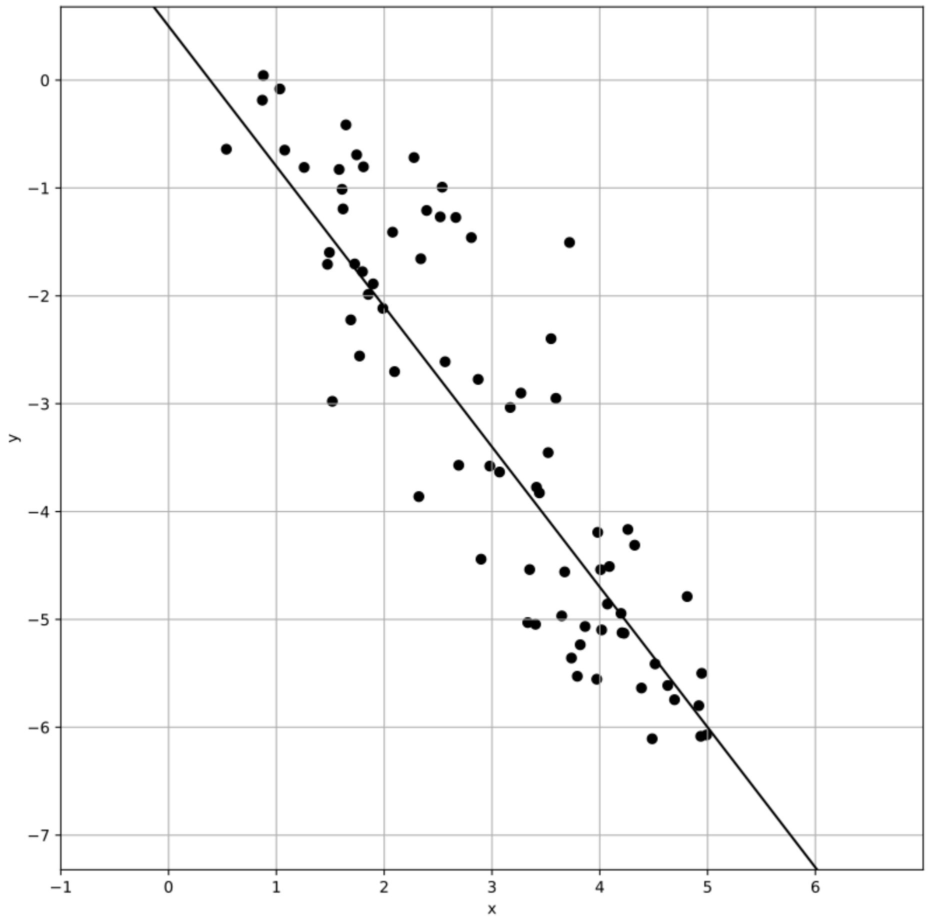
Quick Wins : Remember that the growth rate needs to be converted into decimal form when using it in the formula; for a 5% growth rate, use 0.05. Also, be careful to count the correct number of years from 2020 to 2025, which is 5 years. Make sure to perform the operations in the correct order, using parentheses to ensure accurate calculations.

Mistake Alert : Students often confuse the number of years when calculating growth, so double-check the time span. Also, ensure the growth rate is used in decimal form instead of percentage form. Lastly, pay attention to rounding; depending on the context, you may need to round the final answer to the nearest whole number, as you cannot have a fraction of a member.

SAT Know-How : This problem is a typical example of the 'Problem Solving and Data Analysis' category, specifically focusing on linear and exponential growth. It assesses skills in applying mathematical formulas related to growth rates and understanding how to manipulate exponential equations. Mastering these types of problems is crucial for success on the SAT, as they test both conceptual understanding and practical application of mathematical principles.

The formula for exponential growth is: Final Amount = Initial Amount × (1 + Growth Rate)^n
In this problem, Initial Amount = 2000, Growth Rate = 0.05, and n = 5 years.
Calculate the growth factor for one year: 1 + 0.05 = 1.05
Raise the growth factor to the power of 5 years: 1.05^5
Calculate 1.05^5: 1.05 × 1.05 × 1.05 × 1.05 × 1.05 = 1.2762815625
Multiply this factor by the initial membership: 2000 × 1.2762815625 = 2552.563125
Round the result to the nearest whole number since the number of members must be a whole number: 2553

7. Which of the following equations best represents the line of best fit shown for the given scatter plot?



A. y = -1.3x - 0.5

B. y = 1.3x + 0.5

C. y = -1.3x + 0.5

D. y = 1.3x - 0.5

Answer

C

Solution

Concept Check : The intent of the question is to assess the student's ability to interpret scatter plots and determine the equation of the line of best fit. Students are expected to understand concepts related to linear relationships and how to analyze data visually represented in a scatter plot.

Solution Strategy : To approach this problem, students should first look at the scatter plot to observe the overall trend of the data points. They need to identify whether the relationship appears to be positive, negative, or possibly nonlinear. Then, they should recall how to derive the equation of a line in slope-intercept form ($y = mx + b$), where m is the slope and b is the y-intercept. If the graph provides specific data points, they could use those to calculate the slope and y-intercept, or they could estimate these values visually based on the distribution of the points.

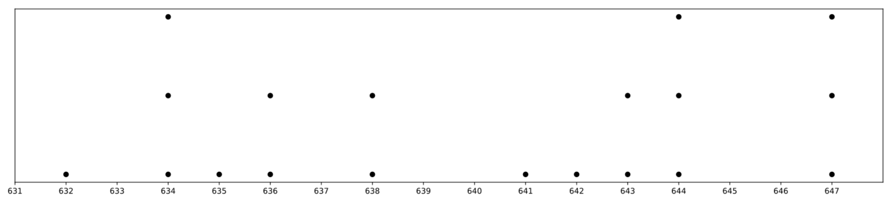
Quick Wins : When analyzing the scatter plot, pay attention to the direction of the data points and any clusters. Estimate the slope by selecting two points on the line of best fit, and use the formula (change in y)/(change in x) to find it. Remember to consider the y-intercept as the point where the line crosses the y-axis. If multiple equations are provided, evaluate each one by substituting a few x-values to see which one gives outputs that align with the visual trend of the scatter plot.

Mistake Alert : Be cautious with visual estimation; it can be easy to misjudge the precise location of data points or the line of best fit. Also, don't forget to check for outliers that might skew your perception of the overall trend. When calculating the slope, ensure that you are using the correct coordinates for your chosen points, and double-check your arithmetic to avoid simple calculation errors.

SAT Know-How : This problem falls under the category of Problem Solving and Data Analysis and specifically focuses on interpreting scatter plots to find the line of best fit. It assesses skills in understanding linear relationships, applying knowledge of slope and intercept, and making estimations based on visual data. Developing proficiency in these areas can significantly enhance your problem-solving abilities on the SAT.

The slope of the line of best fit is -1.3, which means that the line decreases as x increases.
The y-intercept of the line of best fit is 0.5, which means that the line crosses the y-axis at the point (0, 0.5).
We need to find an equation in the format $y = mx + b$, where m is the slope and b is the y-intercept.
Now, let's compare each option:
A) $y = -1.3x - 0.5$: This equation has a slope of -1.3 and a y-intercept of -0.5.
B) $y = 1.3x + 0.5$: This equation has a slope of 1.3 and a y-intercept of 0.5.
C) $y = -1.3x + 0.5$: This equation has a slope of -1.3 and a y-intercept of 0.5, which matches the given conditions.
D) $y = 1.3x - 0.5$: This equation has a slope of 1.3 and a y-intercept of -0.5.
The correct equation is C) $y = -1.3x + 0.5$ because it matches both the given slope and y-intercept.

8. A cybersecurity team has recorded the lengths of various incidents over time, represented by the original dataset. After adding a new incident that lasted 303 days, how does the mean and median of the new dataset compare to those of the original dataset?



A. The mean of the new dataset is less than the mean of the original dataset, and the median of the new dataset is less than the median of the original dataset.

B. The mean of the new dataset is greater than the mean of the original dataset, and the median of the new dataset is greater than the median of the original dataset.

C. The mean of the new dataset is equal to the mean of the original dataset, and the median of the new dataset is less than the median of the original dataset.

D. The mean of the new dataset is less than the mean of the original dataset, while the median of the new dataset is greater than the median of the original dataset.

Answer

A

Solution

Concept Check : The question aims to assess the student's understanding of measures of central tendency, specifically mean and median, and how they are affected by the inclusion of new data points. The student should know how to calculate these values and the implications of adding an outlier or extreme value to a dataset.

Solution Strategy : To approach this problem, the student should first recall the definitions and calculation methods for both the mean and median. They should consider how the addition of a new incident (303 days) compares to the original dataset's values. They should analyze whether 303 days is significantly higher or lower than the previous incidents to predict how it will affect the mean and median. Additionally, they should think about how to recalculate these statistics after including the new data point.

Quick Wins : When calculating the mean, remember that it is the sum of all values divided by the number of values. For the median, ensure you understand that it is the middle value when the data is ordered. If the new data is an outlier, it could significantly affect the mean while having a smaller impact on the median. It can be helpful to sketch a simple number line to visualize where the new data point falls in relation to the existing data.

Mistake Alert : Be cautious about jumping to conclusions based solely on intuition. Always perform the calculations to verify your predictions regarding the mean and median. Additionally, be aware that the mean can be skewed by extreme values, while the median remains more stable. Double-check the ordering of data when finding the median, and ensure you account for the correct number of data points after adding the new incident.

SAT Know-How : This problem falls under Problem Solving and Data Analysis, focusing on understanding the center, spread, and shape of distributions. It assesses the student's ability to analyze how a new data point influences the mean and median of a dataset. Mastery of these concepts is crucial for effective data interpretation, a key skill tested in the SAT.

1. Calculate the original dataset's mean.
- Multiply each data point by its frequency: $(632\times1) + (634\times3) + (635\times1) + (636\times2) + (638\times2) + (641\times1) + (642\times1) + (643\times2) + (644\times3) + (647\times3) = 12159$.
- Total Frequency is 30.
- Original mean = $\frac{12159}{19} = 639.9473$
2. Calculate the new dataset's mean by adding 303.
- New dataset total sum = 12159 + 303 = 12462.
- New total frequency = 19 + 1 = 20.
- New mean = $\frac{12462}{20} = 623.1$
3. Calculate the median for the original dataset.
- Arrange the data points in order, identify the median position: At position 10 (for 19 items) ->641.
4. Calculate the median for the new dataset.
- New dataset: [303, 632, 634, 634, 634, 635, 636, 636, 638, 638, 641, 642, 643, 643, 644, 644, 644, 647, 647, 647].
- With 20 items, the median is the mean of 10th and 11th items ->639.5.
5. Compare means and medians.
- Original mean 639.9473 becomes 623.1, which is less.
- Original median 641 becomes 639.5, which is less.

9. What is the median of the data set shown? data set = [56, 44, 90, 95, 41, 22, 69, 19, 4, 43, 28]

Answer

43

Solution

Concept Check : The intent of this question is to assess the student's understanding of how to find the median in a data set. It requires knowledge of basic statistics, specifically the concept of median, which is the middle value of a sorted data set. The student should be familiar with the steps of ordering the data and determining the median based on whether the number of data points is odd or even.

Solution Strategy : To approach this problem, the student should first sort the data set in ascending order. This is crucial because the median is determined by the position of the numbers in a sorted list. Once the data is sorted, the student will identify the middle number(s) to find the median. If there is an odd number of values, the median is the middle number; if there is an even number of values, the median is the average of the two middle numbers.

Quick Wins : 1. Always sort the data set first; this is essential for accurately finding the median. 2. Count the total number of data points to determine if it's odd or even. 3. If the count is odd, the median is the value at the $\frac{n+1}{2}$ position in the sorted list. If it's even, the median will be the average of the values at the $\frac{n}{2}$ and $(\frac{n}{2} + 1)$ positions. 4. Use a calculator for averaging if needed, to ensure accuracy.

Mistake Alert : Be careful not to skip the sorting step; finding the median without sorting will lead to an incorrect answer. Additionally, double-check the count of data points to ensure you are applying the correct method (odd vs. even). When calculating the average of two numbers, make sure to add them correctly and divide by 2 accurately.

SAT Know-How : This problem falls under the category of 'Problem Solving and Data Analysis', specifically focusing on the center of distribution by finding the median. It assesses the student's ability to organize data, understand statistical concepts, and perform basic arithmetic operations. Mastering how to find the median is a critical skill in data analysis, and practicing such problems enhances overall proficiency in handling statistical data on the SAT.

Step 1: Arrange the data set in ascending order: [4, 19, 22, 28, 41, 43, 44, 56, 69, 90, 95]
Step 2: Identify the middle number. Since the data set contains 11 numbers, the 6th

number in the ordered list is the median.
Step 3: The 6th number in the ordered list is 43.

10. A company has a total of 750 robotic arms in its production line. If 25% of the robotic arms are being used for assembly, how many robotic arms are currently in use for assembly?

A. 185

B. 186

C. 187

D. 188

Answer

C

Solution

Concept Check : The intent of this question is to assess the student's understanding of percentages and their ability to apply this concept in a real-world context. The student is expected to know how to calculate a percentage of a total amount, which is a fundamental skill in problem-solving and data analysis.

Solution Strategy : To solve this problem, the student should first recognize that they need to find 25% of the total number of robotic arms, which is 750. This can be done by converting the percentage into a decimal or fraction and then multiplying it by the total amount. The thought process should include understanding that 'percent' means 'per hundred' and how to apply this concept in multiplication to find the portion in use.

Quick Wins : A helpful way to calculate percentages is to convert the percentage into a fraction or decimal. For example, 25% can be expressed as 0.25 or 25/100. After conversion, multiplying this by the total number of robotic arms (750) will yield the answer. Breaking down the calculation step-by-step can help ensure accuracy. Additionally, think of percentages in terms of parts of a whole to visualize the problem better.

Mistake Alert : Students should be careful not to confuse the percentage with the actual number of robotic arms. It's easy to miscalculate if you forget to convert percentages properly or make arithmetic errors during multiplication. Also, ensure that you are applying the percentage to the correct total amount, and double-check your calculations to avoid simple mistakes.

SAT Know-How : This problem falls under the category of 'Problem Solving and Data Analysis' and specifically focuses on the concept of percentages. It assesses the student's ability to apply basic arithmetic operations and understand percentages in a practical scenario. Mastering such calculations is essential for the SAT, as they often involve real-life applications that require accurate interpretation and computation.

Step 1: Understand the problem.
We have 750 robotic arms, and 25% of them are used for assembly.
Step 2: Calculate 25% of 750.
Step 3: Convert 25% to a decimal. 25% $= \frac{25}{100} = 0.25$.
Step 4: Multiply the total number of robotic arms by this decimal to find how many are used for assembly.
$750 \times 0.25 = 187.5$.

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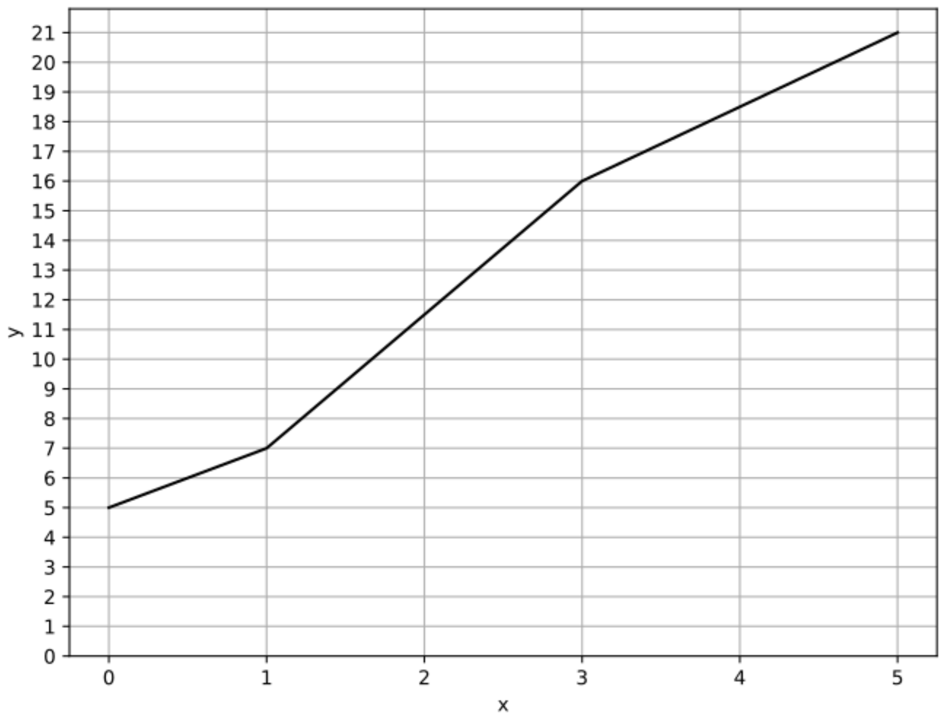
Problem Solving and Data Analysis

2024.10

SAT Math Problem Solving and Data Analysis

1. The graph shows the estimated number of users (y, in thousands) of a cryptocurrency platform x years after launch, where x ranges from 0 to 5. which of the following best describes the increase in the estimated number of users from x = 1 to x = 3?



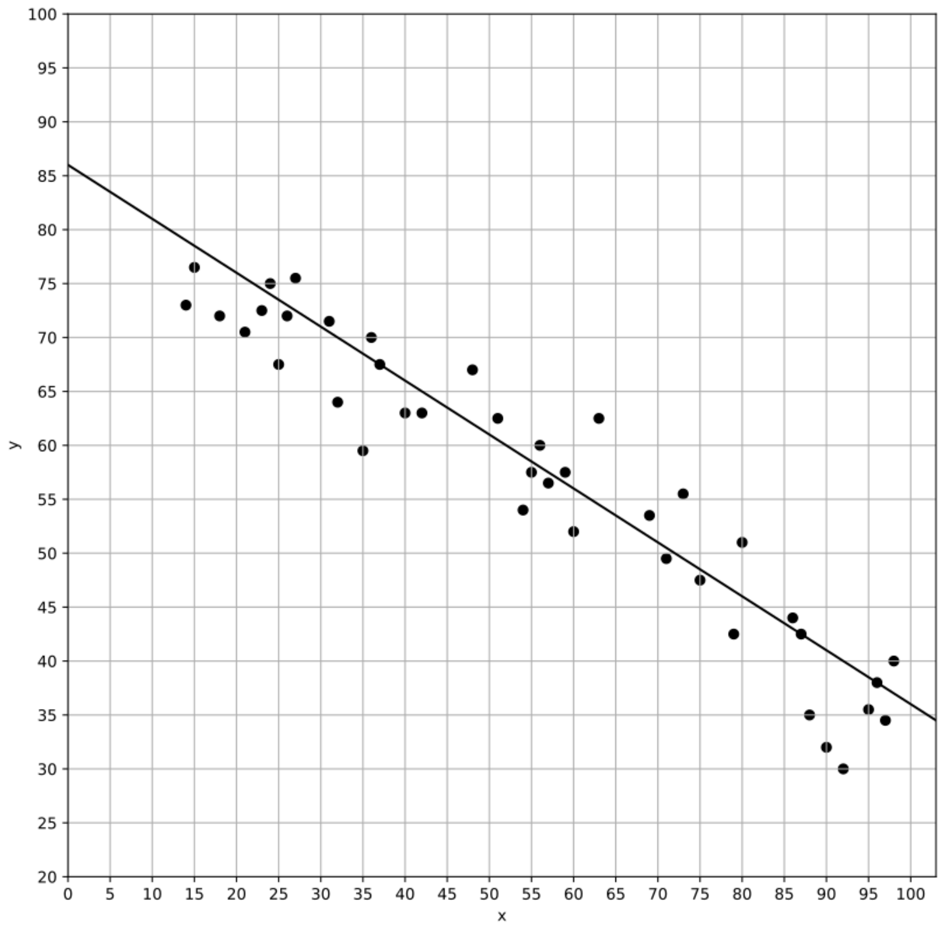
A. Approximately 1.5 times

B. Approximately 2 times

C. Approximately 2.3 times

D. Approximately 3 times

2. The scatterplot shows the relationship between two variables, x and y. If each y-coordinate in the dataset is increased by 11, which of the following is closest to the new y-coordinate of the y-intercept for the line of best fit?



A. 64

B. 75

C. 86

D. 97

3. A car is traveling at a speed of 21 meters per second. Convert this speed to miles per hour, rounding to the nearest tenth. (Use 1 mile = 1,609 meters.)

A. 18.1

B. 32.4

C. 47.0

D. 62.5

4. The positive number a is 2000% of the number c, and c is 10% of the number b. If a - b = wc where w is a constant, what is the value of w?

5. In a city, the number of students enrolled in public schools increased from 2,068,500 to 2,147,166 over a period of 21 years. On average, how many students were added to the enrollment each year?

A. 3,420

B. 3,576

C. 3,746

D. 3,933

6. A certain organization had a membership of 2,000 members in the year 2020. If the membership grows at a steady rate of 5% each year, how many members will the organization have in the year 2025?

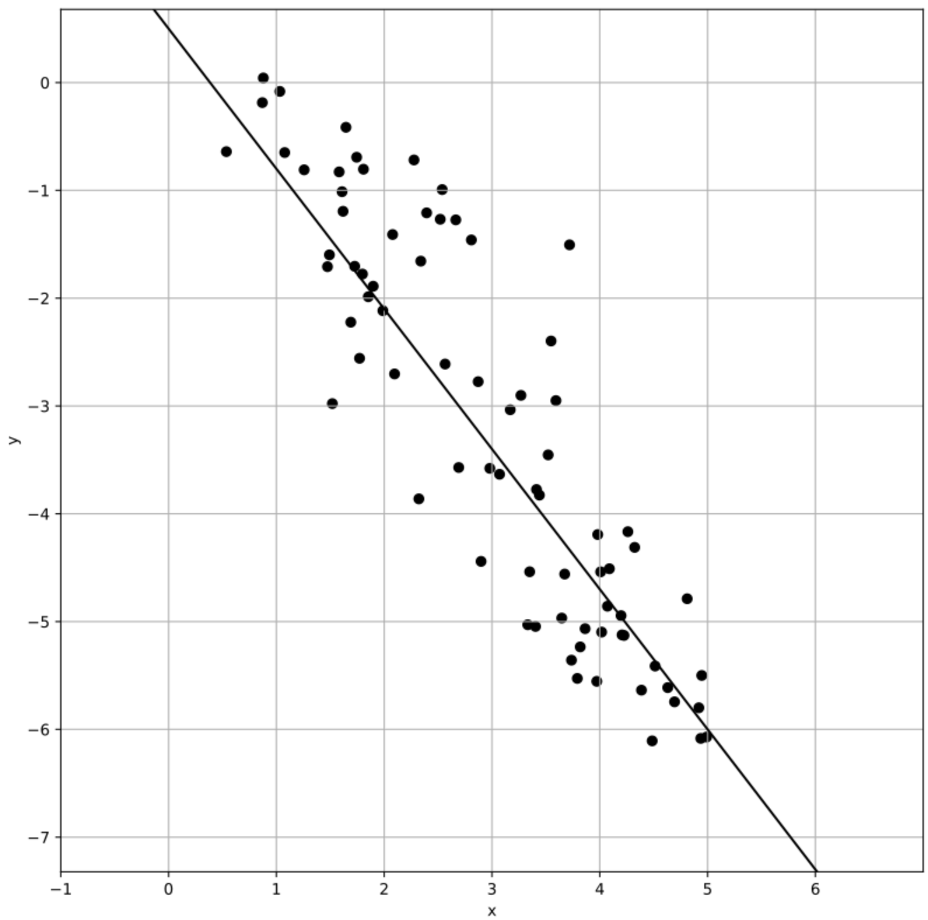
A. 2100

B. 2431

C. 2553

D. 2680

7. Which of the following equations best represents the line of best fit shown for the given scatter plot?



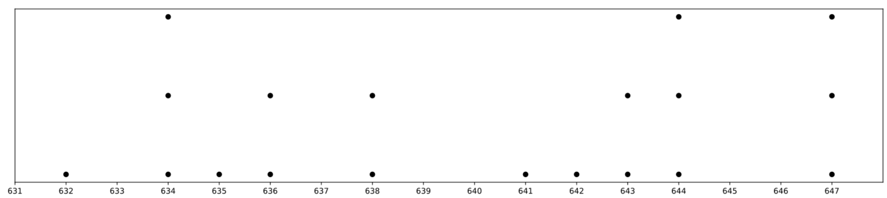
A. $y = -1.3x - 0.5$

B. $y = 1.3x + 0.5$

C. $y = -1.3x + 0.5$

D. $y = 1.3x - 0.5$

8. A cybersecurity team has recorded the lengths of various incidents over time, represented by the original dataset. After adding a new incident that lasted 303 days, how does the mean and median of the new dataset compare to those of the original dataset?



A. The mean of the new dataset is less than the mean of the original dataset, and the median of the new dataset is less than the median of the original dataset.

B. The mean of the new dataset is greater than the mean of the original dataset, and the median of the new dataset is greater than the median of the original dataset.

C. The mean of the new dataset is equal to the mean of the original dataset, and the median of the new dataset is less than the median of the original dataset.

D. The mean of the new dataset is less than the mean of the original dataset, while the median of the new dataset is greater than the median of the original dataset.

9. What is the median of the data set shown? data set = [56, 44, 90, 95, 41, 22, 69, 19, 4, 43, 28]

10. A company has a total of 750 robotic arms in its production line. If 25% of the robotic arms are being used for assembly, how many robotic arms are currently in use for assembly?

A. 185

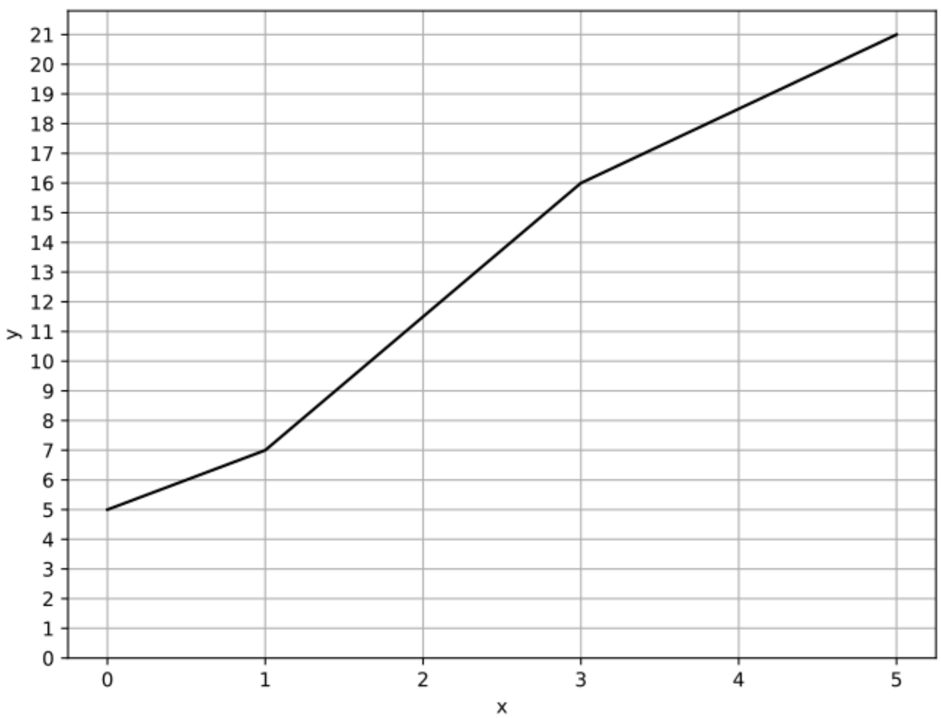
B. 186

C. 187

D. 188

SAT Math Problem Solving and Data Analysis Solutions

1. The graph shows the estimated number of users (y, in thousands) of a cryptocurrency platform x years after launch, where x ranges from 0 to 5. which of the following best describes the increase in the estimated number of users from x = 1 to x = 3?



A. Approximately 1.5 times

B. Approximately 2 times

C. Approximately 2.3 times

D. Approximately 3 times

Answer

C

Solution

Concept Check : The question expects the student to analyze a line graph and understand linear growth. The focus is on interpreting the data points provided and calculating the increase in users over a specific interval (from x = 1 to x = 3). Students should know how to read graphs and understand the concept of change or increase over time.

Solution Strategy : To solve this problem, students should first identify the values of users at x = 1 and x = 3 from the given data points. Then, they need to calculate the difference in the number of users between these two points. This requires basic subtraction and an understanding of how to interpret the y-axis values in thousands.

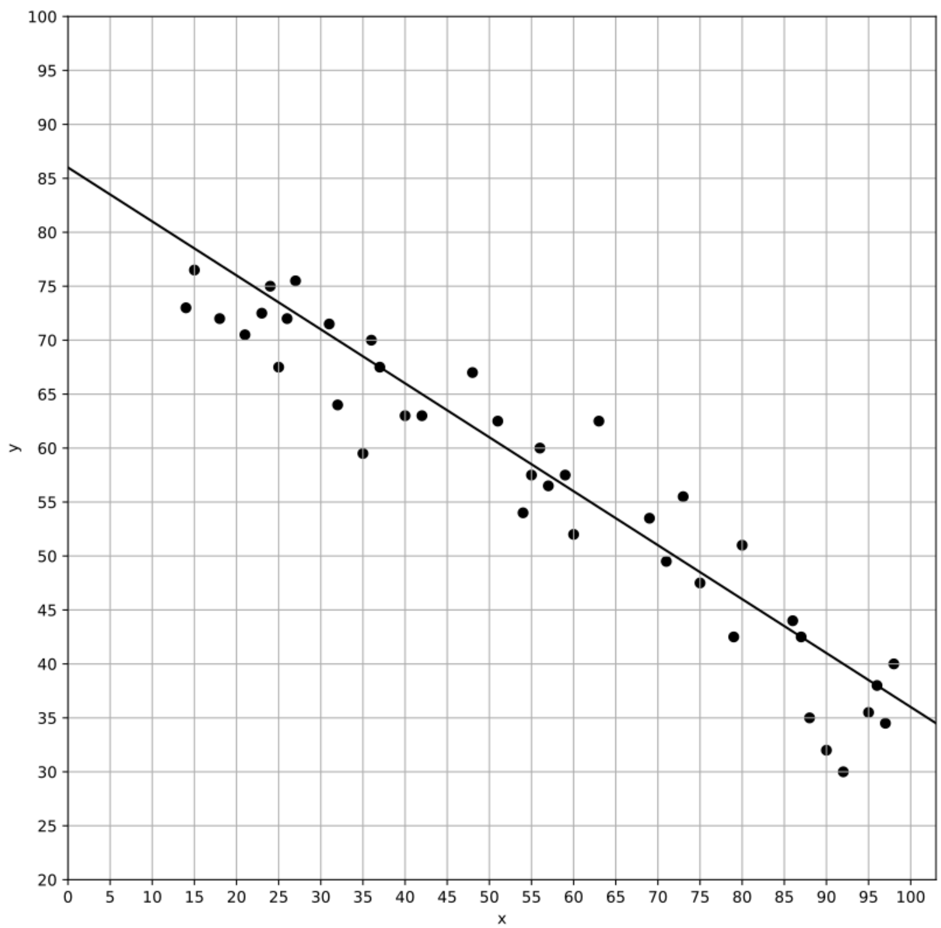
Quick Wins : When analyzing the graph, carefully note the coordinates provided. Always double-check that you are using the correct x-values corresponding to the y-values. It can be helpful to visualize the change by plotting the points on a separate graph if necessary. Also, remember that the y-values are in thousands, so be mindful of the scale when interpreting the numbers.

Mistake Alert : Students may easily make mistakes when reading the graph, especially if they confuse the x and y coordinates. Additionally, be cautious with the units; ensure that you are accounting for the 'thousands' when reporting the final answer. Double-check your arithmetic when calculating the increase to avoid simple calculation errors.

SAT Know-How : This problem falls under the category of Problem Solving and Data Analysis, focusing on linear growth through graph interpretation. It assesses skills in reading and analyzing data points, understanding changes in values, and performing basic arithmetic operations. Mastering these concepts is crucial for success in the SAT, as they frequently appear in various forms in the exam.

First, identify the number of users at x = 1 and x = 3.
At x = 1, the number of users is 7,000 (since y is in thousands).
At x = 3, the number of users is 16,000.
To find how many times the number of users at x = 3 is compared to x = 1, divide the number of users at x = 3 by the number of users at x = 1.
Calculate: 16,000 ÷ 7,000 = 16 ÷ 7
Simplify the fraction $\frac{16}{7}$.
16 ÷ 7 is approximately 2.2857, which suggests to check if it fits one of the option criteria given in the problem.

2. The scatterplot shows the relationship between two variables, x and y. If each y-coordinate in the dataset is increased by 11, which of the following is closest to the new y-coordinate of the y-intercept for the line of best fit?



A. 64

B. 75

C. 86

D. 97

Answer

D

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Solution

Concept Check : The intent of the question is to test the student's understanding of the concepts of slope and y-intercept in the context of linear equations, particularly how changes to the data affect the y-intercept of the line of best fit. Students should know how to apply the formula for a linear equation, $y = mx + b$, where m is the slope and b is the y-intercept, and understand the implications of modifying the y-values in a dataset.

Solution Strategy : To approach this problem, the student should start by recalling that the y-intercept (b) of the line of best fit represents the value of y when x is 0. Initially, the y-intercept is given as 86. The problem states that each y-coordinate is increased by 11, which means you need to add 11 to the original y-intercept to find the new y-intercept. This is a straightforward application of the concept that shifting all y-values upward will also shift the y-intercept upward by the same amount.

Quick Wins : Remember that when all y-values are increased by a constant value, the y-intercept will also increase by that same constant. To avoid confusion, write down the original y-intercept, perform the addition clearly, and check your math carefully. Also, be mindful that the slope of the line does not change when only the y-values are adjusted.

Mistake Alert : Be cautious not to confuse the slope with the y-intercept. The slope is not affected by the increase in y-values, so ensure that you are only altering the y-intercept. Double-check your addition to avoid simple arithmetic mistakes, and remember to maintain clarity in distinguishing between the original and new values.

SAT Know-How : This problem falls under the category of Problem Solving and Data Analysis, specifically focusing on scatter plots and linear relationships. It assesses the student's ability to manipulate and understand linear equations and the effects of changing data values on the line of best fit. Mastering such concepts is crucial for solving similar SAT problems efficiently and accurately.

The equation for the original line of best fit is $y = -0.5x + 86$.
Increasing each y-coordinate by 11 affects the entire line, including the y-intercept.
The new y-intercept can be found by adding 11 to the original y-intercept.
New y-intercept = $86 + 11 = 97$.

3. A car is traveling at a speed of 21 meters per second. Convert this speed to miles per hour, rounding to the nearest tenth. (Use 1 mile = 1,609 meters.)

A. 18.1

B. 32.4

C. 47.0

D. 62.5

Answer

C

Solution

Concept Check : The intent of this question is to assess the student's understanding of unit conversion, specifically converting speed from meters per second to miles per hour. The student is expected to know the relationship between meters and miles and the process of converting units using multiplication or division.

Solution Strategy : To solve this problem, the student should first identify the conversion factor needed to convert meters to miles. Then, they should calculate the speed in miles per hour by multiplying the speed in meters per second by the appropriate conversion factor to account for both the distance (meters to miles) and the time (seconds to hours). This involves recognizing that there are 3600 seconds in an hour, which will play a crucial role in the conversion process.

Quick Wins : Start by writing down the conversion factors clearly: 1 mile = 1,609 meters and 1 hour = 3600 seconds. This will help in setting up the conversion correctly. Remember that you need to multiply the speed in meters per second by the number of seconds in an hour (3600) to get the speed in miles per hour. It might be helpful to do the calculations step by step to avoid confusion.

Mistake Alert : Be careful not to confuse the conversion factors. It's easy to misplace the values when multiplying or dividing. Additionally, remember to round the final result to the nearest tenth, as specified in the problem. Double-check your calculations to ensure you haven't made any arithmetic errors during the conversion process.

SAT Know-How : This problem is an example of a unit conversion question within the Problem Solving and Data Analysis category of the SAT. It assesses the student's ability to apply conversion factors accurately and perform calculations involving different units of measurement. Mastering this type of problem enhances your problem-solving skills and helps you manage time effectively during the exam.

To convert from meters per second to miles per hour, use the conversion factors given.

First, convert meters to miles: 21 m/s × (1 mile / 1,609 meters) = $\frac{21}{1{,}609}$ miles/second.

Now, convert seconds to hours: $\frac{21}{1{,}609}$ miles/second × (3,600 seconds / 1 hour) = $\frac{21 \times 3{,}600}{1.609}$ miles/hour.

Calculate the above expression: $\frac{21 \times 3{,}600}{1.609} = \frac{75{,}600}{1{,}609} \approx 46.9857$ miles/hour.

Round 46.9857 to the nearest tenth: 47.0 miles/hour.

4. The positive number a is 2000% of the number c, and c is 10% of the number b. If a - b = wc where w is a constant, what is the value of w?

Answer

10

Solution

Concept Check : The intent of the question is to assess the student's understanding of percentages and their ability to translate percentage relationships into algebraic equations. The student is expected to know how to express one quantity as a percentage of another and manipulate these relationships to find a constant value.

Solution Strategy : To approach the problem, the student should first convert the percentage statements into equations. For instance, since a is 2000% of c, this can be expressed as a = 20c. Next, since c is 10% of b, it can be rewritten as c = 0.1b or b = 10c. After establishing these relationships, the student can substitute these expressions into the equation a - b = wc and solve for w while keeping track of the relationships between a, b, and c.

Quick Wins : A helpful tip is to clearly define each variable and write down each relationship step-by-step. This will help in avoiding confusion. It can also be useful to create a visual representation or chart of the relationships between a, b, and c. Remember to keep track of the units and ensure that when you substitute values, they align correctly with the equations. Lastly, double-check each percentage conversion to ensure accuracy.

Mistake Alert : Be careful with the percentage conversions; it's easy to miscalculate when changing percentages to decimal or whole numbers. Ensure that when you express a as a function of c and b as a function of c, the values are consistent. Additionally, when manipulating the equation a - b = wc, watch for signs (positive/negative) and double-check your substitutions to avoid algebraic errors.

SAT Know-How : This problem is a classic example of problem-solving and data analysis involving percentages. It assesses skills in translating word problems into algebraic equations and manipulating those equations to solve for an unknown constant. Mastering this type of problem enhances one's ability to handle real-world mathematical scenarios, reflecting a critical skill needed for success on the SAT.

Step 1: Replace a with 20c in the equation a - b = wc.
Step 2: Replace b with 10c in the equation.
Step 3: The equation becomes 20c - 10c = wc.
Step 4: Simplify the left side: 10c = wc.
Step 5: Divide both sides by c (assuming $c \neq 0$): 10 = w.

5. In a city, the number of students enrolled in public schools increased from 2,068,500 to 2,147,166 over a period of 21 years. On average, how many students were added to the enrollment each year?

A. 3,420

B. 3,576

C. 3,746

D. 3,933

Answer

C

Solution

Concept Check : The intent of the question is to assess the student's understanding of rates and proportions, specifically how to calculate the average rate of change over a specified period. Students should be familiar with the concept of finding the difference between two values and dividing that by the number of years to find the average increase per year.

Solution Strategy : To approach this problem, students should first find the total increase in the number of students by subtracting the initial enrollment from the final enrollment. Then, they will divide that total increase by the number of years (21 years) to determine the average number of students added each year. This involves basic arithmetic operations and an understanding of averages.

Quick Wins : When calculating the average increase, remember the formula: Average Increase = (Final Value - Initial Value) / Number of Years. Make sure to clearly perform each arithmetic step to avoid confusion. It can also be helpful to write down the numbers involved before performing calculations to keep track of your work.

Mistake Alert : Be cautious with your subtraction; ensure you accurately calculate the total increase. Also, remember to divide by the correct number of years (21) and not to confuse this with the total number of students. Double-check your arithmetic to avoid simple errors.

SAT Know-How : This problem falls under the category of Problem Solving and Data Analysis, focusing on ratios, rates, and proportions. It assesses skills in calculating average rates of change, a fundamental concept in analyzing data over time. Mastering this type of problem helps students build confidence in handling real-world data and prepares them for similar questions on the SAT.

Calculate the total increase in the number of students: Final number - Initial number.
Increase in students = 2,147,166 - 2,068,500 = 78,666.
Now, find the average annual increase by dividing the total increase by the number of years.
Average annual increase = $\frac{78666}{21}$.
Perform the division to find the average: 78,666 ÷ 21 = 3,746.
Therefore, the average number of students added to the enrollment each year is 3,746.

6. A certain organization had a membership of 2,000 members in the year 2020. If the membership grows at a steady rate of 5% each year, how many members will the organization have in the year 2025?

A. 2100

B. 2431

C. 2553

D. 2680

Answer

C

Solution

Concept Check : The question intends to assess the student's understanding of exponential growth, specifically how to apply the formula for exponential growth to determine the future value based on a given growth rate. The student should recognize that a constant percentage growth leads to exponential growth, which involves the concept of compounding over time.

Solution Strategy : To approach this problem, the student should first identify the initial value (the number of members in 2020) and the growth rate. They need to understand that the formula for exponential growth can be expressed as $A = P(1 + r)^n$, where A is the amount after n years, P is the initial amount, r is the growth rate, and n is the number of years. The student should calculate how many years pass from 2020 to 2025 and input the values into the formula to find the total membership in 2025.

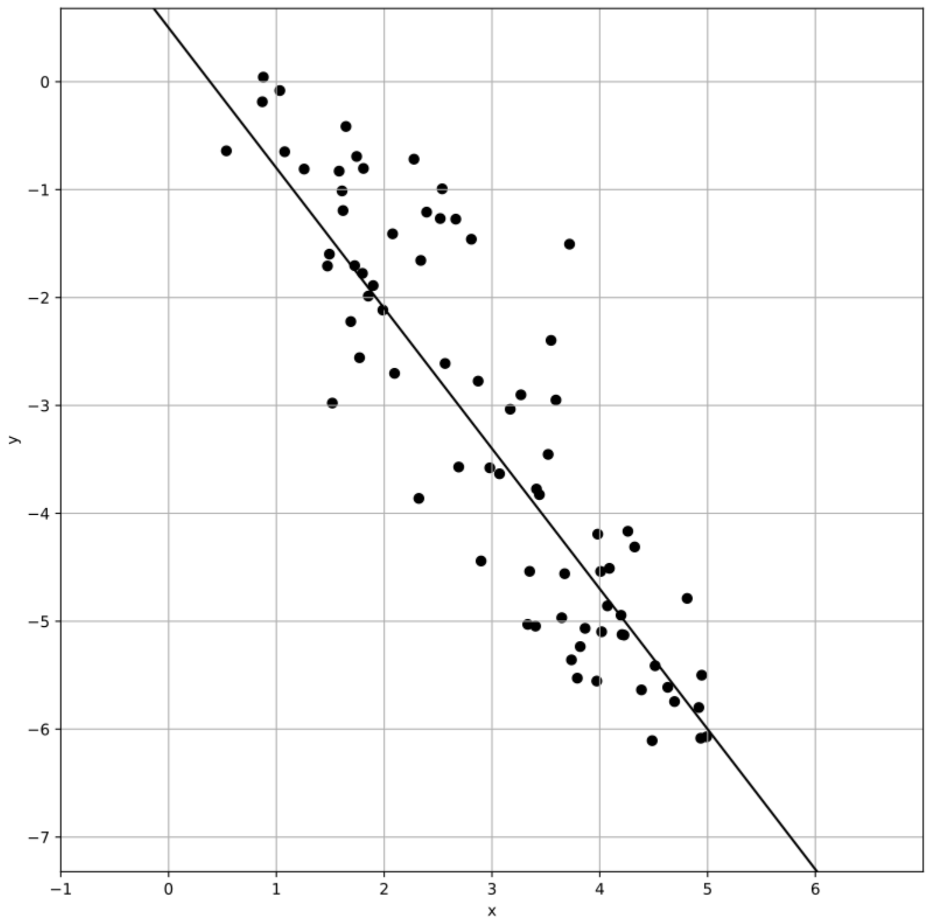
Quick Wins : Remember that the growth rate needs to be converted into decimal form when using it in the formula; for a 5% growth rate, use 0.05. Also, be careful to count the correct number of years from 2020 to 2025, which is 5 years. Make sure to perform the operations in the correct order, using parentheses to ensure accurate calculations.

Mistake Alert : Students often confuse the number of years when calculating growth, so double-check the time span. Also, ensure the growth rate is used in decimal form instead of percentage form. Lastly, pay attention to rounding; depending on the context, you may need to round the final answer to the nearest whole number, as you cannot have a fraction of a member.

SAT Know-How : This problem is a typical example of the 'Problem Solving and Data Analysis' category, specifically focusing on linear and exponential growth. It assesses skills in applying mathematical formulas related to growth rates and understanding how to manipulate exponential equations. Mastering these types of problems is crucial for success on the SAT, as they test both conceptual understanding and practical application of mathematical principles.

The formula for exponential growth is: Final Amount = Initial Amount × (1 + Growth Rate)^n
In this problem, Initial Amount = 2000, Growth Rate = 0.05, and n = 5 years.
Calculate the growth factor for one year: 1 + 0.05 = 1.05
Raise the growth factor to the power of 5 years: 1.05^5
Calculate 1.05^5: 1.05 × 1.05 × 1.05 × 1.05 × 1.05 = 1.2762815625
Multiply this factor by the initial membership: 2000 × 1.2762815625 = 2552.563125
Round the result to the nearest whole number since the number of members must be a whole number: 2553

7. Which of the following equations best represents the line of best fit shown for the given scatter plot?



A. y = -1.3x - 0.5

B. y = 1.3x + 0.5

C. y = -1.3x + 0.5

D. y = 1.3x - 0.5

Answer

C

Solution

Concept Check : The intent of the question is to assess the student's ability to interpret scatter plots and determine the equation of the line of best fit. Students are expected to understand concepts related to linear relationships and how to analyze data visually represented in a scatter plot.

Solution Strategy : To approach this problem, students should first look at the scatter plot to observe the overall trend of the data points. They need to identify whether the relationship appears to be positive, negative, or possibly nonlinear. Then, they should recall how to derive the equation of a line in slope-intercept form ($y = mx + b$), where m is the slope and b is the y-intercept. If the graph provides specific data points, they could use those to calculate the slope and y-intercept, or they could estimate these values visually based on the distribution of the points.

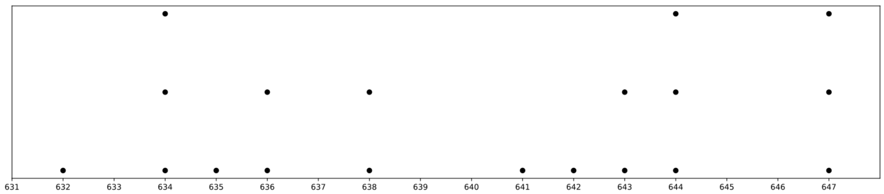
Quick Wins : When analyzing the scatter plot, pay attention to the direction of the data points and any clusters. Estimate the slope by selecting two points on the line of best fit, and use the formula (change in y)/(change in x) to find it. Remember to consider the y-intercept as the point where the line crosses the y-axis. If multiple equations are provided, evaluate each one by substituting a few x-values to see which one gives outputs that align with the visual trend of the scatter plot.

Mistake Alert : Be cautious with visual estimation; it can be easy to misjudge the precise location of data points or the line of best fit. Also, don't forget to check for outliers that might skew your perception of the overall trend. When calculating the slope, ensure that you are using the correct coordinates for your chosen points, and double-check your arithmetic to avoid simple calculation errors.

SAT Know-How : This problem falls under the category of Problem Solving and Data Analysis and specifically focuses on interpreting scatter plots to find the line of best fit. It assesses skills in understanding linear relationships, applying knowledge of slope and intercept, and making estimations based on visual data. Developing proficiency in these areas can significantly enhance your problem-solving abilities on the SAT.

The slope of the line of best fit is -1.3, which means that the line decreases as x increases.
The y-intercept of the line of best fit is 0.5, which means that the line crosses the y-axis at the point (0, 0.5).
We need to find an equation in the format $y = mx + b$, where m is the slope and b is the y-intercept.
Now, let's compare each option:
A) $y = -1.3x - 0.5$: This equation has a slope of -1.3 and a y-intercept of -0.5.
B) $y = 1.3x + 0.5$: This equation has a slope of 1.3 and a y-intercept of 0.5.
C) $y = -1.3x + 0.5$: This equation has a slope of -1.3 and a y-intercept of 0.5, which matches the given conditions.
D) $y = 1.3x - 0.5$: This equation has a slope of 1.3 and a y-intercept of -0.5.
The correct equation is C) $y = -1.3x + 0.5$ because it matches both the given slope and y-intercept.

8. A cybersecurity team has recorded the lengths of various incidents over time, represented by the original dataset. After adding a new incident that lasted 303 days, how does the mean and median of the new dataset compare to those of the original dataset?



A. The mean of the new dataset is less than the mean of the original dataset, and the median of the new dataset is less than the median of the original dataset.

B. The mean of the new dataset is greater than the mean of the original dataset, and the median of the new dataset is greater than the median of the original dataset.

C. The mean of the new dataset is equal to the mean of the original dataset, and the median of the new dataset is less than the median of the original dataset.

D. The mean of the new dataset is less than the mean of the original dataset, while the median of the new dataset is greater than the median of the original dataset.

Answer

A

Solution

Concept Check : The question aims to assess the student's understanding of measures of central tendency, specifically mean and median, and how they are affected by the inclusion of new data points. The student should know how to calculate these values and the implications of adding an outlier or extreme value to a dataset.

Solution Strategy : To approach this problem, the student should first recall the definitions and calculation methods for both the mean and median. They should consider how the addition of a new incident (303 days) compares to the original dataset's values. They should analyze whether 303 days is significantly higher or lower than the previous incidents to predict how it will affect the mean and median. Additionally, they should think about how to recalculate these statistics after including the new data point.

Quick Wins : When calculating the mean, remember that it is the sum of all values divided by the number of values. For the median, ensure you understand that it is the middle value when the data is ordered. If the new data is an outlier, it could significantly affect the mean while having a smaller impact on the median. It can be helpful to sketch a simple number line to visualize where the new data point falls in relation to the existing data.

Mistake Alert : Be cautious about jumping to conclusions based solely on intuition. Always perform the calculations to verify your predictions regarding the mean and median. Additionally, be aware that the mean can be skewed by extreme values, while the median remains more stable. Double-check the ordering of data when finding the median, and ensure you account for the correct number of data points after adding the new incident.

SAT Know-How : This problem falls under Problem Solving and Data Analysis, focusing on understanding the center, spread, and shape of distributions. It assesses the student's ability to analyze how a new data point influences the mean and median of a dataset. Mastery of these concepts is crucial for effective data interpretation, a key skill tested in the SAT.

1. Calculate the original dataset's mean.
- Multiply each data point by its frequency: $(632\times1) + (634\times3) + (635\times1) + (636\times 2) + (638\times2) + (641\times1) + (642\times1) + (643\times2) + (644\times3) + (647\times3) = 12159$.
- Total Frequency is 30.
- Original mean = $\frac{12159}{19}$ = 639.9473
2. Calculate the new dataset's mean by adding 303.
- New dataset total sum = 12159 + 303 = 12462.
- New total frequency = 19 + 1 = 20.
- New mean = $\frac{12462}{20}$ = 623.1
3. Calculate the median for the original dataset.
- Arrange the data points in order, identify the median position: At position 10 (for 19 items) ->641.
4. Calculate the median for the new dataset.
- New dataset: [303, 632, 634, 634, 634, 635, 636, 636, 638, 638, 641, 642, 643, 643, 644, 644, 644, 647, 647, 647].
- With 20 items, the median is the mean of 10th and 11th items ->639.5.
5. Compare means and medians.
- Original mean 639.9473 becomes 623.1, which is less.
- Original median 641 becomes 639.5, which is less.

9. What is the median of the data set shown? data set = [56, 44, 90, 95, 41, 22, 69, 19, 4, 43, 28]

Answer

43

Solution

Concept Check : The intent of this question is to assess the student's understanding of how to find the median in a data set. It requires knowledge of basic statistics, specifically the concept of median, which is the middle value of a sorted data set. The student should be familiar with the steps of ordering the data and determining the median based on whether the number of data points is odd or even.

Solution Strategy : To approach this problem, the student should first sort the data set in ascending order. This is crucial because the median is determined by the position of the numbers in a sorted list. Once the data is sorted, the student will identify the middle number(s) to find the median. If there is an odd number of values, the median is the middle number; if there is an even number of values, the median is the average of the two middle numbers.

Quick Wins : 1. Always sort the data set first; this is essential for accurately finding the median. 2. Count the total number of data points to determine if it's odd or even. 3. If the count is odd, the median is the value at the $\frac{n+1}{2}$ position in the sorted list. If it's even, the median will be the average of the values at the $\frac{n}{2}$ and $(\frac{n}{2} + 1)$ positions. 4. Use a calculator for averaging if needed, to ensure accuracy.

Mistake Alert : Be careful not to skip the sorting step; finding the median without sorting will lead to an incorrect answer. Additionally, double-check the count of data points to ensure you are applying the correct method (odd vs. even). When calculating the average of two numbers, make sure to add them correctly and divide by 2 accurately.

SAT Know-How : This problem falls under the category of 'Problem Solving and Data Analysis', specifically focusing on the center of distribution by finding the median. It assesses the student's ability to organize data, understand statistical concepts, and perform basic arithmetic operations. Mastering how to find the median is a critical skill in data analysis, and practicing such problems enhances overall proficiency in handling statistical data on the SAT.

Step 1: Arrange the data set in ascending order: [4, 19, 22, 28, 41, 43, 44, 56, 69, 90, 95]
Step 2: Identify the middle number. Since the data set contains 11 numbers, the 6th

number in the ordered list is the median.
Step 3: The 6th number in the ordered list is 43.

10. A company has a total of 750 robotic arms in its production line. If 25% of the robotic arms are being used for assembly, how many robotic arms are currently in use for assembly?

A. 185

B. 186

C. 187

D. 188

Answer

C

Solution

Concept Check : The intent of this question is to assess the student's understanding of percentages and their ability to apply this concept in a real-world context. The student is expected to know how to calculate a percentage of a total amount, which is a fundamental skill in problem-solving and data analysis.

Solution Strategy : To solve this problem, the student should first recognize that they need to find 25% of the total number of robotic arms, which is 750. This can be done by converting the percentage into a decimal or fraction and then multiplying it by the total amount. The thought process should include understanding that 'percent' means 'per hundred' and how to apply this concept in multiplication to find the portion in use.

Quick Wins : A helpful way to calculate percentages is to convert the percentage into a fraction or decimal. For example, 25% can be expressed as 0.25 or 25/100. After conversion, multiplying this by the total number of robotic arms (750) will yield the answer. Breaking down the calculation step-by-step can help ensure accuracy. Additionally, think of percentages in terms of parts of a whole to visualize the problem better.

Mistake Alert : Students should be careful not to confuse the percentage with the actual number of robotic arms. It's easy to miscalculate if you forget to convert percentages properly or make arithmetic errors during multiplication. Also, ensure that you are applying the percentage to the correct total amount, and double-check your calculations to avoid simple mistakes.

SAT Know-How : This problem falls under the category of 'Problem Solving and Data Analysis' and specifically focuses on the concept of percentages. It assesses the student's ability to apply basic arithmetic operations and understand percentages in a practical scenario. Mastering such calculations is essential for the SAT, as they often involve real-life applications that require accurate interpretation and computation.

Step 1: Understand the problem.
We have 750 robotic arms, and 25% of them are used for assembly.
Step 2: Calculate 25% of 750.
Step 3: Convert 25% to a decimal. 25% $= \frac{25}{100} = 0.25$.
Step 4: Multiply the total number of robotic arms by this decimal to find how many are used for assembly.
$750 \times 0.25 = 187.5$.

